Supporting Information for

Assessing minimum PDC volume from a large explosive volcanic eruption impacting critical infrastructures: an example from Aso Caldera (Japan)

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Additional Supporting Information (Files uploaded separately)

Caption for Text S1 to S2

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Text S1. Notes on the analytic solution of box model equations for gravity-driven particle currents with constant volume.

We summarize the physical equations and analytic solutions of three versions of the box model equations, suitable for the integral formulation of axisymmetric gravity-driven particle currents with constant volume. The first model is based on a simple constant resisting stress, while the second and third models assume flow dilution by particle deposition. The third model is characterized by assuming an interstitial fluid lighter than the ambient fluid. All the calculations are performed on a flat topography. Ambient fluid entrainment and cooling effects are not considered. All particles are assumed to deposit at the same velocity.

Text S2. Log of scientific email exchanges between co-authors and other colleagues: December 2018 – February 2019.

We summarize the knowledge exchanges that involved five of the co-authors, concerned with assessing the probability of an As04-scale future eruption and four other colleagues providing detailed volcanological support. All names have been anonymized. Note that the marker sites (MS) were called "target sites" (TS) in this document.

Table S3. Numerical results of the minimum PDC volume and mass needed to reach the MS2, with and without consideration of topographic effects (see Section 4.4). Estimates based on 100,000 statistical samples.

No topographic effects modelled				
$MinVol$**: Minimum PDC volume [km^3] required to reach the MS2**				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	29.6	294	504	1650
Model 2a: Elicited inputs	11.3	61.8	86.2	250
Model 2b: Modified inputs*	8.08	18.5	19.2	33.2
Model 2c: Elicited inputs	7.55	48.3	71.3	222
$MinMass$: **Minimum PDC mass [10^{12} kg] required to reach the MS2**				
Model	**5%ile**	**50%ile**	**Mean**	**95%ile**
Model 1: Elicited inputs	31.4	303	500	1580
Model 2a: Elicited inputs	19.2	106	147	431
Model 2b: Modified inputs*	13.9	31.6	32.8	56.2
Model 2c: Elicited inputs	12.6	82.4	123	385
With topographic effects included				
Minimum PDC volume [km^3] required to reach the MS2				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	156	1550	2660	8710
Model 2a: Elicited inputs	49.6	271	378	1100
Model 2b: Modified inputs*	35.4	81.0	84.4	146
Model 2c: Elicited inputs	33.1	212	313	974
Minimum PDC mass [10^{12} kg] required to reach the MS2				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	165	1600	2640	8340
Model 2a: Elicited inputs	84.1	465	646	1890
Model 2b: Modified inputs*	60.9	139	144	247
Model 2c: Elicited inputs	55.5	361	540	1690

* Modified ϕ_0 and w_s based on MDR modelling and Sauter diameter of analogues (see Table 1).

Table S4. Numerical results of the probability that a PDC derived from a caldera-forming eruption similar to Aso-4 reaches the MS2, with and without topographic effects. For each scenario, we present the values of the cumulative curves displayed in Figure 4 at the central point of the variation range of the PDC mass, while between parentheses we include the results at the extremes of these variation ranges. We test the mass of PDC overflow outside of the caldera of Aso-4 and 1/10 of that estimate, representing the Aso-4T unit (volumes per Takarada and Hoshizumi, 2020). Estimates based on 100,000 statistical samples.

Aso-4T		
Model	**TE^{+}: No**	**TE^{+}: Yes**
Model 1: Elicited inputs	15.2% (7.2 – 23.4%)	1.2% (0.0 – 3.0%)
Model 2a: Elicited inputs	39.2% (19.6 – 54.2%)	4.3% (0.5 – 9.3%)
Model 2b: Modified inputs*	100.0% (75.9 – 100.0%)	13.0% (0.5 – 36.3%)
Model 2c: Elicited inputs	48.4% (28.0 – 63.3%)	9.2% (2.7 – 16.4%)
Aso-4, total PDC overflow		
Model	**TE^{+}: No**	**TE^{+}: Yes**
Model 1: Elicited inputs	79.0% (59.0 – 89.9%)	30.0% (15.3 – 41.5%)
Model 2a: Elicited inputs	100.0% (94.5 – 100.0%)	71.2% (45.9 – 84.0%)
Model 2b: Modified inputs*	100.0% (100.0 – 100.0%)	100.0% (100.0 – 100.0%)
Model 2c: Elicited inputs	100.0% (96.3 – 100.0%)	77.4% (55.3 – 87.7%)

$^{+}$ TE: Topographic effects.

* Modified ϕ_0 and w_s based on MDR modelling and Sauter diameter of analogues (see Table 1).

Table S5. Numerical results of the minimum PDC volume and mass needed to reach the MS_3, with and without consideration of topographic effects (see Section 4.4). Estimates based on 100,000 statistical samples.

No topographic effects modelled				
$MinVol$**: Minimum PDC volume [km^3] required to reach the MS3**				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	36.7	365	625	2050
Model 2a: Elicited inputs	13.7	74.8	104	303
Model 2b: Modified inputs*	9.78	22.4	23.3	40.2
Model 2c: Elicited inputs	9.14	58.4	86.3	269
$MinMass$: **Minimum PDC mass [10^{12} kg] required to reach the MS3**				
Model	**5%ile**	**50%ile**	**Mean**	**95%ile**
Model 1: Elicited inputs	38.9	375	619	1960
Model 2a: Elicited inputs	23.2	128	178	521
Model 2b: Modified inputs*	16.8	38.3	39.6	68.0
Model 2c: Elicited inputs	15.3	99.6	149	465
With topographic effects included				
Minimum PDC volume [km^3] required to reach the MS3				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	137	1360	2330	7640
Model 2a: Elicited inputs	44.2	241	337	978
Model 2b: Modified inputs*	31.6	72.1	75.1	130
Model 2c: Elicited inputs	29.5	188	279	867
Minimum PDC mass [10^{12} kg] required to reach the MS3				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	145	1400	2310	7320
Model 2a: Elicited inputs	74.9	414	575	1680
Model 2b: Modified inputs*	54.2	123	128	220
Model 2c: Elicited inputs	49.4	322	481	1500

* Modified ϕ_0 and w_s based on MDR modelling and Sauter diameter of analogues (see Table 1).

Table S6. Numerical results of the probability that a PDC derived from a caldera-forming eruption similar to Aso-4 reaches the MS_3, with and without topographic effects. For each scenario, we present the values of the cumulative curves displayed in Figure 4 at the central point of the variation range of the PDC mass, while between parentheses we include the results at the extremes of these variation ranges. We test the mass of PDC overflow outside of the caldera of Aso-4 and 1/10 of that estimate, representing the Aso-4T unit (volumes per Takarada and Hoshizumi, 2020). Estimates based on 100,000 statistical samples.

Aso-4T		
Model	**TE^+: No**	**TE^+: Yes**
Model 1: Elicited inputs	12.0% (5.5 – 18.7%)	1.7% (0.1 – 3.6%)
Model 2a: Elicited inputs	32.9% (14.6 – 46.7%)	5.5% (0.9 – 11.5%)
Model 2b: Modified inputs*	98.9% (58.2 – 100.0%)	18.3% (1.2 – 45.7%)
Model 2c: Elicited inputs	41.9% (22.6 – 56.0%)	10.9% (3.5 – 19.1%)
Aso-4, total PDC overflow		
Model	**TE^+: No**	**TE^+: Yes**
Model 1: Elicited inputs	72.0% (53.0 – 84.8%)	33.9% (17.7 – 45.1%)
Model 2a: Elicited inputs	99.6% (90.4 – 100.0%)	75.4% (50.4 – 87.1%)
Model 2b: Modified inputs*	100.0% (100.0 – 100.0%)	100.0% (100.0 – 100.0%)
Model 2c: Elicited inputs	99.9% (93.0 – 100.0%)	80.5% (59.7 – 90.2%)

$^+$ TE: Topographic effects.

* Modified ϕ_0 and w_s based on MDR modelling and Sauter diameter of analogues (see Table 1).

Table S7. Numerical results of the minimum PDC volume and mass needed to reach the MS_4, with and without consideration of topographic effects (see Section 4.4). Estimates based on 100,000 statistical samples.

No topographic effects modeled				
$MinVol$**: Minimum PDC volume [km^3] required to reach MS4**				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	18.3	182	312	1020
Model 2a: Elicited inputs	7.38	40.3	56.2	163
Model 2b: Modified inputs*	5.27	12.0	12.5	21.7
Model 2c: Elicited inputs	4.93	31.5	46.5	145
$MinMass$: **Minimum PDC mass [10^{12} kg] required to reach MS4**				
Model	**5%ile**	**50%ile**	**Mean**	**95%ile**
Model 1: Elicited inputs	19.4	187	309	978
Model 2a: Elicited inputs	12.5	69.1	96.1	281
Model 2b: Modified inputs*	9.06	20.6	21.4	36.7
Model 2c: Elicited inputs	8.26	53.7	80.3	251
With topographic effects included				
Minimum PDC volume [km^3] required to reach MS4				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	44.8	446	763	2500
Model 2a: Elicited inputs	16.3	89.3	125	362
Model 2b: Modified inputs*	11.7	26.7	27.8	48.0
Model 2c: Elicited inputs	10.9	69.8	103	321
Minimum PDC mass [10^{12} kg] required to reach MS4				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	47.5	458	756	2400
Model 2a: Elicited inputs	27.7	153	213	623
Model 2b: Modified inputs*	20.1	45.7	47.4	81.3
Model 2c: Elicited inputs	18.3	119	178	556

Table S8. Numerical results of the probability that a PDC derived from a caldera-forming eruption similar to Aso-4 reaches the MS4, with and without topographic effects. We test the mass of PDC overflow outside of the caldera of Aso-4 and 1/10 of that estimate, representing the Aso-4T unit (volumes per Takarada and Hoshizumi, 2020). Estimates based on 100,000 statistical samples.

Aso-4T		
Model	**TE+: No**	**TE+: Yes**
Model 1: Elicited inputs	25.3% (12.9 – 37.1%)	9.5% (4.1 – 15.0%)
Model 2a: Elicited inputs	55.2% (32.6 – 71.5%)	27.4% (10.7 – 40.4%)
Model 2b: Modified inputs*	100.0% (98.7 – 100.0%)	93.6% (42.3 – 100.0%)
Model 2c: Elicited inputs	64.1% (41.5 – 77.7%)	36.1% (18.1 – 49.6%)
Aso-4, total PDC overflow		
Model	**TE+: No**	**TE+: Yes**
Model 1: Elicited inputs	91.5% (74.1 – 97.0%)	65.2% (47.4 – 78.8%)
Model 2a: Elicited inputs	100.0% (99.6 – 100.0%)	98.4% (86.0 – 100.0%)
Model 2b: Modified inputs*	100.0% (100.0 – 100.0%)	100.0% (100.0 – 100.0%)
Model 2c: Elicited inputs	100.0% (99.9 – 100.0%)	99.2% (89.3 – 100.0%)

+ TE: Topographic effects.

* Modified and ws based on MDR modeling and Sauter diameter of analogues (see Table 1).

Table S9. Numerical results of the minimum PDC volume and mass needed to reach the MS_5, with and without consideration of topographic effects (see Section 4.4). Estimates based on 100,000 statistical samples.

No topographic effects modeled				
$MinVol$**: Minimum PDC volume [km^3] required to reach MS5**				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	20.8	207	354	1160
Model 2a: Elicited inputs	8.26	45.1	63.0	183
Model 2b: Modified inputs*	5.91	13.5	14.1	24.3
Model 2c: Elicited inputs	5.52	35.3	52.1	162
$MinMass$: **Minimum PDC mass [10^{12} kg] required to reach MS5**				
Model	**5%ile**	**50%ile**	**Mean**	**95%ile**
Model 1: Elicited inputs	22.0	213	351	1110
Model 2a: Elicited inputs	14.0	77.5	108	315
Model 2b: Modified inputs*	10.1	23.1	23.9	41.1
Model 2c: Elicited inputs	9.25	60.2	89.9	281
With topographic effects included				
Minimum PDC volume [km^3] required to reach MS5				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	59.1	588	1010	3300
Model 2a: Elicited inputs	20.9	114	159	463
Model 2b: Modified inputs*	14.9	34.2	35.6	61.4
Model 2c: Elicited inputs	14.0	89.2	132	411
Minimum PDC mass [10^{12} kg] required to reach MS5				
Model	**5%ile**	**50%ile**	**mean**	**95%ile**
Model 1: Elicited inputs	62.6	605	998	3160
Model 2a: Elicited inputs	35.5	196	272	797
Model 2b: Modified inputs*	25.7	58.5	60.6	104
Model 2c: Elicited inputs	23.4	152	228	711

Table S10. Numerical results of the probability that a PDC derived from a caldera-forming eruption similar to Aso-4 reaches the MS5, with and without topographic effects. We test the mass of PDC overflow outside of the caldera of Aso-4 and 1/10 of that estimate, representing the Aso-4T unit (volumes per Takarada and Hoshizumi, 2020). Estimates based on 100,000 statistical samples.

Aso-4T		
Model	**TE+: No**	**TE+: Yes**
Model 1: Elicited inputs	22.2% (11.2 – 33.5%)	6.7% (2.7 – 11.0%)
Model 2a: Elicited inputs	50.5% (29.0 – 67.2%)	20.0% (6.8 – 32.4%)
Model 2b: Modified inputs*	100.0% (95.7 – 100.0%)	77.3% (24.8 – 98.6%)
Model 2c: Elicited inputs	59.9% (37.8 – 74.3%)	28.5% (12.9 – 41.3%)
Aso-4, total PDC overflow		
Model	**TE+: No**	**TE+: Yes**
Model 1: Elicited inputs	88.8% (69.8 – 95.7%)	57.2% (39.8 – 69.4%)
Model 2a: Elicited inputs	100.0% (98.9 – 100.0%)	94.7% (79.1 – 99.6%)
Model 2b: Modified inputs*	100.0% (100.0 – 100.0%)	100.0% (100.0 – 100.0%)
Model 2c: Elicited inputs	100.0% (99.5 – 100.0%)	96.6% (83.6 – 99.9%)

+ TE: Topographic effects.

* Modified and ws based on MDR modeling and Sauter diameter of analogues (see Table 1).

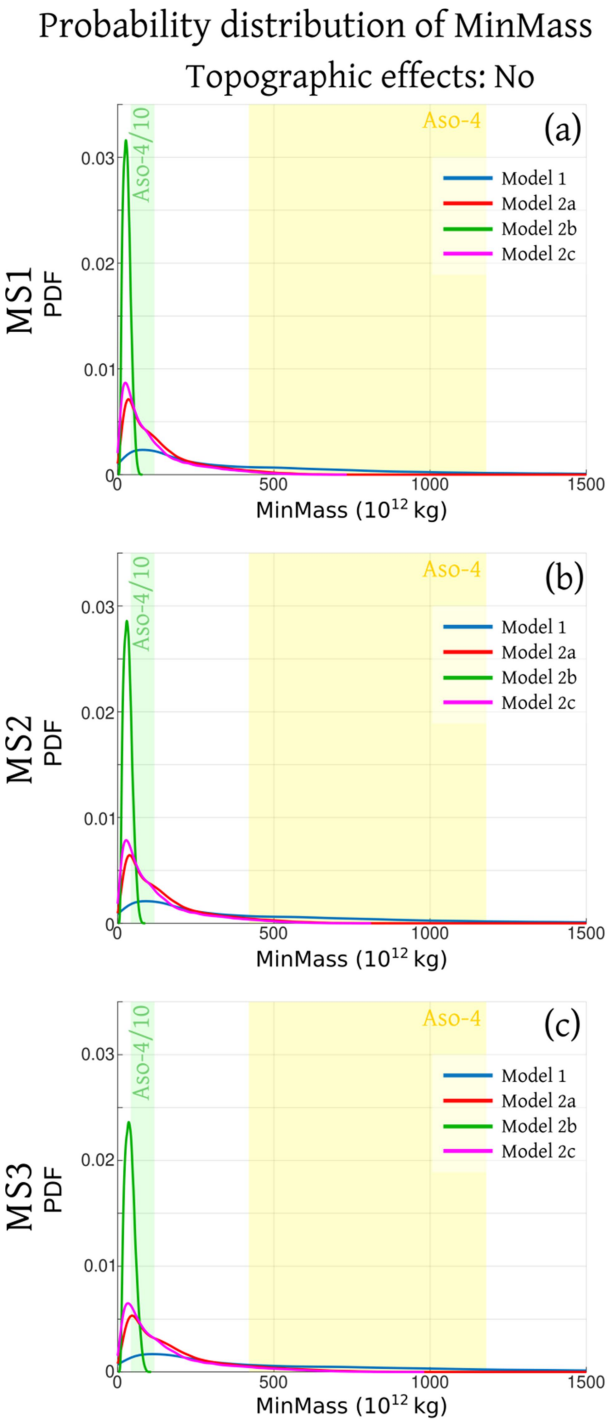

Figure S11. Probability density functions of the variable MinMass, calculated using Models 1, 2a, 2b, 2c and their combination. MinMass represents the mass of pyroclasts in a PDC flow required to invade the marker sites MS1-MS3, related to maximum runout distances equal to those of the marker sites. Estimates of the mass associated with the PDCs produced during the largest caldera-forming eruption of Aso are included: Aso-4/10, representing the Aso-4T unit, in green; Aso-4, total PDC overflow, in yellow (volumes per Takarada and Hoshizumi, 2020). Plots based on 100,000 statistical samples.

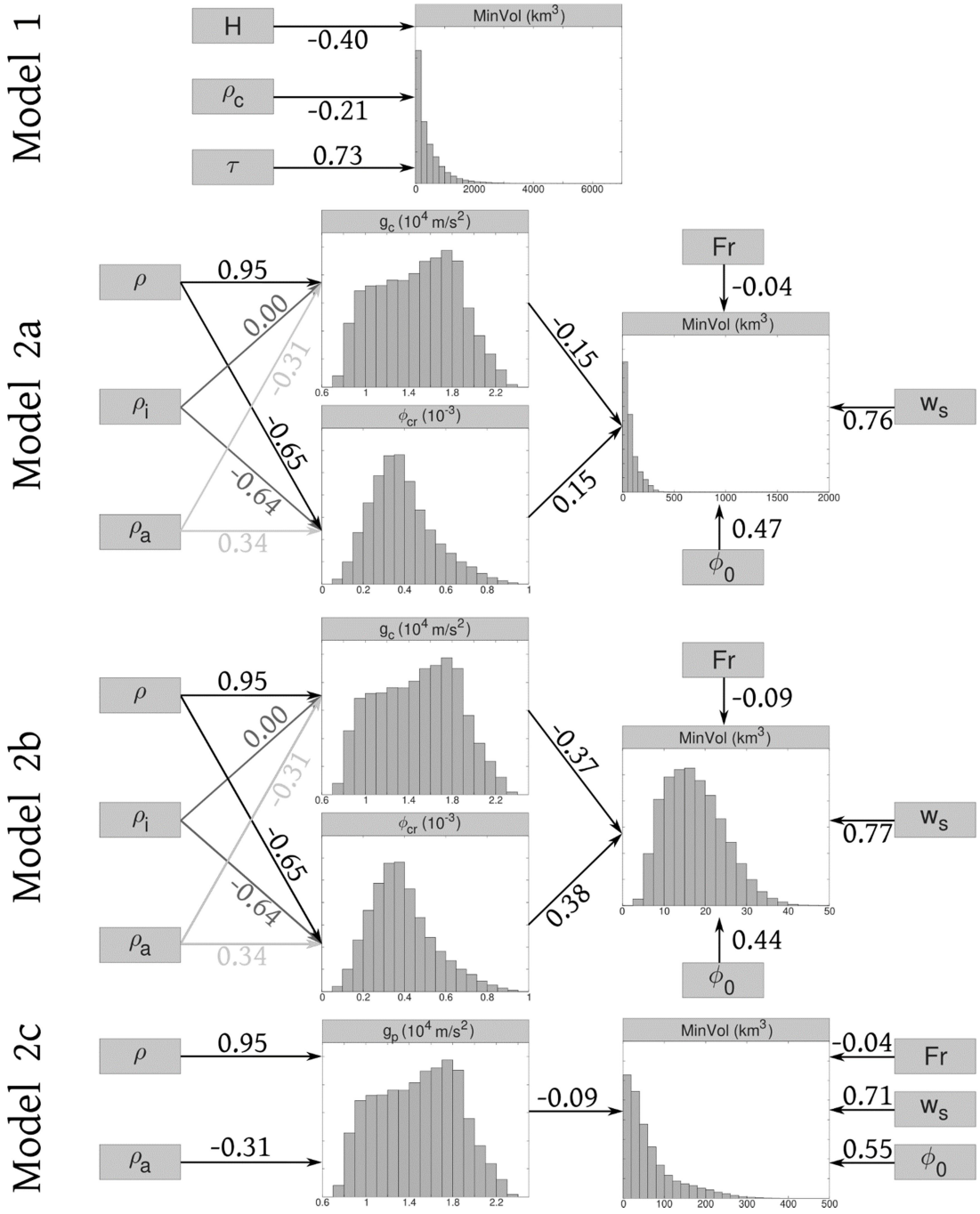

Figure S12. Sensitivity analysis of the parameter MinVol in the different models used in this work. Gray boxes indicate the input parameters, and the arrows indicate the correlation coefficients. BBN based on 100,000 statistical samples.