Dynamic Risk Assessment of Compound Hazards Based on VFS-IEM-IDM: A Case Study of Typhoon-rainstorm Hazards in Shenzhen, China

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Abstract. Global warming has led to an increasing occurrence of compound hazards and an accurate risk assessment of such hazards is of great importance to urban emergency management. Due to the interrelations between multiple hazards, the risk assessment of a compound hazard is facing several challenges: (1) the evaluation of hazard level needs to take into account the correlations between compound hazards drivers, (2) usually only a small number of data samples are available for estimating the joint probability distribution of the compound hazard drivers and the loss caused by the hazards, (3) the temporal dynamics of the occurrences of compound hazards needs to be considered in the process of the risk assessment. To deal with these challenges, this paper proposes an integrated risk assessment model VFS-IEM-IDM to quantify the dynamic risk of compound hazards based on Variable Fuzzy Set (VFS), Information Entropy Method (IEM), and Information Diffusion Method (IDM). For the first challenge, VFS-IEM-IDM measures the effect of the compound hazard drivers via the use of relative membership degree and analyses the correlation between drivers with the entropy weight method, which are combined to evaluate compound hazards level. To address the second challenge, VFS-IEM-IDM applies the normal diffusion function to estimate the probability distribution of the compound hazard and the corresponding loss vulnerability curve. To deal with the third challenge, VFS-IEM-IDM assesses the risk of a compound hazard in different months based on the definition of probabilistic risk. To evaluate the proposed risk assessment model VFS-IEM-IDM, we use the typhoon-rainstorm hazards occurred in Shenzhen, China, as a case study and show that VFS-IEM-IDM can effectively estimate the typhoon-rainstorm compound hazard level and assess the dynamic risk of the compound hazards.

Keywords: Compound hazards; Fuzzy dynamic risk; Variable fuzzy set; Information diffusion; typhoon-rainstorm

1 Introduction

With global climate change, many cities have suffered extreme natural hazards more frequently (Ming et al. (2022)). People and their properties have been exposed to various hazards simultaneously or successively worldwide. In the literature, there has been an increasing interest in the research of assessing multi-hazard risks (Choi et al. (2021)). A compound hazard is a typical multi-hazard problem that involves the concurrence of multiple hazard drivers, such as heavy rainfall, extreme wind intensity, and storm surge. For example, typhoons and rainstorms are two different types of natural hazards that can cause significant damages. When these two types of hazards simultaneously occur, compound hazards are produced, leading to more severe catastrophes than the individual hazards. Therefore, the risk assessment of such compound hazards needs to take into account the interrelations between the individual hazards.

The risk of a hazard is defined as the potential consequences brought by the disasters and can be quantified by the probability of losses (He et al. (2020)). Risk assessment is a technique that uses the relevant hazard data to estimate the likelihood that natural hazards may occur and further assess their economic losses (Huang et al. (2018)). Traditional methods of risk assessment mainly utilize geographic information systems to get risk maps (Gigovic et al. (2017)) or rely on information diffusion methods to deal with the problem of data sparsity (Gong et al. (2020)). These risk assessment methods (Julia et al. (2021); Zhou et al. (2020)) are mostly applied to individual hazards, while the risk assessment of compound hazards is not simply the aggregation of the assessment results of the individual hazards but needs to consider the interrelations between them (Kappes et al. (2012)).

There are many research works discussing the risk assessment of multi-hazards. They classify the relationship between the individual hazards in the scenarios of multi-hazards into three categories: mutually amplified hazards, mutually exclusive hazards, and non-influential stakes (Wang et al. (2020)). The existing methods and technologies relevant to the risk assessment of multi-hazards have been reviewed in (Khan et al. (2020)). For example, a Cloquet integral multiple linear regression model has been proposed to overcome the problem of nonlinear additivity of mutually amplified hazards for hazard level evaluation (He et al. (2020)). An information diffusion method has been used to assess the risk of multiple hazards quantitatively and evaluate the risk of loss of human lives from meteorological hazards in China (Xu et al. (2016)). A quantitative approach of multi-hazard risk assessment based on vulnerability distribution and joint return period of hazards is proposed to assess the risk of crop losses in the Yangtze River Delta region of China (Ming et al. (2015)). However, all of these works do not consider the correlations between the occurrences of the individual hazards, such as the co-appearance of typhoon-rainstorm hazards. Furthermore, there is little research focusing on typhoon-induced risk assessment in the literature, and temporal dynamics are rarely considered in risk assessments.

Compound hazards, a sub-group of 'multi-hazards', are considered as the combination of multiple hazard drivers that contribute to societal or environmental risks. The characteristics of compound hazards include: (1) two or more extreme events occurring simultaneously or successively, (2) combinations of extreme events with underlying conditions that amplify the impact, and (3) combinations of events that are not themselves extreme but lead to an extreme event or impact when combined (Jennifer et al. (2021)). Here, we explicitly consider compound hazards for the case when two or more individual extreme events occurring at the same place and at the same time, such as the extreme precipitation, winds, and ocean waves. In this paper, we define the risk of a compound hazard as a scene in the future associated with some adverse incidents caused by cascading hazards systems, where there are strong connections between different hazard drivers. Compared with the risk assessment of multi-hazards in the literature (Xu et al. (2016); Huang et al. (2018)), assessing the risk of compound hazards needs to obtain an integrated hazard level without losing any correlated information between the individual hazards.

While there have been many attempts to assess the risk of multi-hazards, most of the existing methods have limitations in dealing with compound hazards (Ming et al. (2022); Huang et al. (2018)). Firstly, the correlation between the hazard drivers is often ignored. Considering that the disaster control engineering system is a synthesis of multi-dimensional factors, the potential inter-dependencies of the drivers will affect the joint probability and the economic losses of compound hazards. Secondly, the relationship between the hazards, i.e., vulnerability and exposure analysis, cannot be modeled effectively when the data is sparse. Thirdly, most of the existing risk analysis frameworks for compound hazards are based on either qualitative or semi-quantitative methods. Moreover, the temporal dynamics of the occurrences of compound hazards are often not considered.

To address the first limitation, researchers have applied variable fuzzy set (VFS) methods to deal with the multi-factor problem. Some researchers have shown that the relative membership function can be used to evaluate the relations between multiple indicators in risk assessment (Chen (2006)). A fuzzy method (Li et al. (2012)) is proposed to solve the flood risk assessment problems with interval boundaries and this integrated model improves the reliability of single hazard risk assessment. VFS has also been used to evaluate the synthetic hazards level of Nagapattinam district with the north-east monsoon rainfall's data sets (Beaula et al. (2013)). In this paper, we propose to combine VFS with the information entropy method (IEM) to assess the hazard level of compound hazards such that the correlations between the hazard drivers can be captured.

To deal with the second limitation, the information diffusion method (IDM) is commonly used to model the physical relationship between different attributes. In many cases, it is difficult to collect compound hazards data, and the historical data is often sparse. To this end, many fuzzy probabilistic models have been proposed to enhance the accuracy of the risk assessment results (Mehran et al. (2017)). Fuzzy probabilistic models are used to model uncertainties related to hazards and the randomness due to environmental, natural, or period changes. The main feature of the fuzzy probabilistic models is to transform the raw data points into fuzzy sets to partly fill the gap caused by data sparsity and improve the estimation accuracy between the inputs and the outputs. One of the most powerful techniques is IDM (Huang (1997, 2002)), which helps extract useful underlying information from the hazards data sets. Researchers have done a simulation study on IDM and demonstrated the benefit of information distribution for probability estimation (Huang (2000)). The capability of IDM in dealing with the problem of data sparsity has been well studied in the literature (Li (2013)). In this paper, we construct a normal information diffusion estimator (IDM) to analyze the probability function and vulnerability curve of compound hazards.

As for the third limitation, preliminary attempts have been made to develop quantitative multi-hazard risk assessment frameworks (Huang et al. (2018)). The probabilistic risk model combined with the concept of dynamic risk assessment has been proposed to estimate the flooding risk (Huang (2015)). In this paper, we present the definition of dynamic compound hazards risk and then propose a method to assess the compound hazards risk quantitatively which also takes into account of the temporal dynamics of the occurrences of the hazards.

The main contributions of this paper are summarized as follows.

1. We propose a model, named Variable Fuzzy Set and Information Diffusion (VFS-IEM-IDM), to assess the dynamic risk of compound hazards, which takes into account the interrelations between the hazard drivers, deals with the problem of data sparsity, and considers the temporal dynamics of the occurrences of the compound hazards.
2. We simplify the procedures of calculating relative membership degree to improve the efficiency of compound hazards level evaluation, and we also use a predictive cumulative logistic model to verify the evaluation results.
3. To examine the efficacy of the proposed model VFS-IEM-IDM, a case study of the typhoon-rainstorm hazards occurred in Shenzhen, China is presented.

The rest of this paper is organized as follows. Section 2 introduces the basic concepts and definitions in this paper. In section 3, we present the dynamic compound hazards risk assessment model, namely VFS-IEM-IDM. Section 4 provides an evaluation of the VFS-IEM-IDM with a case study of typhoon-rainstorm hazards occurred in Shenzhen, China. In section 5, we discuss the results of the case study obtained at different stages of VFS-IEM-IDM. Finally, conclusions are drawn in Section 6.

2 Preliminaries

2.1 Basic concepts

Variable fuzzy set is used to express the fuzzy effect of the hazard drivers by relative membership degree (RMD) functions, and then the compound effects between different drivers can be modeled. This method provides an enhanced implementation of the compound hazards level evaluation process and can reflect the coupled characteristics of compound hazards. *Information entropy* is based on the entropy coefficient calculation process, which is used to measure the importance of the individual hazard drivers and determine the weight of different drivers. *Information diffusion* is a function learning method with high estimation accuracy from a small data set, which makes full use of the diffusion information given by the data samples to estimate the probability density of the data samples or the relationship between the data samples without the knowledge of the distribution from which the data samples were drawn. This method is applied to estimate the probability distribution p (hazard potential) of the occurrence of hazards, and the causal relationship f (hazard vulnerability).

2.2 Dynamic compound hazards risk

From the previous studies, risks could be classified into four categories: pseudo risk, probability risk, fuzzy risk, and uncertainty risk (Huang et al. (2018)). Existing hazard risk assessment models are often qualitative or semi-quantitative, which cannot estimate directly economic losses from the joint impact of several hazards. Probability risk is estimated by integrating the probability distribution p of the occurrence of hazards, and the causal relationship f between the economic loss and the hazard attributes. As a result, the probability risk could be quantified as the expected value of economic losses, i.e., the integration of hazard potential with hazard vulnerability.

Though these four types of risks have been investigated by many researchers, there are few research on dynamic compound hazards risk. In this paper, *compound risk* is defined as a scene in the future associated with some adverse incident caused by cascading hazards systems, where there are strong connections between different hazards and the hazard level is influenced by many drivers. Furthermore, as proposed by Huang (2015), the concept of compound risk could be extended to *dynamic compound risk* if the impact of occurrence time on risk assessment is taken into consideration. To assess the risk of a compound hazard, the probability distribution p of the occurrences of the compound hazard will be estimated with probability models, and the causal relationship f between the hazards attributes and the losses is captured by a fuzzy model. The compound hazards risk is defined as follows:

$$Risk = p(\Phi; X) \cdot f(\Phi'; X), \tag{1}$$

where $X = \{x_{ij} | i = 1, 2, \cdots, N; j = 1, 2, \cdots, J\}$ represents the data samples with the sample size N and the number of compound hazards attributes J, Φ and Φ' denote a set of hazard attributes which reflects the characteristics of the compound hazard. For example, the risk of the compound hazard typhoon-rainstorm can be assessed by 3 hazard attributes including hazard occurrence time ϕ_1, compound hazards level ϕ_2, and economic losses ϕ_3. The dynamic compound risk is derived by integrating the conditional probability distribution $p(X; \Phi)$ where $\Phi = (\phi_1, \phi_2)$ with the hazards vulnerability $f(X; \Phi')$ where $\Phi' = (\phi_1, \phi_2, \phi_3)$.

3 Dynamic Risk Assessment of Compound Hazards: VFS-IEM-IDM

Risk assessment of compound hazards should consider the correlation between the compound hazard drivers, the problem of data sparsity, and the dynamic property of hazard occurrences. This section introduces VFS-IEM-IDM, a risk assessment model for compound hazards which combines the variable fuzzy sets theory with the information diffusion method to assess the dynamic risk of compound hazards when only a small set of data samples is available. Fig. 1 shows the workflow of VFS-IEM-IDM which mainly consists of two components. With individual hazard level and historical records of hazard drivers as inputs, the first component VFS-IEM combines variable fuzzy set methods with information entropy methods to provide a comprehensive evaluation of the compound hazards level (Section 3.1). Based on the compound hazards levels and historical records of risk assessment attributes, the second component VFS-IDM adapts normal information diffusion methods to quantify the dynamic risk of the compound hazards in terms of the direct economic losses (Section 3.2). The boxes colored blue represent the results obtained by the VFS-IEM-IDM model.

3.1 Compound hazards level evaluation: VFS-IEM

For the compound hazards risk assessment, the correlation between the compound hazards drivers should be considered. Fortunately, the variable fuzzy set (VFS) theory which considers the contributions of multiple related drivers and decreases the fuzziness by using membership functions (Chen (2006)), provides an appropriate tool for evaluating the compound hazards level.

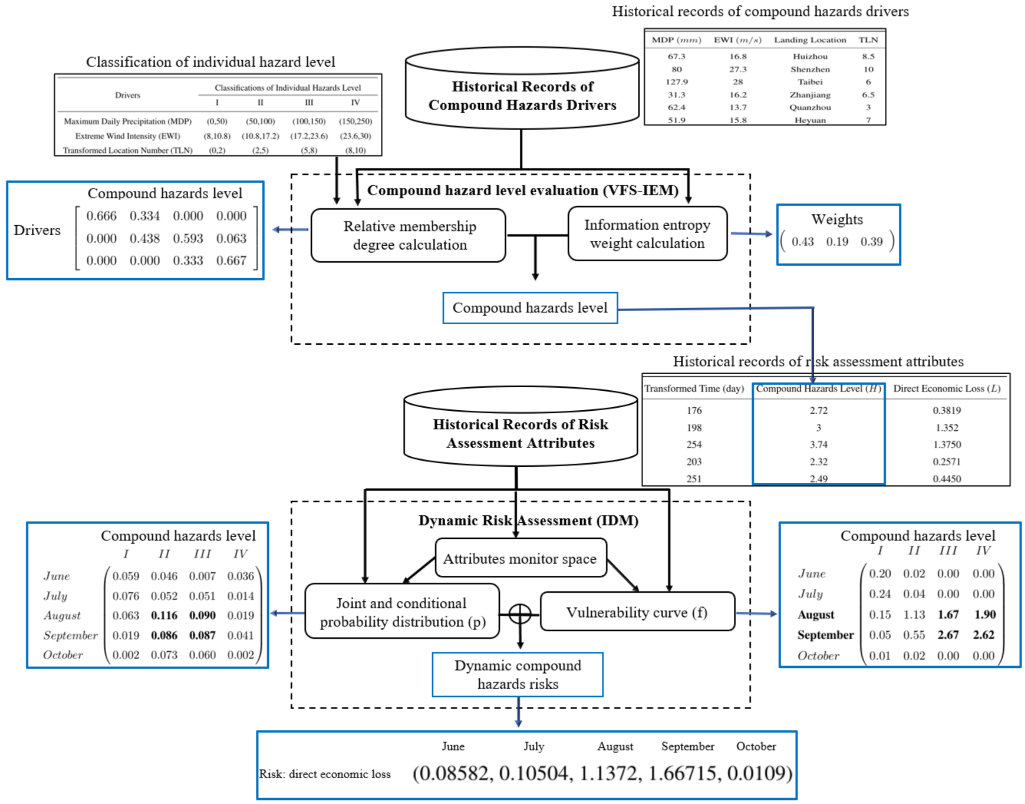

Figure 1. Workflow and illustration of the VFS-IEM-IDM dynamic compound hazards risk assessment model based on case study. Based on the historical records of compound hazards, our proposal provides an enhanced implementation of the compound hazards level evaluation and then estimates the probability distribution and the corresponding loss vulnerability curve of compound hazards attributes to calculate the dynamic compound hazards risk. The blue boxes represent the results obtained by the VFS-IEM-IDM model

Based on VFS, the fuzzy set intervals given by the individual hazard level classification can be used to assess the compound hazards level. For example, suppose we have two fuzzy set intervals $I_{ab} = (a,b)$ and $I_{cd} = (c,d)$ where $a,b,c,d \in \mathcal{R}$, in which I_{cd} is an extended fuzzy set interval based on I_{ab}, as shown in Fig. 2. Specifically, the relative membership degree (RMD) function $\mu(u)$, which determines the probability of a hazard driver u belonging to different hazard level intervals, is applied to evaluate the contributions of compound hazards related drivers. Since the calculation of RMD is complicated and time consuming, we use different locations of the balance boundaries matrix M (Wang et al. (2014)) and the value of driver u to simplify the calculation process. Firstly, we use the interval (a_{rl}, b_{rl}) to define the balance boundaries matrix $M = [m_{rl}]$, which is shown in Eq. 2.

$$m_{rl} = \frac{L-l}{L-1} a_{rl} + \frac{l-1}{L-1} b_{rl}, \tag{2}$$

where $r = 1,2,\ldots,R$ and R indicates the number of hazard drivers, $l = 1,2,\ldots,L$ and L denotes the number of compound hazards levels. Then, we compare the relative locations of u with m_{rl} in the interval (a_{rl}, b_{rl}) and (c_{rl}, d_{rl}). RMD can then be constructed by the ratio $\frac{u-a_{rl}}{m_{rl}-a_{rl}}$ as follows:

$$\begin{cases} \mu(u)_{rl} = 0.5(1+\left(\frac{u-a_{rl}}{m_{rl}-a_{rl}}\right)^q) & u \in (a_{rl}, m_{rl}) \\ \mu(u)_{rl} = 0.5(1-\left(\frac{u-a_{rl}}{c_{rl}-a_{rl}}\right)^q) & u \in (c_{rl}, a_{rl}) \end{cases}. \tag{3}$$

It can be seen that RMD is influenced by the hyper-parameter q and the position between the hazard driver value u and the level interval I_{ab}, I_{cd}, and the value of m_{rl}. In this paper, guided by the procedure of calculating RMD in the literature (Fang et al. (2019)), we simplify the procedure of calculating relative membership degree to improve the efficiency of compound hazards level evaluation. Firstly, the intervals I_{ab}, I_{cd} of the individual hazard levels and the balance boundaries matrix M are obtained following the VFS theory (Chen (2006)). Secondly, we determine whether the location of u is in the lowest, middle or highest grade of the interval I_{ab}, as shown respectively in Fig.2, Fig. 3, and Fig.4. Finally, according to the location of u, we use one of the three sets of formulas to calculate RMD, as shown in Eq. 4, Eq. 5, and Eq. 6 accordingly.

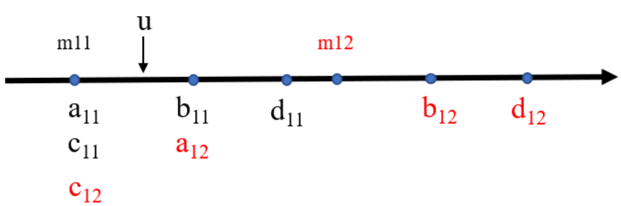

Figure 2. Lowest case: the position between u with parameter m_{11} and fuzzy intervals (a_{11}, b_{11}), (c_{11}, d_{11}). Symbols with different colors indicate different fuzzy intervals.

$$\begin{cases} \mu(u)_r = [\mu(u)_{r1} \quad \mu(u)_{r2} \quad 0 \quad \cdots \quad 0] \\ \mu(u)_{r1} + \mu(u)_{r2} = 1 \\ 0.5 \le \mu(u)_{r1} \le 1 \\ 0 \le \mu(u)_{r2} \le 0.5 \end{cases}. \tag{4}$$

$$\begin{cases} \mu(u)_r = [0 \quad \cdots \quad 0 \quad \mu(u)_{r(L-1)} \quad \mu(u)_{rL}] \\ \mu(u)_{r(L-1)} + \mu(u)_{rL} = 1 \\ 0.5 \le \mu(u)_{rL} \le 1 \\ 0 \le \mu(u)_{r(L-1)} \le 0.5 \end{cases}. \tag{5}$$

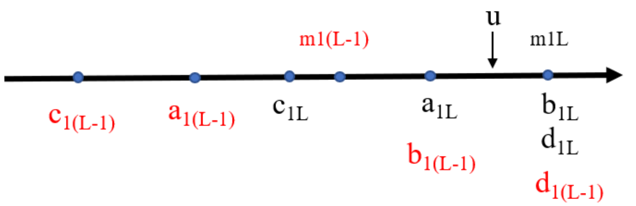

Figure 3. Highest case: the position between u with parameter m_{1L} and fuzzy intervals (a_{1L},b_{1L}), (c_{1L},d_{1L}) . Symbols with different colors indicate different fuzzy intervals.

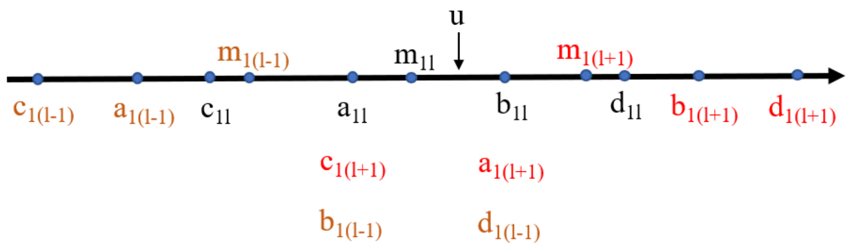

Figure 4. Middle case: the position between u with parameter m_{1l} and fuzzy intervals (a_{1l},b_{1l}), (c_{1l},d_{1l}). Symbols with different colors indicate different fuzzy intervals.

$$\begin{cases} \mu(u)_r = [0 \quad \cdots \quad 0 \quad \mu(u)_{r(l-1)} \quad \mu(u)_{rl} \quad \mu(u)_{r(l+1)} \quad 0 \quad \cdots \quad 0] \\ \mu(u)_{r(l-1)} + \mu(u)_{r(l+1)} = 0.5 \\ 0 \leq \mu(u)_{r(l-1)} \leq 0.5 \\ 0 \leq \mu(u)_{r(l+1)} \leq 0.5 \end{cases} . \tag{6}$$

Following the previous works by Kwakernaak (1978) and Chen (2006), we use the variable fuzzy recognition model to obtain the comprehensive RMD of each driver. Then, the proposed compound hazards level evaluation model can be constructed by Eq. 7.

$$\begin{cases} \nu(u)_l = (1 + (\frac{\sum_{r=1}^{R}(\omega_r(1-\mu(u)_{rl}))^\alpha}{\sum_{r=1}^{R}(\omega_r\mu(u)_{rl})^\alpha})^{\frac{\beta}{\alpha}})^{-1} \\ \hat{\nu}(u)_l = \frac{\nu(u)_l}{\sum_{l=1}^{L}\nu(u)_l} \\ H = (1 \quad 2 \ldots L) \cdot \hat{\nu}(u) \end{cases} . \tag{7}$$

where α and β are two hyper-parameters, w_r indicates the weight of each hazard driver, $\nu(u)_l$ is the weighted RMD of different hazard drivers and H is the compound hazards level. The weights of the individual hazard drivers w_r are obtained via the use of information entropy (Liu et al. (2010)) as shown in Eq. 8:

$$\begin{cases} \hat{g}_{rl} = v_{rl} / \sum_{l=1}^{L} v_{rl} \\ g_r = -1/ln(N) \cdot \sum_{l=1}^{L} (\hat{g}_{rl} ln \hat{g}_{rl}) \\ \omega_r = (1 - g_r)/(R - \sum_{r=1}^{R} g_r) \end{cases} . \tag{8}$$

where v_{rl} is defined as the measured value from the lth level for the rth driver and N denotes the sample size. The detailed procedure of VFS-IEM is shown in Algorithm 1.

Algorithm 1 VFS-IEM Compound Hazards Level Evaluation

Input:

The compound hazards driver fuzzy set $\boldsymbol{U} = \{u_{ir}, r = 1, 2, \cdots, 3 | i = 1, 2, \cdots, N\}$;

Individual hazard level assessment matrix, $V = [v_{rl}]$, $r = 1, 2, 3;\ l = 1, 2, 3, 4$.

Output:

Comprehensive value of compound hazards level.

```
Identification of interval I_ab = [(a,b)_rl] and the extended interval I_cd = [(c,d)_rl] based on assessment matrix V;
Define the balance boundaries matrix M = [m_rl] by Eq. 2;
Calculate the weight of each driver Ω = [ω_1, ω_2, ω_3] by Eq.8;
for i = 1 to N do
    for each u_ir ∈ U do
        if u_ir locates in the lowest grade of the interval I_ab, i.e., a_r1 < u_ir < b_r1 then
            Calculate RMD μ(u)_r with Eq.4;
        else if u_ir locates in the highst grade, i.e., a_rL < u_ir < b_rL then
            Calculate RMD μ(u)_r with Eq.5;
        else
            Calculate RMD μ(u)_r with Eq.6;
        end if
    end for
    The relative membership matrix of each sample is denoted as μ(u) = [μ(u)_1, μ(u)_2, μ(u)_3];
    Combine μ(u) with weights Ω and integrate the ranking level, and calculate the comprehensive compound hazards level for each
    sample with Eq.7.
end for
```

3.2 Dynamic risk assessment model: IDM

To assess the dynamic risk of compound hazards, especially when the data sets are sparse, the information diffusion method (IDM) which belongs to the fuzzy theory can be used to extract useful underlying information from the limited data samples to estimate the probability function p and vulnerability curve f. According to Huang (1997), J-dimension normal information diffusion function $\Gamma(x_i;S^k)$ (shown in Eq. 9) is more powerful to improve the precision of the estimators. Therefore, this paper adapts the normal information diffusion estimator to approximate the dynamic compound hazards risk.

$$\Gamma(x_{ij};s_j^{k_j}) = exp(-\frac{(x_{ij}-s_j^{k_j})^2}{2\sigma_{x_j}^2}),\ k_j = 1,2,\cdots,K_j; i = 1,2,\cdots,N.$$

$$\Gamma(x_i;S^K) = \prod_{j=1}^{J}\Gamma(x_{ij};s_j^{k_j}), \quad S^K = \{s_j^{k_j}|j = 1,2,\cdots,J\} \tag{9}$$

$$\sigma_{x_j} = \begin{cases} 0.6841(b-a), & for \quad N=5; \\ 0.5404(b-a), & for \quad N=6; \\ 0.4482(b-a), & for \quad N=7; \\ 0.3839(b-a), & for \quad N=8; \\ 2.6581(b-a)/(N-1), & for \quad N\geq 9. \end{cases} \tag{10}$$
$$where \quad b = \max_{1\leq i\leq N}\{x_{ij}\}, \quad a = \min_{1\leq i\leq N}\{x_{ij}\}.$$

where N is the sample size of $X = \{x_{ij}|i = 1,2,\cdots,N; j = 1,2,\cdots,J\}$, K_j is the number of diffusion points of a given monitor set s_j, and σ_{x_j} is the diffusion coefficient with respect to different attributes j. Based on the normal estimator, the research by Huang (2002) has shown how to determine the coefficients (shown in Eq. 10). This approximate reasoning of information diffusion is used to estimate probabilities and fuzzy relationships from a small dataset for risk assessment (Huang et al. (2018)). As an example, we use a 2-dimension normal estimator to calculate the discrete probability density function. For the given compound hazards attributes monitor set $S^K = \{(s_1^{k_1}, s_2^{k_2})|1<k_1<K_1, 1<k_2<K_2\}$, we estimate the discrete probability matrix $P = p_{k_1,k_2}$ and the conditional probability distribution $\hat{P} = p_{s_2|s_1}(s_2^{k_2}|s_1)$:

$$p_{k_1,k_2} = \frac{\sum_{i=1}^{N}\Gamma(x_i;S^K)}{\sum_K\sum_{i=1}^{N}\Gamma(x_i;S^K)} \tag{11}$$

$$p_{s_2|s_1}(s_2^{k_2}|s_1) = \frac{p_{k_1,k_2}}{\sum_{k_2=1}^{K_2}p_{k_1,k_2}}, k_2 = 1,2,\cdots K_2. \tag{12}$$

Similarly, we can calculate the three dimensional diffusion function for the compound hazards attributes set $S^K = \{(s_1^{k_1}, s_2^{k_2}, s_3^{k_3})|1<k_1<K_1, 1<k_2<K_2, 1<k_3<K_3\}$. Suppose s_3 corresponds to the attribute of economic loss, the vulnerability curve between the set of causes s_1, s_2 and the consequence s_3 can be estimated by the following fuzzy membership function:

$$\varpi(s_3^{k_3}) = \max_{(k_1,k_2)}\{\min\{\Gamma(x_i;(s_1^{k_1},s_2^{k_2})), R^{k_3}\}\}, k_3 = 1,2,\cdots,K_3 \quad R^{k_3} \text{ is the } \mathrm{k_3}^{\mathrm{th}} \text{ slice of } R.$$

where the fuzzy relationship model $R = [r_{k_1,k_2,k_3}]$ (shown in Eq. 13) is defined by the 3-dimension information diffusion function $\Gamma(x_i; S^K)$.

$$r_{k_1,k_2,k_3} = \frac{\sum_{i=1}^{n} \Gamma(x_i; (s_1^{k_1}, s_2^{k_2}, s_3^{k_3}))}{\max\limits_{1 \le k_3 \le K_3} \sum_{i=1}^{n} \Gamma(x_i; (s_1^{k_1}, s_2^{k_2}, s_3^{k_3}))}. \tag{13}$$

Then the vulnerability curve $f = f_{s_1^{k_1}, s_2^{k_2}}$, is defined as follows.

$$f_{s_1^{k_1}, s_2^{k_2}} = \frac{\sum_{k_3=1}^{K_3} \varpi(s_3^{k_3}) \cdot s_3^{k_3}}{\sum_{k_3=1}^{K_3} \varpi(s_3^{k_3})}, k_1 = 1, 2, \cdots, K_1; k_2 = 1, 2, \cdots, K_2. \tag{14}$$

Based on the VFS-IDM risk assessment model, the dynamic compound hazards risk (Direct Economic Losses) can be obtained via Eq. 15 where the risk is quantified as the expected value of the conditional probability distribution p and the vulnerability distribution f. The detailed procedure of IDM is shown in Algorithm 2.

$$Risk = \sum_{k_2=1}^{K_2} p_{k_1,k_2} \cdot f_{s_1^{k_1}, s_2^{k_2}} \tag{15}$$

Algorithm 2 IDM Dynamic Risk Assessment Model

Input:

Compound hazards data samples $X = \{(x_{i1}, x_{i2}, x_{i3}) | i = 1, 2, \cdots, N\}$, where x_{ij} is the risk attributes of compound hazards;

Coefficients of diffusion function $\Sigma = (\sigma_{x_1}, \sigma_{x_2}, \sigma_{x_3})$.

Output:

Dynamic compound hazards risk.

Compound hazards level evaluation by Algorithm 1;

Monitor space of different attributes $S^K = \{(s_j^{k_j}), 1 < k_j < K_j | j = 1, 2, 3\}$;

for sample index $i = 1$ to N, each x_i **do**

  Construct normal information diffusion functions based on the universes of monitor space and Eq. 9;

end for

Estimate the joint and conditional probability distribution based on Eq. 11 and Eq. 12;

Determine the fuzzy cause relationship based on Eq.13, and estimate the vulnerability curve by Eq.14;

Derive the dynamic risk (direct economic loss) of compound hazards by Eq. 15.

4 Case Study

In this section, we evaluate VFS-IEM-IDM with a case study of typhoon-rainstorm compound hazards that occurred in Shenzen, China. Shenzhen is located in the east bank of the Zhujiang River and is surrounded by Daya Bay and Dapeng Bay, where the climate is subtropical and maritime. Typhoon-rainstorms are the most frequently occurred hazards in Shenzhen. According to the collected data as shown in Table A1, from 1980 to 2016 the direct economic losses of the Typhoon and Rainstorm hazards in Shenzhen on average exceeded 360 million RMB per year. Also, Zhou has investigated the number of death caused by Typhoon and Rainstorm hazards was 3.4 annually and about $149,000$ people were affected (Zhou et al. (2017)). Accurate assessments of the typhoon-rainstorm risk are crucial to determine whether or not the early warning systems are working and implemented effectively.

4.1 Classifications of individual hazard level

The typhoon-rainstorm compound hazards are usually characterized by three drivers, i.e., Maximum Daily Precipitation (MDP), Extreme Wind Intensity (EWI), and landing location. To better measure the impact of typhoon landing on the typhoon-rainstorm compound hazards level, the landing location is converted into Transformed Location Number (TLN) via circle distance calculation where the big value represents the typhoon landed to Shenzhen is more closer. Based on the data provided by the Meteorological Bureau of Shenzhen Municipality (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/), the values of the three drivers are segmented into four intervals in terms of four individual hazard levels, i.e., I, II, III and IV. A higher hazard level indicates a more severe consequence.

Table 1. Classification standards of individual hazard level.

Drivers	Classifications of Individual Hazards Level			
	I	II	III	IV
Maximum Daily Precipitation (MDP)	(0,50)	(50,100)	(100,150)	(150,250)
Extreme Wind Intensity (EWI)	(8,10.8)	(10.8,17.2)	(17.2,23.6)	(23.6,30)
Transformed Location Number (TLN)	(0,2)	(2,5)	(5,8)	(8,10)

Based on the segmentation of the four individual hazard levels, we also classify the typhoon-rainstorm compound hazards into four levels, i.e., I, II, III, IV, where a higher compound hazard level indicates that the corresponding compound hazard is of greater severity. As described in Section 3.1, the VFS-IEM compound hazards level evaluation model (Algorithm 1) can be applied to obtain the comprehensive value H which is then used to derive the compound hazards level based on the classification criteria of the typhoon-rainstorm compound hazards.

4.2 Calculation of relative membership degree

The relative membership degree is determined by the individual hazard level classifications. According to the value segmentation shown in Table 1, we have the different fuzzy intervals for four different hazard levels. Then, for three hazard drivers, the interval criterion matrix I_{ab} can be expressed as

$$I_{ab} = \begin{bmatrix} (0,50) & (50,100) & (100,150) & (150,250) \\ (8,10.8) & (10.8,17.2) & (17.2,23.6) & (23.6,30) \\ (0,2) & (2,5) & (5,8) & (8,10) \end{bmatrix} = [(a,b)_{rl}],$$

Further, the corresponding interval I_{cd} for different hazard level is defined as

$$I_{cd} = \begin{bmatrix} (0,100) & (0,150) & (50,250) & (100,250) \\ (8,17.2) & (8,23.6) & (10.8,30) & (17.2,30) \\ (0,5) & (0,8) & (2,10) & (5,10) \end{bmatrix} = [(c,d)_{rl}],$$

and we define the balance boundaries matrix M:

$$M = \begin{bmatrix} 0 & 66.7 & 133.3 & 250 \\ 8 & 12.9 & 21.5 & 30 \\ 0 & 3 & 7 & 10 \end{bmatrix} = [m_{rl}].$$

In the end, the relative membership degree matrix can be calculated by Eqs. 4, 5 and 6 respectively. Taking sample point $\bar{u}$ = (MDP=33.4, EWI=18, TL=9) for example, we obtain the relative membership degree matrix $\mu(\bar{u})$ shown as below, in which the matrix element represents the probability of each drivers belonging to the four individual hazards level.

$$\mu(\bar{u}) = \begin{bmatrix} 0.666 & 0.334 & 0.000 & 0.000 \\ 0.000 & 0.438 & 0.593 & 0.063 \\ 0.000 & 0.000 & 0.333 & 0.667 \end{bmatrix}.$$

4.3 Typhoon-rainstorm hazards level

To derive the compound hazards level, the information entropy method is used to obtain the weight of each hazard driver. We have the weight Ω shown as follows where the element in Ω implies that the Maximum Daily Precipitation and Location play the main role in determining the typhoon-rainstorm hazards level.

$$\Omega = \begin{bmatrix} 0.43 & 0.19 & 0.39 \end{bmatrix}.$$

Based on the VFS-IEM compound hazards level evaluation model (Algorithm 1), we obtain the comprehensive value H of typhoon-rainstorm hazards. Then, guided by the domain experts, we have the classification criteria of the typhoon-rainstorm compound hazard level in Shenzhen: $H \in [1,2)$ for level I, $H \in [2,2.7)$ for level II, $H \in [2.7,3.5)$ for level III, and $H \in [3.5,4]$ for level IV. For the case (MDP=33.4, EWI=18, TL=9), the value of the typhoon-rainstorm hazards level H is obtained. When the hyper-parameters $\alpha = \beta = 1$, $H = 2.75$. When $\alpha = \beta = 2$, $H = 2.18$. Furthermore, we take the average of $H = 2.75$ and $H = 2.18$ to obtain the final compound hazards level value, i.e., $H = 2.47$ which corresponds to the compound hazard level II. The results of other typhoon-rainstorm cases can be found in Appendix Table B1.

Table 2: Transformed typhoon-rainstorm hazard data sets in Shenzhen.

Year	Date	Transformed Time (day)	Compound Hazards Level (H)	Direct Economic Loss (L)
2009	0627	176	2.72	0.3819
	0719	198	3	1.352
	0915	254	3.74	1.3750
2010	0724	203	2.32	0.2571
	0912	251	2.49	0.4450
	0922	261	2.74	0.9831
2011	0624	173	1.93	0.0765
	0930	269	2.72	0.4013
2012	0630	179	2.31	0.2895
	0724	203	3.95	2.48
	0817	226	2.56	0.7648
2013	0615	164	1.94	0.1527
	0702	181	1.99	0.1894
	0802	211	1.53	0.0452
	0814	223	2.13	0.1423
	0922	261	3.06	1.2351
2014	0718	197	1.83	0.0841
	0916	255	2.48	0.7682
	0823	232	2.92	0.7410
	1004	273	2.96	0.8352
2016	0802	211	3.68	2.1521
	0818	227	1.88	0.0251
	1018	287	2.28	0.2362
	1021	290	3.11	0.9341
2017	0612	161	3.67	2.058
	0723	202	2.11	0.2461
	0823	232	2.46	1.31
	0827	236	3.2	1.613

	0903	242	3.03	1.8872
	1016	285	2.48	0.5902
2018	0606	155	2.47	0.6952
	0718	197	1.58	0.0267
	0811	220	2.45	0.5241
	0916	255	3.93	2.226
2019	0703	182	1.49	0.0528
	0811	210	3.02	0.8182
	0824	233	2.9	0.8391
	0902	241	1.8	0.0725

4.4 Dynamic typhoon-rainstorm hazards risk

Based on the data in Table 2, we obtain three typhoon-rainstorm hazard attributes including direct economic loss L (Billions), hazards level H and hazards occurrence day. Then the dynamic risk of compound hazards can be calculated by 38 data points:

$$x_{ij} = \{(x_{i1}, x_{i2}, x_{i3}), i = 1, 2, \cdots, 38\} = \{(172, 2.72, 0.3819), \ldots, (241, 1.8, 0.0725)\}.$$

where x_{i1}, x_{i2} represents the time attribute of the typhoon-rainstorm hazards and the compound hazard level respectively, and x_{i3} is the direct economic losses caused by the typhoon-rainstorm hazards. Then the diffusion coefficients can be calculated by Eq. 10, shown as follows.

$$\begin{cases} \sigma_{x_1} = 2.6581 \cdot (290 - 155)/(38 - 1) = 10 \\ \sigma_{x_2} = 2.6581 \cdot (3.95 - 1.37)/(38 - 1) = 0.19 \\ \sigma_{x_3} = 2.6581 \cdot (2.48 - 0.0251)/(38 - 1) = 0.1764 \end{cases}.$$

Following Algorithm. 2, we use the information diffusion method to estimate the conditional probability and vulnerability distribution of the typhoon-rainstorm hazards. In this case study, we define the following monitor space: $T = (164, 194, 224, 254, 284)$ corresponds to months $(June, July, August, September, October)$, $H = (1.8, 2.4, 3.0, 3.6)$ corresponds to the compound hazards levels (I, II, III, IV), and $L = (0.1, 0.4, 0.7, 1.0, 1.3, 1.6, 1.9, 2.2)$ corresponds to the direct economic losses. Then we can calculate the joint probability density function P and the conditional probability function $\hat{P}$ as follows:

$$P = \begin{array}{l} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{array}{c} \begin{array}{cccc} \;\;I\;\; & \;\;II\;\; & \;\;III\;\; & \;\;IV\;\; \end{array} \\ \begin{pmatrix} 0.059 & 0.046 & 0.007 & 0.036 \\ 0.076 & 0.052 & 0.051 & 0.014 \\ 0.063 & \mathbf{0.116} & \mathbf{0.090} & 0.019 \\ 0.019 & \mathbf{0.086} & \mathbf{0.087} & 0.041 \\ 0.002 & 0.073 & 0.060 & 0.002 \end{pmatrix} \end{array}$$

$$\hat{P} = \begin{array}{l} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{array}{c} \begin{array}{cccc} \;\;I\;\; & \;\;II\;\; & \;\;III\;\; & \;\;IV\;\; \end{array} \\ \begin{pmatrix} \mathbf{0.398} & 0.311 & 0.049 & 0.243 \\ \mathbf{0.393} & 0.268 & 0.266 & 0.073 \\ 0.218 & \mathbf{0.402} & \mathbf{0.312} & 0.0689 \\ 0.080 & 0.370 & \mathbf{0.373} & 0.177 \\ 0.012 & \mathbf{0.539} & 0.437 & 0.012 \end{pmatrix} \end{array}$$

From the results above, it can be seen that the typhoon-rainstorm with hazard level III occur more frequently and they are most likely to occur in August and September. Furthermore, the vulnerability distribution f between the hazard level H and the direct economic losses L over the time attribute T can be calculated by the 3-dimension diffusion estimator (shown in Eq. 13). The fuzzy causal relationship which takes the time attribute T, hazards level H as the inputs and the loss L as the output is denoted as matrix R. Then the discrete vulnerability curve f in terms of the direct economic loss is evaluated by Eq. 14.

$$f = \begin{array}{l} \\ June \\ July \\ \textbf{August} \\ \textbf{September} \\ October \end{array} \begin{array}{c} \begin{array}{cccc} \;I\; & \;II\; & \;III\; & \;IV\; \end{array} \\ \begin{pmatrix} 0.20 & 0.02 & 0.00 & 0.00 \\ 0.24 & 0.04 & 0.00 & 0.00 \\ 0.15 & 1.13 & \mathbf{1.67} & \mathbf{1.90} \\ 0.05 & 0.55 & \mathbf{2.67} & \mathbf{2.62} \\ 0.01 & 0.02 & 0.00 & 0.00 \end{pmatrix} \end{array}$$

It can be seen that most of the economic losses caused by the typhoon-rainstorm hazards are concentrated in August and September. Dynamic compound hazards risks can be quantified as the expected value of the damages caused by the compound hazards and the result is:

$$Risk = (0.08582, 0.10504, 1.1372, 1.66715, 0.0109) \tag{16}$$

where the elements of the vector denote the estimated economic losses caused by the typhoon-rainstorm hazards from June to October.

5 Discussion

5.1 Compound hazards level prediction

The proposed VFS-IEM-IDM model provides a comprehensive evaluation of the compound hazards level, but the relationship between the hazards level and the hazard drivers is unclear. To find more information from the results of the compound hazards level evaluation model, we build a predictive model (shown in Eq. 17) to shed light on the relationship between the compound hazards levels.

Since the compound hazards level $H \in (I, II, III, IV)$ is ordinal data (monotone trend and proportional odds), the cumulative logistic model (shown in Eq. 18) can be used to predict the compound hazards level. Let the response be the compound hazards level $H = I, II, III, IV$ with probability $\pi_h(U), h = 1, \cdots, 4$ under the covariate compound hazard drivers U. So the cumulative probability of H is less than or equal to level h, i.e., the probabilities of compound hazards belonging to different level categories, is given by

$$P(H \leq h \mid U) = \pi_1(U) + \cdots + \pi_4(U), \quad h = 1, \ldots, 4.$$

According to the research by Alan (1980), the cumulative logistic model can be replaced by

$$\operatorname{logit}(P(H \leq h \mid U)) = \log \frac{P(H \leq h \mid U)}{1 - P(H \leq h \mid U)} = \alpha_h + \boldsymbol{\beta}^T \boldsymbol{U}, \quad h = 1, \ldots, 3. \tag{17}$$

where the log-odds measures how likely the response H is to be in category h or below versus in a category higher than h. In this paper, the typhoon-rainstorm hazards level prediction problem can be solved by using the VAGM package (Thomas (2010)) and the result is given by

$$\operatorname{logit}(P(H \mid (MDP, EWI, TLN))) = 5.07(7.32, 11.15) - 0.12MDP - 0.66EWI - 0.91TLN \tag{18}$$

where the different intercept coefficients denote the main effects of different hazard drivers compared to the reference compound hazard level IV. The rationality of this model is judged by LR-test (p-value<0.001) and the predictive performance $R^2 = 0.898$ which shows that the model is well fitted and can be used to predict the compound hazards level.

5.2 The superiority of the normal diffusion estimator

One advantage of using the information diffusion method to assess the risk of compound hazards is that it does not need to know the type of distribution from which the given samples are drawn and the function form of the causal relationship which are constructed by the joint probability distribution and the vulnerability distribution. More importantly, it can provide a more accurate evaluation when the compound hazards data set is sparse. The performance of the IDM estimation procedure has been well studied in the literature. For example, Huang (2000) shows the efficiency of IDM is about 35% higher than the histogram estimator, and the estimation error is reduced by 23.2% when the data sets are small. Therefore, the assessed compound hazards risk is more reliable and accurate using a normal diffusion estimator. However, if the size of the data samples is large it is unnecessary to replace the statistics with the information diffusion method (Li et al. (2012)).

5.3 Results

For the dynamic risk assessment of typhoon-rainstorm hazards, this paper proposes a hybrid model VFS-IEM-IDM and provides extensive assessment results based on a case study. From the compound hazards level evaluation model VFS-IEM, we show that the probability of the occurrences of type II and III hazard levels is the highest in Shenzhen, which shows that the emergency management department should prepare more effective emergency plans in advance to reduce the occurrences of the secondary hazards. From the dynamic risk assessment model IDM, it can be found that the hazards occurrence probability of different hazard levels is different and the hazards with levels II and III are most likely to occur in August and September. Furthermore, considering the occurrence of the hazards with different hazard levels for each month, the probability of hazards with level I occurring in June and July is the highest, and the hazards with level II mostly occur in August and October, and the hazards with level III is most likely to occur in September. From the perspective of hazard losses, the difference between the direct economic losses caused by the typhoon-rainstorms of the same hazard level each month indicates that the impacts of the typhoon-rainstorm hazards on the economy are not the same. Besides, for the same month, the influence of economic losses decreases gradually when the compound hazards level rises. This indicates that the capacity of typhoon-rainstorm hazard resistance in Shenzhen is reliable, and the ability to cope with sudden compound hazards is relatively strong under the existing emergence management system. The dynamic compound hazards risk of the typhoon-rainstorm hazards in Shenzhen shows that the risk value of these compound hazards in each month is different and the highest risk value appears in August and September. On average, the occurrence of the typhoon-rainstorm hazards brought Shenzhen 114 million RMB and 167 million RMB losses in these two months respectively, which is in line with the actual situation.

6 Conclusions

Compound hazards risk assessment is a complex multi-criteria problem and is crucial to the success of strategic decision-making in emergency management. Traditional statistical methods are often inaccurate when only a small set of data samples is available, and little research discusses the correlations of compound hazards drivers and considers the dynamics of the occurrences of the compound hazards. In this paper, we first present the definition of dynamic compound hazards risk and then propose the Variable Fuzzy Set (VFS) and Information Entropy Method (IEM) model to evaluate the compound hazards level by considering the correlations of different hazards drivers. Based on the results obtained by VFS-IEM, we apply the information diffusion method (IDM) to estimate the compound hazards probability and vulnerability distribution with the hazard occurrence time and the corresponding losses, and then the dynamic risk is calculated by the probabilistic model.

There are mainly three aspects of innovations in this paper. Firstly, based on the definition of compound hazards risk, we consider the temporal dynamics and introduce the concept of dynamic compound hazards risk. Secondly, considering that compound hazards have many drivers for the hazard level evaluation, a hybrid model of Variable Fuzzy Sets and the Information Entropy Method has been proposed to improve the accuracy of compound hazards level evaluation. Thirdly, according to the concept of dynamic compound hazards risk, we apply the Information Diffusion Method to estimate the hazards probability and the vulnerability distribution. The proposed model VFS-IEM-IDM can be used to deal with the problem of data sparsity in dynamic compound hazard risk assessment. By evaluating the expected value of the conditional probability distribution and the vulnerability distribution, we quantify the typhoon-rainstorm dynamic risk. Furthermore, VFS-IEM-IDM can be extended to other compound hazards that occur in urban cities such as flooding. As a case study, we show that the occurrences of the typhoon-rainstorm hazards bring Shenzhen 114 million RMB and 167 million RMB losses in August and September, respectively.

Dynamic risk assessment is a relatively new topic and there are many issues that need further improvement. In this paper, the weights of different types of hazard drivers is subjective, and the results of the vulnerability curve have not considered the development of the affected areas. There are also some subjective issues regarding the processing of the data sets. In future work, we will explore techniques to deal with these two issues and further improve the assessment accuracy.

Code and data availability. The data and code used in the study are available at https://github.com/GongWenwuu/VFS-IEM-IDM.git.

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Author contributions. Wenwu Gong and Lili Yang conceived the research framework and developed the methodology. Wenwu Gong was responsible for the code compilation, data analysis, and graphic visualization. Wenwu Gong and Jie Jiang had done the first draft writing. Lili Yang managed the implementation of the research activities and revised the manuscript. All the authors discussed the results and contributed to the final version of the paper.

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Appendix A: Data Source

For the typhoon-rainstorm dynamic compound hazards risk assessment, the useful data sets were collected from the Meteorological Bureau of Shenzhen Municipality (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/nianduqihougongbao/) and TYPHOON ONLINE (http://typhoon.nmc.cn/web.html), have been sorted out in Table A1. In this table, MDP denotes Maximum Daily Precipitation, EWI denotes Extreme Wind Intensity, DEL denotes Direct Economic Loss, and the Transformed Location Number (TLN) denotes the Typhoon Landing Location which is determined by radio distance transform using expert knowledge.

Table A1: Data sets of typhoon-rainstorm hazards in Shenzhen.

Hazards ID	Impact Date	MDP (mm)	EWI (m/s)	Landing Location	TLN	DEL (Billion)
0904	0627	67.3	16.8	Huizhou	8.5	0.3819
0906	0719	80	27.3	Shenzhen	10	1.152
0915	0912	127.9	28	Taibei	6	1.075
1003	0724	31.3	16.2	Zhanjiang	6.5	0.2571
1010	0912	62.4	13.7	Quanzhou	3	0.345
1011	0922	51.9	15.8	Heyuan	7	0.2983
1105	0624	41.7	14	Yangjiang	4.5	0.0765
1006	0930	53.0	15.2	Wenchang	2.5	0.8243
1206	0630	33.6	16.8	Zhuhai	6.5	0.6873
1208	0724	152.3	23.9	Taishan	7	2.241
1213	0817	46.1	13.5	Zhanjiang	3	0.9153
	0615	36.5	8.4	Wenchang	4	0.3621
1306	0702	38.6	10.9	Zhanjiang	3	0.2561
1309	0802	40.7	10.7	Wenchang	3	0.0851
1311	0814	47.8	14.2	Yangxi	3	0.6413
1319	0922	72.4	21.6	Shanwei	8.5	1.152
201409	0718	31.6	14.7	Wenchang	2.5	0.0841
201415	0916	73.5	18.9	Xuwen	2.5	0.9641
201517	0823	69.4	13.6	Shanwei	10	1.041
201522	1004	108.5	13.5	Zhanjiang	5.5	0.9631
201604	0802	166	19.2	Shenzhen	10	2.31
201608	0818	45.5	9.1	Zhanjiang	5.5	0.0314
201621	1018	117.6	12.3	Wanning	1.5	0.421

201622	1021	83.7	18.8	Shanwei	7.5	0.8721
1702	0612	161.8	16.9	Shenzhen	10	2.109
1707	0723	33.4	10.6	Xianggang	9	0.5315
1713	0823	56.3	23.4	Zhuhai	8.5	1.328
1714	0827	114.5	17.5	Jiangmen	8.5	1.741
1716	0903	82.4	14.4	Shanwei	7.5	0.9631
1720	1016	40	20.3	Zhanjiang	7.5	0.7341
1804	0606	97.2	8.8	Xuwen	8.5	0.9267
1809	0718	50.7	11.1	Wanning	1.5	0.0267
1816	0811	45.3	10.8	Yangjiang	7	0.5241
1822	0916	173.5	30	Taishan	7.5	2.361
1904	0703	48.8	11	Wanning	1.5	0.0672
1907	0811	99.1	14.1	Wenchang	5.5	0.9561
1911	0824	49.4	12.7	Zhangzhou	6	0.5931
1914	0902	52.2	11.3	Wanning	1	0.0751

Appendix B: Comprehensive Compound Hazards Level

Based on the VFS-IEM model, this paper takes the average of $\alpha=\beta=1$ and $\alpha=\beta=2$ to denote the final typhoon-rainstorm hazards level. The following Table B1 has shown that the whole results of compound hazards degree value.

Table B1: Comprehensive compound hazards level in ShenZhen

Time	$\alpha=\beta=1$	$\alpha=\beta=2$	Average Level (H)	typhoon-rainstorm hazards Level (H)
20090627	3.07	2.36	2.72	III
0719	3.34	2.65	3.00	III
0915	3.93	3.55	3.74	IV
20100724	2.67	1.96	2.32	II
0912	2.68	2.29	2.49	III
0922	3.02	2.45	2.74	III
20110624	2.12	1.73	1.93	I
0930	2.87	2.57	2.72	III
20120630	2.66	1.95	2.31	II
0724	3.97	3.93	3.95	IV
0817	2.8	2.32	2.56	II
20130615	2.08	1.79	1.94	I
0702	2.28	1.7	1.99	I
0802	1.65	1.4	1.53	I
0814	2.22	2.03	2.13	II
0922	3.44	2.67	3.06	III
20140718	1.93	1.73	1.83	I
0916	2.65	2.3	2.48	II
0823	3.19	2.64	2.92	III
1004	3	2.91	2.96	III
20160802	3.66	3.69	3.68	IV
0818	1.96	1.8	1.88	I
1018	2.52	2.03	2.28	II
1021	33.1	2.91	3.11	III
20170612	3.69	3.83	3.76	IV
0723	2.52	1.7	2.11	II
0823	2.89	2.03	2.46	II

0827	3.35	3.04	3.2	III
0903	3.22	2.83	3.03	III
1016	2.95	2	2.48	II
20180606	2.75	2.18	2.17	II
0718	1.57	1.45	1.51	I
0811	2.72	2.17	2.45	II
0916	3.87	3.98	3.93	IV
20190703	1.52	1.48	1.5	I
0811	3.25	2.79	3.02	III
0824	2.96	2.83	2.9	III
0902	1.93	1.67	1.8	I