Revision R2 of: *The effect of slab touchdown on anticrack arrest in propagation saw tests*

Dear Anonymous Referee 2 Thank you for your thorough review and constructive feedback on our manuscript. In this document, we address each remark (*blue, italic*) point-by-point (black). We hope that these modifications adequately address the concerns raised and improve the manuscript. We are grateful for the guidance that the reviewers' expertise has provided and are confident that these changes have strengthened our submission.

Referee #2

In general, the authors have done a decent job of addressing my questions and comments. However, I feel that a few important points were overlooked, and I would like to bring them to their attention. Once these are addressed, I am confident that the paper will be of high quality and suitable for publication in NHESS.

Remark 2.1 *Comparison with Benedetti: While a brief discussion of the developed model with that of Benedetti is provided in Section 2.2, no quantitative comparison is offered. The text states that stage A and the transition from A to B are similar to Benedetti, but no further comparison is made beyond this observation. This is surprising, as major revisions were requested and the effort required to make such a comparison seems minimal. I would appreciate it if a more detailed comparison in different stages (graphical, e.g. in case similar to Fig. 6) could be included, ideally in the main paper, or at least in a supplement/appendix or peer-reviewed document. This would further highlight the advantages of their model compared to Benedetti's.*

In the revised document at the beginning of section 2, we have detailed the differences of the modeling approach. To expand on this, we have added the following section to the appendix:

A Additional comparisons with previous models

Benedetti et al. (2019) adopt a modeling approach similar to that of the present work. The key distinction lies in the present model's explicit consideration of weak-layer compliance. The impact of this difference is illustrated in Fig. A1. For homogeneous slabs with a rigid weak layer, Benedetti et al. (2019) derive the following unsupported lengths at the transitions between stages A and B (initial contact),

$$\Lambda_{\mathrm{A}\to\mathrm{B}} = \sqrt[4]{\frac{2E_{\mathrm{sl}}h^3t}{3\,q_{\mathrm{n}}}}, \tag{A1}$$

and between stages B and C (full contact),

$$\Lambda_{\mathrm{B}\to\mathrm{C}} = \sqrt{3}\,\Lambda_{\mathrm{A}\to\mathrm{B}}. \tag{A2}$$

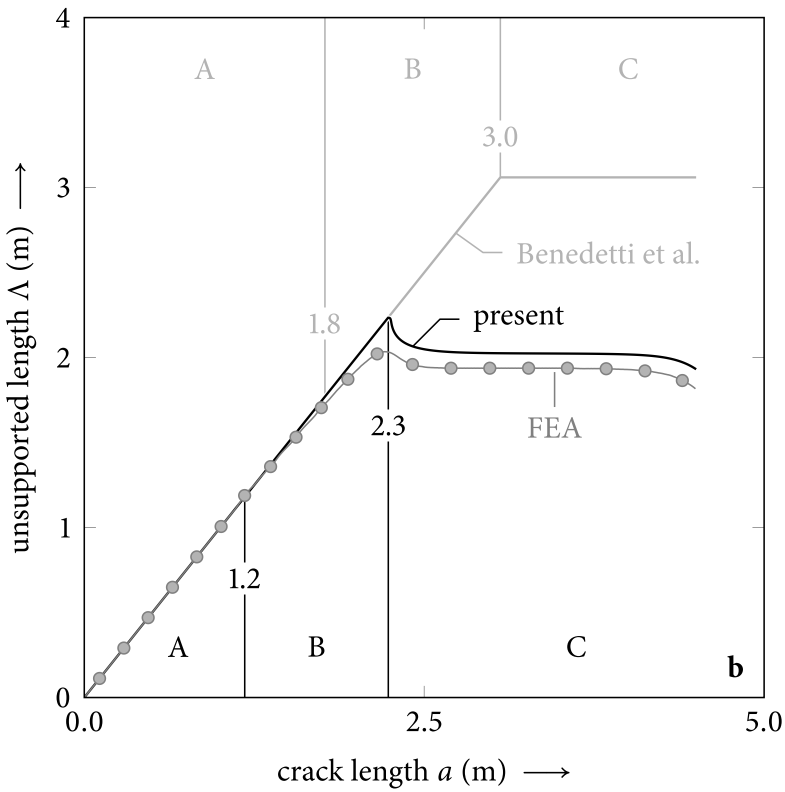

Figure A1. Comparison of unsupported length and stages between Benedetti et al. (2019) (gray), the present work (black), and the FEA reference solution (circles). Material properties and dimensions are given in **??**.

The assumption of a rigid weak layer in these equations results in significantly larger unsupported lengths compared to both the present model and the finite-element reference model (Fig. A1). Additional details in the model assumptions are listed in Table A1.

Table A1. Comparison between Benedetti et al. (2019) and the present model

	Benedetti et al.	Present Model
Slab kinematics	Homogeneous Euler–Bernoulli beam theory	First-order shear deformation theory to account for short beams
Slab layering	Not accounted for	Laminated plate theory, capturing bending–extension coupling and the influence of slab layering on bending and extensional stiffnesses
Weak layer compliance	Not accounted for	Weak layer compliance of intact weak layer as well as in touchdown domain considered via a Winkler foundation in the model
Stress analysis	Rigid beam segment assumption, leading to linear stress distribution along the length	Weak interface solution of shear and normal stresses based on the set of differential equations of the slab deformation (bending deformation, shear deformation, and axial extension). Stress results validated with numerical references models.
Fracture mechanics analyses	None	Energy release rate of cracks within the weak interface obtained with general mode I and mode II equations for weak interface models. Energy release rate results validated with numerical reference models.
Stages		
Stage I / Stage A	Bending of slab	Bending of slab and weak layer deformation, energy release rate of weak-layer crack propagation
Stage II / Stage B	No moment introduced at contact point, initial-contact length is a function of (isotropic) slab properties	No moment introduced at contact point, initial-contact length is a function of layered slab and weak-layer properties
Stage III / Stage C	Slab face is vertical at the contact point (transition from II to III is discontinuous), full-contact length is a function of (isotropic) slab properties	Moments introduced by the slab resting on the collapsed weak layer are represented by rotational springs, whose stiffness is obtained from a full slab deformation analysis by the principle of virtual forces, full-contact length is a function of layered slab and weak-layer properties, energy release rate of weak-layer crack propagation

Remark 2.2 *The response to my question regarding stress is disappointing. The authors begin by suggesting that it is common for some people to misunderstand basic mechanics, followed by a lengthy discussion on how stresses at crack tips in elastic continuum-based numerical methods are infinite, in contrast to analytical interface-based models. This is a well-known fact, and I believe there is no need to elaborate on it. Additionally, as the authors are surely aware, plasticity is a simple and effective option (among others) that addresses this issue in continuum-based models, providing a realistic way to simulate the fracture*

process zone. However, the authors do not employ such a model here. In their approach, as in many previous analytical models, finite stress values are obtained at the crack tip, which are crucial for calculating the energy release rate (stress^2 divided by constant stiffness). I feel the sentence: "However, due to their limited applicability in the analysis of fracture phenomena, we chose not to present details of their calculation." very surprising as their calculation is crucial to this paper. Therefore, I would like to reiterate my request: please provide the formula used to calculate this important quantity, as it underpins the entire paper. This could be included in an appendix or supplement. Additionally, please provide the metrics $\tau c/\tau s$ *and* $\sigma c/\sigma s$ *, as they would allow researchers using similar models to compare results more effectively.*

Thanks for raising this point. We will make it more clear in the revised document. The core point is that the energy release rate is in general not a function of the local stresses but of the change of the total potential $\mathcal{G} = -\mathrm{d}\Pi/\mathrm{d}A$ at change of crack length. Only in elastic interface models with a simplified continuum in the interface this computation is reduced to Krenk's formula (eq. 11 in the manuscript). In modeling real-world phenomena, all approaches rely on certain assumptions, some of which can lead to unphysical results. A well-known example is beam theory, a fundamental concept in structural mechanics. While the constitutive equation for shear deformation predicts a constant shear stress distribution through the cross-section of a beam, this is incorrect. Engineers account for this by determining shear stresses from local equilibrium conditions, which yield the correct quadratic distribution. A similar principle applies in the present case. Finite crack-tip stresses do not accurately represent the underlying physics of the crack problem. Recognizing this, we do not use stresses to model failure but instead focus on back-calculating the relevant governing properties of the physical process—analogous to how civil engineers avoid assuming constant shear stresses when designing structures. In this context, the energy release rate (ERR) is the fundamental quantity governing crack growth. The presence of stresses in the equation for energy release rate is a consequence of the model assumptions, particularly the weak interface. The derivation of ERR remains consistent across various approaches, including Krenk's method, calculus of variations, and J-integral evaluations. Given this well-established theoretical foundation, our choice to use energy release rate rather than stress-based criteria is both justified and necessary for accurately describing the physical process under investigation. The stress analysis is grounded in the local deformation of the slab above the weak layer. Therefore, the ODEs of the slab deformation are governing the stress solution. We have made this more clear in the paper. We have also added this in the comparison in the supplementary material.

Remark 2.3 *This comment arose after a second reading of the paper. Specifically, the fact that the ratio Gc/Gs remains below one (indicating that the energy release rate exceeds toughness) calls into question the conclusion that crack arrest is not induced by spatial variability. While this may be true (though yet to be fully explained), the findings of the paper, particularly the above point, prevent the authors from definitively concluding this. Spatial variability, especially in weak layer strength, could still explain why Gc/Gs < 1 and in the absence of other explanation, this seems to be a reasonable conclusion. I believe this is a crucial point that warrants further discussion. In fact, the DEM simulations conducted by the authors, or other recently developed continuum models (e.g., MPM or FEM), could help to either support or challenge this conclusion.*

We agree that spatial variability might be an important point to understand conditions of crack arrest. We will study this in future works. We have rephrased the paragraph in the manuscript.

References

Benedetti, L., Gaume, J., and Fischer, J.-T.: A mechanically-based model of snow slab and weak layer fracture in the Propagation Saw Test, International Journal of Solids and Structures, 158, 1–20, https://doi.org/10.1016/j.ijsolstr.2017.12.033, https://linkinghub.elsevier.com/retrieve/pii/S0020768317305358, 2019.