Dynamic Risk Assessment of Compound Hazards Based on VFS-IEM-IDM: A Case Study of ~~Typhoon-Rainstorm~~ Typhoon-rainstorm Hazards in Shenzhen, China

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Abstract. ~~Typhoons and rainstorms are types of natural hazards that can cause significant impacts. These individual hazards may also occur simultaneously to produce compound hazards, leading to increased losses. The accurate~~ Global warming has led to an increasing occurrence of compound hazards and an accurate risk assessment of such hazards is of great importance to urban emergency management. Due to the interrelations between multiple hazards, the risk assessment of ~~such compound hazards faces several challengesdue to the uncertainties in multiple hazards level evaluation, and the incomplete information in historical data sets. In this paper, to~~ a compound hazard is facing several challenges: (1) the evaluation of hazard level needs to take into account the correlations between compound hazards drivers, (2) usually only a small number of data samples are available for estimating the joint probability distribution of the compound hazard drivers and the loss caused by the hazards, (3) the temporal dynamics of the occurrences of compound hazards needs to be considered in the process of the risk assessment. To deal with these challenges, ~~we propose a~~ this paper proposes an integrated risk assessment model ~~called~~ VFS-IEM-IDM ~~based on the~~ to quantify the dynamic risk of compound hazards based on Variable Fuzzy Set (VFS), Information Entropy Method (IEM), and Information Diffusion Method ~~. In particular~~(IDM). For the first challenge, VFS-IEM-IDM ~~provides a comprehensive evaluation~~ measures the effect of the compound ~~hazards level, and a predictive cumulative logistic model is used to verify the results. Furthermore~~hazard drivers via the use of relative membership degree and analyses the correlation between drivers with the entropy weight method, which are combined to evaluate compound hazards level. To address the second challenge, VFS-IEM-IDM applies ~~a normal information diffusion estimator~~ the normal diffusion function to estimate the ~~conditional probability distribution and the vulnerability distribution of the compound hazards~~ probability distribution of the compound hazard and the corresponding loss vulnerability curve. To deal with the third challenge, VFS-IEM-IDM assesses the risk of a compound hazard in different months based on the ~~hazards level, the hazards occurrence time, and the corresponding losses. To examine the efficacy of~~ definition of probabilistic risk. To evaluate the proposed risk assessment model VFS-IEM-IDM, ~~a case study of the Typhoon-Rainstorm hazards that~~ we use the typhoon-rainstorm hazards occurred in Shenzhen, China~~is presented. The risk assessment results indicate that hazards of level mostly occur in August and October, while hazards of level often occur in September. The~~ , as a case study and show that VFS-IEM-IDM can effectively estimate the typhoon-rainstorm compound hazard level and assess the dynamic risk of the ~~Typhoon-Rainstorm hazardsdiffers in each month and in August and September the risk gets the highest value, and the estimated economic losses are around 114 million RMB and 167 million RMB respectively~~compound hazards.

~~Key words~~Keywords: Compound hazards~~risk~~; Fuzzy dynamic risk; Variable fuzzy set; Information diffusion; ~~Typhoon-Rainstorm~~ typhoon-rainstorm

1 Introduction

~~Assessing risk is an effective way to reduce the negative impacts on natural hazards and plays an increasingly important role in helping the decision maker in emergency management. With the~~ With global climate change, many cities have suffered extreme natural hazards more frequently ~~and many people's lives are under threat. Located in the southern part of China, Shenzhen is a coastal city with a low latitude, where Typhoon and Rainstorm hazards have severely restricted the sustainable development of the local economy and society. Furthermore, the development of the Guangdong-HongKong-Macao Greater Bay Area highly relies on timely and effective emergency plans which are often determined by the efficiency of the risk assessment.~~ (Ming et al. (2022)). People and their properties have been exposed to various hazards simultaneously or successively worldwide. In the literature, there has been an increasing interest in the research of assessing multi-hazard risks Choi et al. (2021). A compound hazard is a typical multi-hazard problem that involves the concurrence of multiple hazard drivers, such as heavy rainfall, extreme wind intensity, and storm surge. For example, typhoons and rainstorms are two different types of natural hazards that can cause significant damages. When these two types of hazards simultaneously occur, compound hazards are produced, leading to more severe catastrophes than the individual hazards. Therefore, the risk assessment of such compound hazards needs to take into account the interrelations between the individual hazards.

The risk of a hazard is defined as the potential consequences brought by the disasters and can be quantified by the probability of losses (He et al. (2020)). Risk assessment is a technique that uses ~~hazards~~ the relevant hazard data to estimate the ~~probability~~ likelihood that natural hazards ~~occur and~~ may occur and further assess their economic losses (Huang et al. (2018)). Traditional methods of risk assessment mainly utilize ~~Geographic Information System (GIS)~~ geographic information systems to get risk maps (Gigovic et al. (2017)) ~~,~~ or rely on information diffusion ~~method (IDM)~~ methods to deal with ~~incomplete data sets~~ the problem of data sparsity (Gong et al. (2020)). These ~~relevant~~ risk assessment methods (Julia et al. (2021); Zhou et al. (2020)) ~~have became more comprehensive and mature in single hazardsevaluation. However,~~ are mostly applied to individual hazards, while the risk assessment of compound hazards is not simply the ~~multi-hazard risk assessment is not the~~ aggregation of ~~their individual assessment results but considers the connections among different hazards~~ the assessment results of the individual hazards but needs to consider the interrelations between them (Kappes et al. (2012))~~, so the assessment results for multiple hazards are often inaccurate and insufficient. Furthermore, there is little research focusing on Typhoon-induced risk assessment in the literature and many aspects such as dynamic risk assessment are not considered.~~ .

There are many research works discussing the ~~multi-hazard risk assessment which have been reviewed the relevant literature Choi et al. (2021). Furthermore, Wang et al. (2020) clarified~~ risk assessment of multi-hazards. They classify the relationship between ~~hazards in multi-hazard scenarios by dividing them~~ the individual hazards in the scenarios of multi-hazards into three categories: mutually amplified hazards, mutually exclusive hazards, and non-influential ~~hazards. Khan et al. (2020) presented an analysis of the~~ stakes (Wang et al. (2020)). The existing methods and technologies ~~that are relevant to multi-hazard scenarios. Huang et al. (2018) developed information diffusion technique to construct a joint probability distribution and a vulnerability distribution for assessing the flood and earthquake risks. Xu et al. (2016) also used the~~ relevant to the risk assessment of multi-hazards have been reviewed in (Khan et al. (2020)). For example, a Cloquet integral multiple linear regression model has been proposed to overcome the problem of nonlinear additivity of mutually amplified hazards for hazard level evaluation (He et al. (2020)). An information diffusion method has been used to assess the risk of multiple hazards quantitatively and ~~evaluated~~ evaluate the risk of loss of human lives from meteorological hazards in China ~~. Ming et al. (2015) proposed a~~ (Xu et al. (2016)). A quantitative approach of multi-hazard risk assessment based on vulnerability distribution and joint return period of hazards is proposed to assess the risk of crop losses in the Yangtze River Delta region of China (Ming et al. (2015)). However, all of these works ~~focus on integrating the risks caused by single hazards and ignoring these truly correlations between hazards occurrence~~do not consider the correlations between the occurrences of the individual hazards, such as the co-appearance of ~~Typhoon-Rainstorm hazards. In this paper, we aim at the multi-hazards and investigate new methods for multiple hazards level evaluation and dynamic risk assessment of compound hazards~~typhoon-rainstorm hazards. Furthermore, there is little research focusing on typhoon-induced risk assessment in the literature, and temporal dynamics are rarely considered in risk assessments.

Compound hazards, a sub-group of ~~the term 'multi-hazard', can be~~ 'multi-hazards', are considered as the combination of multiple hazard drivers that contribute to societal ~~risk (Jennifer et al. (2021)), within which two associated hazards impacting the same time and place~~or environmental risks. The characteristics of compound hazards include: (1) two or more extreme events occurring simultaneously or successively, (2) combinations of extreme events with underlying conditions that amplify the impact, and (3) combinations of events that are not themselves extreme but lead to an extreme event or impact when combined (Jennifer et al. (2021)). Here, we explicitly consider compound hazards for the case when two or more individual extreme events occurring at the same place and at the same time, such as the extreme precipitation, winds, and ocean waves. In this paper, we define the ~~compound hazards risk~~ risk of a compound hazard as a scene in the future associated with some adverse ~~incident~~ incidents caused by cascading hazards systems, where there are strong connections ~~among different hazard indicators~~between different hazard drivers. Compared with the ~~multi-hazard risk assessment~~ risk assessment of multi-hazards in the literature (Xu et al. (2016); Huang et al. (2018)), ~~the risk assessment~~ assessing the risk of compound hazards ~~obtains the comprehensive hazards~~ needs to obtain an integrated hazard level without losing any correlated information ~~and often reflects the property of hazard-induced. Risk assessment of compound hazardshas been studied by He et al. (2020), who presented the Cloquet integral multiple linear regression model to overcome the problems of nonlinear additivity of couple hazards. But this method only provides the magnification coefficients to assess the risks of compound hazards and neglects the changing of time span. Here, there are some problems remaining to be solved. On the one hand, the collected data for assessing the compound hazards risk is often incomplete such that the results may not be reliable. On the other hand, the change of month in which the compound hazards occur also has impacts on the risk assessment and is often ignored. In this paper, we emphasize that risk assessment of compound hazards should deal with the uncertainties caused by multi-indicators, the unknown probability distributions, the incomplete information in historical data sets, and the dynamic property of hazards occurrence~~between the individual hazards.

~~Some research based on variable fuzzy sets~~ While there have been many attempts to assess the risk of multi-hazards, most of the existing methods have limitations in dealing with compound hazards (Ming et al. (2022); Huang et al. (2018)). Firstly, the correlation between the hazard drivers is often ignored. Considering that the disaster control engineering system is a synthesis of multi-dimensional factors, the potential inter-dependencies of the drivers will affect the joint probability and the economic losses of compound hazards. Secondly, the relationship between the hazards, i.e., vulnerability and exposure analysis, cannot be modeled effectively when the data is sparse. Thirdly, most of the existing risk analysis frameworks for compound hazards are based on either qualitative or semi-quantitative methods. Moreover, the temporal dynamics of the occurrences of compound hazards are often not considered.

To address the first limitation, researchers have applied variable fuzzy set (VFS) ~~theory, introduced by Chen (2006),~~ methods to deal with the multi-factor problem. Some researchers have shown that the relative membership function can be used to evaluate the ~~multi-indicators assessment problems. Li et al. (2012) proposed the fuzzy comprehensive assessment method~~ relations between multiple indicators in risk assessment (Chen (2006)). A fuzzy method (Li et al. (2012)) is proposed to solve the flood risk assessment problems with interval boundaries and this integrated model improves the reliability of single hazard risk assessment. ~~Beaula et al. (2013) used variable fuzzy sets~~ VFS has also been used to evaluate the synthetic hazards level of Nagapattinam district with the north-east monsoon rainfall~~''~~’s data sets ~~. Similarly, the variable fuzzy set theory can be used to obtain the comprehensive evaluation of compound hazards.~~ (Beaula et al. (2013)). In this paper, we propose to combine ~~the VFS with~~ VFS with the information entropy method (IEM) to assess ~~compound hazards indicators and obtain a comprehensive risk assessment~~the hazard level of compound hazards such that the correlations between the hazard drivers can be captured.

To deal with the second limitation, the information diffusion method (IDM) is commonly used to model the physical relationship between different attributes. In many cases, it is difficult to collect compound hazards data~~sets, such that the information carried by~~ , and the historical data is often ~~incomplete. Therefore, the traditional models often give an unreliable estimation result, and~~ sparse. To this end, many fuzzy probabilistic models have been proposed to enhance the accuracy of ~~risk assessment Mehran et al. (2017)~~the risk assessment results (Mehran et al. (2017)). Fuzzy probabilistic models are used to model uncertainties related to hazards and the randomness due to environmental, natural, or ~~time span changing~~period changes. The main feature of the fuzzy probabilistic ~~model is to change the traditional~~ models is to transform the raw data points into fuzzy ~~set for partly filling~~ sets to partly fill the gap caused by data ~~incompleteness~~ sparsity and improve the estimation accuracy between ~~inputs and outputs. The most powerful technique is the information diffusion method (IDM~~ the inputs and the outputs. One of the most powerful techniques is IDM (Huang (1997, 2002)), which helps extract useful underlying information from the ~~hazard~~ hazards data sets. ~~Research by Huang (1997, 2002) has given many results about IDM and there are many papers have shown the capability of information diffusion method to deal with incomplete data sets (Huang (2009); Li (2013); Huang et al. (2018)~~Researchers have done a simulation study on IDM and demonstrated the benefit of information distribution for probability estimation (Huang (2000)). The capability of IDM in dealing with the problem of data sparsity has been well studied in the literature (Li (2013)). In this paper, we ~~introduce the information diffusion method to deal with the incomplete data problem and combine the variable fuzzy sets theory to carry out dynamic risk assessments of compound hazards~~construct a normal information diffusion estimator (IDM) to analyze the probability function and vulnerability curve of compound hazards.

As for the third limitation, preliminary attempts have been made to develop quantitative multi-hazard risk assessment frameworks (Huang et al. (2018)). The probabilistic risk model combined with the concept of dynamic risk assessment has been proposed to estimate the flooding risk (Huang (2015)). In this paper, we present the definition of dynamic compound hazards risk and then propose a method to assess the compound hazards risk quantitatively which also takes into account of the temporal dynamics of the occurrences of the hazards.

The main contributions of this paper are summarized as ~~following two-folds.~~ follows.

1. ~~1) For technological innovation, we propose a hybrid~~ We propose a model, named ~~as~~ Variable Fuzzy Set and Information Diffusion ~~Method~~ (VFS-IEM-IDM), to assess ~~compound hazards risk dynamically. Furthermore, we simplify the calculation procedures of~~ the dynamic risk of compound hazards, which takes into account the interrelations between the hazard drivers, deals with the problem of data sparsity, and considers the temporal dynamics of the occurrences of the compound hazards.

2. We simplify the procedures of calculating relative membership degree to improve the efficiency ~~and accuracy~~ of compound hazards level evaluation, and we also use a predictive cumulative logistic model to verify the evaluation results.

3. ~~2)~~ To examine the efficacy of the proposed model VFS-IEM-IDM, a case study of the ~~Typhoon-Rainstorm hazards that~~ typhoon-rainstorm hazards occurred in Shenzhen, China is presented.

The rest of this paper is organized as follows. Section 2 introduces the basic concepts and ~~presents~~ definitions in this paper. In section 3, we present the dynamic compound hazards risk assessment model~~(~~, namely VFS-IEM-IDM~~). Section 3 illustrates how the proposed model can be used to assess the dynamic risk of Typhoon-Rainstorm hazards~~ . Section 4 provides an evaluation of the VFS-IEM-IDM with a case study of typhoon-rainstorm hazards occurred in Shenzhen, China. ~~Section 4 discusses the comprehensive evaluation of the compound hazards level, conditional probability distribution, vulnerability distribution and the dynamic expectation risk of the Typhoon-Rainstorm hazards to show the effectiveness~~ In section 5, we discuss the results of the case study obtained at different stages of VFS-IEM-IDM. Finally, ~~we conclude the paper in Section 5.~~ conclusions are drawn in Section 6.

2 ~~Dynamic Risk Assessment of Compound Hazards Based on VFS-IEM-IDM~~ Preliminaries

~~Risk assessment of compound hazards should consider the uncertainties caused by multi-indicator, incomplete information contained in historical data sets, and~~

2.1 Basic concepts

Variable fuzzy set is used to express the fuzzy effect of the ~~impact of internal attribute changes on the hazards. This section introduces VFS-IEM-IDM which combines the variable fuzzy sets theory with information diffusion method to assess the dynamic risk of~~ hazard drivers by relative membership degree (RMD) functions, and then the compound effects between different drivers can be modeled. This method provides an enhanced implementation of the compound hazards level evaluation process and can reflect the coupled characteristics of compound hazards~~when the given data sets are incomplete. The proposed~~

~~VFS-IEM-IDM model consists of VFS-IEW dimension reduction model to obtain the comprehensive evaluation of compound hazardslevel (Section 2.2), and VFS-IDM dynamic risk assessment model~~ . *Information entropy* is based on the entropy coefficient calculation process, which is used to measure the importance of the individual hazard drivers and determine the weight of different drivers. *Information diffusion* is a function learning method with high estimation accuracy from a small data set, which makes full use of the diffusion information given by the data samples to estimate the ~~expectation risk of direct economic losses (Section 2.3).~~

2.2 ~~Dynamic compound hazards risk~~

~~Risk is assumed to be the possible scene~~ probability density of the data samples or the relationship between the data samples without the knowledge of the distribution from which the data samples were drawn. This method is applied to estimate the probability distribution p (hazard potential) of the occurrence of ~~a harmful event.~~ hazards, and the causal relationship f (hazard vulnerability).

2.2 Dynamic compound hazards risk

From the previous studies, ~~the type of risk~~ risks could be classified into four categories: pseudo risk, probability risk, fuzzy risk, and uncertainty risk (Huang et al. (2018)). ~~In the case of that we can estimate~~ Existing hazard risk assessment models are often qualitative or semi-quantitative, which cannot estimate directly economic losses from the joint impact of several hazards. Probability risk is estimated by integrating the probability distribution ~~$p(x)$ (hazard potential)~~ p of the occurrence of ~~a hazard with respect to its magnitude x, and we can estimate the relationship $f(x)$ (hazard vulnerability) between the magnitude and hazard level, a~~ hazards, and the causal relationship f between the economic loss and the hazard attributes. As a result, the probability risk could be quantified as the expected value of economic losses, i.e.,

$$Risk\ =\ Hazard\,Potential \times\ Vulnerability.$$

the integration of hazard potential with hazard vulnerability.

Though these four types of risks have been investigated by many researchers, there ~~is little~~ are few research on dynamic compound hazards risk~~(Huang et al. (2018))~~. In this paper, ~~we give a definition of dynamic compound hazards risk and illustrate how to assess this kind of risks.~~

~~*The compound risk* is~~ *compound risk* is defined a scene in the future associated with some adverse incident caused by cascading hazards systems, where there are strong connections between different hazards and the hazard level is influenced by many ~~indicators~~drivers. Furthermore, ~~Huang (2015) mentioned that it could extent~~ as proposed by Huang (2015), the concept of compound risk could be extended to *dynamic compound risk* if the impact of occurrence time on risk assessment ~~has been~~ is taken into consideration. To ~~evaluate the compound hazards risk , the most important things are to estimate probability distribution $p(x)$~~ assess the risk of a compound hazard, the probability distribution p of the occurrences of ~~compound hazards by using~~ the compound hazard will be estimated with probability models, and the ~~input-output relationship $f(x)$~~ causal relationship f between the hazards ~~level and losses by using fuzzy models~~attributes and the losses is captured by a fuzzy model. The compound ~~risk, quantified as the economic losses~~ hazards risk is defined as follows:

$$Risk = p(\Phi; X) \cdot f(\Phi'; X), \tag{1}$$

where $X = \{x_{ij} | i = 1,2,\cdots,N; j = 1,2,\cdots,J\}$ represents the data samples with the sample size N and the number of compound hazards ~~, is given by Eq. 1.~~

$$\cancel{Risk = \int p(x) \cdot f(x)\, dx = \sum_{j=1}^{J} p(x; att_j) \cdot f(x; att_j),}$$

~~where vector att_j denotes the hazards indicator for different index j and reflects the internal attribute changes of compound hazards~~attributes J, Φ and Φ' denote a set of hazard attributes which reflects the characteristics of the compound hazard. For example, the ~~Typhoon-Rainstorm risk is influenced by different indicators att = (~~risk of the compound hazard typhoon-rainstorm can be assessed by 3 hazard attributes including hazard occurrence time ϕ_1, compound hazards level ~~, economic losses) and the~~ ϕ_2, and economic losses ϕ_3. The dynamic compound risk ~~can be assessed by integrating~~ is derived by integrating the conditional probability distribution ~~$p(x; att)$ with hazards vulnerability $f(x; att)$ of Typhoon-Rainstorm hazards~~ $p(X; \Phi)$ where $\Phi = (\phi_1, \phi_2)$ with the hazards vulnerability $f(X; \Phi')$ where $\Phi' = (\phi_1, \phi_2, \phi_3)$.

2.3 ~~VFS-IEW dimension reduction model~~

~~For the compound hazards risk assessment system, the randomness and fuzziness caused by multi-indicators evaluation should be dealt with properly. Variable fuzzy sets theory (VFST) , which deals with randomness and fuzziness, provide~~

3 Dynamic Risk Assessment of Compound Hazards: VFS-IEM-IDM

Risk assessment of compound hazards should consider the correlation between the compound hazard drivers, the problem of data sparsity, and the dynamic property of hazard occurrences. This section introduces VFS-IEM-IDM, a risk assessment model for compound hazards which combines the variable fuzzy sets theory with the information diffusion method to assess the dynamic risk of compound hazards when only a small set of data samples is available. Fig. 1 shows the workflow of VFS-IEM-IDM which mainly consists of two components. With individual hazard level and historical records of hazard drivers as inputs, the first component VFS-IEM combines variable fuzzy set methods with information entropy methods to provide a comprehensive evaluation of the compound hazards level (Section 3.1). Based on the compound hazards levels and historical records of risk assessment attributes, the second component VFS-IDM adapts normal information diffusion methods to quantify the dynamic risk of the compound hazards in terms of the direct economic losses (Section 3.2). The boxes colored blue represent the results obtained by the VFS-IEM-IDM model.

3.1 Compound hazards level evaluation: VFS-IEM

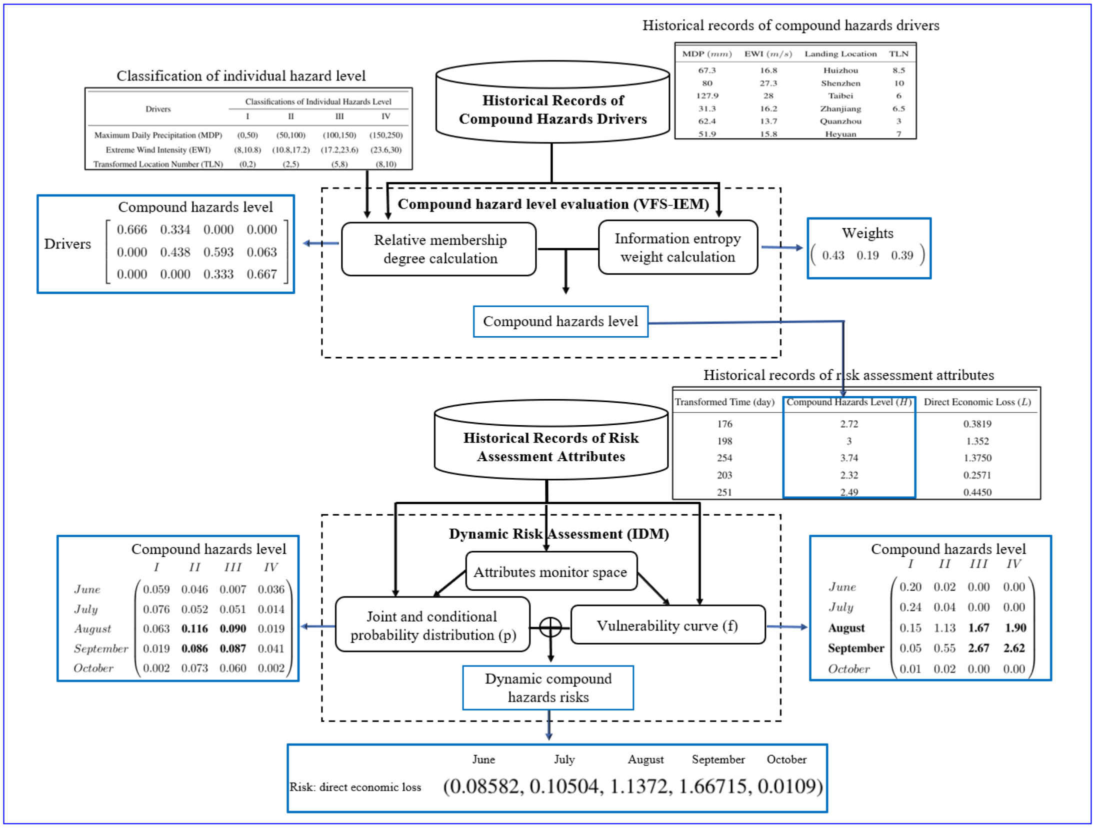

Figure 1. Workflow of the VFS-IEM-IDM dynamic compound hazards risk assessment model based on case study. Based on the historical records of compound hazards, our proposal provides an enhanced implementation of the compound hazards level evaluation and then estimates the probability distribution and the corresponding loss vulnerability curve of compound hazards attributes to calculate the dynamic compound hazards risk. The blue boxes represent the results obtained by the VFS-IEM-IDM model

For the compound hazards risk assessment, the correlation between the compound hazards drivers should be considered. Fortunately, the variable fuzzy set (VFS) theory which considers the contributions of multiple related drivers and decreases the fuzziness by using membership functions (Chen (2006)), provides an appropriate tool for ~~solving~~ evaluating the compound hazards level~~evaluation. The Variable Fuzzy Sets-Information Entropy Weight (VFS-IEW) dimension reduction model has been proposed in this section.~~.

~~In this paper, we define interval $I_0 = [a,b]$ as the attracting sets of variable fuzzy sets (VFS) U and extends I_0 to interval $I = [c,d]$ on the real axis. For $u \in U$, the elements in interval I_0 satisfy $\mu_A(u) > \mu_A^c(u)$. In VFST, $\mu_A(u)$ denotes~~ Based on VFS, the fuzzy set intervals given by the individual hazard level classification can be used to assess the compound hazards level. For example, suppose we have two fuzzy set intervals $I_{ab} = (a,b)$ and $I_{cd} = (c,d)$ where $a,b,c,d \in \mathcal{R}$, in which I_{cd} is an extended fuzzy set interval based on I_{ab}, as shown in Fig. 2. Specifically, the relative membership degree (RMD) ~~and the core idea is to determine the RMD of each sample point by transferring fuzzy set U into real value. Wang et al. (2014) has~~

~~defined the balance boundaries matrix and illustrates the~~ function $\mu(u)$, which determines the probability of a hazard driver u belonging to different hazard level intervals, is applied to evaluate the contributions of compound hazards related drivers. Since the calculation of RMD ~~as a~~ is complicated and time consuming~~problem. We apply~~, we use different locations of the balance boundaries matrix ~~$M=\{M_{rl}\}$ (shown in Eq. 2) to locate the eigenvalue x and defines the relative membership degree functions (~~M (Wang et al. (2014)) and the value of driver u to simplify the calculation process. Firstly, we use the interval (a_{rl}, b_{rl}) to define the balance boundaries matrix $M=[m_{rl}]$, which is shown in Eq. ~~3) to evaluate the comprehensive value of compound hazards level.~~ 2.

$$\cancel{M} m_{rl} = \frac{L-l}{L-1}a_{rl} + \frac{l-1}{L-1}b_{rl} \cancel{= (M_{rl})}, \tag{2}$$

where ~~r stands for the assessment indicator set,~~ $r=1,2,\ldots,R$ and R indicates the number of hazard drivers, ~~l denotes the comprehensive level,~~ $l=1,2,\ldots,L$ ~~. Compared with~~ and L denotes the number of compound hazards levels. Then, we compare the relative locations of ~~sample points and parameter M_{rl}, RMD calculation can be solved~~ u with m_{rl} in the interval (a_{rl}, b_{rl}) and (c_{rl}, d_{rl}). RMD can then be constructed by the ratio ~~$\frac{x-a}{M-a}$, i.e.~~ $\frac{u-a_{rl}}{m_{rl}-a_{rl}}$ as follows:

$$\begin{cases} \mu(u)_{rl} = 0.5\left(1+\left(\frac{u-a_{rl}}{m_{rl}-a_{rl}}\right)^{q}\right) & u \in (a_{rl}, m_{rl}) \\ \mu(u)_{rl} = 0.5\left(1-\left(\frac{u-a_{rl}}{c_{rl}-a_{rl}}\right)^{q}\right) & u \in (c_{rl}, a_{rl}) \end{cases}. \tag{3}$$

It can be seen that ~~the RMD is affected by~~ RMD is influenced by the hyper-parameter ~~p~~ q and the position between ~~sample point x with parameters a, b, c, d, and M~~the hazard driver value u and the level interval I_{ab}, I_{cd}, and the value of m_{rl}. In this paper, ~~the characteristics for different locations of x with respect to the class interval u have been used to classify RMD calculation: judge~~ guided by the procedure of calculating RMD in the literature (Fang et al. (2019)), we simplify the procedure of calculating relative membership degree to improve the efficiency of compound hazards level evaluation. Firstly, the intervals I_{ab}, I_{cd} of the individual hazard levels and the balance boundaries matrix M are obtained following the VFS theory (Chen (2006)). Secondly, we determine whether the location of ~~x~~ u is in the lowest, middle or highest grade of the ~~class interval u or not.~~ interval I_{ab}, as shown respectively in Fig.2~~Fig. 4 have shown three types of RMD calculation and the detailed induction can be referred by Fang et al. (2019).~~, Fig. 3, and Fig.4. Finally, according to the location of u, we use one of the three sets of formulas to calculate RMD, as shown in Eq. 4, 5, and Eq. 6 accordingly.

$$\begin{cases} \mu(u)_{r} = [\mu(u)_{r1} \quad \mu(u)_{r2} \quad 0 \quad \cdots \quad 0] \\ \mu(u)_{r1} + \mu(u)_{r2} = 1 \\ 0.5 \le \mu(u)_{r1} \le 1 \\ 0 \le \mu(u)_{r2} \le 0.5 \end{cases}. \tag{4}$$

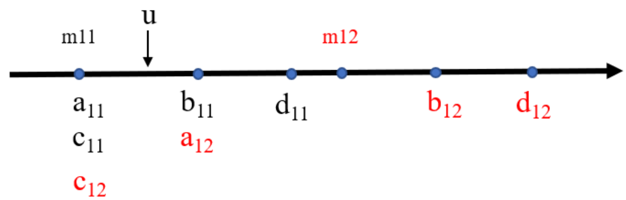

Figure 2. Lowest ~~case~~ case: the position between ~~the random point $u_1{}^t = x$~~ u with parameter ~~M_{11}~~ m_{11} and ~~zones $[a_{11}, b_{11}]$~~ fuzzy intervals (a_{11}, b_{11}), ~~$[c_{11}, d_{11}]$~~ (c_{11}, d_{11}). Symbols with different colors indicate different fuzzy intervals.

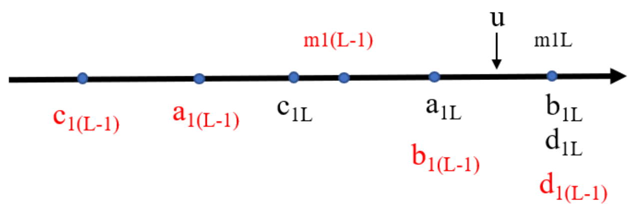

Figure 3. Highest case: the position between ~~the random point $u_1{}^t = x$~~ u with parameter ~~M_{1L}~~ m_{1L} and ~~zones $[a_{1L}, b_{1L}]$~~ fuzzy intervals (a_{1L}, b_{1L}), ~~$[c_{1L}, d_{1L}]$~~ (c_{1L}, d_{1L}) . Symbols with different colors indicate different fuzzy intervals.

$$\begin{cases} \mu(u)_r = [0 \quad \cdots \quad 0 \quad \mu(u)_{r(L-1)} \quad \mu(u)_{rL}] \\ \mu(u)_{r(L-1)} + \mu(u)_{rL} = 1 \\ 0.5 \leq \mu(u)_{rL} \leq 1 \\ 0 \leq \mu(u)_{r(L-1)} \leq 0.5 \end{cases} . \tag{5}$$

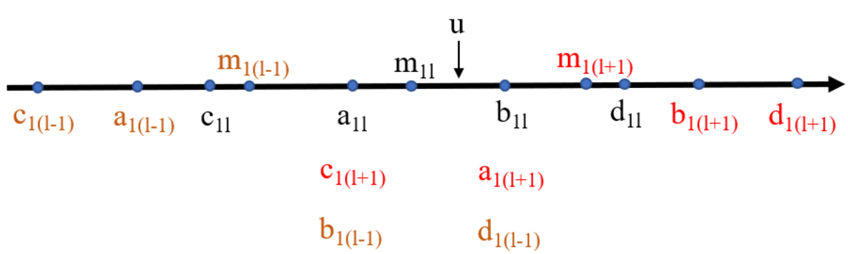

Figure 4. Middle case: the position between ~~the random point $u_1{}^t = x$~~ u with parameter ~~M_{1l}~~ m_{1l} and ~~zones $[a_{1l}, b_{1l}]$~~ fuzzy intervals (a_{1l}, b_{1l}), ~~$[c_{1l}, d_{1l}]$~~ (c_{1l}, d_{1l}). Symbols with different colors indicate different fuzzy intervals.

$$\begin{cases} \mu(u)_r = [0 \quad \cdots \quad 0 \quad \mu(u)_{r(l-1)} \quad \mu(u)_{rl} \quad \mu(u)_{r(l+1)} \quad 0 \quad \cdots \quad 0] \\ \mu(u)_{r(l-1)} + \mu(u)_{r(l+1)} = 0.5 \\ 0 \leq \mu(u)_{r(l-1)} \leq 0.5 \\ 0 \leq \mu(u)_{r(l+1)} \leq 0.5 \end{cases} . \tag{6}$$

Following the previous works by Kwakernaak (1978) and Chen (2006), ~~the proposed variable fuzzy set dimension reduction model~~ we use the variable fuzzy recognition model to obtain the comprehensive RMD of each driver. Then, the proposed compound hazards level evaluation model can be constructed by Eq. 7. ~~It indicates that the proposed model is affected by hyper-parameter α, β and the multi-indicators are transferred into a single degree value so as to obtain the comprehensive assessment results.~~

$$\begin{cases} \nu_A(u)_l = [1 + (\frac{\sum_{r=1}^{R}[\omega_r(1-\mu_A(u)_{rl})]^\alpha}{\sum_{r=1}^{R}[\omega_r \mu_A(u)_{rl}]^\alpha})^{\frac{\beta}{\alpha}}]^{-1} \\ \nu_A^o(u)_l = \frac{\nu_A(u)_l}{\sum_{l=1}^{L}\nu_A(u)_l} \\ H = (1 \quad 2 \ldots L) \cdot (\nu_A^o(u)_l)^T \end{cases} .$$

$$\begin{cases} \nu(u)_l = (1 + (\frac{\sum_{r=1}^{R}(\omega_r(1-\mu(u)_{rl}))^\alpha}{\sum_{r=1}^{R}(\omega_r \mu(u)_{rl})^\alpha})^{\frac{\beta}{\alpha}})^{-1} \\ \hat{\nu}(u)_l = \frac{\nu(u)_l}{\sum_{l=1}^{L}\nu(u)_l} \\ H = (1 \quad 2 \ldots L) \cdot \hat{\nu}(u) \end{cases} . \tag{7}$$

where ~~$\nu_A^o(u)$ is the normalized process of RMD~~ α and β are two hyper-parameters, w_r indicates the weight of each hazard driver, $\nu(u)_l$ is the weighted RMD of different hazard drivers and H is the ~~comprehensive value (a real value, can be transferred to hazards level). Further, the weight of indicators in this VFS-IEW model can be calculated by information entropy weight~~ compound hazards level. The weights of the individual hazard drivers w_r are obtained via the use of information entropy (Liu et al. (2010)) ~~(~~as shown in Eq. 8~~).~~ :

$$\begin{cases} \hat{g}_{rl} = v_{rl} / \sum_{l=1}^{L} v_{rl} \\ g_r = -1/ln(N) \cdot \sum_{l=1}^{L}(\hat{g}_{rl} ln \hat{g}_{rl}) \\ \omega_r = (1 - g_r)/(R - \sum_{r=1}^{R} g_r) \end{cases} . \tag{8}$$

~~We now present the main steps of VFS-IEW model and the corresponding algorithm (~~where v_{rl} is defined as the measured value from the lth level for the rth driver and N denotes the sample size. The detailed procedure of VFS-IEM is shown in Algorithm 1~~) as follows. **Step-1:** Initialize the variable fuzzy sets and the balance boundaries M_{rl}. **Step-2:** Repeat the relative membership degree calculation. **Step-3:** Calculate the information entropy weight ω_r. **Step-4:** Return the comprehensive degree value.~~

Algorithm 1 ~~VFS-IEW Dimension Reduction Model for~~ VFS-IEM Compound Hazards Level Evaluation

Input:

The ~~assessment object set $D = \{U_t = (u_r)^t, r = 1, 2, \cdots, R | t = 1, 2, \cdots, T\}$, where u_r is the eigenvalue~~compound hazards driver fuzzy set $U = \{u_{ir}, r = 1, 2, \cdots, 3 | i = 1, 2, \cdots, N\}$;

~~Assessment criteria matrix, $V = \{(v_{rl}), r = 1, 2, \cdots, R;\ l = 1, 2, \cdots, L\}$~~Individual hazard level assessment matrix, $V = [v_{rl}]$, $r = 1, 2, 3;\ l = 1, 2, 3, 4$.

Output:

Comprehensive value of compound hazards level.

Identification of ~~attracting sets $I_{ab} = ([a, b]_{rl})$~~ interval $I_{ab} = [(a, b)_{rl}]$ and the extended ~~intervals $I_{cd} = ([c, d]_{rl})$~~ interval $I_{cd} = [(c, d)_{rl}]$ based on assessment ~~criteria~~ matrix V;

Define the balance boundaries matrix ~~$M = \{(M_{rl}), r = 1, 2, \cdots, R;\ l = 1, 2, \cdots, L\}$~~ $M = [m_{rl}]$ by Eq. 2;

Calculate the ~~information entropy weight ω_r~~ weight of each driver $\Omega = [\omega_1, \omega_2, \omega_3]$ by Eq.8;

for $i = 1$ to N **do**

  for each ~~$u_r{}^t$~~ $u_{ir} \in$ ~~U_t~~U **do**

    if u_{ir} locates in the lowest grade of the interval I_{ab}, i.e., $a_{r1} < u_{ir} < b_{r1}$ **then**

      ~~RMD $\mu_A(u)_r{}^t$ has the expression given by~~ Calculate RMD $\mu(u)_r$ with Eq.4;

    else if u_{ir} locates in the highst grade, i.e., $a_{rL} < u_{ir} < b_{rL}$ **then**

      ~~The value of RMD $\mu_A(u)_r{}^t$ is given by~~ Calculate RMD $\mu(u)_r$ with Eq.5;

    else

      ~~RMD $\mu_A(u)_r{}^t$ have the expression given by~~ Calculate RMD $\mu(u)_r$ with Eq.6;

    end if

  end for

  The relative membership matrix of each sample ~~can be denoted as $\mu_A(u)^t = (\mu_A(u)_{rl}{}^t)$~~is denoted as $\mu(u) = [\mu(u)_1, \mu(u)_2, \mu(u)_3]$;

  Combine ~~$\mu_A(u)^t$ with weights ω_r~~ $\mu(u)$ with weights Ω and integrate the ranking level, ~~the comprehensive degree value~~ and calculate the comprehensive compound hazards level for each sample ~~is given by~~ with Eq.7.

end for

3.2 Dynamic risk assessment model: IDM

~~In order to~~ To assess the dynamic risk of compound hazards, especially when the ~~recorded~~ data sets are ~~incomplete,~~ sparse, the information diffusion method (IDM) which belongs to ~~fuzzy sets~~ the fuzzy theory can be used to extract useful underlying information from the limited data samples to estimate the ~~relationships behind the incomplete data~~probability function p and vulnerability curve f. According to ~~the research by Huang (1997),~~ Huang (1997), J-dimension normal information diffusion function ~~$\mu(X^t{}_i; S_o)$~~ $\Gamma(x_i; S^k)$ (shown in Eq. 9) is more powerful to improve the precision of ~~estimators. So~~the estimators. Therefore, this paper adapts the normal information diffusion estimator to approximate the dynamic compound hazards risk~~as~~

~~follows.~~ .

$$\mu\Gamma(X^t x_{ij} i; S_o s_j^{k_j}) = \prod_{o=1}^{3} exp(-\frac{(x_{ot}-s_o)^2}{2h_s^2}\frac{(x_{ij}-s_j^{k_j})^2}{2\sigma_{x_j}^2}), i\ k_j = 1, 2, \cdots, n K_j; t\ i = 1, 2, \cdots, T N.$$

$$\Gamma(x_i; S^K) = \prod_{j=1}^{J} \Gamma(x_{ij}; s_j^{k_j}), \quad S^K = \{s_j^{k_j} | j = 1, 2, \cdots, J\} \tag{9}$$

~~where T is the different month value, S_o denotes monitor space, and h_s~~

$$\sigma_{x_j} = \begin{cases} 0.6841(b-a), & for \quad N = 5; \\ 0.5404(b-a), & for \quad N = 6; \\ 0.4482(b-a), & for \quad N = 7; \\ 0.3839(b-a), & for \quad N = 8; \\ 2.6581(b-a)/(N-1), & for \quad N \geq 9. \end{cases} \tag{10}$$

$$where \quad b = \max_{1 \leq i \leq N} \{x_{ij}\}, \quad a = \min_{1 \leq i \leq N} \{x_{ij}\}.$$

where N is the sample size of $X = \{x_{ij} | i = 1, 2, \cdots, N; j = 1, 2, \cdots, J\}$, K_j is the number of diffusion points of a given monitor set s_j, and σ_{x_j} is the diffusion coefficient with respect to different attributes j. Based on ~~this~~ the normal estimator, the research by Huang (2002) has shown how to determine the coefficients (shown in Eq. 10)~~and~~ . This approximate reasoning of information diffusion is used to estimate probabilities and fuzzy relationships from a small dataset for risk assessment (Huang et al. (2018)). As an example, we use a 2-dimension normal estimator to calculate the discrete probability density function~~can be estimated by matrix $P = \{p_{jk}\}$.~~

$$p_{jk} = \frac{\sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k)}{\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k)}, j = 1, 2, \cdots, J; k = 1, 2, \cdots, K.$$

~~where u_j and v_k are the hazard indicator vectors. Further, the~~ . For the given compound hazards attributes monitor set $S^K = \{(s_1^{k_1}, s_2^{k_2}) | 1 < k_1 < K_1, 1 < k_2 < K_2\}$, we estimate the discrete probability matrix $P = p_{k_1,k_2}$ and the conditional probability distribution ~~of the given compound hazards risk indicator u has the expression of Eq. 12:~~ $\hat{P} = p_{s_2|s_1}(s_2^{k_2}|s_1)$:

$$p_{v|u_j}(v_k|u_j)_{k_1,k_2} = \frac{p_{jk}}{\sum_{k=1}^{K} p_{jk}}, k = 1, 2, \cdots K. \frac{\sum_{i=1}^{N} \Gamma(x_i; S^K)}{\sum_K \sum_{i=1}^{N} \Gamma(x_i; S^K)} \tag{11}$$

$$p_{s_2|s_1}(s_2^{k_2}|s_1) = \frac{p_{k_1,k_2}}{\sum_{k_2=1}^{K_2} p_{k_1,k_2}}, k_2 = 1, 2, \cdots K_2. \tag{12}$$

~~For the two dimensional input risk indicator set (time and hazard level value) $A=\{(x_{1t},x_{2t})|t=1,2,\cdots,T\}$ with diffusion function $\mu_A(u_j,v_k)$, the fuzzy relationship (vulnerability distribution) between A and fuzzy output (economic losses indicator f_m) $B=R_f$~~ Similarly, we can calculate the three dimensional diffusion function for the compound hazards attributes set $S^K = \{(s_1^{k_1}, s_2^{k_2}, s_3^{k_3})|1<k_1<K_1, 1<k_2<K_2, 1<k_3<K_3\}$. Suppose s_3 corresponds to the attribute of economic loss, the vulnerability curve between the set of causes s_1, s_2 and the consequence s_3 can be estimated by ~~membership function $\mu_B(f_m)$:~~ the following fuzzy membership function:

$$\mu_B\varpi(f_m s_3^{k_3}) = \max_{\substack{u_j\in U\\ v_k\in V}\ (k_1,k_2)}\{\min\{\mu_A \Gamma(u_j x_i;(s_1^{k_1}, v_k s_2^{k_2})), R_f^{k_3}\}\}, m k_3 = 1,2,\cdots, M K_3 \quad R^{k_3} \text{ is the } \mathrm{k}_3^{\text{th}} \text{ slice of } R.$$

where the fuzzy relationship model ~~$R_f=\{(r_{jkm})\}$ (~~ $R=[r_{k_1,k_2,k_3}]$ (shown in Eq. 13) is ~~given by the three-dimension information diffusion matrix $\mu(X_t;u_j,v_k,f_m)$.~~ defined by the 3-dimension information diffusion function $\Gamma(x_i;S^K)$.

$$r_{jkm\,k_1,k_2,k_3} = \frac{\sum_{i=1}^{n}\mu(X^t{}_i;u_j,v_k,f_m)}{\max\limits_{1\le m\le M}\sum_{i=1}^{n}\mu(X^t{}_i;u_j,v_k,f_m)}\frac{\sum_{i=1}^{n}\Gamma(x_i;(s_1^{k_1},s_2^{k_2},s_3^{k_3}))}{\max\limits_{1\le k_3\le K_3}\sum_{i=1}^{n}\Gamma(x_i;(s_1^{k_1},s_2^{k_2},s_3^{k_3}))}. \tag{13}$$

Then the ~~weighted value $f(u_j,v_k)$, represented as vulnerability distribution~~ vulnerability curve $f=f_{s_1^{k_1},s_2^{k_2}}$, is defined as follows.

$$f(u_j,v_k)_{s_1^{k_1},s_2^{k_2}} = \frac{\sum_{m=1}^{M}\mu_B(f_m)\cdot f_m}{\sum_{m=1}^{M}\mu_B(f_m)}\frac{\sum_{k_3=1}^{K_3}\varpi(s_3^{k_3})\cdot s_3^{k_3}}{\sum_{k_3=1}^{K_3}\varpi(s_3^{k_3})}, j k_1 = 1,2,\cdots, J K_1; k_2 = 1,2,\cdots,K_2. \tag{14}$$

Based on the VFS-IDM risk assessment model, the dynamic compound hazards risk (Direct Economic Losses) ~~is shown in~~ can be obtained via Eq. 15 where the risk is quantified as the expected value of the conditional probability distribution ~~and vulnerability distribution. The proposed algorithm, which can be used to deal with incomplete information risk assessment, is given by~~ p and the vulnerability distribution f. The detailed procedure of IDM is shown in Algorithm 2.

$$Risk_{u_j} = \sum_{k=1}^{K}{}_{k_2=1}^{K_2} p_{v|u_j}(v_k|u_j)_{k_1,k_2}\cdot f(u_j,v_k)_{s_1^{k_1},s_2^{k_2}} \tag{15}$$

Algorithm 2 ~~VFS-IDM~~ IDM Dynamic Risk Assessment Model ~~of Compound Hazards~~

Input:

~~Sample set $D = \{X^t{}_i = (x^t{}_{1i}, x^t{}_{2i}, x^t{}_{3i}) | i = 1, 2, \cdots, n; t = 1, 2, \cdots, T$~~Compound hazards data samples $X = \{(x_{i1}, x_{i2}, x_{i3}) | i = 1, 2, \cdots, N\}$, where ~~$x^t{}_{oi}, o = 1, 2, 3$ is the related factor~~ x_{ij} is the risk attributes of compound hazards~~(results given by Algorithm 1)~~;

~~Universes of monitor space $S = \{(s_{ot}), l = 1, 2, \cdots, L | o = 1, 2, 3\}$, where the length L varies from different universes;~~Coefficients of diffusion function ~~$H = (h_1, h_2, h_3)$~~$\Sigma = (\sigma_{x1}, \sigma_{x2}, \sigma_{x3})$.

Output:

Dynamic compound hazards risk.

~~Identification of the comprehensive value of compound hazards level by VFS-IEW;~~ Compound hazards level evaluation by Algorithm 1;

~~Based~~ Monitor space of different attributes $S^K = \{(s_j^{k_j}), 1 < k_j < K_j | j = 1, 2, 3\}$;

for sample index $i = 1$ to N, each x_i **do**

  Construct normal information diffusion functions based on the universes of monitor space ~~, employing the normal diffusion function in~~ and Eq. 9~~to construct information diffusion matrix of sample D~~;

end for

Estimate the joint and conditional probability distribution based on Eq. 11 and Eq. 12;

Determine the ~~input-output sets and model the fuzzy~~ fuzzy cause relationship based on Eq.13, ~~then~~ and estimate the vulnerability ~~distribution~~ curve by Eq.14;

~~The~~ Derive the dynamic risk (~~Direct Economic Loss~~direct economic loss) of compound hazards ~~can be quantified~~ by Eq. 15.

4 Case Study

~~This sectionuses the Typhoon-Rainstorm~~ In this section, we evaluate VFS-IEM-IDM with a case study of typhoon-rainstorm compound hazards that occurred in Shenzen, China~~, as an example to show how the proposed VFS-IEM-IDM model can be used to dynamically assess the risk of compound hazards~~. Shenzhen is located in the east bank of the Zhujiang River and is surrounded by Daya Bay and Dapeng Bay, where the climate is ~~a subtropical maritime and Typhoon-Rainstorms are undoubtedly~~ subtropical and maritime. typhoon-rainstorms are the most frequently occurred hazards in Shenzhen. According to the collected data ~~(see Table A1)~~as shown in Table A1, from 1980 to 2016 ~~, on average, the directed~~ the direct economic losses of the Typhoon and Rainstorm hazards in Shenzhen on average exceeded 360 million RMB per year~~, the~~. Also, Zhou has investigated the Typhoon and Rainstorm hazards cause the number of death was 3.4 ~~deaths~~ annually and about 149,000 people were affected (Zhou et al. (2017)). ~~The assessment results of the Typhoon-Rainstorm dynamic risk are the basis~~ Accurate assessments of the typhoon-rainstorm risk are crucial to determine whether or not the early warning systems are ~~worked~~ working and implemented effectively.

~~Since the Typhoon-Rainstorm~~

4.1 Classifications of individual hazard level

The typhoon-rainstorm compound hazards are ~~characteried by three indicators (~~usually characterized by three drivers, i.e., Maximum Daily Precipitation (MDP), Extreme Wind Intensity ~~,~~ (EWI), and landing location. To better measure the impact of typhoon landing on the typhoon-rainstorm compound hazards level, the landing location is converted into Transformed Location Number ~~), the variable fuzzy set dimension reduction model can be used to get more precise comprehensive hazard level. This paper has outlined the index classification criteria (shown in Table 1), which is guided by Shenzhen Climate Bulletin~~ (TLN) via circle distance calculation where the big value represents the typhoon landed to Shenzhen is more closer. Based on the data provided by the Meteorological Bureau of Shenzhen Municipality (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/)~~.~~ ~~We also classify the Typhoon-Rainstorm compound hazard level into four types, and which is related to our final results.~~ , the values of the three drivers are segmented into four intervals in terms of four individual hazard levels, i.e., I, II, III and IV. A higher hazard level indicates a more severe consequence.

Table 1. Classification standards of individual hazard level.

Drivers	Classifications of Individual Hazards Level			
	I	II	III	IV
Maximum Daily Precipitation (MDP)	(0,50)	(50,100)	(100,150)	(150,250)
Extreme Wind Intensity (EWI)	(8,10.8)	(10.8,17.2)	(17.2,23.6)	(23.6,30)
Transformed Location Number (TLN)	(0,2)	(2,5)	(5,8)	(8,10)

Based on ~~expert experiences and the relevant government documents, the classification results of Typhoon-Rainstorm hazard level (H) in Shenzhen express type as $H \in [1.5, 2)$, type as $H \in [2, 2.7)$, type as $H \in [2.7, 3.5)$, and type as $H \in [3.5, 4]$. This paper uses the dimension reduction model~~ the segmentation of the four individual hazard levels, we also classify the typhoon-rainstorm compound hazards into four levels, i.e., I, II, III, IV, where a higher compound hazard level indicates that the corresponding compound hazard is of greater severity. As described in Section 3.1, the VFS-IEM compound hazards level evaluation model (Algorithm 1) ~~to get~~ can be applied to obtain the comprehensive value H ~~and transfers them into different hazard levels based on Typhoon-Rainstorm classification standards.~~ which is then used to derive the compound hazards level based on the classification criteria of the typhoon-rainstorm compound hazards.

4.2 Calculation of relative membership degree

The relative membership degree is determined by the individual hazard level classifications. According to the ~~classification results~~ value segmentation shown in Table 1, ~~the~~ we have the different fuzzy intervals for four different hazard levels. Then, for three hazard drivers, the interval criterion matrix I_{ab} can be expressed as

$$I_{ab} = \begin{bmatrix} (0,50) & (50,100) & (100,150) & (150,250) \\ (8,10.8) & (10.8,17.2) & (17.2,23.6) & (23.6,30) \\ (0,2) & (2,5) & (5,8) & (8,10) \end{bmatrix} = [((a,b)_{rl})],$$

Further, the corresponding interval I_{cd} for different hazard level is defined as

$$I_{cd} = \begin{bmatrix} (0,100) & (0,150) & (50,250) & (100,250) \\ (8,17.2) & (8,23.6) & (10.8,30) & (17.2,30) \\ (0,5) & (0,8) & (2,10) & (5,10) \end{bmatrix} = [((c,d)_{rl})],$$

and we define the balance boundaries matrix M ~~is defined in Eq. ??~~ :

$$M = \begin{bmatrix} 0 & 66.7 & 133.3 & 250 \\ 8 & 12.9 & 21.5 & 30 \\ 0 & 3 & 7 & 10 \end{bmatrix} = (M[m_{rl})].$$

~~Then the~~ In the end, the relative membership degree matrix can be calculated by Eqs. 4, 5 and 6 respectively.

Taking sample point ~~$(MDP = 33.4, EWI = 18, TL = 9)$~~ $\bar{u}$ = (MDP=33.4, EWI=18, TL=9) for example, we obtain the relative membership degree matrix ~~is expressed by Eq. ?? where the matrix value~~ $\mu(\bar{u})$ shown as below, in which the matrix element represents the probability of each ~~indicator~~ drivers belonging to the ~~different compound~~ four individual hazards level.

$$\mu_{A}(u\bar{u}) = \begin{bmatrix} 0.666 & 0.334 & 0.000 & 0.000 \\ 0.000 & 0.438 & 0.593 & 0.063 \\ 0.000 & 0.000 & 0.333 & 0.667 \end{bmatrix}.$$

~~To get the comprehensive hazard~~

4.3 Typhoon-rainstorm hazards level

To derive the compound hazards level, the information entropy method ~~can be used to get~~ is used to obtain the weight of each ~~indicator ω, which~~ hazard driver. We have the weight Ω shown as follows where the element in Ω implies that the Maximum Daily Precipitation and Location play the main role in determining the ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards level.

$$\omega\Omega = [\ 0.43 \quad 0.19 \quad 0.39\ ].$$

~~Then by Algorithm 1,~~ Based on the VFS-IEM compound hazards level evaluation model (Algorithm 1), we obtain the comprehensive value ~~of the Typhoon-Rainstorm hazards level $(MDP = 33.4, EWI = 18, TL = 9)$ is $H = 2.75$ when the hyper-parameter~~ H of typhoon-rainstorm hazards. Then, guided by the domain experts, we have the classification criteria of the typhoon-rainstorm compound hazard level in Shenzhen: $H \in [1,2)$ for level I, $H \in [2,2.7)$ for level II, $H \in [2.7,3.5)$ for level III, and $H \in [3.5,4]$ for level IV. For the case (MDP=33.4, EWI=18, TL=9), the value of the typhoon-rainstorm hazards level H

is obtained. When the hyper-parameters $\alpha = \beta = 1$, ~~and $H = 2.18$ when the hyper-parameter~~ $H = 2.75$. When $\alpha = \beta = 2$~~. To be more general, this paper takes~~ , $H = 2.18$. Furthermore, we take the average of $H = 2.75$ and $H = 2.18$ to obtain the final compound hazards level value, i.e., ~~$H = 2.47$, Type~~ $H = 2.47$ which corresponds to the compound hazard level II. The results of other ~~Typhoon-Rainstorm comprehensive hazard levels~~ typhoon-rainstorm cases can be found in Appendix ~~(see Table ??).~~ Table ??.

Table 2: Transformed Typhoon-Rainstorm typhoon-rainstorm hazard data sets in Shenzhen.

~~Time~~ Year	Date	Transformed Time (~~T~~day)	~~Comprehensive Hazard~~ Compound Hazards Level (H)	Direct Economic Los
~~20090627~~ 2009	0627	176	2.72	0.3819
	0719	198	3	1.352
	0915	254	3.74	1.3750
~~20100724~~ 2010	0724	203	2.32	0.2571
	0912	251	2.49	0.4450
	0922	261	2.74	0.9831
~~20110624~~ 2011	0624	173	1.93	0.0765
	0930	269	2.72	0.4013
~~20120630~~ 2012	0630	179	2.31	0.2895
	0724	203	3.95	2.48
	0817	226	2.56	0.7648
~~20130615~~ 2013	0615	164	1.94	0.1527
	0702	181	1.99	0.1894
	0802	211	1.53	0.0452
	0814	223	2.13	0.1423
	0922	261	3.06	1.2351
~~20140718~~ 2014	0718	197	1.83	0.0841
	0916	255	2.48	0.7682
	0823	232	2.92	0.7410
	1004	273	2.96	0.8352
~~20160802~~ 2016	0802	211	3.68	2.1521
	0818	227	1.88	0.0251
	1018	287	2.28	0.2362
	1021	290	3.11	0.9341
~~20170612~~ 2017	0612	161	3.67	2.058
	0723	202	2.11	0.2461

	0823	232	2.46	1.31
	0827	236	3.2	1.613
	0903	242	3.03	1.8872
	1016	285	2.48	0.5902
~~20180606~~ 2018	0606	155	2.47	0.6952
	0718	197	1.58	0.0267
	0811	220	2.45	0.5241
	0916	255	3.93	2.226
~~20190703~~ 2019	0703	182	1.49	0.0528
	0811	210	3.02	0.8182
	0824	233	2.9	0.8391
	0902	241	1.8	0.0725

~~From~~

4.4 Dynamic typhoon-rainstorm hazards risk

Based on the data in Table B1, ~~the sample observations on~~ we obtain three typhoon-rainstorm hazard attributes including direct economic loss L (Billions)~~over each comprehensive compound~~ , hazards level H ~~are written as~~ and hazards occurrence day. Then the dynamic risk of compound hazards can be calculated by 38 data points:

$$\cancel{Sample}\, x_{ij} = \{(\cancel{t_1}\, x_{i1}, \cancel{d_1}\, x_{i2}, \cancel{l_1}\, x_{i3}), \ldots i=1, \cancel{(t_i}\, 2, \cancel{d_i} \cdots, \cancel{l_i), \ldots, (t_{38}, d_{38}, l_{38})}\, 38\} = \{(172, 2.72, 0.3819), \ldots, (241, 1.8, 0.0725)\}.$$

where ~~t_i, d_i~~ x_{i1}, x_{i2} represents the time ~~dimension of the Typhoon-Rainstorm hazard and the comprehensive value of the hazards~~ attribute of the typhoon-rainstorm hazards and the compound hazard level respectively, and ~~l_i~~ x_{i3} is the direct economic losses caused by the ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards. Then the diffusion coefficients can be calculated by Eq. 10, ~~written as~~ shown as follows.

$$\begin{cases} \sigma_{x_1} = 2.6581 \cdot (290 - 155)/(38-1) = 10 \\ \sigma_{x_2} = 2.6581 \cdot (3.95 - 1.37)/(38-1) = 0.19 \\ \sigma_{x_3} = 2.6581 \cdot (2.48 - 0.0251)/(38-1) = 0.1764 \end{cases} .$$

Following Algorithm. 2~~outlines how to~~ , we use the information diffusion method to estimate the conditional probability and vulnerability distribution of the ~~Typhoon-Rainstorm hazards. Then by the 2-dimensional normal diffusion estimator, the joint probability density function P (Eq. ??) and conditional probability function P_{con} (Eq. ??) can be evaluated.~~ typhoon-rainstorm hazards. In this paper, we ~~denote the monitor space $T=(t=164,t=194,t=224,t=254,t=284)$ as~~ define the following monitor space: $T=(164,194,224,254,284)$ corresponds to months $(June, July, August, September, October)$ ~~and $H=(d=1.8,d=2$~~ ~~as comprehensive~~, $H=(1.8,2.4,3.0,3.6)$ corresponds to the compound hazards levels (I,II,III,IV) ~~.~~, and $L=(0.1,0.4,0.7,1.0,1.3,1.6$ corresponds to the direct economic losses. Then we can calculate the joint probability density function P and the conditional probability function $\hat{P}$ as follows:

$$
P = \begin{array}{c} \\ June \\ July \\ August \\ September \\ October \end{array}
\begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\
\begin{pmatrix}
0.059 & 0.046 & 0.007 & 0.036 \\
0.076 & 0.052 & 0.051 & 0.014 \\
0.063 & \mathbf{0.116} & \mathbf{0.090} & 0.019 \\
0.019 & \mathbf{0.086} & \mathbf{0.087} & 0.041 \\
0.002 & 0.073 & 0.060 & 0.002
\end{pmatrix} \end{array}
$$

$$
\cancel{P_{con}}\hat{P} = \begin{array}{c} \\ June \\ July \\ August \\ September \\ October \end{array}
\begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\
\begin{pmatrix}
\mathbf{0.398} & 0.311 & 0.049 & 0.243 \\
\mathbf{0.393} & 0.268 & 0.266 & 0.073 \\
0.218 & \mathbf{0.402} & \mathbf{0.312} & 0.0689 \\
0.080 & 0.370 & \mathbf{0.373} & 0.177 \\
0.012 & \mathbf{0.539} & 0.437 & 0.012
\end{pmatrix} \end{array}
$$

From the results above, it can be seen that the ~~Typhoon-Rainstorm hazard level of~~ typhoon-rainstorm with hazard level III occur more frequently and they are most likely to occur in August and September.

~~The vulnerability distribution $f(x)$ between the comprehensive value~~ Furthermore, the vulnerability distribution f between the hazard level H and the direct economic losses L over the time ~~dimension~~ attribute T can be calculated by the 3-dimension diffusion estimator ~~. The fuzzy~~ (shown in Eq. 13). The fuzzy causal relationship which takes ~~time dimension~~ the time attribute T, hazards level H as ~~input~~ the inputs and the loss L as the output ~~can be~~ is denoted as matrix ~~R_f.~~

~~The discrete vulnerability distribution~~ R. Then the discrete vulnerability curve f in terms of the direct economic loss is evaluated by Eq. 14~~and the results are shown in Eq. ??.~~.

$$
f = \begin{array}{c} \\ June \\ July \\ \mathbf{August} \\ \mathbf{September} \\ October \end{array}
\begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\
\begin{pmatrix}
0.20 & 0.02 & 0.00 & 0.00 \\
0.24 & 0.04 & 0.00 & 0.00 \\
0.15 & 1.13 & \mathbf{1.67} & \mathbf{1.90} \\
0.05 & 0.55 & \mathbf{2.67} & \mathbf{2.62} \\
0.01 & 0.02 & 0.00 & 0.00
\end{pmatrix} \end{array}
$$

It can be seen that ~~Shenzhen,~~ most of the economic losses caused by the ~~Typhoon-Rainstorm hazards is~~ typhoon-rainstorm hazards are concentrated in August and September.

$$
f(x;t,h) = \begin{array}{l} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{pmatrix} 0.20 & 0.02 & 0.00 & 0.00 \\ 0.24 & 0.04 & 0.00 & 0.00 \\ 0.15 & 1.13 & 1.67 & 1.90 \\ 0.05 & 0.55 & 2.67 & 2.62 \\ 0.01 & 0.02 & 0.00 & 0.00 \end{pmatrix} \end{array}
$$

Dynamic compound hazards risks can be quantified as the expected value of ~~hazards influence~~ the damages caused by the compound hazards and the result is~~shown as Eq. ??~~ :

$$Risk = (0.08582, 0.10504, 1.1372, 1.66715, 0.0109) \tag{16}$$

where the elements of ~~vector denotes~~ the vector denote the estimated economic losses caused by the ~~Typhoon-Rainstorm hazards in different months.~~

$$Risk = \begin{pmatrix} 0.08582 & 0.10504 & 1.1372 & 1.66715 & 0.0109 \end{pmatrix}.$$

typhoon-rainstorm hazards from June to October.

5 Discussion

~~Dimension reduction model VFS-IEW presents the comprehensive value of~~

5.1 Compound hazards level evaluation

The proposed VFS-IEM-IDM model provides a comprehensive evaluation of the compound hazards level, but the relationship between the hazards level and the ~~indicators are not clear~~hazard drivers is unclear. To find more information from the results of ~~VFS-IEM, this paper has built~~ the compound hazards level evaluation model, we build a predictive model ~~to shield the light on compound hazards relationship and predict the Typhoon-Rainstorm hazards level.~~ (shown in Eq. 17) to shed light on the relationship between the compound hazards levels.

Since the compound hazards level ~~is an~~ $H \in (I, II, III, IV)$ is ordinal data (monotone trend and proportional odds), the cumulative logistic model (shown in Eq. ~~17~~18) can be used to predict the compound hazards level. ~~The probabilities of different order categoriesgiven by cumulative logistic model are~~ Let the response be the compound hazards level $H = I, II, III, IV$ with probability $\pi_h(U), h = 1, \cdots, 4$ under the covariate compound hazard drivers U. So the cumulative probability of H is less than or equal to level h, i.e., the probabilities of compound hazards belonging to different level categories, is given by

$$P(\cancel{Y}H \le \cancel{j}h \mid \cancel{x}U) = \pi_1(\cancel{x}U) + \cdots + \pi_{\cancel{j}4}(\cancel{x}U), \quad \cancel{j}h = 1,\ldots,\cancel{J}.4.$$

According to the research by Alan (1980), the cumulative logistic model ~~is defined as~~ can be replaced by

$$\operatorname{logit}\left(P(H \le h \mid U)\right) = \log \cancel{\frac{P(Y \le j \mid x)}{1-P(Y \le j \mid x)}} \frac{P(H \le h \mid U)}{1-P(H \le h \mid U)} = \cancel{=}\alpha_{\cancel{j}h} + \boldsymbol{\beta}^T \boldsymbol{U}, \quad \cancel{j}h = 1,\ldots,\cancel{J-1}.3. \tag{17}$$

~~The Typhoon-Rainstorm hazard~~ where the log-odds measures how likely the response H is to be in category h or below versus in a category higher than h. In this paper, the typhoon-rainstorm hazards level prediction problem can be solved by using the VAGM package (Thomas (2010)) and the result is given by

$$\operatorname{logit}\left(P(H \mid (MDP, EWI, TLN))\right) = 5.07(7.32, 11.15) - 0.12MDP - 0.66EWI - 0.91TLN, \tag{18}$$

where the different ~~intercepts denote the different main-effects of hazard levels~~ intercept coefficients denote the main effects of different hazard drivers compared to the reference ~~category, i.e.,~~ compound hazard level IV. The rationality of this model is judged by LR-test (p-value<0.001) and the predictive performance $R^2 = 0.898$ which shows that the model is well fitted and can be used ~~as~~ to predict the compound hazards ~~prediction~~level.

~~One advantage~~

5.2 The superiority of the normal diffusion estimator

One advantage of using the information diffusion ~~technique~~ method to assess the risk of ~~an~~ compound hazards is that it does not need to know ~~(1) the distribution type of the population from which~~ the type of distribution from which the given samples are drawn ~~, (2)~~ and the function form of the causal relationship ~~,~~ which are constructed by the joint probability distribution and the vulnerability distribution. ~~Moreover, researchers have done simulation study on IDM and demonstrate the benefit of information distribution for probability estimation (Huang (2000); Li et al. (2012)) by minimizing the mean integrated square error (Kullback-Leibler divergence error) between the estimator and the true density~~More importantly, it can provide a more accurate evaluation when the compound hazards data set is sparse. The performance of ~~this non-parametric estimation procedure is studied well by Huang (2000) which shows the work efficiency~~ the IDM estimation procedure has been well studied in the literature. For example, Huang (2000) shows the efficiency of IDM is about 35% higher than ~~histogram estimator (HE) and the performance is improved to reduce the error~~ the histogram estimator, and the estimation error is reduced by 23.2% when the data sets are ~~incomplete~~small. Therefore, the assessed compound hazards risk is more reliable and accurate ~~.~~

using a normal diffusion estimator. However, if the size of the data samples is large it is unnecessary to replace the statistics with the information diffusion method (Li et al. (2012)).

5.3 Results

For the dynamic risk assessment of ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards, this paper proposes a hybrid model VFS-IEM-IDM and provides extensive assessment results based on a case study. From the ~~dimension reduction~~ compound hazards level evaluation model VFS-IEM, ~~this paper shows~~ we show that the probability of the occurrences of type II and III hazard levels is the highest in Shenzhen~~. The~~ , which shows that the emergency management department should prepare more effective emergency plans in advance to reduce the occurrences of the secondary hazards. From the dynamic risk assessment model ~~VFS-IEM-IDM~~ IDM, it can be found that the hazards occurrence probability of different hazard levels is different and the ~~type hazards of~~ hazards with levels II and III ~~hazards level~~ are most likely to occur in August and September. ~~Also~~Furthermore, considering the occurrence of ~~different hazard level for each months~~the hazards with different hazard levels for each month, the probability of ~~hazard~~ hazards with level I occurring in June and July is the highest, and the ~~hazard~~ hazards with level II mostly ~~occurs~~ occur in August and October, and the ~~type~~ hazards with level III ~~hazard level~~ is most likely to occur in September. From the perspective of hazard losses, the ~~different~~ difference between the direct economic losses caused by the ~~Typhoon-Rainstorms~~ typhoon-rainstorms of the same hazard level ~~in~~ each month indicates that the impacts of the ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards on the economy are not the same. Besides, for the same month, the influence of economic ~~loss~~ losses decreases gradually when the compound hazards level rises. This indicates that the capacity of ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazard resistance in Shenzhen is reliable, and the ability to ~~copy with the~~ cope with sudden compound hazards ~~are~~ is relatively strong under the existing emergence management system. The dynamic compound hazards risk of ~~Typhoon-Rainstorm~~ the typhoon-rainstorm hazards in Shenzhen shows that the risk value of ~~this~~ these compound hazards in each month is different and the highest risk value appears in August and September. On average, the occurrence of ~~Typhoon-Rainstorm~~ the typhoon-rainstorm hazards brought Shenzhen 114 million RMB and 167 million RMB losses in these two months respectively, which is in line with the actual situation.

6 Conclusions

~~Risk assessment is an important step~~ Compound hazards risk assessment is a complex multi-criteria problem and is crucial to the success of strategic decision-making in emergency management~~, but few~~ . Traditional statistical methods are often inaccurate when only a small set of data samples is available, and little research discusses the ~~uncertainties~~ correlations of compound hazards ~~evaluation and considers dynamic risk assessments when the data sets are incomplete~~drivers and considers the dynamics of the occurrences of the compound hazards. In this paper, we first present the definition of dynamic compound hazards risk ~~, and then~~ and then propose the Variable Fuzzy Set (VFS) ~~theory is employed to evaluate the relative membership degree,~~ and Information Entropy Method (IEM) ~~is applied to obtain the weights of criteria indicators for~~ model to evaluate the compound hazards level ~~evaluation~~by considering the correlations of different hazards drivers. Based on the results obtained by VFS-IEM, we apply the information diffusion method (IDM) to estimate the ~~conditional probability distribution~~ compound hazards probability and vulnerability distribution with the hazard occurrence time and the corresponding losses~~. Then~~ , and then the dynamic risk is ~~assessed using fuzzy probabilistic risk to improve the accuracy of risk assessment. The innovations of this paper are: () Based~~ calculated by the probabilistic model.

There are mainly three aspects of innovations in this paper. Firstly, based on the definition of compound hazards risk, we ~~take time dimension into consideration to~~ consider the temporal dynamics and introduce the concept of dynamic compound hazards risk. ~~() Considering~~ Secondly, considering that compound hazards have ~~different measurement indicators for the comprehensive~~ many drivers for the hazard level evaluation, a hybrid model of Variable Fuzzy Sets and the Information Entropy Method has been proposed to improve the accuracy of compound hazards level evaluation. ~~() According~~ Thirdly, according to the concept of dynamic compound hazards risk, we apply the Information Diffusion Method to estimate the ~~conditional probability distribution~~ hazards probability and the vulnerability distribution~~using the comprehensive hazard levels, hazards occurrence time and the losses of compound hazards~~. The proposed model VFS-IEM-IDM can be used to deal with the problem of ~~incomplete and limited information in dynamic~~ data sparsity in dynamic compound hazard risk assessment. ~~()~~ By evaluating the expected value of the conditional probability distribution and the vulnerability distribution, we quantify the ~~Typhoon-Rainstorm dynamic riskwhich shows that~~ typhoon-rainstorm dynamic risk. Furthermore, VFS-IEM-IDM can be extended to other compound hazards that occur in urban cities such as flooding. As a case study, we show that the occurrences of the ~~occurrence of Typhoon-Rainstorm hazards brings~~ typhoon-rainstorm hazards bring Shenzhen 114 million RMB and 167 million RMB losses in August and September~~respectively. These risk assessment results obtained by VFS-IEM-IDM model are in line with the actual situation and can be used to guide the emergency management in Shenzhen~~ , respectively.

Dynamic risk assessment is a relatively new topic and ~~we have proposed a hybrid model to assess the compound hazards risk, but there are somewhere need to be improved. On one hand, the weight calculation~~ there are many issues that need further improvement. In this paper, the weights of different types of ~~hazards indicators is subjective~~hazard drivers is subjective, and the results of the vulnerability curve have not ~~taken the changes in the internal attributes~~ considered the development of the affected ~~area into consideration. On the other hand, there~~ areas. There are also some subjective issues regarding ~~how to process the data sets, so maybe we can consider adopting a more scientific method to process the original data to obtain more scientific conclusions~~the processing of the data sets. In future work, we will explore techniques to deal with these two issues and further improve the assessment accuracy.

Code and data availability. The data and code used in the study are available at https://github.com/GongWenwuu/VFS-IEM-IDM.git.

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Author contributions. ~~GWW and YLL~~ Wenwu Gong and Lili Yang conceived the research framework and developed the methodology. ~~GWW~~ Wenwu Gong was responsible for the code compilation, data analysis, and graphic visualization. ~~GWW and JJ~~ Wenwu Gong and Jie Jiang had done the first draft writing. ~~YLL~~ Lili Yang managed the implementation of the research activities and revised the manuscript. All the authors discussed the results and contributed to the final version of the paper.

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Appendix A: Data Source

~~For the Typhoon-Rainstorm~~ For the typhoon-rainstorm dynamic compound hazards risk assessment, the useful data sets ~~, collected from~~ were collected from the Meteorological Bureau of ~~ShenZhen Municipality (~~Shenzhen Municipality (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/nianduqihougongbao/) and TYPHOON ONLINE (http://typhoon.nmc.cn/web.html), have been sorted out in Table A1. In this table, MDP denotes ~~as~~ Maximum Daily Precipitation, EWI denotes ~~as~~ Extreme Wind Intensity, DEL denotes ~~as~~ Direct Economic Loss, and the Transformed Location Number (TLN) denotes ~~as~~ the Typhoon Landing Location which is determined by radio distance transform using ~~expertise~~ expert knowledge.

Table A1: Data sets of ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards in Shenzhen.

Hazards ~~Number~~ ID	Impact ~~Time~~ Date	MDP (mm)	EWI (m/s)	Landing Location	TLN	DEL (Billion)
0904	0627	67.3	16.8	Huizhou	8.5	0.3819
0906	0719	80	27.3	Shenzhen	10	1.152
0915	0912	127.9	28	Taibei	6	1.075
1003	0724	31.3	16.2	Zhanjiang	6.5	0.2571
1010	0912	62.4	13.7	Quanzhou	3	0.345
1011	0922	51.9	15.8	Heyuan	7	0.2983
1105	0624	41.7	14	Yangjiang	4.5	0.0765
1006	0930	53.0	15.2	Wenchang	2.5	0.8243
1206	0630	33.6	16.8	Zhuhai	6.5	0.6873
1208	0724	152.3	23.9	Taishan	7	2.241
1213	0817	46.1	13.5	Zhanjiang	3	0.9153
	0615	36.5	8.4	Wenchang	4	0.3621
1306	0702	38.6	10.9	Zhanjiang	3	0.2561
1309	0802	40.7	10.7	Wenchang	3	0.0851
1311	0814	47.8	14.2	Yangxi	3	0.6413
1319	0922	72.4	21.6	Shanwei	8.5	1.152
201409	0718	31.6	14.7	Wenchang	2.5	0.0841
201415	0916	73.5	18.9	Xuwen	2.5	0.9641
201517	0823	69.4	13.6	Shanwei	10	1.041
201522	1004	108.5	13.5	Zhanjiang	5.5	0.9631
201604	0802	166	19.2	Shenzhen	10	2.31
201608	0818	45.5	9.1	Zhanjiang	5.5	0.0314
201621	1018	117.6	12.3	Wanning	1.5	0.421

201622	1021	83.7	18.8	Shanwei	7.5	0.8721
1702	0612	161.8	16.9	Shenzhen	10	2.109
1707	0723	33.4	10.6	Xianggang	9	0.5315
1713	0823	56.3	23.4	Zhuhai	8.5	1.328
1714	0827	114.5	17.5	Jiangmen	8.5	1.741
1716	0903	82.4	14.4	Shanwei	7.5	0.9631
1720	1016	40	20.3	Zhanjiang	7.5	0.7341
1804	0606	97.2	8.8	Xuwen	8.5	0.9267
1809	0718	50.7	11.1	Wanning	1.5	0.0267
1816	0811	45.3	10.8	Yangjiang	7	0.5241
1822	0916	173.5	30	Taishan	7.5	2.361
1904	0703	48.8	11	Wanning	1.5	0.0672
1907	0811	99.1	14.1	Wenchang	5.5	0.9561
1911	0824	49.4	12.7	Zhangzhou	6	0.5931
1914	0902	52.2	11.3	Wanning	1	0.0751

Appendix B: Comprehensive Compound Hazards Level

Based on the ~~Dimension Reduction Model~~ VFS-IEM model, this paper takes the average of $\alpha = \beta = 1$ and $\alpha = \beta = 2$ to denote the final ~~Typhoon-Rainstorm~~ typhoon-rainstorm hazards level. The following Table **??** has shown that the whole results of ~~comprehensive~~ compound hazards degree value.

Table B1: Transformed typhoon-rainstorm hazard data sets in Shenzhen.

Year	Date	Transformed Time (day)	Compound Hazards Level (H)	Direct Economic Loss (L)
2009	0627	176	2.72	0.3819
	0719	198	3	1.352
	0915	254	3.74	1.3750
2010	0724	203	2.32	0.2571
	0912	251	2.49	0.4450
	0922	261	2.74	0.9831
2011	0624	173	1.93	0.0765
	0930	269	2.72	0.4013
2012	0630	179	2.31	0.2895
	0724	203	3.95	2.48
	0817	226	2.56	0.7648
2013	0615	164	1.94	0.1527
	0702	181	1.99	0.1894
	0802	211	1.53	0.0452
	0814	223	2.13	0.1423
	0922	261	3.06	1.2351
2014	0718	197	1.83	0.0841
	0916	255	2.48	0.7682
	0823	232	2.92	0.7410
	1004	273	2.96	0.8352
2016	0802	211	3.68	2.1521
	0818	227	1.88	0.0251
	1018	287	2.28	0.2362
	1021	290	3.11	0.9341
2017	0612	161	3.67	2.058
	0723	202	2.11	0.2461

	0823	232	2.46	1.31
	0827	236	3.2	1.613
	0903	242	3.03	1.8872
	1016	285	2.48	0.5902
2018	0606	155	2.47	0.6952
	0718	197	1.58	0.0267
	0811	220	2.45	0.5241
	0916	255	3.93	2.226
2019	0703	182	1.49	0.0528
	0811	210	3.02	0.8182
	0824	233	2.9	0.8391
	0902	241	1.8	0.0725