



# Brief Communication: towards a universal formula for the probability of tornadoes

Roberto Inghrosso<sup>1</sup>, Piero Lionello<sup>2</sup>, Mario Marcello Miglietta<sup>3</sup>, and Gianfausto Salvadori<sup>4</sup>

<sup>1</sup>Department of Earth and Atmospheric Sciences, University of Quebec in Montréal, 201, av. du President Kennedy, Montréal, H3C 3P8, Canada

<sup>2</sup>Dipartimento di Scienze Ambientali e Biologiche, Università del Salento, via per Monteroni 165, Lecce, 73100, Italy

<sup>3</sup>ISAC-CNR, Istituto di Scienze dell'Atmosfera e del Clima, Consiglio Nazionale delle Ricerche, corso Stati Uniti 4, Padua, 35127, Italy

<sup>4</sup>Dipartimento di Matematica e Fisica, Università del Salento, Provinciale Lecce-Arnesano, P.O.Box 193, Lecce, 73100, Italy

**Correspondence:** Piero Lionello (piero.lionello@unisalento.it)

**Abstract.** A methodological approach is proposed to provide an analytical (exponential-like) expression for the probability of occurrence of tornadoes as a function of the convective available potential energy and the wind shear (or, alternatively, the storm relative helicity). The resulting expression allows to compute the probability of tornado occurrence using variables that are computed by weather prediction and climate models, thus compensating for the lack of resolution needed to resolve these phenomena in numerical simulations.

## 1 Introduction

Tornadoes are rapidly rotating columns of air (American Meteorological Society, 2020), extending vertically from the surface to the base of a cumuliform cloud, and represent one of the most severe weather phenomena in terms of victims and damages. Considering only the USA, every year about 500 tornadoes (Kunkel et al, 2013) of intensity EF1 (Enhanced Fujita scale, Fujita (1971); Potter (2007)) or stronger occur, producing an average of 125 victims and huge devastation (Ashley, 2007). The very fine spatial and temporal scale of tornadoes (typically with a diameter of less than 2 km and a duration of less than 1000 s) require resolutions that are orders of magnitude smaller than those currently available in operational weather prediction and climate models (Yokota et al, 2018). Further, the chaotic dynamics of these vortices limit their deterministic prediction (Markowski, 2020). Consequently, climatological studies focused on the identification of the environmental conditions favourable to tornado-spawning severe convective storms. Several thermodynamics and kinematic meteorological parameters have been analysed, either individually or considering combined instability indices, to identify the conditions most favourable to the genesis of tornadoes (Brooks et al, 2003; Romero et al, 2007; Taszarek et al, 2018, 2020; Inghrosso et al, 2020; Bagaglini et al, 2021). This approach is consistent with the basic idea that tornadoes result from a multi-stage process, which takes into account that the tilting of the horizontal vorticity near the ground by a violent updraft plays a basic role (Rotunno, 2013; Davies-Jones, 2015). Such a conceptual model is used here as a background framework for introducing an analytical formula for the probability of tornado occurrence. A previous study defined a tornado index limited to the USA based on a Poisson



regression between the observed U.S. climatology of tornadoes and monthly averaged environmental parameters from reanalysis (Tippett et al, 2012). Other studies limited their conclusions to the identification of the conditions that are associated with mesoscale convective hazards (Brooks, 2013; Diffenbaugh et al, 2013). The expression that we propose in this study is meant to provide a tool for supporting tornadoes warning in operational weather predictions and estimating changes of frequency of tornado occurrence in climate projections.

## 2 Data and Methods

Our analysis is based on tornadoes that occurred in the USA (dataset provided by the Storm Prediction Center-SPC, <https://www.spc.noaa.gov/wcm/#dat>) and in Europe (dataset provided by the European Severe Weather Database (ESWD), <https://www.essl.org>, managed by the European Severe Storm Laboratory (ESSL), Dotzek et al (2009)). We considered only tornadoes of category 2 or higher, following the idea that weak events might have an uncertain signature in the environmental conditions and their reporting in official databases is less accurate. A total number of 3073 tornadoes have been considered in this study (2632 for the USA and 441 for Europe, see Supplementary Material for density plots) during the period 2000-2018.

The hourly fields of ERA5 (ECMWF ReAnalysis 5, (Hersbach et al, 2020)) are used to establish a statistical link between the occurrence of tornadoes and a set of suitable meteorological variables: the updraft maximum parcel vertical velocity (WMAX), which depends on the Convective Available Potential Energy CAPE, the mid-level wind shear ( $WS_{700}$ ), the low-level storm relative helicity ( $SRH_{900}$ ), and the lifting condensation level (LCL, Kaltenböck et al (2009)). The Supplementary Material reports the actual expressions defining the variables used in this study. The values of these variables have been extracted in the period 2000–2018 in all cells where at least one tornado occurred, considering the hourly reanalysis fields at a 25 km resolution. The values corresponding to the occurrence of tornadoes have been selected considering the time step closest to the recorded time of the tornado onset in the database.

The univariate analysis of the (conditional) probability  $P$  of tornado occurrence is carried out by partitioning the observed range spanned by each variable into 17 equi-probable sub-intervals (bins). Such a number has been chosen as a compromise between the need of a number of bins sufficient for robust regressions and of a number of observations in each bin sufficient for a robust statistical analysis. An empirical estimate of the probability of tornado occurrence, conditional to the fact that the value of the variable lies in a given bin, is computed as the relative frequency of tornadoes in the bin. Its uncertainty is estimated via a suitable Bootstrap (Monte Carlo) procedure. An analytical expression of  $y = \log_{10}P$  is found by a simple linear regression for  $WS_{700}$ ,  $SRH_{900}$ , and LCL, and by a non-linear regression for WMAX (see the Supplementary Materials).

## 3 Results

The univariate analysis shows that all the four variables considered in our study (i.e. WMAX,  $WS_{700}$ ,  $SRH_{900}$ , LCL) are significantly linked to the formation of tornadoes. However, the formulas involving  $WS_{700}$  and WMAX, i.e.



$$\log_{10} P = -6.8 + 0.11 WS_{700} \quad (1)$$

$$\log_{10} P = -6.9 + \frac{WMAX}{3 + 0.32 WMAX} \quad (2)$$

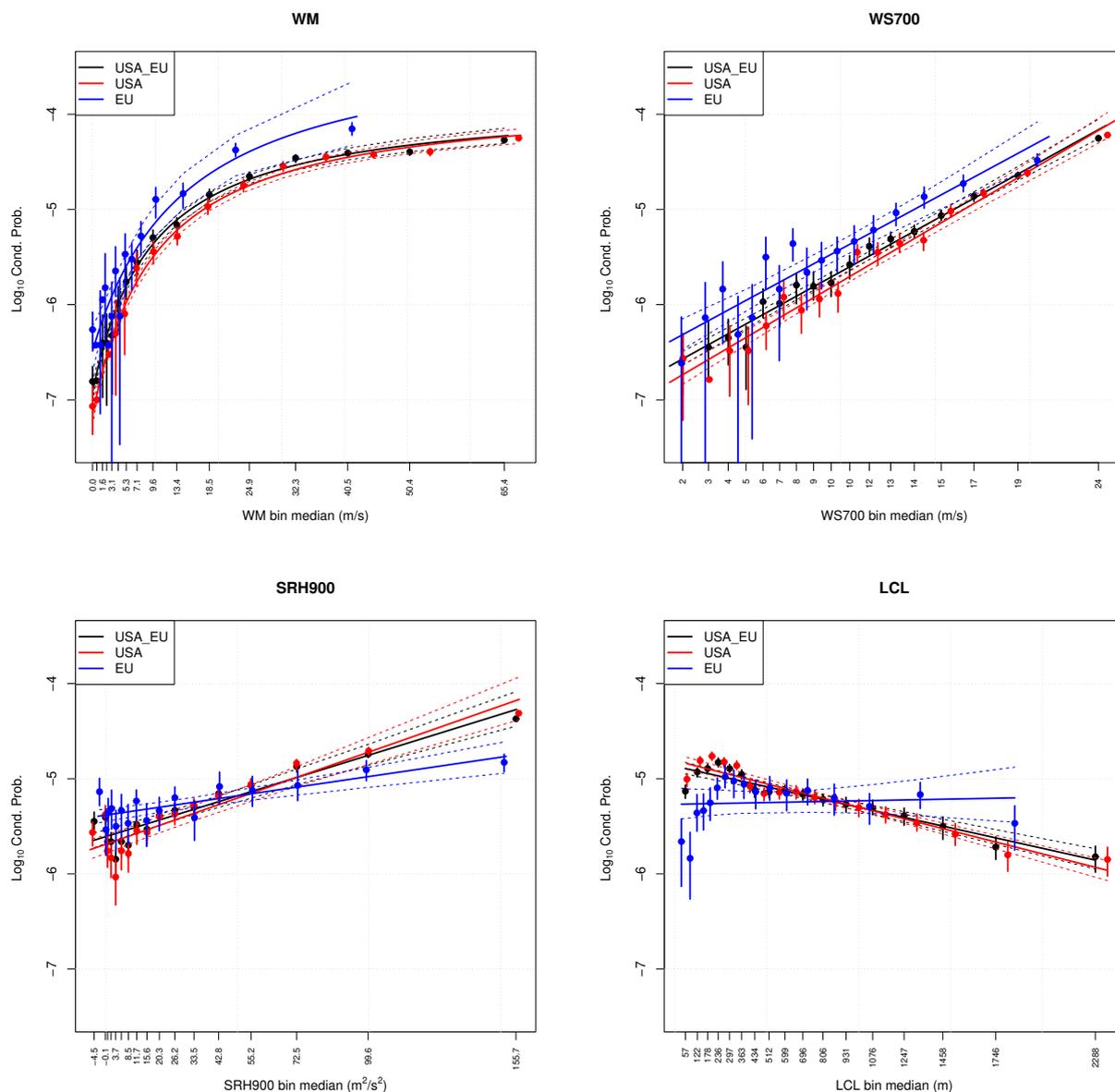
55 describe a range of probabilities (from  $10^{-7}$  to  $10^{-4}$ ) wider than that spanned by  $SRH_{900}$  and LCL. In the case of  $WS_{700}$ ,  
the probability increases exponentially over all the whole range. Instead, the behaviour of  $\log_{10} P$  as a function of WMAX  
is non-linear and shows a hyper-exponential increase of  $P$  for low values ( $WMAX < 10$  m/s), when the probability is small  
(about  $10^{-7}$ ); in the intermediate range the growth gradually slows down, and  $P$  becomes quasi-constant for large values  
( $WMAX > 30$  m/s), where the probability tends to  $\approx 10^{-4}$ . For LCL and  $SRH_{900}$ , the exponential decrease and increase,  
60 respectively, only describes a narrow range of probability (approximately from  $10^{-6}$  to  $10^{-5}$ ). In other words, variations of  
these two variables do not allow to discriminate among low and high probability of occurrence of tornadoes as effectively as  
in the case of  $WS_{700}$  and WMAX (see Fig. 1).

Concerning the bivariate analysis (i.e., considering the joint behavior of pairs of predictors), in analogy with the univariate  
case, a  $17 \times 17$  grid matrix is constructed to partition the whole two-dimensional domain in cells. The empirical estimate of the  
65 (conditional) probability  $P$  of tornado occurrence, provided that the pair of variables lie in a given cell, is empirically computed  
as above via the estimate of a relative frequency of occurrence. Six different bivariate analyses are carried out considering all  
possible pair combinations of WMAX,  $WS_{700}$ ,  $SRH_{900}$  and LCL. For the bivariate probability, non-linear expressions have  
been adopted for all the pairs of variables involving WMAX, and a multiple linear expression for the remaining pairs (see the  
Supplementary Materials). The values of the parameters of the bivariate probability functions have been found by a regression  
70 of the proposed expressions over the empirical probabilities.

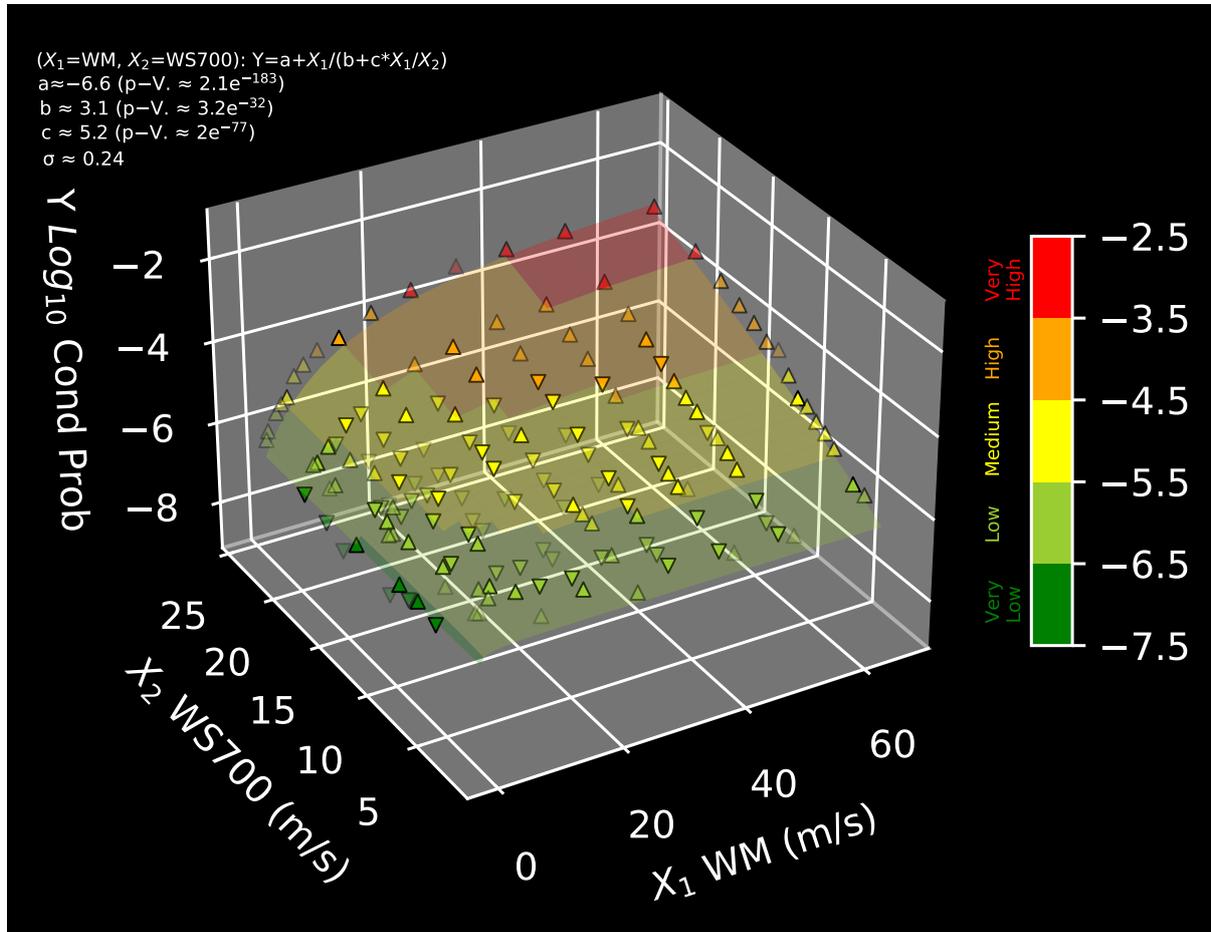
Considering the bivariate expression of  $P$  as a function of the pairs (WMAX, LCL) and ( $WS_{700}$ ,  $SRH_{900}$ ), the second  
variable lacks significance, meaning that it provides information analogue to the first one of the pair (in fact, they are fairly  
correlated), but the first variable provides more (univariate) informative details than the second one in terms of the range of  $P$ .  
Considering the pairs (WMAX,  $SRH_{900}$ ), ( $WS_{700}$ , LCL) and ( $SRH_{900}$ , LCL), the probability of tornadoes significantly depends  
75 on both variables, but they describe variation of  $P$  only over 2–3 orders of magnitude, whereas using the pair (WMAX,  $WS_{700}$ )  
shown in Fig. 2 it is possible to discriminate between conditions where the probability ranges from  $10^{-7}$  to  $10^{-3}$  (see the  
Supplementary Materials section for the figures regarding all the other pairs). In conclusion, a valuable fit of the probability of  
occurrence of tornadoes over the range  $10^{-7}$ – $10^{-3}$  is

$$\log_{10} P = -6.6 + \frac{WMAX}{3.1 + 5.2 WMAX/WS_{700}} \quad (3)$$

80 All parameters of the univariate fits in Fig. 1 and bivariate ones in Fig. 2 are statistically significant and significantly different  
from zero, since the p-values of the corresponding tests are (much) smaller than 1%. For all univariate linear regressions, the



**Figure 1.** Univariate probability distribution for WMAX, WS<sub>700</sub>, SRH<sub>900</sub> and LCL. Markers and whiskers denote the empirical probabilities with uncertainty range. Lines denote the empirical estimates (continuous) with uncertainty ranges (dashed). Different colours represent values based on the full dataset (USA&EU, black), the USA data only (red), and the European data only (EU, blue). Uncertainty ranges correspond to a 95% confidence level.



**Figure 2.** Bivariate probability distribution for  $(X_1 = WMAX, X_2 = WS700)$ . The coloured surface shows the empirical fit of  $y = \log_{10} P$ . Upward/downward triangles represent empirical estimates located above/below the fitted surface. All values are reported according to the colour bar.

adjusted  $R^2$  is larger than 90%, and, in general, the goodness of the fits is visually confirmed by the overwhelming fraction (from 90% to 100%) of probability values within the 95% confidence bands. In the bivariate case, considering the multiple linear regressions of the pairs  $(WS_{700}, SRH_{900})$ ,  $(WS_{700}, LCL)$ , and  $(SRH_{900}, LCL)$ ,  $R^2$  is, respectively, 70%, 72%, and 54%:  
 85 in general, these are smaller than in the single-variable case, but this is justified by the fact that the residual variances  $\sigma$ 's are about three times larger than those estimated in the univariate case. For the three pairs involving WMAX,  $R^2$  cannot be used to assess the goodness-of-fit because the regression is non-linear. However, a slice-analysis of the fits (see the Supplementary Material for details) shows that the proposed models provide valuable fits over the whole domain of interest.



#### 4 Discussion

90 Further investigations are required to ensure the validity of the expressions in Eqs. 1, 2, and 3 in different environmental and geomorphological conditions. Hypothesis testing the similarity of the populations of tornado probabilities  $P_{EU}$  and  $P_{USA}$ , obtained using only EU and only USA data, respectively, has been carried out by using a Kolmogorov-Smirnov-like (KS) approach (Lopes, 2011) adopting the metric  $d_0 = \max|P_{EU} - P_{USA}|$ . The significance level of the difference is assessed by computing the fraction of statistics exceeding  $d_0$  using a Monte Carlo permutation procedure. Considering the univariate  
95 models, the null hypothesis that  $P_{EU}$  and  $P_{USA}$ , as a function of WMAX and WS<sub>700</sub>, are statistically compatible cannot be rejected at 95% and 99% levels (suggesting that Eqs. 1 and 2 are acceptable in different geographical domains), whereas it is rejected at a level larger than 99% for  $P_{EU}$  and  $P_{USA}$  as a function of SRH<sub>900</sub> and LCL. Considering the bivariate conditional probabilities, the null hypothesis - that  $P_{EU}$  and  $P_{USA}$  are statistically compatible - could not be rejected (at a 90% level) only for the pair (WMAX, SRH<sub>900</sub>). In this case, the overall conditional probability (combining USA and EU data) is:

$$100 \log_{10} P = -6.6 + 0.34 WMAX^{0.37} |SRH_{900}|^{0.12} \quad (4)$$

For all other pairs the null hypothesis could be rejected at the 99% level.

Possible explanations of the lack of compatibility between conditional probabilities obtained using the EU and USA datasets alone could be: different tornadoes damage reporting practices (leading to different counting and attributions of tornadoes to the EF/F scale), and different meteorological and/or morphological conditions in the two domains. In spite of this limitations,  
105 and the need for further investigations, the proposed statistical models suitably fit the conditional probabilities of tornado occurrence. In particular, Eq. 3 has the merit of fitting the bulk of all available data, and Eqs. 1, 2 and 4 of being robust with respect to the considered geographical domains.

The formulas of Eqs. 1-4, and particularly the bivariate expressions of Eq. 3 and Eq. 4, outline a new statistical tool that can be used for diagnosing the likelihood of tornadoes with potential applications to short-medium range weather predictions and  
110 future changes of their frequency in climate projections.

*Sample availability* The list of tornadoes in the USA can be freely downloaded at <https://www.spc.noaa.gov/wcm/#dat> The list of tornadoes in Europe have been obtained from <https://www.essl.org>. ERA-5 data can be freely downloaded from <https://cds.climate.copernicus.eu/cdsapp#!/home>

*Author contributions.* RI has been responsible for data collecting, processing and plotting, PL for the coordination of the study, MMM for  
115 the meteorological analysis, GS for the statistical analysis and the computation of the probability of occurrence of tornadoes. All the Authors wrote and contributed to the final manuscript.

<https://doi.org/10.5194/nhess-2023-19>  
Preprint. Discussion started: 7 February 2023  
© Author(s) 2023. CC BY 4.0 License.



*Competing interests.* PL is Editor for this journal

*Acknowledgements.* The Authors gratefully acknowledge useful discussions and suggestions by Prof. F. Durante (University of Salento, Lecce, Italy)



## 120 References

- American Meteorological Society (2020) Glossary of Meteorology. <http://https://glossary.ametsoc.org/wiki/Tornado>, Last accessed on 2022-08-22
- Ashley WS (2007) Spatial and temporal analysis of tornado fatalities in the United States: 1880–2005. *Weather and Forecasting* 22(6):1214–1228. <https://doi.org/10.1175/2007WAF2007004.1>
- 125 Bagaglini L, Ingrassio R, Miglietta MM (2021) Synoptic patterns and mesoscale precursors of Italian tornadoes. *Atmospheric Research* 253:105,503. <https://doi.org/https://doi.org/10.1016/j.atmosres.2021.105503>
- Brooks HE (2013) Severe thunderstorms and climate change. *Atmospheric Research* 123:129–138. <https://doi.org/10.1016/j.atmosres.2012.04.002>
- Brooks HE, Lee JW, Craven JP (2003) The spatial distribution of severe thunderstorm and tornado environments from global reanalysis data. *Atmospheric Research* 67–68:73–94. [https://doi.org/10.1016/S0169-8095\(03\)00045-0](https://doi.org/10.1016/S0169-8095(03)00045-0)
- 130 Davies-Jones R (2015) A review of supercell and tornado dynamics. *Atmospheric Research* 158-159:274–291. <https://doi.org/https://doi.org/10.1016/j.atmosres.2014.04.007>
- Diffenbaugh NS, Scherer M, Trapp RJ (2013) Robust increases in severe thunderstorm environments in response to greenhouse forcing. *Proceedings of the National Academy of Sciences* 110(41):16,361–16,366. <https://doi.org/10.1073/pnas.1307758110>
- 135 Dotzek N, Groenemeijer P, Feuerstein B, et al (2009) Overview of essl’s severe convective storms research using the european severe weather database eswd. *Atmospheric Research* 93(1):575–586. <https://doi.org/https://doi.org/10.1016/j.atmosres.2008.10.020>, 4th European Conference on Severe Storms
- Fujita TT (1971) Proposed characterization of tornadoes and hurricanes by area and intensity. *Satellite and Mesometeorology Research Project* 91:42 pp.
- 140 Hersbach H, Bell B, Berrisford P, et al (2020) The ERA5 global reanalysis. *Quarterly Journal of the Royal Meteorological Society* 146(730):1999–2049. <https://doi.org/10.1002/qj.3803>
- Ingrassio R, Lionello P, Miglietta MM, et al (2020) A Statistical Investigation of Mesoscale Precursors of Significant Tornadoes: The Italian Case Study. *Atmosphere* 11(3):301. <https://doi.org/10-1002/joc.5526>
- Kaltenböck R, Diendorfer G, Dotzek N (2009) Evaluation of thunderstorm indices from ECMWF analyses, lightning data and severe storm reports. *Atmospheric Research* 93(1):381–396. <https://doi.org/https://doi.org/10.1016/j.atmosres.2008.11.005>, 4th European Conference on Severe Storms
- 145 Kunkel KE, Karl TR, Brooks H, et al (2013) Monitoring and understanding trends in extreme storms: State of knowledge:. *Bulletin of the American Meteorological Society* 94(4):499–514. <https://doi.org/10.1175/BAMS-D-11-00262.1>
- Lopes R (2011) Kolmogorov-Smirnov Test. In: *International Encyclopedia of Statistical Science*. Springer Berlin Heidelberg
- 150 Markowski PM (2020) What is the intrinsic predictability of tornadic supercell thunderstorms? *Monthly Weather Review* 148(8):3157–3180. <https://doi.org/10.1175/MWR-D-20-0076.1>
- Potter S (2007) Fine-Tuning Fujita: After 35 years, a new scale for rating tornadoes takes effect. *Weatherwise* 60(2):64–71. <https://doi.org/10.3200/WEWI.60.2.64-71>
- Romero R, Gayà M, Doswell CA (2007) European climatology of severe convective storm environmental parameters: A test for significant tornado events. *Atmospheric Research* 83(2):389–404. <https://doi.org/https://doi.org/10.1016/j.atmosres.2005.06.011>, european Conference on Severe Storms 2004



- Rotunno R (2013) The Fluid Dynamics of Tornadoes. *Annual Review of Fluid Mechanics* 45(1):59–84. <https://doi.org/10.1146/annurev-fluid-011212-140639>
- 160 Taszarek M, Brooks HE, Czernecki B, et al (2018) Climatological Aspects of Convective Parameters over Europe: A Comparison of ERA-Interim and Sounding Data. *Journal of Climate* 31(11):4281–4308. <https://doi.org/10.1175/JCLI-D-17-0596.1>, <https://doi.org/10.1175/JCLI-D-17-0596.1>
- Taszarek M, Allen JT, Púčik T, et al (2020) Severe convective storms across Europe and the United States. part ii: ERA5 environments associated with lightning, large hail, severe wind, and tornadoes. *Journal of Climate* 33(23):10,263 – 10,286. <https://doi.org/10.1175/JCLI-D-20-0346.1>
- 165 Tippett MK, Sobel AH, Camargo SJ (2012) Association of u.s. tornado occurrence with monthly environmental parameters. *Geophysical Research Letters* 39(2). <https://doi.org/https://doi.org/10.1029/2011GL050368>
- Yokota S, Niino H, Seko H, et al (2018) Important factors for tornadogenesis as revealed by high-resolution ensemble forecasts of the Tsukuba supercell tornado of 6 may 2012 in Japan. *Monthly Weather Review* 146(4):1109 – 1132. <https://doi.org/10.1175/MWR-D-17-0254.1>