Since the other reports are already available, I am only providing additional comments.

Thank you for your time in reviewing the manuscript.

Mathematical questions:

1. In equation (4), why is it \epsilon^3?

Looking at the Supplementary Materials, Eq. (1) can be splitted into the sum of three terms, as explained in Eq. (2). To approximate the integral between 0 and ε, we use the Taylor expansions to the third order of $\frac{sin(ma)}{ma}$ and $cos(mx)$, respectively . It follows that $\frac{sin(ma)}{ma} \approx 1 + O(m^2)$ and $cos(mx) \approx 1 + O(m^2)$. Finally, the integral a $\int_0^{\varepsilon}(1 + O(m^2))(1 + O(m^2))dm$ can be approximated by $\varepsilon + O(\varepsilon^3)$.

2. In equation (5), the last parenthesis should be after dm.

Thank you for noticing it. This is actually a typo error that will be fixed.

3. I don't understand equation (6).

Equation (6) defines the maximum spatial frequency of the function $cos(mUx_p)sin(mUa)$, at the numerator of the integrand. The individual frequencies are given by $\omega_1 = \frac{Ux_p}{2\pi}$ for the cosine, and $\omega_2 = \frac{Ua}{2\pi}$ for the sine. For a given U value, the maximum spatial frequency is given by $\omega_{max} = Umax(\frac{Ux_p}{2\pi}, \frac{Ua}{2\pi})$. The idea is to take a number of points in the integral support to be applied in the quadrature formulae used to approximate the integral in Eq. (5).

4. I don't understand equation (7).

The Nyquist theorem states that a sinusoidal function can be regenerated with no loss of information as long as it is sampled at a frequency greater than or equal to twice per cycle. As the integrand of Eq(5) is a product of sinusoidal functions, in Eq (7) we take the maximum frequency between the one computed in Eq. (5) and a certain N_s value, which is supposed to be high enough, in order to take the best representation of our integrand for the following quadrature formulae.

5. Figure 3: it is misleading because in the text the authors mention two quadrature formulas and they mention three in the figure. Why is GAQ the groundtruth?

The GAQ here mentioned is the “Global Adaptive Quadrature” (Shampine, L.: Vectorized adaptive quadrature in MATLAB, Journal of Computational and Applied Mathematics, 211, 131–140, https://doi.org/https://doi.org/10.1016/j.cam.2006.11.021, 2008). This GAQ method uses adaptive integration points that are very convenient for our case and we used it with a tolerance of 10^{-8} . Of course, this method is more expensive (in terms of computational cost) than Gauss-Legendre of Filon methods, but the results can be used as our reference solution. We will specify this is the text.

6. In equation (9), why is it \epsilon^4?

This results from the natural extension to the 2D case of the reasoning applied to the 1D case (question 1).

There are several awkward sentences. Examples are:

1. The last sentence of the abstract

2. The sentence on lines 58/59

Thank you for saying this. We will try to rephrase those sentences in a clearer way.

The last author is missing in the reference Kervella and Dutykh (2007). In the main text, it should read Kervella et al. (2007). Please replace >> and << by their LaTeX notation: \gg and \ll. I would replace the first sentence of Section 2 by: Let R denote the set of real numbers. We consider a domain D \in R. Trigonometric functions inside equations should be written \cosh, \cos, \sin, \max, etc.

Thanks for noticing it. All these points will be addressed in an updated version of the manuscript.