

Response to Referee 2 Comments on:

Probabilistic Flood Inundation Mapping through Copula Bayesian Multi-Modelling of Precipitation Products

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We appreciate the time spent in the review process of the article manuscript. The feedback provided for the submission is of high relevance in order to improve the quality of our manuscript. Enclosed, please find the revised manuscript with all the necessary corrections implemented in accordance with the comments and suggestions in tracked and cleaned versions. The following are point-by-point answers, in blue color, to the comments made by the referee.

Answers to referee #2:

1. Although the technical details of the copula functions have been presented in the literature, it will be helpful for readers to catch the meanings of new variables presented in this article if these variables are illustrated clearly. For example, what are the terms in the copula function on the right-hand side of Equation (3)?

Response: To address this comment, we have further extended the description of copulas for better understanding of the readers. Specifically, in response to the suggestions provided, in Section 2.2 we have expanded the description of key terms in equations, the formulation of copulas and implementation as Cop-BMA. We added references of original formulation from Sklar (1959) and previous Cop-BMA methodology from Madadgar et al., (2014) and Abbaszadeh et al. (2022) to provide readers with additional context and theoretical explanations. These additions aim to facilitate a deeper understanding and address questions that may arise. The modified text and equations added reads as follows:

“Copulas are functions in the unit cube, which can link multi-dimensional distributions to their one-dimensional marginals (Sklar, 1959), they provide a flexible and powerful tool for modelling the dependency structure between variables, regardless of their individual marginal distributions and model dependency. This is particularly valuable in scenarios where the relationships between variables are complex and may not follow a simple linear pattern. Cop-BMA modifies the BMA predictive distribution through relaxing the assumption on parametric posterior distribution $g(y^t | M_i^t, \sigma_i^2)$ replaced with a group of multivariate copula functions. Multiple copula functions have been applied to postprocess hydrological forecasts (Abbaszadeh et al., 2022a; He et al., 2018; Madadgar et al., 2014; Madadgar and Moradkhani, 2014), and are used in this study for the estimation of water surface elevation posterior distribution. Equation 1 is modified to incorporate copula functions replacing the posterior distribution $p(y^t | M_i^t, Y)$ following the procedure from Abbaszadeh et al. (2022a). Supported

by Sklar theorem copulas can express the joint behaviour among correlated variables through their marginal CDFs in equation 3.

$$P(x_1, \dots, x_n) = C[P(x_1), \dots, P(x_n)] = C(u_1, \dots, u_n) \quad (3)$$

where C is the Cumulative Distribution Function (CDF) of the copula and $P(x_i)$ is the marginal distribution of x_i denoted as u_i for the interval $[0, 1]$. Using the PDF of copula, the joint probability density function of the variables involved can be defined as follows:

$$p(x_1, \dots, x_n) = c(u_1, \dots, u_n) \prod_{i=1}^n p(x_i) \quad (4)$$

The conditional probability distribution of x_1 given x_2 is defined in equation 5:

$$p(x_1|x_2) = \frac{p(x_1, x_2)}{p(x_2)} \quad (5)$$

Considering the copula joint probability from equation 4, the equation 5 can be expressed as:

$$p(x_1|x_2) = \frac{p(x_1, x_2)}{p(x_2)} = \frac{c(u_1, u_2) \cdot p(x_1) \cdot p(x_2)}{p(x_2)} = c(u_1, u_2) \cdot p(x_1) \quad (6)$$

Since u_1 and u_2 are the observations (y^t) and simulations (M_k^t) respectively, the posterior distribution in equation 1 is replaced with the conditional probability distribution from equation 6 as:

$$p(y^t | M_1^t, M_2^t, \dots, M_k^t, Y) = \sum_{i=1}^k w_i \cdot p(y^t | M_i^t, Y) = \sum_{i=1}^k w_i \cdot c(u_{y^t}, u_{M_i^t}) \cdot p(y^t) \quad (7)$$

Where $c(u_{y^t}, u_{M_i^t})$ represents the PDF of the copula function. To estimate weight w_i , it is required to maximize the log likelihood function of the vector of parameter $\theta = \{w_i, i = 1, \dots, k\}$ as:

$$l(\theta) = \log \left(\sum_{i=1}^k w_i \sum_{t=1}^T c(u_{y^t}, u_{M_i^t}) \cdot p(y^t) \right) \quad (8)$$

2. As shown in Equation (4), the form of the likelihood function (a product of the probability density at different time steps) is valid when the temporal predictions are independent. But in this study, we may not assume the “hourly” output to be independent of each other. So what are the potential impacts of autocorrelation of the target variables, y , on the Cop-BMA results?
Response: We consider that hydrodynamic simulations resulting from the different QPEs are independent of each other as these products come from different sources and methodologies, with both different spatial and temporal resolutions. For readers interested in a more detailed analysis, we refer to the original BMA and Cop-BMA research article by Madadgar and Moradkhani (2014) in the manuscript. Additionally, equation 4 (equation 8 in recent edited version) was edited as the summation over time term was typed in an incorrect position as shown in the comment 1.

3. It is not clear how to transform the rainfall into runoff in the HEC-RAS 2D model. Also, it seems to be unfair to compare the performance of different precipitation products since the infiltration process was not considered in the hydrodynamic modeling process. In addition, in Line 453, why do the infiltration processes mainly impact the “initial” water surface elevation results? How about the “continuous” loss during the flood event?
Response: We appreciate the reviewer's feedback regarding infiltration processes, and we acknowledge the importance of considering these hydrological processes in hydrodynamic models. It is commonly assumed, especially for events like Hurricane Harvey in highly urbanized areas such as Houston, that the soil was completely saturated due to previous rainfall and the percentage of impervious cover is significant. While this assumption simplifies the analysis by neglecting infiltration processes, it may not fully capture the dynamic nature of soil properties and runoff transformation in hydrodynamic simulations as mentioned in the manuscript. In light of this, we recognize the need to explore infiltration processes more comprehensively in future research. We have outlined this aspect in Section 5 for the benefit of the readers as future research topics. This includes testing different infiltration methods

directly within the HR2D model, such as Deficit and Constant, SCS Curve Number, and Green-Ampt, across various storm events in rural areas with more diverse land cover. The additional text reads as follows:

“The HEC-RAS model can also incorporate the impact of infiltration during flood events. This involves testing various infiltration methods, such as Deficit and Constant, SCS Curve Number, and Green-Ampt, across different storm events in rural areas with diverse land cover.”

4. Manning values are important parameters in flood modeling and the values will change with the water depth. As presented in Line 192, the Manning values “are further adjusted during the calibration period, 7 days before the occurrence of Hurricane Harvey”, would it be better or necessary to calibrate the roughness parameters based on a similar flood event?

Response: Previous research conducted by our research group, as outlined in Muñoz et al. (2022) focused on the estimation of Manning roughness coefficients using Latin Hypercube Sampling (LHS) within the study area considering the extreme water levels generated at the peak of Hurricane Harvey for calibration procedure. The values obtained from this research were thoroughly tested and, where necessary, slightly modified to suit the specific requirements of the current study using HR2D. We cite this previous work as a guiding reference to the readers for the methodology employed and the selection of Manning coefficients in our study. Modified text in Section 3.1 reads as follows:

“In a previous research conducted by Muñoz et al. (2022), they used Latin Hypercube Sampling and tested various Manning roughness values for different land cover categories during Hurricane Harvey event. We use their calibrated parameters as a reference for HR2D model setup. These values are slightly adjusted during the calibration period, 7 days before the occurrence of Hurricane Harvey”.

5. For the application of Cop-BMA, it is like a trial-and-error procedure to select an appropriate marginal distribution and a copula function for the target variable. Is there any general guidance or suggestion for interested readers if they want to apply the framework to the other areas or variables?

Response: We appreciate the reviewer's constructive suggestion. To provide clarity and guidance to interested readers, we have incorporated a description of the criteria used in the selection process in Section 4.1 of the manuscript. The selection of marginal distribution is based on minimizing the sum of squared error (SSE) using the Maximum Likelihood Estimation (MLE). Modified section reads as follows:

“Parameter estimation for each distribution is performed using the Maximum Likelihood Estimation (MLE) technique. To identify the most suitable marginal distribution, the sum of squared errors (SSE) is employed to facilitate the selection process, choosing the distribution that provide the lower SSE value.”

To select the appropriate copula, we calculated the Akaike information criterion (AIC) and cross-validation criterion (xv-CIC) for each copula and then chose the one with the lowest AIC and highest xv-CIC. Details of this analysis have been included in Table S2 and can be found in the supplementary materials. We recommend interested readers follow this procedure when applying the framework to other studies. Added text to Section 4.1 reads as follows:

“Fitting and selection process was conducted using Akaike Information Criterion (AIC) and copula cross-validation criterion (xv-CIC) (Grønneberg and Hjort, 2014) using copula package implemented in R (Hofert et al., 2023), where the copula fit with lowest value of AIC and higher xv-CIC was selected. Table 4 shows the selected copulas for the seven QPEs evaluated in HR2D simulations over the three clusters. Calculated values for AIC and xv-CIC are presented in Table S2 in the supplementary material.”

6. Some cases in Figures 4 and 6 show that if all the members in the precipitation ensembles consistently overestimated (e.g., NOAA 8770613 and USGS 08074710) or underestimated (e.g., USGS 08072050) the peak WSE, Global Cop-BMA did not help at all. Any comments on that?

Response: These discrepancies primarily arise from errors in the model structure and the parameterization of the hydrodynamic model. Factors such as assumptions within the governing equations of the HR2D model, infiltration methods, and the absence of full bathymetry datasets are identified as significant contributors to these differences in WSE values. In Section 4 of the manuscript, we discussed this issue. In our ongoing research, we aim to address these discrepancies by exploring alternative numerical models for evaluation and testing purposes. By incorporating additional modeling frameworks, we seek to refine and improve the accuracy of our simulation results.

Additionally, when we use the Cop-BMA methodology, the highest weight is assigned to the product that provides the closest results to the observations. However, in cases where all model members generate overestimated or underestimated results, there may not be a meaningful improvement. We incorporate this important remark of Global Cop-BMA methodology in Section 4.1, the added text reads as follows:

“It is important to highlight that if all models consistently overestimate or underestimate, Global Cop-BMA may not lead to significant improvement in the result (e.g., NOAA 8770613, USGS 08074710, USGS 08072050 in Figure 4; and Coastal cluster in Figure 6). Despite its advanced weighting mechanism, Global Cop-BMA's effectiveness relies on the diversity and accuracy of the model ensemble. Therefore, while it enhances the integration of diverse model outputs, its capability to improve results may be limited when all models exhibit similar differences compared to the observations at certain sections of the hydrograph.”

7. Line 20 and Line 382, could you provide quantitative results to measure the degree of improvement due to the application of the Cop-BMA approach?

Response: We appreciate this comment which shows general improvements in the proposed methodology. We have added this information to the abstract, providing an overview of the performance achieved through the application of the Cop-BMA approach specially with NSE metric results. Additionally, in Section 4.1 of the manuscript, detailed results and NSE, KGE, RMSE and MBE performance metrics from the utilization of the Cop-BMA approach have been elaborated as discussion of the results obtained in the boxplots plotted in Figure 8. This analysis offers a comprehensive examination of the quantitative outcomes derived from our study, thereby facilitating a more thorough evaluation of the effectiveness and impact of the Cop-BMA methodology. The modified text in the manuscript reads as follows:

“Figure 8 provides a comprehensive overview of collective performance metrics of the HR2D model across the seven QPE simulations, rain gauges simulation, and the Global Cop-BMA multi-modelling for the seven QPEs evaluated at 30 validation stations over the 11-day simulation period. In general, the inundation modelling driven by different products consistently exhibits NSE performance with mean values ranging between 0.695 and 0.846. In terms of KGE performance, the interquartile ranges for QPEs display broader ranges, and the medians for Daymet and ERA5 products fall below 0.8 in contrast to other simulations. Notably, the Cop-BMA approach exhibits slightly higher performance metrics compared to the QPE products, NSE has an average of 0.858 and its total variability is lower compared to single precipitation products. KGE metric has a similar result with an average value of 0.852. The Averaged RMSE for Cop-BMA is 0.561m which is smaller than all the single QPE except for the rain gauges simulation which is only 3 centimetres lower. The averaged MBE for single QPEs ranged between -0.018 and 0.23m, while the Global Cop-BMA method results in an averaged value of 0.049m.”

8. As discussed by the authors, the final flood inundation maps could not be validated effectively because of the scarcity of spatial observed data. Is it possible that the performance of a model member in the ensemble would be better than that of Cop-BMA in terms of the inundation extents, even though its performance in the WSE comparison at one gauge location is not the best?

Response: It is worth noting that while a flood inundation map provided by a single QPE may potentially exhibit greater accuracy compared to one generated by Cop-BMA, the primary advantage of using Cop-BMA lies in its ability to generate probabilistic flood inundation maps while considering uncertainties associated with various QPE sources. Additionally, the QPE offering the highest accuracy is not consistently a single product; it may vary across different case studies and flood events. Therefore, employing a BMA-based approach could be a viable strategy to achieve high accuracy while addressing sources of uncertainty. Additional text in Section 4.1 reads as follows:

“The validation tasks were primarily focused on assessing the performance of model outputs at validation stations, as depicted in Figure 5. This approach enabled us to calculate the performance metrics of WSE over a well-distributed network of stations with remarkable temporal resolution. Data collected from these validation stations sufficiently capture the hydrograph behaviour within the study domain and enables us to quantify flood extents in a probabilistic manner using the HR2D model incorporated with the Cop-BMA method. It is worth noting that while a flood inundation map provided by a single QPE may potentially exhibit greater accuracy compared to one generated by Cop-BMA, the primary advantage of using Cop-BMA lies in its ability to generate probabilistic flood inundation maps while considering uncertainties associated with various QPE sources. Additionally, the QPE offering the highest accuracy is not consistently a single product; it may vary across different study cases and flood event characteristics. Therefore, employing a BMA-based approach could be a viable strategy to achieve high accuracy while accounting for uncertainties.”

9. Minor Issues

- Line 42-43, in BMA applications, I think it is the conditional PDF (the second term on the right-hand side of Equation (1)) rather than the “data” that is assumed to follow a Gaussian distribution. In other words, the pattern of model residuals follows a Gaussian distribution in BMA.

Response: Usually, the conditional PDF of the data is assumed to follow a Gaussian distribution in hydrologic-hydrodynamic applications. This phrase was adjusted to be more concise.

- It would be better if more information can be added to Table 1 or Figure 2. For example, the temporal resolution of discharge and WSE data, the start and end date of the simulation period, indicating which stations are used as boundary conditions and validation, etc. Also, the information of USGS 08074710 was not included in Table 1.

Response:

- o Section 3.1 was complemented with a brief description of simulation time window in HR2D model and mention the hourly outputs. Information is incorporated to the manuscript as follows:

“For unsteady flow analysis in HR2D setup, an hourly simulation time window is defined between August 16/2017 to September 3/2017.”

- o Within the description of BCs in Section 3.1 the hourly temporal resolution of these inputs was incorporated, except for QPEs which are explained in Table 2.
 - o We include in the Supplementary data document Table S1 with the stations used for validation with details and referred in the manuscript.
 - o Figure 5 in Section 4.1 shows stations used for validation during the hurricane event, station USGS 08074710 is part of this validation set, not as BC.
- Line 73: two brackets were used for the reference. **Response:** Two brackets were deleted.
 - Please add the units of SSE in Table 3. **Response:** Units added to table.

Additional references included to manuscript:

Grønneberg, S. and Hjort, N. L.: The Copula Information Criteria, Scandinavian J Statistics, 41, 436–459, <https://doi.org/10.1111/sjos.12042>, 2014.

Sklar, M.: Fonctions de répartition à N dimensions et leurs marges, Annales de l’ISUP, 229–231, 1959.

Thank you again for your constructive comments.

Sincerely,

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