https://doi.org/10.5194/nhess-2024-226
Preprint. Discussion started: 21 March 2025

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Implementing the Equations of Motion in the Energy Line Principle to Simulate the Runout Zones of Gravitational Natural Hazards

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Abstract. The mitigation of the risk from gravitational natural hazards involves a variety of measures in engineering and management. The basis for any measures is the identification of hazard prone areas by considering past events and model simulations. Various models are available for the simulation of gravitational natural hazards but their application is often a trade off between simplicity and accuracy. A physical interpretation of the well established energy line principle allows to formulate the corresponding equations of motion and to introduce a model that is still simple but more accurate. The equations of motion are derived by the application of the Lagrange formalism for a friction block that slides on an inclined plane. Furthermore, a numerical algorithm based on the Euler method is set up to solve the equations of motion on a digital terrain model. This model is applied and compared to the energy line principle based on the equation of energy and to past events in two case studies for rockfall and landslide. The outcomes show that the formulation of the energy line principle with the equations of motion corresponds well to the formulation based on the equation of energy but allows a more differentiated simulation of the runout zone that reproduces better the past events. However, there are artifacts from the numerical solution of the equations of motion that require a deeper theoretical investigation and also uncertainties in the past events that must be worked through in a more elaborated empirical study. Nevertheless, the concept of this approach to enhance the performance of the energy line principle simply by another perspective to the same physical concept is thus proved.

1 Introduction

Gravitational natural hazards in terms of rapid gravity driven mass movements such as rockfalls, landslides and consequent mudflows, can pose a substantial risk to settlement zones and critical infrastructure in steep environments. In Italy, the available data about the affected damage and the economic consequences highlight the impact of these phenomena. Data from the last decade reveal that 9.6% of the population resides in landslide prone areas, which results in over 1000 fatalities and the displacement of more than 138743 individuals from 1973 to 2022 (Bianchi and Salvati, 2024). The national territory affected spans 24770 km^2 (8.4% of Italy's total area) (Iadanza et al., 2021). Economic losses from hydrogeological instabilities, including

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gravitational natural hazards, reached €46 billion from 2010 to 2023, emphasizing the urgency for enhanced prevention and intervention strategies (Bellicini et al., 2024). Despite increased funding for landslide prevention, which indicates an increase in the awareness, a comprehensive and scalable hazard assessment approach remains elusive, highlighting a critical research gap.

In Italy, by law, natural hazard management strategies fall under the responsibility of the MATTM (Ministry of Environment and the Protection of Land and Sea), while the DPC (National Civil Protection Service) is responsible for emergency response. Measures to mitigate gravitational natural hazards include structural solutions like rockfall barriers and gabions, as well as ecological engineering and forestry interventions (Gallozzi et al., 2020), but also preventive urban and territorial planning (Brandolini et al., 2012). However, the effectiveness of these strategies is hindered by methodological inconsistencies in the PAI (Hydrogeological Assessment Plan), which forms the basis for hazard identification and mitigation planning. Basin authorities employ varied methods, combining expert assessments with statistical analyses of past events (Trigila et al., 2021). They are documented in the IFFI (Italian Landslide Inventory), which provide an empirical basis about gravitational natural hazards involving mass movements of rock and soil (Trigila and Iadanza, 2007). This methodological diversity and the absence of standardized simulation models to complement the empirical basis result in an uneven and incomplete national understanding, complicating resource allocation and hazard mitigation efforts. Addressing this gap requires scalable, accurate, and unified modeling approaches that integrate empirical data and predictive capabilities.

Various models to simulate rapid gravity driven mass movements exist, each with strengths and weaknesses that involve trade offs in practical applications. Preliminary analysis often relies on models such as ELINE (May and Dorren, 2019), which are easy to parameterize and fast to execute but provide only approximate outcomes. This approach is based on a failure zone definition using slope inclination criteria and a runout zone simulation using the energy line principle in the form of an equation of energy. The application of the energy line principle allows a simple parameterization only by using the energy line angle (Evans and Hungr, 1993), which can be estimated by the vertical fall height and the horizontal runout distance from past events. However, the outcomes are only a rough estimates whether a location can be reached and, if so, with what energy. For in depth analysis, there are models available such as RAMMS (Noel et al., 2023), which provides accurate outcomes but are challenging to parameterize and costly to execute. This approach is based on a complex equation of motion for rapid mass movements, which enables a detailed simulation of the runout zone but require an elaborate parameterization and high computational effort. Therefore its application is time consuming and susceptible to uncertainties in the estimation of the parameters that are crucial for the quality of the simulation. This trade off between simplicity and accuracy is particularly relevant for analysis on large scales which are needed for the identification of areas prone to gravitational natural hazards.

To address the trade off between simplicity and accuracy in the modeling of gravitational natural hazards, this study explores the physical interpretation of the energy line principle, building on the established correspondence between the energy line angle and the kinetic friction coefficient (Jaboyedoff and Labiouse, 2011). This allows to reformulate the energy line principle

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through a system of differential equations corresponding to the equations of motion equivalent to the integral equation corresponding to the equation of energy used in the established models. Unlike the formulation based on the equation of motion, which merely compares energies at boundary points, the formulation based on the equations of motion captures dynamic interactions of forces and allows to calculate trajectory lines. This adaptation represents a simplified version of the Salm model Salm (1993), assuming a discrete block mass and excluding turbulent friction effects. The resulting model holds the potential to significantly enhance accuracy in simulating runout zones, while maintaining the simplicity of the established energy line principle by requiring only a single parameter.

This study presents the model BLOCKSLIDE, grounded in the equations of motion of the energy line principle, to simulate the dynamics of discrete block masses governed by sliding friction laws. This model is designed to bridge the gap between simplicity and accuracy, providing a straightforward and robust approach for predicting runout zones of gravitational natural hazards across large scales. It offers a significant improvement in accuracy over established approaches like ELINE but keeps its simplicity, while maintaining a computational efficiency unmatched by more complex models such as RAMMS. To proof its effectiveness, BLOCKSLIDE is applied to two study sites in Northern Italy to simulate rockfall and landslide dynamics. The simulations are compared to the outcomes of ELINE as well as past events from historical data in the IFFI, enabling a critical evaluation of both formulations of the energy line principle and their alignment with empirical evidence. This study aims to demonstrate the potential of the physical interpretation of the energy line principle for advancing hazard modeling frameworks and contributing to more effective mitigation planning on a regional scale in national concepts.

2 Methods

2.1 Derivation of the equations of motion

The energy line principle captures the dynamics of rapid gravity driven mass movements under the assumption that it is governed by three principal forces. First the inertial force that describes the tendency of a mass in its state of motion. Second the gravitational force, which leads to a downslope force and a normal force that depend from the slope inclination angle α (°) and the gravitation acceleration g (m*s^{-2}). Third the frictional force that opposes the direction of motion with a magnitude proportional to the normal force, which further depends from the kinetic friction coefficient μ (-) that corresponds to the tangent of the energy line angle. Therefore the energy line principle can be considered as a generalization of the dynamics of a friction block that slides on an inclined plane to an arbitrary surface (fig 1). Those considerations are the baseline for the equations of motion for the energy line principle and therefore the physical foundation of the model BLOCKSLIDE.

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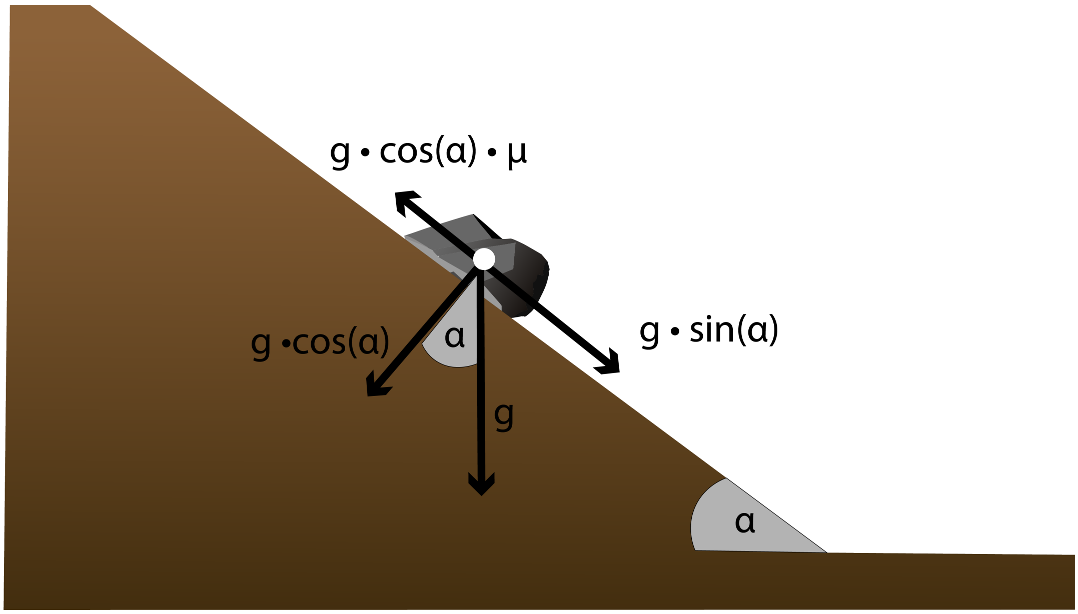

Figure 1. Illustration of the physical principle that explains the energy line principle and provides the baseline for the equations of motion. The illustrated forces (arrows) are the gravitational force (g), the downslope force ($g*sin(\alpha)$), the normal force ($g*cos(\alpha)$) and the frictional force ($g*\mu*cos(\alpha)$)

For the derivation of the equations of motion, a Lagrangian formulation is used since it provides deeper insights and allows an easier application of constraints. This approach is based on the stationary action principle, which states that the path taken by the mass movement minimizes the action (Landau and Lifshitz, 1976). This requires the definition of a Lagrange function L (m^2*s$-^2$) that accounts for the inertial forces and the gravitational forces (eq 1). It is defined as the difference of the kinetic energy to the potential energy and depends from the spatial coordinates x, y and z (m), their derivatives with respect to the time coordinate t (s) and the gravitational acceleration. Since the energy line principle is not conservative because energy is dissipated, the derivation of the equations of motion requires further a Rayleigh function R (m^2*s$-^3$) that accounts for the frictional forces (Minguzzi, 2015). For the energy line principle they can be parameterized by the kinetic friction coefficient and do only depend from the direction but not of the value of the velocity (eq 2). Both, the Lagrange function and the Rayleigh function are formulated without the mass since in this context it is only a proportional constant.

$$L = \frac{(\frac{dx}{dt})^2}{2} + \frac{(\frac{dy}{dt})^2}{2} + \frac{(\frac{dz}{dt})^2}{2} - g*z \tag{1}$$

$$R = \frac{g*\mu*((\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2)^{\frac{1}{2}}}{(1+(\frac{\partial h}{\partial x})^2 + (\frac{\partial h}{\partial y})^2)^{\frac{1}{2}}} \tag{2}$$

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It is assumed that the trajectories of the motion follow the terrain surface, which is in line with the definition of the potential energy in common applications of the energy line principle. This can be implemented as a constraint that sets the vertical coordinate equal to the terrain head h (m), which is a function of the horizontal coordinates (eq 3). This results in an expression for the vertical velocity as a function of the horizontal velocities and the slope gradients (eq 4). The expression for the square of the vertical velocity is simplified by neglecting the mixed terms to avoid complex interconnection in the equations of motion (eq 5).

$$z = h(x,y) \tag{3}$$

$$\frac{dz}{dt} = (\frac{dx}{dt}) * (\frac{\partial h}{\partial x}) + (\frac{dy}{dt}) * (\frac{\partial h}{\partial y}) \tag{4}$$

$$(\frac{dz}{dt})^2 = (\frac{dx}{dt})^2 * (\frac{\partial h}{\partial x})^2 + (\frac{dy}{dt})^2 * (\frac{\partial h}{\partial y})^2 \tag{5}$$

With the application of this constraint, the vertical coordinate can be eliminated respectively expressed by the horizontal coordinates. This leads to adapted expressions for the Lagrange function and the Rayleigh function, which are together the two fundamental components that define the equation of motion for the energy line principle (eq 6 and 7).

$$L = \frac{(\frac{dx}{dt})^2 * (1 + (\frac{\partial h}{\partial x})^2)}{2} + \frac{(\frac{dy}{dt})^2 * (1 + (\frac{\partial h}{\partial y})^2)}{2} - g * h(x,y) \tag{6}$$

$$R = \frac{g * \mu * ((\frac{dx}{dt})^2 * (1 + (\frac{\partial h}{\partial x})^2) + (\frac{dy}{dt})^2 * (1 + (\frac{\partial h}{\partial y})^2))^{\frac{1}{2}}}{(1 + (\frac{\partial h}{\partial x})^2 + (\frac{\partial h}{\partial y})^2)^{\frac{1}{2}}} \tag{7}$$

The equation of motion follow from the derivatives the Lagrange function and the Rayleigh function according to the calculus of variation. The general formulation leads to two equations, one for each horizontal component, since with the horizontal coordinates there are two dependent variables (eq 8 and 9).

$$\frac{d(\frac{\partial L}{\partial(\frac{dx}{dt})})}{dt} = \frac{\partial L}{\partial x} - \frac{\partial R}{\partial(\frac{dx}{dt})} \tag{8}$$

$$\frac{d(\frac{\partial L}{\partial(\frac{dy}{dt})})}{dt} = \frac{\partial L}{\partial y} - \frac{\partial R}{\partial(\frac{dy}{dt})} \tag{9}$$

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The specific formulation of the equation of motion results from the assessment of the Lagrange function and the Rayleigh function (eq 10 and 11). This provides a relation between the horizontal accelerations that reflect the inertial forces on the left hand side to the gravitational forces reflected by the first term and frictional forces reflected by the second term on the right hand side.

$$\frac{d^2x}{dt^2} = -\frac{g*(\frac{\partial h}{\partial x})}{1+(\frac{\partial h}{\partial x})^2} - \frac{g*\mu*(\frac{dx}{dt})}{(1+(\frac{\partial h}{\partial x})^2+(\frac{\partial h}{\partial y})^2)^{\frac{1}{2}}*((\frac{dx}{dt})^2*(1+(\frac{\partial h}{\partial x})^2)+(\frac{dy}{dt})^2*(1+(\frac{\partial h}{\partial y})^2))^{\frac{1}{2}}} \tag{10}$$

$$\frac{d^2y}{dt^2} = -\frac{g*(\frac{\partial h}{\partial y})}{1+(\frac{\partial h}{\partial y})^2} - \frac{g*\mu*(\frac{dy}{dt})}{(1+(\frac{\partial h}{\partial x})^2+(\frac{\partial h}{\partial y})^2)^{\frac{1}{2}}*((\frac{dx}{dt})^2*(1+(\frac{\partial h}{\partial x})^2)+(\frac{dy}{dt})^2*(1+(\frac{\partial h}{\partial y})^2))^{\frac{1}{2}}} \tag{11}$$

This system of differential equations corresponds to the equations of motion for the energy line principle. The solution of these equations provides the trajectories of a motion governed inertial forces, gravitational forces and frictional forces according to the friction law on which the energy line principle is based. This can be shown by formulating the Lagrange function and the Rayleigh function with the arc coordinate of the trajectory in the horizontal plane. Assuming that the trajectory approximately follows the terrain gradient, the established equation of energy for the energy line principle results exactly. This assumption is valid if the gravitational forces are greater than the inertial forces. Therefore the underlying physical concepts are equivalent to the established equation of energy of the energy line principle, but the equations of motion are able to capture further aspects of the dynamic of rapid gravity driven mass movements.

2.2 Algorithm for the numerical solution

The application of the BLOCKSLIDE on a digital terrain model requires an algorithm for numerical solution of the equations of motion. For this prototype of the model, the most simple linear approaches are implemented for the discretization of the system of differential equations. The discretization involves to break down the continuous differential equations into discrete difference equations to allow their numerical solution, which transforms the infinitesimal changes into calculations over finite intervals of space and time. For stability and accuracy in the numerical solution, the discretization must consider the characteristic scales of the model (Barenblatt, 1996). In this simple model they are identified as the characteristic length λ (m) equal to the cell size of the digital terrain model and the characteristic time τ (s) that yields from combining the cell size with the gravitational acceleration (eq 12).

$$g = \frac{\lambda}{\tau^2} \tag{12}$$

The derivatives of the terrain head with respect to the horizontal coordinates are numerically represented by a first order central difference calculation (Wilson and Gallant, 2000). This approach uses the values of the terrain head in the adjacent cells, those immediately before and after the cell in question, to calculate the local change in terrain head. It is therefore the simplest possible approach to approximate the slope gradients for both horizontal directions (eq 13 and 14).

$$\frac{dh}{dx} = \frac{h(x+\lambda, y) - h(x-\lambda, y)}{2*\lambda} \tag{13}$$

$$\frac{dh}{dy} = \frac{h(x, y+\lambda) - h(x, y-\lambda)}{2*\lambda} \tag{14}$$

The derivatives of the horizontal coordinates with respect to the time coordinate are numerically represented by a first order time step iteration process, which is also known also Euler method (Press et al., 1986). Those terms correspond to change of the horizontal coordinates and the horizontal velocities v and w (m/s) over time (eq 15, 16, 17 and 18). The time step is capped by the characteristic time, which serves as a maximum limit to ensure the stability of the numerical solution. However, it can vary dynamically throughout the iteration process based on the ratio between the characteristic length and the absolute velocity. This means that the propagation distance during a time step cannot exceed the cell size, which ensures the accuracy of the numerical simulation.

$$\frac{dx}{dt} = \frac{x(t) - x(t-\tau)}{\tau} \tag{15}$$

$$\frac{dy}{dt} = \frac{y(t) - y(t-\tau)}{\tau} \tag{16}$$

$$\frac{d^2x}{dt^2} = \frac{v(t) - v(t-\tau)}{\tau} \tag{17}$$

$$\frac{d^2y}{dt^2} = \frac{w(t) - w(t-\tau)}{\tau} \tag{18}$$

Beside of those definitions for the discrete formulation of the equations of motion, their numerical solutions requires some conditions that control the iteration process. A first condition controls the start of the motion and sets the components of the horizontal velocity equal to zero before the first iteration, which is then only determined by the gravitational forces. A second condition controls the propagation of the motion and sets the component of the horizontal velocity equal to zero if it is about to reverse direction within an iteration. A third condition controls the end of the motion and interrupts the iteration if both horizontal velocity components are equal to zero or if the perimeter extent would be left in the course of an iteration. Those simple conditions maintain the physical consistency without demanding for any auxiliary parameter.

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2.3 Parameterization and outcomes of the model

The simulation of gravitational natural hazards with the energy line principle based on the equations of motion requires the same parameterization like models based on its equation of energy, which is basically an energy line angle β (°) corresponding to the kinetic friction coefficient (eq 19). In the most simple version with a constant value for the energy line angle, the BLOCKSLIDE does not require any assumptions about the mass and therefore the magnitude of the gravitational hazard process. An appropriate value for the energy line angle can be estimated based on literature or on past events. For this case study, the energy line angle is derived from the past events since the simulated runout zone is supposed to be compared to them.

$$\beta = atan(\mu) \tag{19}$$

Furthermore the simulation requires a digital terrain model and a start cell model that define the initialization points for solving the equations of motion numerically. The digital terrain model provides the values of the terrain head for the numerical solution of the equations of motion. For this case study, the current version for Northern Italy from the INGV (National Institute of Geophysics and Vulcanology) with a cell size of 10 m is applied (Tarquini and Nannipieri, 2017). The start cell model is generally derived by modeling the failure zone that initiate the motion of a mass specifically for the considered gravitational hazard process. In this case study that focus on the runout zone, a simple approach based on a slope inclination criteria specified for rockfall and landslide is applied. For the rockfall, all cells steeper than a soil coverage angle of 55 ° according to the model ROCKYFOR3D are defined as failure zone (Dorren, 2016). This angle characterizes the limit below which soil and above which rock covers the terrain surface. For the landslide, all cells on slopes flatter than this value down to a soil stability angle of 35° according to a study of shallow landslides in a neighboring site are defined as failure zone (Crosta and Frattini, 2003). This angle characterizes a generalized limit of the static stability of the soil under critical conditions. With this approach, the entire parameterization of the model is limited to three characteristic angles. Both, the soil coverage angle and the soil stability angle, are adjusted by the cell size to account for the resolution of the digital elevation model with the formula that is applied in the ROCKYFOR3D model (eq 20).

$$\frac{d\alpha}{d\lambda} = -\frac{0.075 * \alpha}{\lambda} \tag{20}$$

The principle outcome of the BLOCKSLIDE are the simulated trajectories. They are in the form of vector data with line geometry so that each simulated trajectory is available as an unique object. Those vector data are furthermore transformed into raster data in order to obtain the extent of the runout zone. According to this definition, each raster cell that is affected by at least one trajectory line accounts to the runout zone, which is in line with the definition of the failure zone, where each start cell is an initialization point for one trajectory line.

2.4 Assessment of the simulations

The simulations with the BLOCKSLIDE are assessed in terms of a proof of concept. This includes three aspects that are tested based on corresponding work hypothesis. The first is the correspondence to the energy line principle in general which assumes that the simulated trajectories of the BLOCKSLIDE have energy line angles according to the kinetic friction coefficient. The second is the correspondence with the energy line principle in its specific application based on the equation of energy, for which the BLOCKSLIDE is compared to the ELINE. The third is the comparison with the past event from the IFFI, for which both models are applied to reproduce the event envelopes from the inventory data.

A fundamental assumption of the energy line principle is, that the mean of the back calculated energy line angles from the simulated trajectories is equal to the kinetic friction coefficient from the model parameterization (Jaboyedoff and Pedrazzini, 2010). This relationship assumes that, on average, the ratio from the vertical fall height to horizontal runout distance between the start point and the end point of the mass movement is determined by the kinetic friction coefficient. For the back calculation of the energy line angle, the vertical fall height and the horizontal runout distance of each simulated trajectory of the BLOCKSLIDE are determined. The simulated trajectories that are constraint by the perimeter extent must be excluded to avoid edge effects. This yields a statistical distribution of back calculated energy line angles, for which the mean value and the standard deviation are calculated. Furthermore the residual deviation is defined based on the difference of the back calculated energy line angles to the kinetic friction coefficient that is defined as expectation value. Those terms are related by the partition of the total sum of squares (Hedges and Olkin, 1985), which allow to discuss the effective and the expected variance of the back calculated energy line angles (eq 21).

$$\sum(\beta - \sum\beta)^2 = \sum(\beta - atan(\mu))^2 + (atan(\mu) - \sum\beta)^2 \tag{21}$$

For the comparison with the energy line principle based on the equation of energy, the model ELINE is applied as an example model. It is also deterministic model based on the energy line principle and therefore conceptually comparable to BLOCKSLIDE. For the parameterization of the ELINE, the same start cells and the same energy line angle are used as for BLOCKSLIDE. For the cone width angle, the default value of 20 ° is applied. The moving mass volume and the moving mass density are not relevant in this context and therefore both set equal to 1. Of the outcomes of the ELINE, only the frequency grid is considered to determine the runout zone that includes all raster cells with a value greater than 0.

The runout zone is the basis to compare BLOCKSLIDE with ELINE and both models to the past events. For both models, the definition of the runout zone is comparable since it includes all raster cells that are affected by the simulation of the rapid gravity driven mass movement. While the simulations with all start cells are used for the model pairing, only those start cells that lie within the event envelopes are used for the pairing with the past events. For the model pairing and also for the pairing with the past events, the intersection area, both difference areas and the union area of the runout zone are determined which define the partition of the set of sub areas a (m^2) (eq 22). Those sub areas allow to calculate a confusion matrix that serves as

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the basis for the assessment of the performance of the models (Kotu and Deshpande, 2019).

$$a(A \cap B) = a(B \cup A) + a(A \setminus B) + a(B \setminus A) \tag{22}$$

These considerations allow the formulation of criteria to test the work hypothesis under examination. The first work hypothesis assumes that the standard deviation and the residual deviation of the back calculated energy line angles from the BLOCKSLIDE are equal, which means that the assumed relationship between kinetic friction coefficient and the energy line angle is valid. The second hypothesis assumes that the intersection area and the union area of the runout zone for the BLOCKSLIDE and ELINE models are equal, which implies that both models simulate the same spatial extents of the runout zone. The third hypothesis assumes that the intersection area and the union area between the past events and the runout zones simulated by BLOCKSLIDE and ELINE are equal, which suggests that the spatial distribution of simulated runout zone aligns with past events. While the first work hypothesis tests a necessary condition for the validity of the model, the second and third work hypothesis investigate additional conditions to explore the quality of the model.

2.5 Characterization of the study sites

Two study sites are chosen in Northern Italy, Belluno in the region of Veneto and Como in the region of Lombardy (fig 2). At both study sites, protection forest afforestation projects were carried out, for which the identification of areas that are prone to gravitational natural hazards is an important basis. Both study sites are located in the Southern foothill of the Alps and include steep slopes that are susceptible to gravitational natural hazards as well as settlements and infrastructure that are potentially endangered from them. The study perimeter is defined along the divides of sub basins derived from the digital terrain model by a deterministc flow routing in order to apply topographic barriers as perimeter borders.

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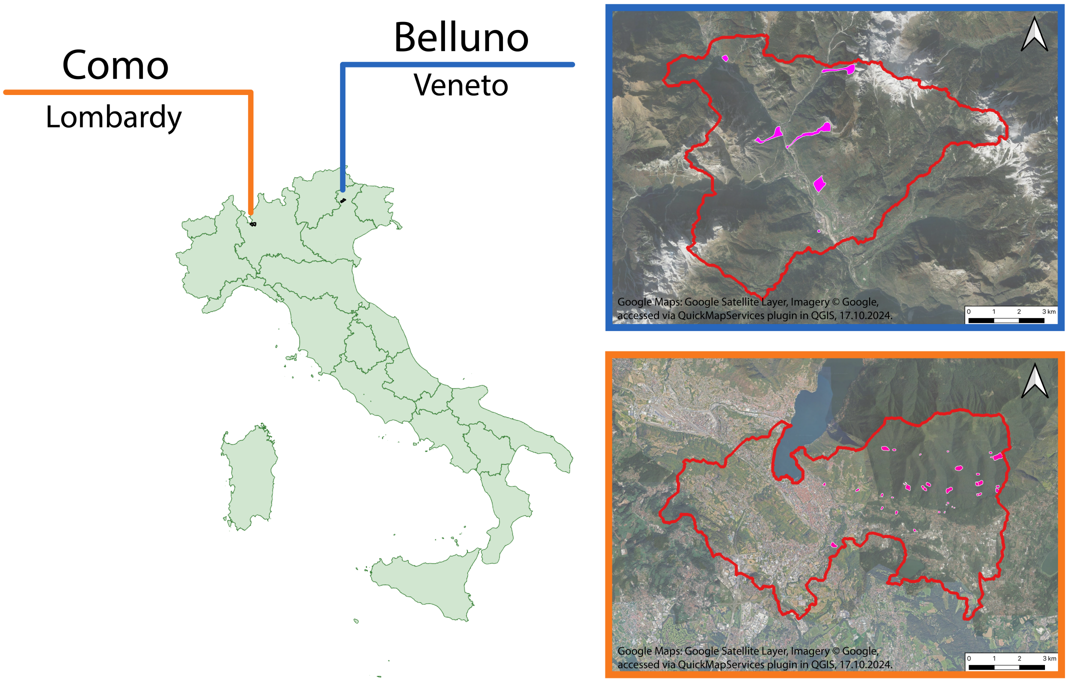

Figure 2. Map for the location of the study sites Belluno (blue) and Como (orange) in Italy (green). The study perimeter (red) and the past events (pink) are shown against the background of © Google Satellite.

For both study sites, the past events documented in the IFFI are selected according to their occurrence and geometry. Because there is no occurrence of rockfall *(Crollo/Ribaltamento)* documented in Como and only a few occurrences of landslides *(Scivolamento rotazionale/traslativo)* documented in Belluno, the assessment is limited to rockfall in Belluno and to landslide in Como. For those past events it must be further ensured, that their geometry covers a failure zone in the upper part and a runout zone in the lower part. Since this differentiation is not given in the data of IFFI, it has been approximated using two criteria. First, the failure zone defined on the slope inclination criteria need to be within the event envelope and secondly, the centroid of the failure zone must be located above than the centroid of the event envelope. The centroid of the event envelope is furthermore used to back calculate the energy line angle by connecting a line from the highest point via the centroid to the lowest point. The energy line angle from the selected past events are an important basis for the parameterization of the models, whereby the rounded mean values are used.

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Study Site	**Event Type**	**Event Count (-)**	**Event Area Mean Value (m^2)**	**Event Area Standard Deviation (m^2)**
Belluno	rockfall *(Crollo/Ribaltamento)*	6	139650	94084
Como	landslide *(Scivolamento rotazionale/traslativo)*	36	14781	15346

Table 1. Details about the frequency and the size of selected past events in the study sites. The past events are taken from the IFFI and selected according to the location of the failure zone.

The relevant properties to characterize the study sites are the geology and the soil since they are related to rockfall and landslide, respectively (tab 2). Both study sites have a carbonate based geological underground and are dominated by cambisol soil cover. The study is in Como is dominated by conglomerate (Ferrario et al., 2015), which is more stratified and less brittle than the dolomite in Belluno (Poli and Zanferrari, 1992). Furthermore the soils in Belluno are more skeletal but less acid than the soils in Como (Costantini et al., 2012).

Study Site	**Study Area (km^2)**	**Geology Type**	**Soil Type**
Belluno	74	Dolomites, eastern Southern Alps	Calcaric Endoleptic Cambisol, Rendzic Leptosol, Rendzic Phaeozem, Haplic Luvisol
Como	68	Turbiditic conglomerates	Dystric, Eutric and Hypereutric Endoleptic Cambisol

Table 2. Characterization of the two study sites is based on official maps about properties of the geology and the soil. The predominant class is indicated for each of the various properties.

The distribution of the past events and the properties of the sites imply a tendency to more rockfall activity in Belluno than in Como but more landslide activity in Como than in Belluno. Since these two aspects coincide, the limitation to rockfall in Belluno and to landslide in Como seems reasonable. Because this study focus on the runout zone but does not look in depth at the failure zone, this remains a working hypothesis that is not further investigated.

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3 Results

3.1 Distribution of the energy line angles

The spatial distribution of the trajectories simulated with the BLOCKSLIDE shows a differentiated picture of the predicted runout zone (fig 3). On the large scale of the total perimeter, there is a differentiated allocation of affected and untouched areas that reflects the regional topography. The close up to the small scale shows a differentiated pattern of convergence and divergence due to the local topography.

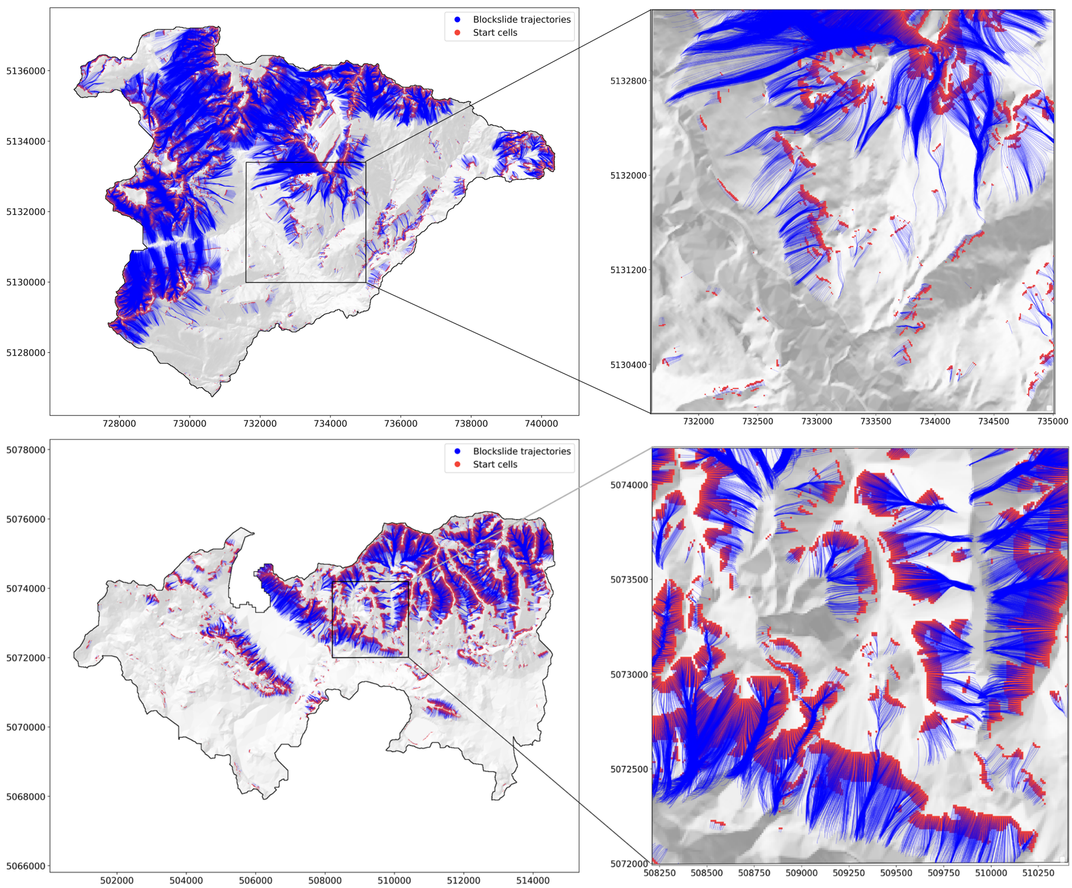

Figure 3. Map of the trajectories lines (blue) and the start cell model (red) simulated with BLOCKSLIDE. They are backed by the hillshade based on the digital terrain model and parted into Belluno (top) and Como (bottom) as well as into the total perimeter (left) and a close up of a representative portion of the map (right).

The frequency distribution of the back calculated energy line angles from the trajectories simulated with the BLOCKSLIDE reveals an unimodal distribution with a peak close to the expectation value defined by the kinetic friction coefficient (fig 4).

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This correspondence is also reflected quantitatively in the partition of the sum of squares (tab 3).

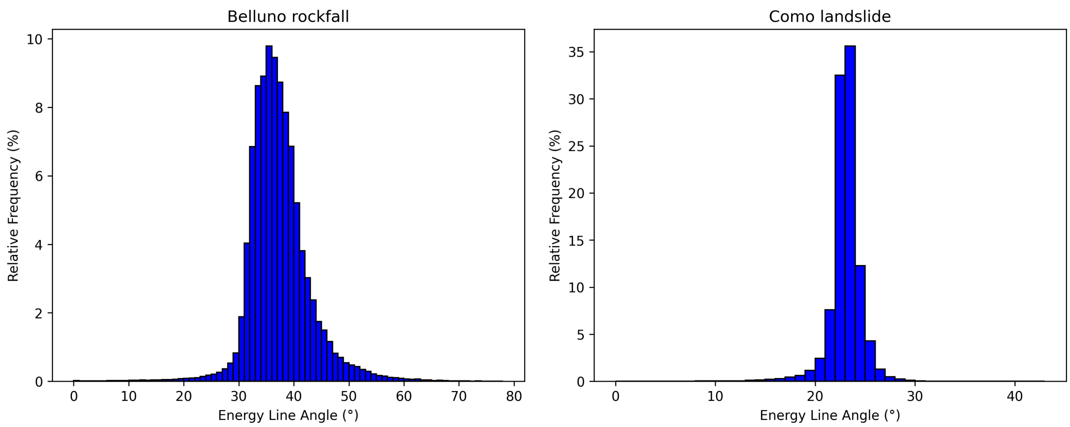

Figure 4. Statistical distribution of the back calculated energy line angle of the simulated trajectories from the BLOCKSLIDE. The underlying sample for the statistical distribution consists of 158403 simulated trajectories for the rockfall in Belluno (left) and 114712 simulated trajectories the landslide in Como (right).

Study site	**Expectation Value (°)**	**Mean Value (°)**	**Standard Deviation (°)**	**Residual Deviation (°)**
Belluno	33	37	6	4
Como	23	23	2	2

Table 3. Statistical assessment of the back calculated energy line angle of the simulated trajectories from the BLOCKSLIDE with the partition of the total sum of squares. The expectation value corresponds to the rounded mean value of the energy line angle back calculated from the past events.

The back calculated energy line angles show a better correspondence to the kinetic friction coefficient in Como than in Belluno. The standardized difference between the mean value and the expectation value is approximately 67 % in Belluno and 0 % in Como. But for both study site, the difference between the mean value and the expectation value is still within the standard deviation.

3.2 Extent of the runout zone

The comparison between BLOCKSLIDE and ELINE shows distinct differences in the extent of the runout zone (fig 5). The runout zone from the ELINE is generally more expansive and less differentiated than the runout zone from the BLOCKSLIDE.

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This trend is confirmed quantitatively by the partion of the sub areas of the runout zone simulated by both models (tab 4).

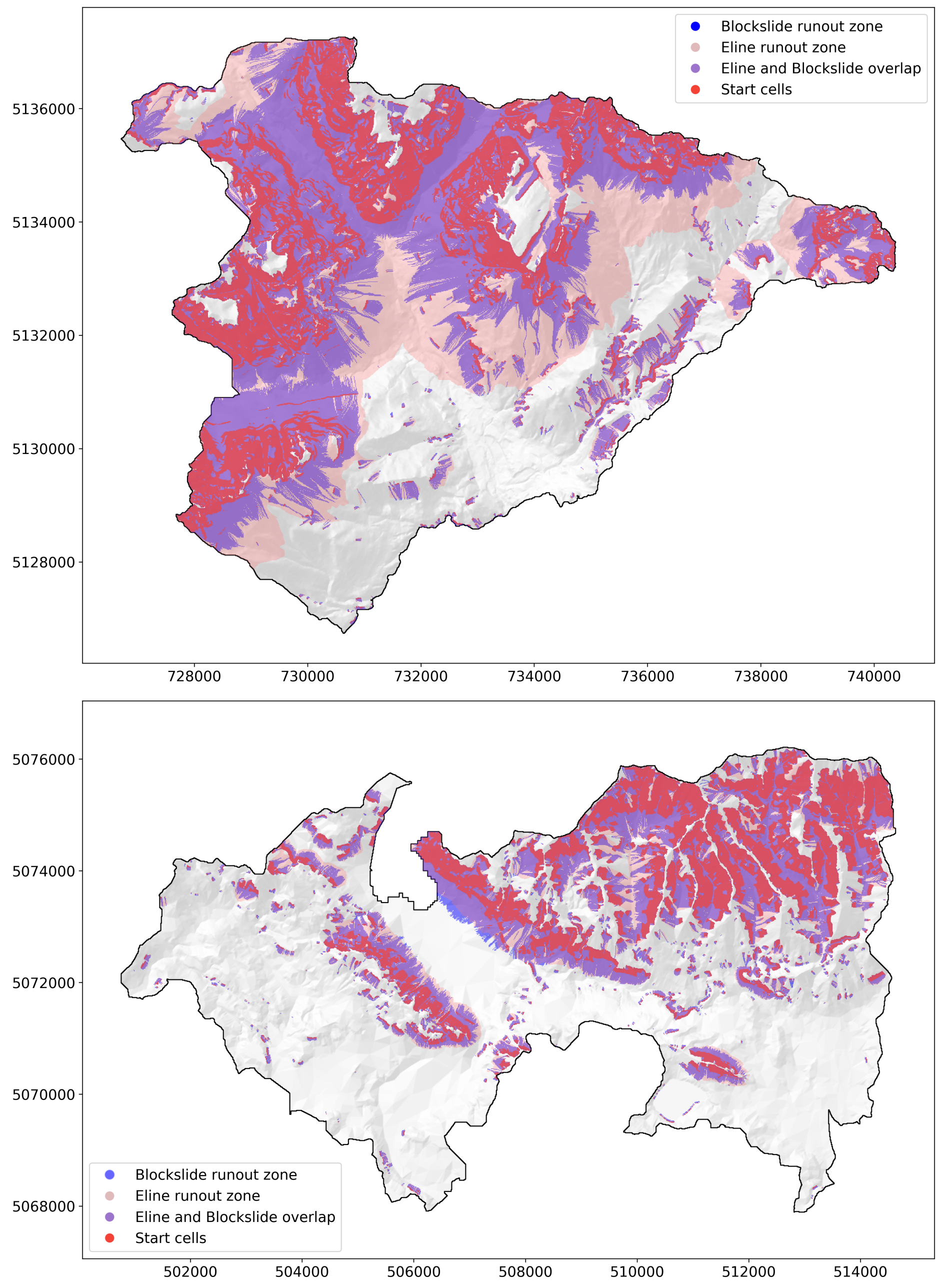

Figure 5. Map of the runout zone simulated with BLOCKSLIDE (purple) and ELINE (pink). They are backed by the hillshade based on the digital terrain model and parted into Belluno (top) and Como (bottom).

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Study site	**BLOCKSLIDE and ELINE (%)**	**ELINE without BLOCKSLIDE (%)**	**BLOCKSLIDE without ELINE (%)**
Belluno	71	27	2
Como	80	12	8

Table 4. Areal assessment of the runout zone simulated by the BLOCKSLIDE and by the ELINE. The comparison is based on the partition of the sub areas with the common intersection (BLOCKSLIDE and ELINE) and the respective differences (ELINE without BLOCKSLIDE, BLOCKSLIDE without ELINE) and the common union (ELINE or BLOCKSLIDE) to normalize the areal values.

The total area covered by both models is 53 km^2 in Belluno and 24 km^2 in Como. For both study sites, the area of BLOCKSLIDE without ELINE is negligibly small. The area of ELINE with BLOCKSLIDE and the area of BLOCKSLIDE and ELINE are significantly greater. For Belluno, the area of ELINE without BLOCKSLIDE is greater than the area of BLOCKSLIDE and ELINE, while for Como the area of BLOCKSLIDE and ELINE is greater than the area of ELINE without BLOCKSLIDE.

3.3 Reproduction of the past events

The simulation of the past events with BLOCKSLIDE and ELINE reveals, that they can only be reproduced to a limited extent using those model approaches (fig 6). The quality of the reproduction varies with past events that match well but also with some that match poorly, whereby the match for the rockfall events is better than for the landslide events. The runout zone from BLOCKSLIDE is more differentiated than the runout zone from ELINE, which, however, approximately balances out in terms of the overall match with the past events. This is also reflected quantitatively by the partition of the sub areas of the past events and the runout zones (tab 5).

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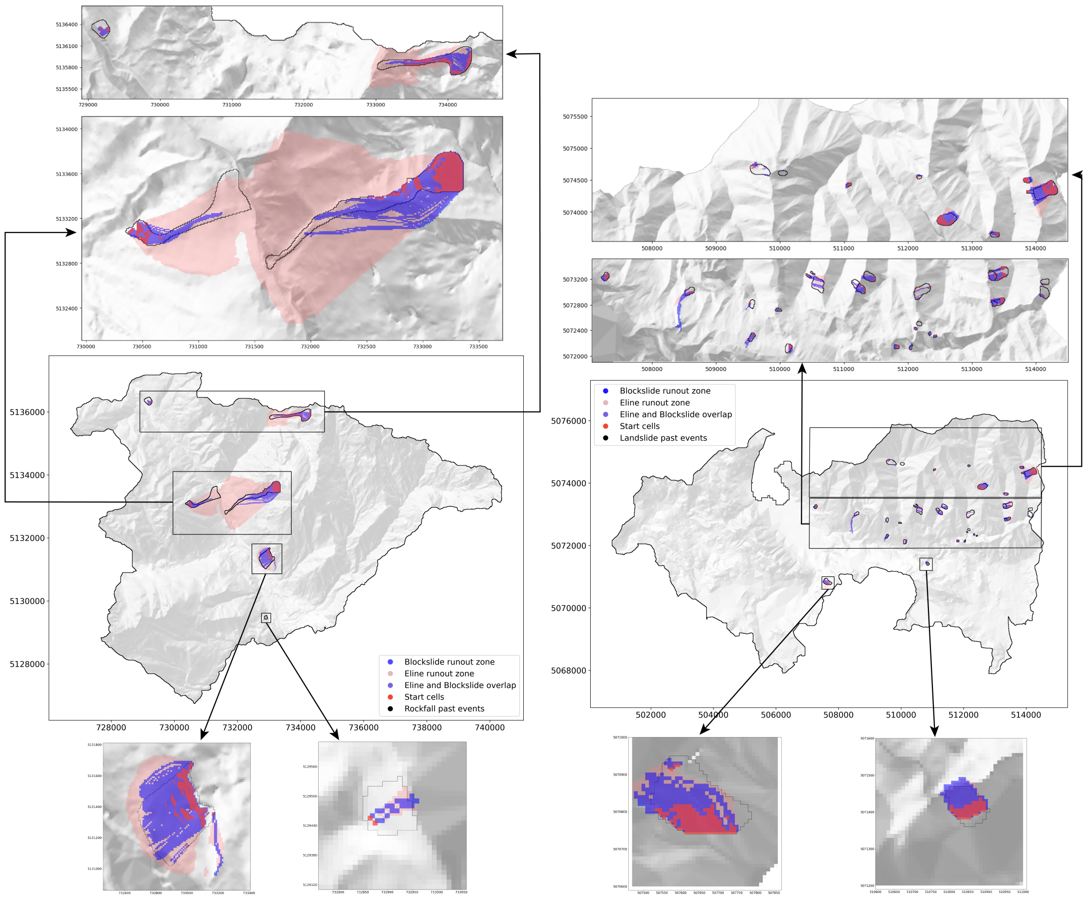

Figure 6. Map with the past events from the IFFI (black line) as well as the runout zones from BLOCKSLIDE (purple) and ELINE (pink). The map is backed by the hillshade based on the digital terrain model and parted into the entire study site of Belluno (left) and Como (right) as well as close ups of the 6 rockfall events in Belluno and the 36 landslide events in Como.

Study site	**Model Approach**	**True Positive Area (%)**	**False Positive Area (%)**	**False Negative Area (%)**
Belluno	BLOCKSLIDE	52	22	26
Belluno	ELINE	11	87	2
Como	BLOCKSLIDE	52	17	31
Como	ELINE	41	33	26

Table 5. Areal comparison to assess the match of the runout zone simulated with BLOCKSLIDE and ELINE to the past events from the IFFI. Based on the partition of the sub areas, the true positive (common intersection past events and runout zone), false positive (difference runout zone without past events) and false negative (difference past events without runout zone) are defined while the true negative is meaningless in this context.

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The total area covered by the simulated runout zone or the past events is approximately 1 km^2 regarding BLOCKSLIDE and 7 km^2 regarding ELINE in Belluno and 1 km^2 regarding BLOCKSLIDE and 1 km^2 regarding ELINE in Como. In both study sites, the true positive area but also the false negative area area greater for the BLOCKSLIDE than for the ELINE while the false positive area is greater for the ELINE than for the BLOCKSLIDE. Those tendencies are more distinct for the rockfall in Belluno than for the landslide in Como.

4 Discussion

4.1 General assumption of the energy line principle

Overall, the back calculated energy line angles from the simulated trajectories of BLOCKSLIDE fit closely to the kinetic friction coefficient used for the model parameterization considering that the difference between the mean value from the expectation value is smaller than the standard deviation. While this difference is negligible for the simulations in Como, it is larger for the simulations in Belluno with a mean value above the expectation value. This means that the first work hypothesis can be only partly accepted.

A crucial aspect to be emphasized is that there is a statistical distribution at all in the back calculated energy line angles from the simulated trajectories with BLOCKSLIDE. This version of the model is completely deterministic without any probabilistic components, so that each repetition with the same inputs results in identical outputs. Therefore, the cause of this statistical distribution can only be found in the interaction between the numerical algorithms and the terrain properties, which is a well known issue in numerical modeling (LeVeque, 1992). The discretization and the conditions for the numerical solution of the equations of motion can lead to random differences in the length values of the trajectory lines in the order of magnitude of the cell size that is used to define the characteristic length. These random differences result in a statistical distribution of the back calculated energy line angles, even though the underlying assumptions are deterministic.

Although random differences between the back calculated energy line angles and the expectation value can explain the statistical distribution, they should lead to the same mean value in a large sample. The fact that the mean value differs from the expectation value indicates also systematic differences due to the interaction between the numerical algorithms and the terrain properties. It is recognized that such systematic differences can occur in numerical modeling and that they can be also site specific (Caers, 2011). They could principally be caused by a coarse numerical discretization over a highly variable terrain and especially be amplified the stop conditions of the numerical algorithm. Similar effects and their impact on the simulation of the runout zone of gravitational natural hazards have been found while using RAMMS (Buehler et al., 2011). Further reasons for this systematic differences found in the simulations with BLOCKSLIDE could be due to cases in which the inertial forces are greater than the gravitational forces as well as the neglect of the mixed terms in the equations of motion. Both issues may cause deviations in the back calculated energy line angles of the trajectories from the kinetic friction coefficient depending from the terrain properties. This shows the need for further theoretical considerations on the physical concepts and numerical algorithms for their implementation.

4.2 Specific application of the energy line principle

The comparison of BLOCKSLIDE to ELINE reveals clear differences in the simulated runout zone based on the same start cell model and the same digital terrain model. The runout zone simulated only by ELINE is smaller than the commonly simulated runout zone but larger than the runout zone simulated only by BLOCKSLIDE, which is negligibly small. The tendency is similar for both study sites, which implies that it is not affected from site specific terrain properties and the value of the kinetic friction coefficient. This rejects the second work hypothesis that both models simulate the same spatial extents of the runout zone.

The runout zone simulated with BLOCKSLIDE shows a more differentiated pattern than the runout zone simulated with ELINE. This can be attributed to the fact that BLOCKSLIDE based on differential equations takes the local topography into account, while ELINE based on an integral equation only considers the global topography. Furthermore the runout zone differs more in terms of lateral spread than in terms of downslope reach, which is a pattern that is also found by a comparison of the energy line principle based on the equation of energy with the more complex model ROCKYFOR3D (Clouet et al., 2012). Those limitations of the energy line principle considering the interaction with topography and the lateral spread are recognized and are reason for its restricted application in preliminary analysis (Volkwein et al., 2011). Considering that both model approaches are based on the same physical concepts, these differences imply that the energy line principle based on the equations of motion leads to more accurate runout zones than the established approach based on the equation of energy and can overcome some of its crucial limitations.

4.3 Empirical review of the different model formulations

Both model approaches, BLOCKSLIDE and ELINE, can only partially reproduce the past events from the IFFI. While the match of BLOCKSLIDE is clearly better considering the true positive area and the false positive area, the match of ELINE is better for the false negative area. Therefore the runout zone simulated with BLOCKSLIDE tends to be too small while the runout zone simulated with ELINE tends to be too large. This rejects the third work hypothesis that the simulated runout zones aligns with the past events.

Overall, BLOCKSLIDE matches significantly better the past events than ELINE. The mismatches occurring with both models are probably due to the general parameterization that is not adapted to each specific past event and the missing information about the division into failure zone and runout zone. The reconstruction of the failure zone is also identified as a possible cause of uncertainties in a study that applies the energy line principle to past events (Marinelli et al., 2022). Furthermore, the past events are directly taken from the IFFI without any adaption of the event envelopes and with the inclusion of failure zone and their average location as the only criterion to check their plausibility. Also another study that compares model simulations with past events revealed such issues due to spatial inaccuracies of the envelopes (Alvioli et al., 2021). Because of these limitations, the false negative area is assumed to be less relevant for the assessment of the accuracy than the false positive area since the past events serve as a basis for a relative, rather than an absolute, comparison.

Nevertheless, the assessment of the accuracy implies that the energy line principle based on the equations of motion provides better reproduction of past events than the established approach based on the equation of energy using the same parameterization. Even if an in depth assessment of past events is still outstanding, this shows that the formulation with the equations of motion entails a significant improvement of the application of the energy line principle.

4.4 Options for applications and improvements

This explorative implementation of the equations of motion for the energy line principle in BLOCKSLIDE shows its capabilities but also that there is room for future improvements. Already this explorative version has the potential for various applications that go beyond the replacement of the established models that use the energy line principle based on the equation of energy. This is because it allows to simulate unique trajectories of rapid gravity driven mass movements while using the same simple parameterization. However, it is important to note that BLOCKSLIDE is still based on the energy line principle with its simple physical concepts, which means that the trajectories should not be considered as effective but as indicative. Therefore it is not supposed to replace model approaches based on more complex physical concepts and challenging applications such as dimensioning of protective measures at small scales. However, it is suitable for simulating runout zones at large scales and applications such as the identification of hazard prone areas and protective forest.

Further improvements of BLOCKSLIDE could include the explicit consideration of the mass, which would allow providing a rough estimate of the kinetic energy. The mass could also be used to define a more appropriate value for the kinetic friction coefficient, which tends to decrease with increasing mass (Corminas, 1996). Furthermore, the kinetic friction coefficient could be parameterized not as single value but as raster data based on terrain properties such as the surface roughness (Haas et al., 2012). This spatial variability of the kinetic friction coefficient would also offers an opportunity to consider the protective effect of forest vegetation. A further improvement could be the integration of probabilistic components. This could be based on similar principles like for the example those applied in the model ROCKYFOR3D (Bourrier et al., 2009). The physical and numerical basis of BLOCKSLIDE would allow to implement these aspects and to enhance its framework for various application in modeling gravitational natural hazards. However, further research on its theoretical functions and comparison with empirical data is essential.

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5 Conclusions

The introduction of the equations of motion for the energy line principle offers a significant improvement of this established approach by increasing its accuracy and allows further applications by the ability to simulate trajectories for rapid gravity driven mass movements. The implementation in BLOCKSLIDE is able to at least partially resolve the trade off issue between a simple parameterization and accurate outcomes in the context of the simulation of gravitational natural hazards. It maintains the simplicity and scalability of the energy line principle but introduces a level of accuracy that is usually achievable only with more complex model approaches. Nevertheless, its limitation is the simple physical concepts of the energy line principle, which models the motion of the mass as a friction block on an inclined plane given by the terrain surface.

With the assessment of BLOCKSLIDE in this case study, some need for future research is revealed on the theoretical as well as on the empirical level. On the theoretical level, the physical concepts and their implementation in the numerical algorithms must be further investigated to better understand how the required assumptions effect the outcomes and interacts with the terrain properties. On the empirical level, the comparison with past events must be conducted with a more elaborated plausibility check to validate the accuracy of the model outcomes. Furthermore, the accuracy of the model must be compared with other established models. Both aspects are crucial to clear on the way towards a practical application.

The possible applications of BLOCKSLIDE are versatile and promise a simpler solution to challenging problems. The focus is on applications on preliminary level and at large scales, where a simple parameterization is still more important than a high accuracy, even though this approach is a good compromise that is still highly accurate in relation to its simplicity. The simulation of trajectories enables more differentiated analyses than is usually possible with the energy line principle and also many other models that are not based on equations of motion. Due to its applicability to various gravitational natural hazards such as rockfall and landslide but also mudflow and avalanche, this model approach is promising to be used as a standard model for preliminary analyses to identify hazard prone areas at large scales.

Data availability. Data will be made available on request.

Author contributions. The authors contributed as following: EM, DM Conceptualization. EM, DM Data curation. EM, DM Formal analysis. FG Funding acquisition. EM, DM Investigation. EM, DM, LD Methodology. EM, DM Project administration. EM, DM Resources. EM, DM Software. EM, DM, FG, LD Supervision. EM, DM Visualization. EM, DM Writing – original draft preparation. EM, DM, FG, LD Writing – review & editing

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Competing interests. The authors declare that they have no conflict of interest.

Acknowledgements. EM, DM, LD and FG thank REACT-EU and PON R&I for financial support. Also EM would like to thank FM for the two significant contributions, without which this work could not have been completed.

References

Alvioli, M., Santangelo, M., Fiorucci, F., Cardinali, M., Marchesini, I., Reichenbach, P., Rossi, M., Guzzetti, F., and Peruccacci, S.: Rockfall susceptibility and network-ranked susceptibility along the Italian railway, Engineering Geology, 293, 72–80, https://doi.org/doi.org/10.1016/j.enggeo.2021.106301, 2021.

Barenblatt, G.: Scaling, self-similarity, and intermediate asymptotics: dimensional analysis and intermediate asymptotics, Cambridge University Press, 1996.

Bellicini, L., D'Alessandris, P., Reggio, P., and Tascedda, M.: Lo stato di rischio del territorio italiano 2023, Tech. rep., Associazione Nazionale Costruttori Edili, 2024.

Bianchi, C. and Salvati, P.: Rapporto Periodico sul Rischio posto alla Popolazione italiana da Frane e Inondazioni, Tech. rep., Consiglio Nazionale delle Ricerche, 2024.

Bourrier, F., Dorren, L., Nicot, F., Berger, F., and Darve, F.: Toward objective rockfall trajectory simulation using a stochastic impact model, Geomorphology, 110, 68–79, https://doi.org/doi.org/10.1016/j.geomorph.2009.03.017, 2009.

Brandolini, P., Cevasco, A., Firpo, M., Robbiano, A., and Sacchini, A.: Geo-hydrological risk management for civil protection purposes in the urban area of Genoa (Liguria, NW Italy), Natural Hazards and Earth System Sciences, 12, 943–959, https://doi.org/doi.org/10.5194/nhess-12-943-2012, 2012.

Buehler, Y., Christen, M., Kowalski, J., and Bartelt, P.: Sensitivity of snow avalanche simulations to digital elevation model quality and resolution, Annals of Glaciology, 52, 72–80, https://doi.org/doi:10.3189/172756411797252121, 2011.

Caers, J.: Modeling Uncertainty in the Earth Sciences, Wiley, 2011.

Clouet, N., Berger, F., and Lievois, J.: Rockfall Modelling and Risk Zoning: A Case Study in the French Alps Using Geomatics, Airborne Laser Scanning, 2D and 3D Runout Models, 12th Congress Interpraevent Conference Proceedings, pp. 479–486, 2012.

Corminas, J.: The angle of reach as a mobility index for small and large landslides, Canadian Geotechnical Journal, 33, 260–271, https://doi.org/doi.org/10.1139/t96-005, 1996.

Costantini, E., L'Abate, G., Barbetti, R., Fantappie, M., Lorenzetti, R., and Magini, S.: Soil map of Italy, scale 1:1.000.000, Tech. rep., Centro Nazionale di Cartografia Pedologica, 2012.

Crosta, G. and Frattini, P.: Distributed modelling of shallow landslides triggered by intense rainfall, Natural Hazards and Earth System Sciences, 3, 81–93, 2003.

Dorren, L.: Rockyfor3D (v5.2) revealed – Transparent description of the complete 3D rockfall model, Ecorisq, 2016.

Evans, S. and Hungr, O.: The assessment of rockfall hazard at the base of talus slopes, Canadian Geotechnical Journal, 30, 620–636, https://doi.org/doi:10.1139/t93-054, 1993.

Ferrario, M., Bonadeo, L., Brunamonte, F., Livio, F., Martinelli, E., Michetti, A., Censi Neri, P., Chiessi, V., Comerci, V., and Hoebig, N.: Late Quaternary environmental evolution of the Como urban area (Northern Italy): A multidisciplinary tool for risk management and urban planning, Engineering Geology, 193, 384–401, https://doi.org/doi.org/10.1016/j.enggeo.2015.05.013, 2015.

Gallozzi, P., Dessi, B., Iadanza, C., Guarneri, E., Marasciulo, T., Miscione, F., Spizzichino, D., Rischia, I., and Trigila, A.: ReNDiS 2020 La difesa del suolo in vent'anni di monitoraggio ISPRA sugli interventi per la mitigazione del rischio idrogeologico - Rapporti 328/20, Tech. rep., Istituto Superiore per la Protezione e la Ricerca Ambientale, 2020.

Haas, F., Heckmann, T., Wichmann, V., and Becht, M.: Runout analysis of a large rockfall in the Dolomites/Italian Alps using LIDAR derived particle sizes and shapes, Earth Surface Processes and Landforms, 37, 1444–1455, https://doi.org/doi.org/10.1002/esp.3295, 2012.

https://doi.org/10.5194/nhess-2024-226
Preprint. Discussion started: 21 March 2025

Hedges, L. and Olkin, I.: Statistical Methods for Meta-Analysis, Academic Press, 1985.

Iadanza, C., Trigila, A., Starace, P., Dragoni, A., Biondo, T., and Roccisano, M.: IdroGEO: A collaborative web mapping application based on REST API services and open data on landslides and floods in Italy, ISPRS International Journal of Geo-Information, 10, https://doi.org/https://doi.org/10.3390/ijgi10020089, 2021.

Jaboyedoff, M. and Labiouse, V.: Technical Note: Preliminary estimation of rockfall runout zones, Natural Hazards and Earth System Sciences, 11, 819–828, https://doi.org/doi:10.5194/nhess-11-819-2011, 2011.

Jaboyedoff, M. and Pedrazzini, A.: The usefulness of the reach angle concept for hazard zoning using statistical approach, Geophysical Research Abstracts, 12, 2010.

Kotu, V. and Deshpande, B.: Data Science: Concepts and Practice, Elsevier, 2019.

Landau, L. and Lifshitz, E.: Mechanics, Pergamon Press, 1976.

LeVeque, R.: Numerical Methods for Conservation Laws, Springer, 1992.

Marinelli, A., Medici, C., Rosi, A., Tofani, V., Bianchini, S., and Casagli, N.: Shallow Landslides and Rockfalls Velocity Assessment at Regional Scale: A Methodology Based on a Morphometric Approach, Geosciences, 12, https://doi.org/doi.org/10.3390/geosciences12040177, 2022.

May, D. and Dorren, L.: ELine – Energy Line Calculation Manual, Ecorisq, 2019.

Minguzzi, E.: Rayleigh's dissipation function at work, European Journal of Physics, 36, https://doi.org/doi:10.1088/0143-0807/36/3/035014, 2015.

Noel, F., Nordang, S., Jaboyedoff, M., Digout, M., Guerin, A., Locat, J., and Matasci, B.: Comparing Flow-R, Rockyfor3D and RAMMS to Rockfalls from the Mel de la Niva Mountain: A Benchmarking Exercise, Geosciences, 13, https://doi.org/doi.org/10.3390/geosciences13070200, 2023.

Poli, M. and Zanferrari, A.: The Agordo Basement (NE Italy): A 500 Ma-Long Geological Record in the Southalpine Crust, Contributions to the Geology of Italy, 5, 283–296, 1992.

Press, W., Teukolsky, S., Vetterling, W., and Flannery, B.: Numerical Recipes: The Art of Scientific Computing, Cambridge University Press, 1986.

Salm, B.: Flow, flow transition and runout distances of flowing avalanches, Annals of Glaciology, 18, 221–226, https://doi.org/doi:10.1017/s0260305500011551, 1993.

Tarquini, S. and Nannipieri, L.: The 10 m-resolution TINITALY DEM as a trans-disciplinary basis for the analysis of the Italian territory: Current trends and new perspectives, Geomorphology, 281, 108–115, https://doi.org/doi.org/10.1016/j.geomorph.2016.12.022, 2017.

Trigila, A. and Iadanza, C.: Rapporto sulle frane in Italia: il Progetto IFFI: metodologia, risultati e rapporti regionali, Tech. rep., Istituto Superiore per la Protezione e la Ricerca Ambientale, 2007.

Trigila, A., Iadanza, C., Lastoria, B., Bussettini, M., and Barbano, A.: Dissesto idrogeologico in Italia: pericolosità e fattori di rischio - Edizione 2021 - Rapporti 356/2021, Tech. rep., Istituto Superiore per la Protezione e la Ricerca Ambientale, 2021.

Volkwein, A., Schellenberg, K., Labiouse, V., Agliardi, F., Berger, F., Bourrier, F., Dorren, L., Gerber, W., and Jaboyedoff, M.: Rockfall characterisation and structural protection – a review, Natural Hazards and Earth System Sciences, 11, 2617–2651, https://doi.org/doi:10.5194/nhess-11-2617-2011, 2011.

Wilson, J. and Gallant, J.: Terrain Analysis: Principles and Applications, Wiley, 2000.