Supplementary material for "Bayesian hierarchical modeling of sea level extremes in the Finnish coastal region"

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1 Model details and prior distributions for the statistical models

This document provides more details on the priors chosen for the models. For the Separate model, we used weakly informative Gaussian priors for the three GEV parameters:

$$
\begin{aligned}
&y_i \sim GEV(\mu_i, \sigma_i, \xi_i)\\
&\mu_i \sim N(800, 10000^2)\\
&\sigma_i \sim N(200, 10000^2)1_{\{\sigma_i > 0\}}\\
&\xi_i \sim N(0, 0.5^2)
\end{aligned}
\tag{1}
$$

In Common, tide gauge specific GEV parameters are assumed to come from the same joint Gaussian distribution. We therefore need to define hyper-priors for six additional parameters and the corresponding hyper-parameters in our model:

$$
\begin{aligned}
&y_i \sim GEV(\mu_i, \sigma_i, \xi_i)\\
&\boldsymbol{\mu} \sim N(\mu_\mu, \sigma_\mu^2)\\
&\boldsymbol{\sigma} \sim N(\mu_\sigma, \sigma_\sigma^2)\\
&\boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2)\\
&\mu_\mu \sim N(800, 10000^2)\\
&\sigma_\mu \sim N(100, 200^2)1_{\{\sigma_\mu > 0\}}\\
&\mu_\sigma \sim N(250, 1000^2)\\
&\sigma_\sigma \sim N(20, 100^2)1_{\{\sigma_\sigma > 0\}}\\
&\xi_\mu \sim N(0, 0.5^2)\\
&\xi_\sigma \sim \log N(\log(0.05), 0.5^2)
\end{aligned}
\tag{2}
$$

In Spline, we used penalised cubic B-splines with first order random walk priors for the spline coefficients, expressed (e.g.) for the spline coefficients of μ as $\alpha_j = \alpha_{j-1} + u_j,\ j > 1$, where $u_j \sim N(0, \tau^2)$ and $\alpha_1 \propto$ const. Let $\boldsymbol{B}$ denote $K \times L$ matrix, where K is the number of stations and $L = 12$ the number of B-spline basis functions. The spline model is expressed as

$$
\begin{aligned}
& y_i \sim GEV(\mu_i, \sigma_i, \xi_i) \\
& \boldsymbol{\mu} = \boldsymbol{B}\boldsymbol{\alpha} \\
& \boldsymbol{\sigma} = \boldsymbol{B}\boldsymbol{\beta} \\
& \boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2) \\
& \alpha_1 \sim N(1000, 10000^2) \\
& \beta_1 \sim N(200, 1000^2) \\
& \alpha_j \sim N(\alpha_{j-1}, \tau_\alpha^2), \quad j = 2, \ldots, L \\
& \beta_j \sim N(\beta_{j-1}, \tau_\beta^2), \quad j = 2, \ldots, L \\
& \tau_\alpha, \tau_\beta \overset{\text{ind}}{\sim} N(0, 100^2) 1_{\{\sigma_\mu > 0\}} \\
& \mu_\xi \sim N(0, 0.5^2) \\
& \sigma_\xi \sim \log N(\log(0.05), 0.5^2)
\end{aligned}
\tag{3}
$$

For the GP model we use squared exponential kernel K based on distance d described earlier. The model specification is the following:

$$
\begin{aligned}
& y_i \sim GEV(\mu_i, \sigma_i, \xi_i) \\
& \boldsymbol{\mu} \sim GP(\mu_\mu, K(\sigma_\mu, \phi_\mu)) \\
& \boldsymbol{\sigma} \sim GP(\mu_\sigma, K(\sigma_\sigma, \phi_\sigma)) \\
& \boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2) \\
& \mu_\mu \sim N(800, 10000^2) \\
& \sigma_\mu \sim \log N(\log(40), 0.5^2) \\
& \phi_\mu \sim \log N(\log(100), 0.1^2) \\
& \mu_\sigma \sim N(200, 1000^2) \\
& \sigma_\sigma \sim \log N(\log(30), 0.5^2) \\
& \phi_\sigma \sim \log N(\log(100), 0.1^2) \\
& \mu_\xi \sim N(0, 0.5^2) \\
& \sigma_\xi \sim \log N(\log(0.05), 0.5^2)
\end{aligned}
\tag{4}
$$