Supplement of

Evolution of multivariate drought hazard, vulnerability and risk in India under climate change
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1. Drought hazard and vulnerability assessment

Drought hazard assessment

Drought hazard forms an important component of drought risk assessment. Here, we assess the country-wide drought hazard based on the deficiencies in precipitation and soil moisture. Therefore, the multivariate standardized drought index (MSDI) of the non-parametric form is computed using the bivariate case of Gringorten plotting position formula (Gringorten, 1963). The steps involved in the calculation of MSDI is presented below

1. The joint probability distribution of the 1-month time scale precipitation \((R)\) and soil moisture \((S)\) is given by

\[
P (R \leq r, S \leq s) = p
\]  \(\ldots(1)\)

where \(p\) represents the joint probability of the precipitation and soil moisture.

2. For the sample size \(n\), the count of occurrence of the pair \((r_i, s_i)\) for \(r_i \leq r_k \) and \(s_i \leq s_k\) is denoted as \(m_k\). This count is used to derive the empirical joint probability for the bivariate case with the Gringorten plotting position (Gringorten, 1963) as

\[
P(r_k, s_k) = \frac{m_k - 0.44}{n + 0.12}
\]  \(\ldots(2)\)

3. The above empirical joint probability is then standardized to obtain the multivariate index MSDI.

\[
MSDI = \varphi^{-1}(p)
\]  \(\ldots(3)\)

where \(\varphi\) is the standard normal distribution function. Since the empirical distributions use ranks of data instead of actual values, the sample size should be sufficiently large.

Weightages and ratings system for the drought index is adopted for drought hazard assessment (Kim et al., 2015). The MSDI series at each region is categorized into four groups similar to Mckee et al. (1993). Further, each category is organised into sub-groups based on the occurrence probabilities of the selected category. While the weightages are assigned to MSDI categories to account for drought magnitude, ratings are assigned to the sub-groups of each MSDI category to account for drought occurrence probability. The total number of ratings in each category is
determined using the prominent k-means data clustering algorithm. The distance between the data points is computed using the squared Euclidean distance metric. To avoid the convergence to local minima, the algorithm is run with 100 random initial seeds with 10000 iterations. The Calinski-Harabasz Index (CHI) (Calinski and Harabasz, 1974) is used to determine the optimum number of clusters and is given by

\[ CHI = \frac{n - K}{K - 1} \times \frac{BGSS}{WGSS} \]  \( \text{... (4)} \)

where \( n \) = number of data points, \( K \) = number of clusters, \( BGSS = \sum_{k=1}^{K} n_k \|G^{(k)} - G\|^2 \) is the between the group scatter, \( G^{(k)} = \) centroid of the \( k \)th cluster, \( G = \) centroid of all the observations, \( WGSS = \sum_{k=1}^{K} WGSS^{(k)} \) is within the group scatter and \( WGSS^{(k)} = \sum_{i \in I_k} \|M_i^{(k)} - G^{(k)}\|^2 \). The k-means clustering algorithm is driven for 1 to \( n \) clusters. The number of clusters that gives highest value of CHI is the optimum number of clusters. These optimum number of clusters is used for assigning ratings. Clusters with higher occurrence probability will be given higher ratings. The categorized weightages and computed ratings are used to calculate the drought hazard for every region as below.

\[ DH = (M_r \times M_w) + (MO_r \times MO_w) + (S_r \times S_w) + (E_r \times E_w) \] \( \text{... (5)} \)

where \( M_r, MO_r, S_r \) and \( E_r \) represent the ratings and \( M_w, MO_w, S_w, E_w \) represent the weights of mild, moderate, severe and extreme categories, respectively.

The \( DH \) values from Eq 5 are standardized as shown below to obtain \( DHI \) that varies between 0 and 1.

\[ DHI = \frac{DH - DH_{min}}{DH_{max} - DH_{min}} \] \( \text{... (6)} \)

**Drought vulnerability assessment**

Drought vulnerability forms another important component of drought risk assessment. Several aggregation techniques have been employed in the past studies to combine the drought vulnerability indicators to assess drought vulnerability. However, we use the robust method –
TOPSIS (Hwang and Yoon, 1981) owing to its lesser rank reversal probabilities (Sahana et al., 2021). The steps involved in drought vulnerability assessment is outlined as below.

1. Standardization of numerical drought vulnerability indicators (irrigation index, water body fraction, groundwater availability, population density and GDP) is carried out such that their values vary between 0 and 1. Suitable weights are assigned to categorical drought vulnerability indicators (LULC, slope and soil texture), following Thomas et al. (2016) and Sahana et al. (2021). This gives the decision matrix $n_{ij}$, where $i = 1,2, ... n$ represents the number of regions and $j = 1,2, ... m$ represents the number of drought vulnerability indicators.

2. The above decision matrix $n_{ij}$ is associated with the indicator weights $w_j$ obtained from the Analytic Hierarchy Process (AHP) method (Sahana et al., 2021). This gives the weighted decision matrix $v_{ij}$

$$v_{ij} = w_j n_{ij} \quad \ldots (7)$$

3. Positive ($A^+$) and Negative ($A^-$) Ideal solution is calculated for each of the indicators.

$$A^+ = (v_1^+, v_2^+, ... v_m^+) = \left[ (\max v_{ij} | j \in I), (\min v_{ij} | j \in J) \right] \quad \ldots (8)$$

$$A^- = (v_1^-, v_2^-, ... v_m^-) = \left[ (\min v_{ij} | j \in I), (\max v_{ij} | j \in J) \right] \quad \ldots (9)$$

where $I$ and $J$ are associated with the benefit and cost criteria respectively. Here population density, LULC, slope and soil texture that bear positive correlation with the drought vulnerability are considered as benefit criteria. On the other hand, irrigation index, groundwater availability, waterbody fraction and GDP that bear negative correlation with drought vulnerability are considered as cost criteria.

4. Positive ($d_i^+$) and negative ($d_i^-$) separation measures for each region $i$ are computed based on $A^+$ and $A^-$

$$d_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^+)^2} \quad \ldots (10)$$

$$d_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2} \quad \ldots (11)$$

5. Relative closeness ($R_i$) of each region to the Positive Ideal Solution is calculated as
\[ R_i = \frac{d_i^-}{d_i^- + d_i^+} \quad \text{... (12)} \]

\( R_i \) signifies vulnerability of region \( i \) to drought. \( R \) varies between 0 and 1. The drought vulnerability index (DVI) is given by \( R \)

\[ DVI = R \quad \text{... (13)} \]
2. Figures

Figure S1. Country-wide average annual precipitation for baseline period and multi-model ensemble mean of average annual precipitation of projected period for different time-slices and RCP scenarios.
Figure S2. Country-wide average annual soil moisture for baseline period and multi-model ensemble mean of average annual soil moisture of projected period for different time-slices and RCP scenarios.
Figure S3. Constant drought vulnerability indicators for drought vulnerability assessment. a) Slope, b) Soil texture.
**Figure S4.** Scatter of meteorological sub-division-wise DHI and DVI for the scenarios a) baseline, b) RCP2.6-SSP2 Near future, c) RCP2.6-SSP2 Far future, d) RCP6.0-SSP2 Near future, e) RCP6.0-SSP2 Far future.
References:


