

1 Reviewer #1

Q1 — The manuscript describes the extension of a depth-averaged flow model that uses Voellmy’s rheology by a physically-based stopping criterion. To be honest, I am not convinced by the manuscript in its present form. There are too many aspects not explained or justified sufficiently.

5 From my point of view, publishing the manuscript as a research paper would require much more explanation of the approach and convincing arguments that the problem addressed here is important for practical applications and that it is a general problem and not just a deficiency of the implementation used here.

Since these points are quite fundamental for me, I do not write line-by-line comments at the moment, but look forward to reviewing a revised version. I would also like to point out that my rating of the manuscript refers to the “worst-case scenario”
10 and would be much higher if the points raised above can be addressed.

A1 — We appreciate you having taken the time to review our manuscript. Please refer to our point-by-point responses below.

Q2 — There is only a vague promise to disclose the codes used for the simulation and for the analysis after acceptance of the
15 paper. This makes assessing the correctness of the manuscript difficult.

A2 — Thank you for your helpful suggestion. We shall add as an Appendix a full description of the code.

Q3 — It does not become clear whether the issue addressed in the paper is a general problem of Voellmy’s rheology (and then
20 also of similar rheologies) of just a deficiency of the implementation used here (and perhaps of some other implementations). In the beginning of Sect. 2.2, it reads as if Voellmy’s rheology itself would not let the material come to rest. This is not true since material will finally come to rest if the tangent of the slope angle of the free surface is smaller than the coefficient of friction μ . This means that spreading of the deposits in the runout zone should stop completely. Otherwise, there may be a problem with the numerical implementation of the friction term. I was involved in two model developments. One of them (based on Gerris,
25 doi 10.5194/nhess-15-671-2015) lets the fluid accelerate first and then uses a fully implicit scheme for the friction term, which leads to a permanent creeping and some artificial spreading of the deposits. The other (MinVoellmy, doi 10.5194/gmd-17-781-2024, already cited) uses a mixed scheme for the friction term and stops reasonably well in the runout zone. I do not know whether the model used here includes a ‘good’ or a ‘bad’ implementation of the friction term.

30 **A3** — Yes, we agree that the material will “finally come to rest” using a Voellmy rheology, when the material reaches sufficiently gentle gradients and provided a proper yielding criterion is implemented. One issue is *the time involved for this to occur* – it is not computationally tractable to wait for all nodes in the model to converge to zero when performing e.g. sensitivity studies. Furthermore, if you let the model run for too long the results become unrealistic due to numerical diffusion, it is important to be able to identify the point at which the flow can be considered as arrested, to discard all results that would

35 be obtained later. Our intention was to start from the point of view that practitioners want to optimise their model run time, rather than increasing it by some arbitrary factor to try to wait for convergence to zero. We note that the original text from the Introduction was not sufficiently clear on this subject:

40 Numerical diffusion affects these numerical models, the extent depending typically on the numerical scheme adopted, the friction law used, and the topographical complexity (see also Hergarten, 2024). In practice, this numerical diffusion can lead to excessive spreading of avalanche deposits, smoothing of fronts, and, relatedly, persistent non-convergence of velocities to zero after apparent macroscopic flow stoppage. These artifacts are generally caused by numerical discretisation of the flow domain and numerical handling of floating point numbers.

We would therefore update this text to discuss the run-time issue. A secondary problem is the influence of numerical diffusion due to complex topographies, which we believe to affect all schemes on complex topographies, and which induces non-physical 45 contributions to the results. We propose making changes to the wording of Section 2.2 as well, to further clarify that “not coming to rest” is true for “complex topographies” and “under tractable computational times”.

Q4 — It is true that some models include an empirical (non-physical) stopping criterion. However, there is no illustration or discussion how strong the effect of such a criterion is practically and whether it might be relevant for hazard assessment.

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A4 — As mentioned in the current version of our manuscript:

55 RAMMS2D, for instance, implements several options for user-defined thresholds to terminate simulations, the most commonly used considering the ratio between current global flow momentum and the maximum global momentum reached during the flow. Zugliani *et al.* (2021) suggest using typical values between 1 and 10 % for this threshold.

Although momentum and velocity are not directly interchangeable, some general insights can still be obtained by considering the above thresholds (for momentum) in the context of velocity. For instance, for a flow with a maximum velocity at the upper bound recorded for avalanches of 100 m/s, a momentum threshold of 10% would imply an average residual velocity on the order of around 10 m/s. Depending on the underlying topography, this could translate into an additional runout distance of tens 60 (or hundreds) of metres, beyond the final state of the simulated avalanche. Even with a momentum threshold of 1%, this would give an average residual velocity on the order of 1 m/s. This could also lead to substantially longer runout distances than what is computed, which might be significant in terms of risk assessments, or influencing where one might want to install mitigation measures. We propose adding text into the manuscript to discuss this in more detail.

65 **Q5** — The stopping criterion (Eq. 6) is not explained completely since a proper definition of $\tau_{b,\text{test}}$ is missing. Is the criterion just that the actual momentum could be consumed entirely in the actual time step, as implemented in MinVoellmy? Or is it something more elaborate?

A5 — We agree that this was a deficiency of the original manuscript. We shall define $\tau_{b,\text{test}}$ mathematically in the Appendix.
70 The idea behind the criterion is that if the resisting stresses (encapsulated in $\tau_c + \rho g_z h \mu$) are greater than the driving stresses (encapsulated in $\tau_{b,\text{test}}$), the flow velocity is set to exactly (integer) zero instead. Without this criterion, a flow velocity of near-zero would be achieved instead, with a stronger asymptotic component than with the criterion.

Q6 — The authors often refer to numerical diffusion as the source of the issues addressed in this manuscript. This is not clear
75 to me, in particular since the simple implementation in MinVoellmy does not include any measures against numerical diffusion, but shows little spreading of the deposits.

A6 — Thank you for pointing out the lack of sufficient clarity here. As mentioned in Hergarten (2024), numerical diffusion is a
key issue for CFD solvers, although this paper also states that for the MinVoellmy implementation, numerical diffusion is not a
80 major issue. However, as far as we can tell, the topographies considered in that manuscript were simple, i.e. were comprised of planes and splines. We do not debate that for simulations on *simple* topographies, most implementations show little spreading of the deposits – which is consistent with the results from our current scheme. The intent of the manuscript is to highlight how spreading continues indefinitely on *complex topographies*, which is an issue that affects multiple numerical schemes into which the Voellmy rheology is implemented. We note that some of the text from the Introduction could probably be refined.
85 For instance:

Numerical diffusion affects these numerical models, the extent depending typically on the numerical scheme adopted, the friction law used, and the topographical complexity (see also Hergarten, 2024). *In practice*, this numerical diffusion can lead to excessive spreading of avalanche deposits, smoothing of fronts, and, relatedly, persistent non-convergence of velocities to zero after apparent macroscopic flow stoppage.

90 We would update “In practice” to explicitly refer to complex topographies, and also clarify the “see also Hergarten, 2024” to give more relevant details, per the above discussion.

We also note that numerical diffusion is evaluated within Figures 9 and 10, and their associated text, in the original version of the manuscript. We propose adding more cross-references throughout the manuscript to help guide the reader.

95 **Q7** — As a second aspect, the manuscript discusses the effect of an additional cohesion term in Voellmy’s rheology. In contrast to the points discussed above, this aspect becomes clear, but is not very surprising. As mentioned above, Voellmy’s rheology without cohesion lets the material move permanently if the fluid surface is steeper than the tangent of the coefficient of friction μ . This results in an ongoing, but small flow into the runout zone and finally lets the runout zone grow a bit. Cohesion just results in an increase of μ by a term $\tau_c/(\rho g_z h)$, which is inversely proportional to the thickness h . So flow on the slope stops
100 if the thickness falls below a threshold that depends on cohesion.

A7 — Yes, this is an accurate summary of this part of the paper. However, the ability of the cohesion term to allow us to isolate the effects of numerical diffusion was valuable. Furthermore, it shows that our numerical scheme is able to properly cause the flow material to converge to zero velocity, even for flows on complex topographies. This implies that flow material with a lower cohesion would also converge to zero velocity, but over a much longer period of time. We would adjust the discussion within the manuscript to put further emphasis of the value of the results relating to cohesion being related to the isolation of numerical diffusion on complex topographies. Furthermore, for practical applications, cohesion “helps” to define flow arrest even in presence of numerical diffusion