Report on the manuscript

Tsunami propagation kernel and its applications

by

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The proposed manuscript deals with the prediction of the coastline inundation through the use of proper analytical kernels for the Linear Shallow Water Equations. The basic idea has been already developed in a previous paper of the author (i.e. Shimozono 2020) and is further inspected here in order to define a more straightforward procedure for the data assignment. In particular, the formulation proposed allows one to assign a boundary datum that includes both incident- and reflected-wave components. In addition, the author proposes the use of a damping factor which accounts for the energy dissipation encountered by the wave during its path toward the shoreline.

My overall opinion is that the manuscript is well written and that the mathematical approach is sound. Further, the proposed analytical solution is compared with measurements from real tsunami and this makes the present contributions interesting for the readers of NHESS. There are, in any case, some aspects that deserve an accurate inspection and a deeper insight. Below, I list the points that have to be addressed before the paper may be accepted.

Major Points

- The author states that the proposed solution applies to a generic datum (that is, a signal that includes both reflected and incident components). In any case, differently from his previous work (Shimozono 2020), he does not provide any evidence of this.

  My personal opinion is that this statement by the author is not correct and this is confirmed by the occurrence of a singular kernel in the equation (14) as a consequence of the assignment in (11). Indeed, in absence of dissipation (namely, $\alpha = 0$), the complex values of p such that $I_0(2s) = 0$

correspond to a null wave elevation datum at the seaward boundary (while in general $G(s) \not\equiv 0$). This occurs when the reflected and incident components of the wave elevation are in opposite phase at $x = 1$. In the same case, the velocity at $x = 1$ is generally different from zero [see the equation (10)].

This points has to be clarified with care. I think that the cause of the presence of singularities in the kernel is simple due to the assignment on variables which do not exclusively represent the whole incident signal.

- The presence of the dissipation (i.e. $\alpha \neq 0$) seems to mitigate somehow the singularity of the kernel, since the poles are not aligned along the imaginary axis [see the equation (19)]. I think some comments about the differences between the inviscid and viscous solutions should be added.

- Section 3.2. As shown in the previous sections, the singularities in the kernel are "accounted for" through the integration along the Bromwich path. This means that the singularities are handled through principal-value integrals. This is a well established procedure. What I do not understand is why the solution for η in the equation (33) is obtained by substituting the equation (32) inside the equation (12) straightforwardly. The solution that is obtained is obviously ill-posed and cannot be accepted in this form.

  This is clear if we observe the Figure 5. For example the panel *(a)* shows an amplitude which is $\mathcal{O}(10^2)$ while the amplitude at the seaward limit is $\mathcal{O}(1)$! More in general, the overall behaviour of the amplitude described in this figure is quite odd and seems not physical.

  Once again, I stress that the occurrence of singularities in the solution for η is here mitigated by the presence of the damping factor [see the expressions for A and B just after the equation (35)]. In fact, if $\alpha = 0$, we would obtain a singular solution of η for those values of λ such that $I_0(2i\lambda) = 0$.

  In conclusion, I think that all this section has to be rewritten by deriving a solution for η obtained through the use of principal-value integrals (as done in the previous sections).

Minor Points

- Page 6, line 144. Maybe $\hat{\Psi}_1$ instead of Ψ_1.

- Page 6, line 154. The author states "*Both* $\Psi_0(x, s)$ *and* $\Psi_1(x, s)$ *have a pole at* $s = 0 \ldots$". Actually, it seems to me that $\Psi_1(x, s)$ has a removable singularity at $s = 0$ [I refer to the second formula in the equation (14)]. Please check again.

- I think that the expressions for $t^{\pm}$ in the equation (24) are simply the two branches of some specific characteristic curves in the (x, t)-plane. Indeed they may be cast in the following compact form:

$$x = [\, t - 2 - T\,(m-1)\,]^2 \,, \tag{1}$$

  where $T = 4$ is the time "period" that takes a signal to travel back and forth in the fluid region.

- Equation (19). Here the author should point out that the damping factor has an upper-limit. Specifically, it should be $\alpha \leq c_1$ where $c_1 \simeq 2.405$ is the first real zero of the Bessel function J_0.

References

T. Shimozono, *Kernel representation of long-wave dynamics on a uniform slope*, Proc. R. Soc. A 476: 20200333.