Responses to the Editor

I have received reviews from the two reviewers. One reviewer suggests including a legend (for plot lines) in Figure 7 (*See our response to the reviewer #2 below*). The other reviewer still has some concerns related to the comparison of new approach with the sinusoidal fitting approach, whose comments are attached below. If the new approach is not always better than the sinusoidal fitting approach, it is important to point out this limitation.

Thank the Editor for the suggestions. We've replied to the reviewer #1 as follows. We also added some sentences to enhance the illustration about the performance of the fitting method and our method. See L389-394.

"While our method may have a more significant degree of uncertainty than linear and sinusoidal fitting, this uncertainty remains within an acceptable range. The fitting method generally performs better, as seen in Figure 8. However, linear and sinusoidal fittings can be unstable in occasional cases. The poor fitting may be addressed by providing better initial guess values, constraining parameter intervals, changing the numerical method, filtering the data, and other approaches. All of these require additional trials. Our proposed method provides another robust and efficient method that can avoid this disadvantage. Users can choose the method that best suits their analysis needs."

Responses to Reviewer #1

I appreciate the authors' efforts in incorporating my previous suggestions and comparing their method with other widely accepted approaches in the atmospheric and climate community. Figures 7 and 8 in the revised manuscript are pivotal; however, I am somewhat surprised by the apparent uncertainty in the sinusoidal fitting. It would be beneficial if the authors could include a figure depicting a synthetic time series where the sinusoidal fitting fails and their proposed method excels.

Thanks. The additional figure suggested by the review has been incorporated in the revised manuscript as Figure 9.

The associated sentences are in L335-342.

"Figure 9a shows a successful fitting curve of (5) (blue solid line), which overlaps with the simulated data (red solid line), when the data length used is 8 years. The resulting long-term trend (blue dashed line) also aligns with that from the evenized SST (red

dashed line, but it is exactly covered by the blue dashed line). However, unexpected fitting failures can cause large deviations (blue line in Fig. 8a), such as in the example when the data length of 7 years is used (Fig. 9b). The fitting curve (blue solid line in Fig. 9b) has a smaller seasonal amplitude and a clear phase shift compared to the simulated data (red solid line in Fig. 9b). The estimated slope of the long-term trend (blue dashed line) is gentler than the known trend. In contrast, the known trend agrees with that estimated using evenized SST (red dashed line)."

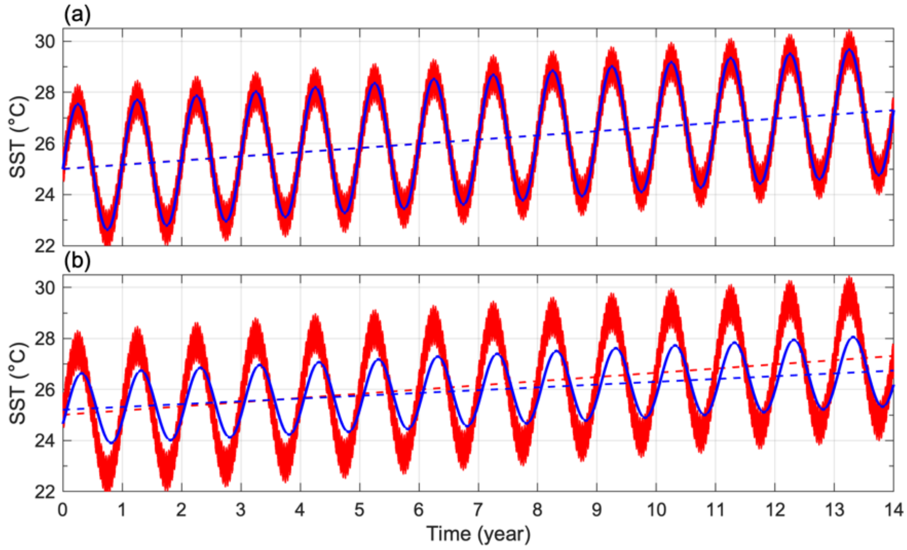

Figure 9: Linear and sinusoidal fitting curve of equation (5) using simulated data lengths of (a) 8 years and (b) 7 years. The red and blue solid lines represent the simulated data and its fitting curve, respectively. The red and blue dashed lines represent the long-term trends from the evenized SST and fitting methods, respectively.

We do not claim that our method is entirely superior to the combination of linear and sinusoidal fitting, which generally performs better, as seen in Figure 8. However, it is well known that small-scale variations and noise in the data can occasionally lead to poor fitting. Poor fitting may be addressed by providing better initial guess values, constraining parameter intervals, changing the numerical method, filtering the data, and other approaches. All of these require additional trials. We also added some associated sentences to strengthen our illustration in L335-342.

Additionally, I suspect that the limitations in fitting accuracy may be due to the model only incorporating once-per-year cycles, whereas actual seasonal cycles might exhibit two or three cycle per year components. Previously, I had suggested experimenting with a larger number of cycles and employing an F-test to determine the necessary number of cycles. Demonstrating that their method maintains superior stability compared to this enhanced sinusoidal fitting approach would strengthen their findings.

Thanks for the further suggestions. The seasonal cycle typically shows a higher SST in summer and a lower SST in winter, resulting in a periodic variation of 365 days. Intraseasonal variations (two or three cycles per year) may be present in the data but usually have a much smaller amplitude than seasonal signals. Figure 8 suggests that a suitable data length for applying our method is longer than 7 years (i.e., 7 cycles). While it may not be as convincing as utilizing the F-test, it remains a good approach. Using synthetic and field data, we have examined various linear regression methods, identified a suitable approach to address the problem of seasonal bias, validated our method, and compared it with two traditional methods. There is always more to explore with the new method, particularly its applications to complex real-sea data other than around Taiwan. As the manuscript content has been a bit lengthy, we intend to save these further tests for our future efforts and possibly for other researchers in our community.

<u>Responses to Reviewer #2</u>
Please consider including a legend (for plot lines) in Figure 7.

Thanks. Adding a legend indicating what the lines and dots mean is a good idea. However, we had a hard time doing this because the lines in Figure 7 require longer explanations, which is different from the other figures. This makes the legend box occupy a considerable amount of space in the figure, making it too complex and busy. Therefore, we decided not to add the legend. We have illustrated the lines in the figure caption.