Dynamic Risk Assessment of Compound Hazards Based on VFS-IEM-IDM: A Case Study of Typhoon-Rainstorm Hazards in Shenzhen, China

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Abstract. Typhoons and rainstorms are types of natural hazards that can cause significant impacts. These individual hazards may also occur simultaneously to produce compound hazards, leading to increased losses. The accurate risk assessment of such compound hazards faces several challenges due to the uncertainties in multiple hazards level evaluation, and the incomplete information in historical data sets. In this paper, to deal with these challenges, we propose a risk assessment model called VFS-IEM-IDM based on the Variable Fuzzy Set and Information Diffusion Method. In particular, VFS-IEM-IDM provides a comprehensive evaluation of the compound hazards level, and a predictive cumulative logistic model is used to verify the results. Furthermore, VFS-IEM-IDM applies a normal information diffusion estimator to estimate the conditional probability distribution and the vulnerability distribution of the compound hazards based on the hazards level, the hazards occurrence time, and the corresponding losses. To examine the efficacy of VFS-IEM-IDM, a case study of the Typhoon-Rainstorm hazards that occurred in Shenzhen, China is presented. The risk assessment results indicate that hazards of level II mostly occur in August and October, while hazards of level III often occur in September. The risk of the Typhoon-Rainstorm hazards differs in each month and in August and September the risk gets the highest value, and the estimated economic losses are around 114 million RMB and 167 million RMB respectively.

Key words: Compound hazards risk; Fuzzy dynamic risk; Variable fuzzy set; Information diffusion; Typhoon-Rainstorm

1 Introduction

Assessing risk is an effective way to reduce the negative impacts on natural hazards and plays an increasingly important role in helping the decision maker in emergency management. With the global climate change, many cities have suffered extreme natural hazards more frequently and many people's lives are under threat. Located in the southern part of China, Shenzhen is a coastal city with a low latitude, where Typhoon and Rainstorm hazards have severely restricted the sustainable development of the local economy and society. Furthermore, the development of the Guangdong-HongKong-Macao Greater Bay Area highly relies on timely and effective emergency plans which are often determined by the efficiency of the risk assessment.

Risk assessment is a technique that uses hazards data to estimate the probability that natural hazards occur and assess their economic losses. Traditional methods of risk assessment mainly utilize Geographic Information System (GIS) to get risk maps (Gigovic et al. (2017)), or rely on information diffusion method (IDM) to deal with incomplete data sets (Gong et al. (2020)). These relevant risk assessment methods (Julia et al. (2021); Zhou et al. (2020)) have became more comprehensive and mature in single hazards evaluation. However, the multi-hazard risk assessment is not the aggregation of their individual assessment results but considers the connections among different hazards (Kappes et al. (2012)), so the assessment results for multiple hazards are often inaccurate and insufficient. Furthermore, there is little research focusing on Typhoon-induced risk assessment in the literature and many aspects such as dynamic risk assessment are not considered.

There are many works discussing the multi-hazard risk assessment ~~and Choi et al. (2021) had~~ which have been reviewed the relevant literature Choi et al. (2021). Furthermore, Wang et al. (2020) clarified the relationship between hazards in multi-hazard scenarios by dividing them into three categories: mutually amplified hazards, mutually exclusive hazards, and non-influential hazards. Khan et al. (2020) presented an analysis of the existing methods and technologies that are relevant to multi-hazard scenarios. Huang et al. (2018) developed information diffusion technique to construct a joint probability distribution and a vulnerability distribution for assessing the flood and earthquake risks. Xu et al. (2016) also used the information diffusion method to assess the risk of multiple hazards quantitatively and evaluated the risk of loss of human lives from meteorological hazards in China. Ming et al. (2015) proposed a quantitative approach of multi-hazard risk assessment based on vulnerability distribution and joint return period of hazards to assess the risk of crop losses in the Yangtze River Delta region of China. However, all of these works focus on integrating the risks caused by single hazards and ignoring these truly correlations between hazards occurrence, such as the co-appearance of Typhoon-Rainstorm hazards. In this paper, we aim at the multi-hazards and investigate new methods for multiple hazards level evaluation and dynamic risk assessment of compound hazards.

Compound hazards, a sub-group of the term 'multi-hazard', can be considered as the combination of multiple drivers that contribute to societal risk (Jennifer et al. (2021)), within which two associated hazards impacting the same time and place. In this paper, we define the compound hazards risk as a scene in the future associated with some adverse incident caused by cascading hazards systems, where there are strong connections among different hazard indicators. Compared with the multi-hazard risk assessment (Xu et al. (2016); Huang et al. (2018)), the risk assessment of compound hazards obtains the comprehensive hazards level without losing any correlated information and often reflects the property of hazard-induced. Risk assessment of compound hazards has been studied by He et al. (2020), who presented the Cloquet integral multiple

linear regression model to overcome the problems of nonlinear additivity of couple hazards. But this method only provides

50　the magnification coefficients to assess the risks of compound hazards and neglects the changing of time span. Here, there are some problems remaining to be solved. On the one hand, the collected data for assessing the compound hazards risk is often incomplete such that the results may not be reliable. On the other hand, the change of month in which the compound hazards occur also has impacts on the risk assessment and is often ignored. In this paper, we emphasize that risk assessment of compound hazards should deal with the uncertainties caused by multi-indicators, the unknown probability distributions, the

55　incomplete information in historical data sets, and the dynamic property of hazards occurrence.

Some research based on variable fuzzy sets (VFS) theory, introduced by Chen (2006), have shown that the relative membership function can be used to evaluate the multi-indicators assessment problems. Li et al. (2012) proposed the fuzzy comprehensive assessment method to solve the flood risk assessment problems with interval boundaries and this integrated model improves the reliability of single hazard risk assessment. Beaula et al. (2013) used variable fuzzy sets to evaluate the synthetic

60　hazards level of Nagapattinam district with the north-east monsoon rainfall's data sets. Similarly, the variable fuzzy set theory can be used to obtain the comprehensive evaluation of compound hazards. In this paper, we propose to combine the VFS with information entropy method (IEM) to assess compound hazards indicators and obtain a comprehensive risk assessment.

In many cases, it is difficult to collect compound hazards data sets, such that the information carried by historical data is often incomplete. Therefore, the traditional models often give an unreliable estimation result, and many fuzzy probabilistic models

65　have been proposed to enhance the accuracy of risk assessment Mehran et al. (2017). Fuzzy probabilistic models are used to model uncertainties related to hazards and the randomness due to environmental, natural, or time span changing. The main feature of the fuzzy probabilistic model is to change the traditional data points into fuzzy set for partly filling the gap caused by data incompleteness and improve the estimation accuracy between inputs and outputs. The most powerful technique is the information diffusion method (IDM), which helps extract useful underlying information from the hazard data sets. Research by

70　Huang (1997, 2002) has given many results about IDM and there are many papers have shown the capability of information diffusion method to deal with incomplete data sets (Huang (2009); Li (2013); Huang et al. (2018). In this paper, we introduce the information diffusion method to deal with the incomplete data problem and combine the variable fuzzy sets theory to carry out dynamic risk assessments of compound hazards.

The main contributions of this paper are summarized as ~~follows~~following two-folds.

75　　– 1) ~~We consider the uncertainties in compound hazards level evaluation and incomplete information in historical data sets, and~~ For technological innovation, we propose a hybrid model, named as Variable Fuzzy Set and Information Diffusion Method (VFS-IEM-IDM), to ~~deal with~~ assess compound hazards risk ~~assessment dynamically.~~

　　– ~~2) To~~ dynamically. Furthermore, we simplify the calculation procedures of relative membership degree to improve the efficiency and accuracy of compound hazards level evaluation~~, the calculation procedures of relative membership degree~~

80　　~~have been categorized into three types~~.

　　– ~~3)~~ 2) To examine the efficacy of the proposed model VFS-IEM-IDM, a case study of the Typhoon-Rainstorm hazards that occurred in Shenzhen, China is presented.

The rest of this paper is organized as follows. Section 2 introduces the basic concepts and presents the dynamic compound hazards risk assessment model (VFS-IEM-IDM). Section 3 illustrates how the proposed model can be used to assess the dynamic risk of Typhoon-Rainstorm hazards in Shenzhen, China. Section 4 discusses the comprehensive evaluation of the compound hazards level, conditional probability distribution, vulnerability distribution and the dynamic expectation risk of the Typhoon-Rainstorm hazards to show the effectiveness of VFS-IEM-IDM. Finally, we conclude the paper in Section 5.

2 Dynamic Risk Assessment of Compound Hazards Based on VFS-IEM-IDM

Risk assessment of compound hazards should consider the uncertainties caused by multi-indicator, incomplete information contained in historical data sets, and the impact of internal attribute changes on the hazards. This section introduces VFS-IEM-IDM which combines the variable fuzzy sets theory with information diffusion method to assess the dynamic risk of compound hazards when the given data sets are incomplete. The proposed VFS-IEM-IDM model consists of VFS-IEW dimension reduction model to obtain the comprehensive evaluation of compound hazards level (Section 2.2), and VFS-IDM dynamic risk assessment model to estimate the expectation risk of direct economic losses (Section 2.3).

2.1 Dynamic compound hazards risk

Risk is assumed to be the possible scene of the occurrence of a harmful event. From the previous studies, the type of risk could be classified into four categories: pseudo risk, probability risk, fuzzy risk, and uncertainty risk (Huang et al. (2018)). In the case of that we can estimate the probability distribution $p(x)$ (hazard potential) of the occurrence of a hazard with respect to its magnitude x, and we can estimate the relationship $f(x)$ (hazard vulnerability) between the magnitude and hazard level, a probability risk could be quantified as the expected value of economic losses, i.e.,

$$Risk = Hazard\ Potential \times Vulnerability. \tag{1}$$

Though these four types of risks have been investigated by many researchers, there is little research on dynamic compound hazards risk (Huang et al. (2018)). In this paper, we give a definition of dynamic compound hazards risk and illustrate how to assess this kind of risks.

The compound risk is a scene in the future associated with some adverse incident caused by cascading hazards systems, where there are strong connections between different hazards and the hazard level is influenced by many indicators. Furthermore, Huang (2015) mentioned that it could extent to *dynamic compound risk* if the impact of occurrence time on risk assessment has been taken into consideration. To evaluate the compound hazards risk, the most important things are to estimate probability distribution $p(x)$ of the occurrences of compound hazards by using probability models, and the input-output relationship $f(x)$ between the hazards level and losses by using fuzzy models. The compound risk, quantified as the economic losses of compound hazards, is given by Eq. 2.

$$Risk = \int p(x) \cdot f(x)\, dx = \sum_{j=1}^{J} p(x; att_j) \cdot f(x; att_j), \tag{2}$$

where vector att_j denotes the hazards indicator for different index j and reflects the internal attribute changes of compound hazards. For example, the Typhoon-Rainstorm risk is influenced by different indicators att = (compound hazards level, economic losses) and the dynamic compound risk can be assessed by integrating conditional probability distribution $p(x; att)$ with hazards vulnerability $f(x; att)$ of Typhoon-Rainstorm hazards.

2.2 VFS-IEW dimension reduction model

For the compound hazards risk assessment system, the randomness and fuzziness caused by multi-indicators evaluation should be dealt with properly. Variable fuzzy sets theory (VFST), which deals with randomness and fuzziness, provide an appropriate tool for solving the compound hazards level evaluation. The Variable Fuzzy Sets-Information Entropy Weight (VFS-IEW) dimension reduction model has been proposed in this section.

In this paper, we define interval $I_0 = [a, b]$ as the attracting sets of variable fuzzy sets (VFS) U and extends I_0 to interval $I = [c, d]$ on the real axis. For $u \in U$, the elements in interval I_0 satisfy $\mu_A(u) > \mu_A^c(u)$. In VFST, $\mu_A(u)$ denotes the relative membership degree (RMD) and the core idea is to determine the RMD of each sample point by transferring fuzzy set U into real value. Wang et al. (2014) has defined the balance boundaries matrix and illustrates the calculation of RMD as a complicated and time consuming problem. We apply the balance boundaries matrix $M = \{M_{rl}\}$ (shown in Eq. 3) to locate the eigenvalue x and defines the relative membership degree functions (shown in Eq. 4) to evaluate the comprehensive value of compound hazards level.

$$M = \frac{L-l}{L-1} a_{rl} + \frac{l-1}{L-1} b_{rl} = (M_{rl}), \tag{3}$$

where r stands for the assessment indicator set, $r = 1, 2, \ldots, R$, l denotes the comprehensive level, $l = 1, 2, \ldots, L$. Compared with relative locations of sample points and parameter M_{rl}, RMD calculation can be solved by the ratio $\frac{x-a}{M-a}$, i.e.

$$\begin{cases} \mu_A(u) = 0.5[1 + \left(\frac{x-a}{M-a}\right)^p] & x \in [a, M] \\ \mu_A(u) = 0.5[1 - \left(\frac{x-a}{c-a}\right)^p] & x \in [c, a] \end{cases}. \tag{4}$$

It can be seen that the RMD is affected by hyper-parameter p and the position between sample point x with parameters a, b, c, d, and M. In this paper, the characteristics for different locations of x with respect to the class interval u have been used to classify RMD calculation: judge whether the location of x is in the lowest or highest grade of the class interval u or not. Fig.1-Fig.3 have shown three types of RMD calculation and the detailed induction can be referred by Fang et al. (2019).

$$\begin{cases} \mu_A(u)_1 = [\mu_A(u)_{11} \quad \mu_A(u)_{12} \quad 0 \quad \cdots \quad 0] \\ \mu_A(u)_{11} + \mu_A(u)_{12} = 1 \\ 0.5 \leq \mu_A(u)_{11} \leq 1 \\ 0 \leq \mu_A(u)_{12} \leq 0.5 \end{cases}. \tag{5}$$
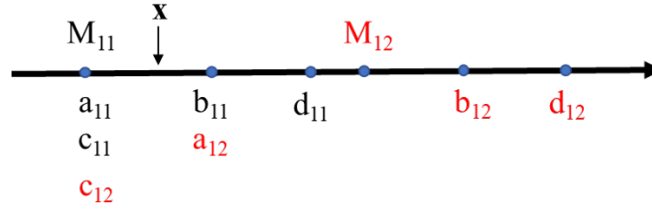
Figure 1. Lowest: the position between the random point $u_1{}^t = x$ with parameter M_{11} and zones $[a_{11}, b_{11}]$, $[c_{11}, d_{11}]$.
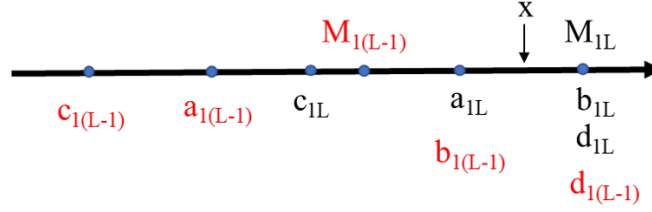

Figure 2. Highest: the position between the random point $u_1{}^t = x$ with parameter M_{1L} and zones $[a_{1L}, b_{1L}]$, $[c_{1L}, d_{1L}]$.

$$\begin{cases} \mu_A(u)_1 = [0 \quad \cdots \quad 0 \quad \mu_A(u)_{1(L-1)} \quad \mu_A(u)_{1L}] \\ \mu_A(u)_{1(L-1)} + \mu_A(u)_{1L} = 1 \\ 0.5 \leq \mu_A(u)_{1L} \leq 1 \\ 0 \leq \mu_A(u)_{1(L-1)} \leq 0.5 \end{cases} \tag{6}$$
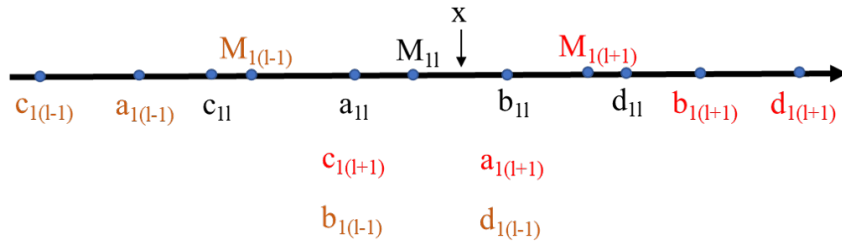

Figure 3. Middle: the position between the random point $u_1{}^t = x$ with parameter M_{1l} and zones $[a_{1l}, b_{1l}]$, $[c_{1l}, d_{1l}]$.

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$$
\begin{cases}
\mu_A(u)_1 = [0 \quad \cdots \quad 0 \quad \mu_A(u)_{1(l-1)} \quad \mu_A(u)_{1l} \quad \mu_A(u)_{1(l+1)} \quad 0 \quad \cdots \quad 0] \\
\mu_A(u)_{1(l-1)} + \mu_A(u)_{1(l+1)} = 0.5 \\
0 \le \mu_A(u)_{1(l-1)} \le 0.5 \\
0 \le \mu_A(u)_{1(l+1)} \le 0.5
\end{cases}
. \tag{7}
$$

Following the previous works by Kwakernaak (1978) and Chen (2006), the proposed variable fuzzy set dimension reduction model can be constructed by Eq. 8. It indicates that the proposed model is affected by hyper-parameter α, β and the multi-indicators are transferred into a single degree value so as to obtain the comprehensive assessment results.

$$
\begin{cases}
\nu_A(u)_l = [1 + (\frac{\sum_{r=1}^{R}[\omega_r(1-\mu_A(u)_{rl})]^\alpha}{\sum_{r=1}^{R}[\omega_r\mu_A(u)_{rl}]^\alpha})^{\frac{\beta}{\alpha}}]^{-1} \\
\nu_A^o(u)_l = \frac{\nu_A(u)_l}{\sum_{l=1}^{L}\nu_A(u)_l} \\
H = (1 \quad 2 \ldots L) \cdot (\nu_A^o(u)_l)^T
\end{cases}
. \tag{8}
$$

where $\nu_A^o(u)$ is the normalized process of RMD and H is the comprehensive value (a real value, can be transferred to hazards level). Further, the weight of indicators in this VFS-IEW model can be calculated by information entropy weight (Liu et al. (2010)) (shown in Eq. 9).

$$
\begin{cases}
f_r{}^t = u_r{}^t / \sum_{t=1}^{T} u_r{}^t \\
h_r = -1/lnn \cdot \sum_{t=1}^{T}(f_r{}^t ln f_r{}^t) \\
\omega_r = (1-h_r)/(R - \sum_{r=1}^{R} h_r)
\end{cases}
. \tag{9}
$$

We now present the main steps of VFS-IEW model and the corresponding algorithm (shown in Algorithm 1) as follows.

- **Step-1:** Initialize the variable fuzzy sets and the balance boundaries M_{rl}.

- **Step-2:** Repeat the relative membership degree calculation.

- **Step-3:** Calculate the information entropy weight ω_r.

- **Step-4:** Return the comprehensive degree value.

2.3 Dynamic risk assessment model

In order to assess the dynamic risk of compound hazards, especially when the recorded data sets are incomplete, information diffusion method (IDM) which belongs to fuzzy sets theory can be used to extract useful underlying information from the samples to estimate the relationships behind the incomplete data. According to the research by Huang (1997), normal information

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Algorithm 1 VFS-IEW Dimension Reduction Model for Compound Hazards

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Input:

1: The assessment object set $D = \{U_t = (u_r)^t, r = 1, 2, \cdots, R | t = 1, 2, \cdots, T\}$, where u_r is the eigenvalue;

2: Assessment criteria matrix, $V = \{(v_{rl}), r = 1, 2, \cdots, R; l = 1, 2, \cdots, L\}$.

Output:

Comprehensive value of compound hazards level.

3: Identification of attracting sets $I_{ab} = ([a, b]_{rl})$ and the extended intervals $I_{cd} = ([c, d]_{rl})$ based on assessment criteria matrix V;

4: Define the balance boundaries matrix $M = \{(M_{rl}), r = 1, 2, \cdots, R; l = 1, 2, \cdots, L\}$ by Eq. 3;

5: Calculate the information entropy weight ω_r by Eq.9;

6: **for** $t = 1$ to T, each U_t **do**

7:      **for** each $u_r{}^t \in U_t$ **do**

8:          **if** $u_r{}^t$ locates in the lowest grade of the class interval I_{ab}, i.e., $a_{r1} < u_r{}^t < b_{r1}$ **then**

9:              RMD $\mu_A(u)_r{}^t$ has the expression given by Eq.5;

10:          **else if** $u_r{}^t$ locates in the highst grade, i.e., $a_{rL} < u_r{}^t < b_{rL}$ **then**

11:              The value of RMD $\mu_A(u)_r{}^t$ is given by Eq.6;

12:          **else**

13:              RMD $\mu_A(u)_r{}^t$ have the expression given by Eq.7;

14:          **end if**

15:      **end for**

16:      The relative membership matrix of each sample can be denoted as $\mu_A(u)^t = (\mu_A(u)_{rl}{}^t)$;

17:      Combine $\mu_A(u)^t$ with weights ω_r and integrate the ranking level, the comprehensive degree value for each sample is given by Eq.8.

18: **end for**

---

diffusion function $\mu(X^t{}_i; S_o)$ (shown in Eq. 10) is more powerful to improve the precision of estimators. So, this paper adapts normal information diffusion estimator to approximate the dynamic compound hazards risk as follows.

$$\mu(X^t{}_i; S_o) = \prod_{o=1}^{3} exp[-\frac{(x_{ot} - s_o)^2}{2h_s^2}], \ i = 1, \cdots, n; \ t = 1, 2, \cdots, T. \tag{10}$$

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$$h = \begin{cases} 0.6841(b-a), & for \quad n = 5; \\ 0.5404(b-a), & for \quad n = 6; \\ 0.4482(b-a), & for \quad n = 7; \\ 0.3839(b-a), & for \quad n = 8; \\ 2.6581(b-a)/(n-1), & for \quad n \geq 9. \end{cases} \tag{11}$$

$$where \quad b = \max_{1 \leq i \leq n} \{x_i\}, \quad a = \min_{1 \leq i \leq n} \{x_i\}.$$

9

where T is the different month value, S_o denotes monitor space, and h_s is the diffusion coefficient. Based on this normal estimator, the research by Huang (2002) has shown how to determine the coefficients (shown in Eq. 11) and the discrete probability density function can be estimated by matrix $P = \{p_{jk}\}$.

$$p_{jk} = \frac{\sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k)}{\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k)}, j = 1, 2, \cdots, J; k = 1, 2, \cdots, K. \tag{12}$$

where u_j and v_k are the hazard indicator vectors. Further, the conditional probability distribution of the given compound hazards risk indicator u has the expression of Eq. 13:

$$p_{v|u_j}(v_k|u_j) = \frac{p_{jk}}{\sum_{k=1}^{K} p_{jk}}, k = 1, 2, \cdots K. \tag{13}$$

For the two dimensional input risk indicator set (time and hazard level value) $A = \{(x_{1t}, x_{2t}) | t = 1, 2, \cdots, T\}$ with diffusion function $\mu_A(u_j, v_k)$, the fuzzy relationship (vulnerability distribution) between A and fuzzy output (economic losses indicator f_m) $B = R_f$ can be estimated by membership function $\mu_B(f_m)$:

$$\mu_B(f_m) = \max_{\substack{u_j \in U \\ v_k \in V}} \{\min\{\mu_A(u_j, v_k), R_f\}\}, m = 1, 2, \cdots, M. \tag{14}$$

where the fuzzy relationship model $R_f = \{(r_{jkm})\}$ (Eq. 15) is given by the three-dimension information diffusion matrix $\mu(X_t; u_j, v_k, f_m)$.

$$r_{jkm} = \frac{\sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k, f_m)}{\max_{1 \leq m \leq M} \sum_{i=1}^{n} \mu(X^t{}_i; u_j, v_k, f_m)}. \tag{15}$$

Then the weighted value $f(u_j, v_k)$, represented as vulnerability distribution, is defined as follows.

$$f(u_j, v_k) = \frac{\sum_{m=1}^{M} \mu_B(f_m) \cdot f_m}{\sum_{m=1}^{M} \mu_B(f_m)}, j = 1, 2, \cdots, J; k = 1, 2, \cdots, K. \tag{16}$$

Based on the VFS-IDM risk assessment model, the dynamic compound hazards risk (Direct Economic Losses) is shown in Eq. 17 where the risk is quantified as the expected value of conditional probability distribution and vulnerability distribution. The proposed algorithm, which can be used to deal with incomplete information risk assessment, is given by Algorithm 2.

$$Risk_{u_j} = \sum_{k=1}^{K} p_{v|u_j}(v_k|u_j) \cdot f(u_j, v_k) \tag{17}$$

3 Case Study

This section uses the Typhoon-Rainstorm compound hazards that occurred in Shenzen, China, as an example to show how the proposed VFS-IEM-IDM model can be used to dynamically assess the risk of compound hazards. Shenzhen is located in the east bank of the Zhujiang River and is surrounded by Daya Bay and Dapeng Bay, where the climate is a subtropical maritime

Algorithm 2 VFS-IDM Dynamic Risk Assessment Model of Compound Hazards

Input:

1: Sample set $D = \{X^t_i = (x^t_{1i}, x^t_{2i}, x^t_{3i})|i = 1, 2, \cdots, n; t = 1, 2, \cdots, T\}$, where $x^t_{oi}, o = 1, 2, 3$ is the related factor of compound hazards (results given by Algorithm 1);

2: Universes of monitor space $S = \{(s_{ol}), l = 1, 2, \cdots, L|o = 1, 2, 3\}$, where the length L varies from different universes;

3: Coefficients of diffusion function $H = (h_1, h_2, h_3)$.

Output:

Dynamic compound hazards risk.

4: Identification of the comprehensive value of compound hazards level by VFS-IEW;

5: **for** Sample index $i = 1$ to n, each X^t_i **do**

6:     Based on the universes of monitor space, employing the normal diffusion function in Eq. 10 to construct information diffusion matrix of sample D;

7: **end for**

8: Estimate the joint and conditional probability distribution based on Eq. 12 and Eq. 13;

9: Determine the input-output sets and model the fuzzy relationship based on Eq.15, then estimate the vulnerability distribution by Eq.16;

10: The dynamic risk (Direct Economic Loss) of compound hazards can be quantified by Eq. 17.

and Typhoon-Rainstorms are undoubtedly the most frequently occurred hazards in Shenzhen. According to the collected data (see Table A1), from 1980 to 2016, on average, the directed economic losses of the Typhoon and Rainstorm hazards in Shenzhen exceeded 360 million RMB per year, the number of death was 3.4 deaths annually and about $149,000$ people were affected (Zhou et al. (2017)). The assessment results of the Typhoon-Rainstorm dynamic risk are the basis to determine whether or not the early warning systems are worked and implemented effectively.

Since the Typhoon-Rainstorm compound hazards are characeried by three indicators , (Maximum Daily Precipitation, Extreme Wind Intensity, Transformed Location Number), the variable fuzzy set dimension reduction model can be used to get more precise comprehensive hazard level. According to the Classification Standards of Rainstorm and Typhoon, this paper outlines This paper has outlined the index classification criteria (shown in Table 1, Guided by TYPHOON ONLINE and the explanation is shown in the Table A1)and the four types of), which is guided by Shenzhen Climate Bulletin (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/). We also classify the Typhoon-Rainstorm hazard levelcompound hazard level into four types, and which is related to our final results.

Based on expert experiences and the relevant government documents, the classification results of Typhoon-Rainstorm hazard level (H) in Shenzhen express type I as $H \in [1.5, 2)$, type II as $H \in [2, 2.7)$, type III as $H \in [2.7, 3.5)$, and type IV as $H \in [3.5, 4]$. This paper uses the dimension reduction model VFS-IEM (Algorithm 1) to get the comprehensive value H and transfers them into different hazard levels based on Typhoon-Rainstorm classification standards. According to the classification results

Table 1. Classification standards of Typhoon-Rainstorm hazards.

Indicators	Compound Hazards Level			
	I	II	III	IV
Maximum Daily Precipitation	(0,50)	(50,100)	(100,150)	(150,250)
~~Strong~~ Extreme Wind Intensity	(8,10.8)	(10.8,17.2)	(17.2,23.6)	(23.6,30)
Transformed Location Number	(0,2)	(2,5)	(5,8)	(8,10)

shown in Table 1, the interval criterion matrix can be expressed as

$$
I_{ab} = \begin{bmatrix} (0,50) & (50,100) & (100,150) & (150,250) \\ (8,10.8) & (10.8,17.2) & (17.2,23.6) & (23.6,30) \\ (0,2) & (2,5) & (5,8) & (8,10) \end{bmatrix} = ((a,b)_{rl}),
$$

$$
I_{cd} = \begin{bmatrix} (0,100) & (0,150) & (50,250) & (100,250) \\ (8,17.2) & (8,23.6) & (10.8,30) & (17.2,30) \\ (0,5) & (0,8) & (2,10) & (5,10) \end{bmatrix} = ((c,d)_{rl}),
$$

and the balance boundaries matrix M is defined in Eq. 18

$$
M = \begin{bmatrix} 0 & 66.7 & 133.3 & 250 \\ 8 & 12.9 & 21.5 & 30 \\ 0 & 3 & 7 & 10 \end{bmatrix} = (M_{rl}). \tag{18}
$$

Then the relative membership degree matrix can be calculated by Eqs. 5, 6 and 7 respectively.

Taking sample point ~~$(MP = 33.4, SWI = 18, TL = 9)$~~ $(MDP = 33.4, EWI = 18, TL = 9)$ for example, the relative membership degree matrix is expressed by Eq. 19 where the matrix value represents the probability of each indicator belonging to the different compound hazards level.

$$
\mu_A(u) = \begin{bmatrix} 0.666 & 0.334 & 0.000 & 0.000 \\ 0.000 & 0.438 & 0.593 & 0.063 \\ 0.000 & 0.000 & 0.333 & 0.667 \end{bmatrix}. \tag{19}
$$

To get the comprehensive hazard level, the information entropy method can be used to get the weight of each indicator ω, which implies that the Maximum Daily Precipitation and Location play the main role in determining the Typhoon-Rainstorm hazards level.

$$
\omega = \begin{pmatrix} 0.43 & 0.19 & 0.39 \end{pmatrix}. \tag{20}
$$

Then by Algorithm 1, the comprehensive value of the Typhoon-Rainstorm hazards level ~~$(MP = 33.4, SWI = 18, TL = 9)$~~ $(MDP = 33.4, EWI = 18, TL = 9)$ is $H = 2.75$ when the hyper-parameter $\alpha = \beta = 1$, and $H = 2.18$ when the hyper-parameter

$\alpha = \beta = 2$. To be more general, this paper takes the average of $H = 2.75$ and $H = 2.18$ to obtain the final compound hazards level, i.e., $H = 2.47$, Type II. The results of other Typhoon-Rainstorm comprehensive hazard levels can be found in Appendix (see Table B1).

Table 2: Transformed Typhoon-Rainstorm hazard data sets in Shenzhen.

Time	Transformed Time (T)	Comprehensive Hazard Level (H)	Direct Economic Loss (L)
20090627	176	2.72	0.3819
0719	198	3	1.352
0915	254	3.74	1.3750
20100724	203	2.32	0.2571
0912	251	2.49	0.4450
0922	261	2.74	0.9831
20110624	173	1.93	0.0765
0930	269	2.72	0.4013
20120630	179	2.31	0.2895
0724	203	3.95	2.48
0817	226	2.56	0.7648
20130615	164	1.94	0.1527
0702	181	1.99	0.1894
0802	211	1.53	0.0452
0814	223	2.13	0.1423
0922	261	3.06	1.2351
20140718	197	1.83	0.0841
0916	255	2.48	0.7682
0823	232	2.92	0.7410
1004	273	2.96	0.8352
20160802	211	3.68	2.1521
0818	227	1.88	0.0251
1018	287	2.28	0.2362
1021	290	3.11	0.9341
20170612	161	3.67	2.058
0723	202	2.11	0.2461
0823	232	2.46	1.31
0827	236	3.2	1.613
0903	242	3.03	1.8872
1016	285	2.48	0.5902
20180606	155	2.47	0.6952

0718	197	1.58	0.0267
0811	220	2.45	0.5241
0916	255	3.93	2.226
20190703	182	1.49	0.0528
0811	210	3.02	0.8182
0824	233	2.9	0.8391
0902	241	1.8	0.0725

From Table 2, the sample observations on direct economic loss L (Billions) over each comprehensive compound hazards level H are written as

225    $Sample = \{(t_1,d_1,l_1),\ldots,(t_i,d_i,l_i),\ldots,(t_{38},d_{38},l_{38})\} = \{(172,2.72,0.3819),\ldots,(241,1.8,0.0725)\}.$

where t_i, d_i represents the time dimension of the Typhoon-Rainstorm hazard and the comprehensive value of the hazards level respectively, and l_i is the direct economic losses caused by the Typhoon-Rainstorm hazards. Then the diffusion coefficients can be calculated by Eq. 11, written as

$$\begin{cases} h_t = 2.6581 \cdot (290 - 155)/(38 - 1) = 10 \\ h_d = 2.6581 \cdot (3.95 - 1.37)/(38 - 1) = 0.19 \\ h_l = 2.6581 \cdot (2.48 - 0.0251)/(38 - 1) = 0.1764 \end{cases}.$$

230   Algorithm. 2 outlines how to use the information diffusion method to estimate the conditional probability and vulnerability distribution of the Typhoon-Rainstorm hazards. Then by the 2-dimensional normal diffusion estimator, the joint probability density function P (Eq. 21) and conditional probability function P_{con} (Eq. 22) can be evaluated. In this paper, we denote the monitor space $T = (t = 164, t = 194, t = 224, t = 254, t = 284)$ as months $(June, July, August, September, October)$ and $H = (d = 1.8, d = 2.4, d = 3.0, d = 3.6)$ as comprehensive hazards levels (I, II, III, IV).

$$235 \quad P = \begin{array}{c} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{pmatrix} I & II & III & IV \\ 0.059 & 0.046 & 0.007 & 0.036 \\ 0.076 & 0.052 & 0.051 & 0.014 \\ 0.063 & 0.116 & 0.090 & 0.019 \\ 0.019 & 0.086 & 0.087 & 0.041 \\ 0.002 & 0.073 & 0.060 & 0.002 \end{pmatrix} \qquad (21)$$

$$P_{con} = . \begin{array}{c} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{matrix} I & II & III & IV \\ \begin{pmatrix} 0.398 & 0.311 & 0.049 & 0.243 \\ 0.393 & 0.268 & 0.266 & 0.073 \\ 0.218 & 0.402 & 0.312 & 0.0689 \\ 0.080 & 0.370 & 0.373 & 0.177 \\ 0.012 & 0.539 & 0.437 & 0.012 \end{pmatrix} \end{matrix} \tag{22}$$

From the results above, it can be seen that the Typhoon-Rainstorm hazard level of III occur more frequently and they are most likely to occur in August and September.

The vulnerability distribution $f(x)$ between the comprehensive value H and the direct economic losses L over the time dimension T can be calculated by the 3-dimension diffusion estimator. The fuzzy relationship which takes time dimension T, hazards level H as input and the loss L as the output can be denoted as matrix R_f.

$$R_f = \begin{pmatrix} & & l=0.1 & 0.4 & 0.7 & 1.0 & l=1.3 & 1.6 & 1.9 & 2.2 \\ & \begin{array}{c} d=1.8 \\ d=2.4 \\ t=164 \\ d=3.0 \\ d=3.6 \end{array} & \begin{bmatrix} 0.80 & 0.43 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.15 & 0.42 & 0.57 & 0.15 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.05 & 0.13 & 0.04 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.08 & 1.00 & 1.00 \end{bmatrix} \\ & \begin{array}{c} d=1.8 \\ d=2.4 \\ t=194 \\ d=3.0 \\ d=3.6 \end{array} & \begin{bmatrix} 1.00 & 0.49 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.45 & 0.82 & 0.10 & 0.01 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.04 & 0.22 & 0.28 & 1.00 & 0.94 & 0.02 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.13 & 0.43 \end{bmatrix} \\ & & \vdots \\ & \begin{array}{c} d=1.8 \\ d=2.4 \\ t=284 \\ d=3.0 \\ d=3.6 \end{array} & \begin{bmatrix} 0.02 & 0.03 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.35 & 1.00 & 0.61 & 0.08 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.15 & 0.56 & 1.00 & 0.19 & 0.03 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.01 & 0.03 & 0.01 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{pmatrix}.$$

The discrete vulnerability distribution in terms of the direct economic loss is evaluated by Eq. 16 and the results are shown in Eq. 23. It can be seen that Shenzhen, most of the economic losses caused by the Typhoon-Rainstorm hazards is concentrated in August and September.

$$
f(x;t,h) = . \begin{array}{c} \\ June \\ July \\ August \\ September \\ October \end{array} \begin{array}{cccc} I & II & III & IV \\ \begin{pmatrix} 0.20 & 0.02 & 0.00 & 0.00 \\ 0.24 & 0.04 & 0.00 & 0.00 \\ 0.15 & 1.13 & 1.67 & 1.90 \\ 0.05 & 0.55 & 2.67 & 2.62 \\ 0.01 & 0.02 & 0.00 & 0.00 \end{pmatrix} \end{array} \tag{23}
$$

Dynamic compound hazards risks can be quantified as the expected value of hazards influence and the result is shown as Eq. 24 where the elements of vector denotes the estimated economic losses caused by the Typhoon-Rainstorm hazards in different months.

$$
Risk = \begin{pmatrix} 0.08582 & 0.10504 & 1.1372 & 1.66715 & 0.0109 \end{pmatrix}. \tag{24}
$$

4 Discussion

Dimension reduction model VFS-IEW presents the comprehensive value of compound hazards level, but the relationship between hazards level and the indicators are not clear. To find more information from the results of VFS-IEM, this paper has built a predictive model to shield the light on compound hazards relationship and predict the Typhoon-Rainstorm hazards level. Since the compound hazards level is an ordinal data (monotone trend and proportional odds), the cumulative logistic model (shown in Eq. 25) can be used to predict the compound hazards level. The probabilities of different order categories given by cumulative logistic model are

$$
P(Y \leq j \mid x) = \pi_1(x) + \cdots + \pi_j(x), \quad j = 1, \ldots, J.
$$

According to the research by Alan (1980), the cumulative logistic model is defined as

$$
\text{logit}[P(Y \leq j \mid x)] = \log \frac{P(Y \leq j \mid x)}{1 - P(Y \leq j \mid x)} = \text{logit}[P(Y \leq j \mid x)] = \alpha_j + \beta^T x, \quad j = 1, \ldots, J-1. \tag{25}
$$

The Typhoon-Rainstorm hazard level prediction problem can be solved by using the VAGM package (Thomas (2010)) and the result is given by

$$
\text{logit}[P(Y \leq j \mid x)] = 5.07(7.32, 11.15) - 0.12MPMDP - 0.66SWIEWI - 0.91TLN, \tag{26}
$$

where the different intercepts denote the different main-effects of hazard levels compared to the reference category, i.e., hazard level IV. The rationality of this model is judged by LR-test (p-value<0.001) and the predictive performance $R^2 = 0.898$ which shows that the model is well fitted and can be used as the compound hazards prediction.

17

One advantage using the information diffusion technique to assess the risk of an compound hazards is that it does not need to know (1) the distribution type of the population from which given samples are drawn, (2) the function form of the causal relationship, which are constructed by the joint probability distribution and the vulnerability distribution. Moreover, researchers have done simulation study on IDM and demonstrate the benefit of information distribution for probability estimation (Huang (2000); Li et al. (2012)) by minimizing the mean integrated square error (Kullback-Leibler divergence error) between the estimator and the true density. The performance of this non-parametric estimation procedure is studied well by Huang (2000) which shows the work efficiency is about 35% higher than histogram estimator (HE) and the performance is improved to reduce the error by 23.2% when data sets are incomplete. Therefore, the assessed compound hazards risk is more reliable and accurate.

For the dynamic risk assessment of Typhoon-Rainstorm hazards, this paper provides extensive assessment results. From the dimension reduction model VFS-IEM, this paper shows that the probability of the occurrences of type II and III hazard levels is highest in Shenzhen. The emergency management department should prepare more effective emergency plans in advance to reduce the occurrences of the secondary hazards. From the dynamic risk assessment model VFS-IEM-IDM , it can be found that the hazards occurrence probability of different hazard levels is different and the type hazards of II and III hazards level are most likely to occur in August and September. Also, considering the occurrence of different hazard level for each months, the probability of hazard level I occurring in June and July is the highest, and the hazard level II mostly occurs in August and October, and the type III hazard level is most likely to occur in September. From the perspective of hazard losses, the different direct economic losses caused by the Typhoon-Rainstorms of the same hazard level in each month indicates that the impacts of the Typhoon-Rainstorm hazards on the economy are not the same. Besides, for the same month, the influence of economic loss decreases gradually when the compound hazards level rises. This indicates that the capacity of Typhoon-Rainstorm hazard resistance in Shenzhen is reliable, and the ability to copy with the sudden compound hazards are relatively strong under the existing emergence system. The dynamic compound hazards risk of Typhoon-Rainstorm hazards in Shenzhen shows that the risk value of this compound hazards in each month is different and the highest risk value appears in August and September. On average, the occurrence of Typhoon-Rainstorm hazards brought Shenzhen 114 million RMB and 167 million RMB losses in these two months respectively, which is in line with the actual situation.

5 Conclusions

Risk assessment is an important step in emergency management, but few research discusses the uncertainties of compound hazards evaluation and considers dynamic risk assessments when the data sets are incomplete. In this paper, we first present the definition of dynamic compound hazards risk, and then Variable Fuzzy Set (VFS) theory is employed to evaluate the relative membership degree, and Information Entropy Method (IEM) is applied to obtain the weights of criteria indicators for compound hazards level evaluation. Based on the results obtained by VFS-IEM, we apply the information diffusion method (IDM) to estimate the conditional probability distribution and vulnerability distribution with the hazard occurrence time and the corresponding losses. Then the dynamic risk is assessed using fuzzy probabilistic risk to improve the accuracy of risk

assessment. The innovations of this paper are: (i) Based on the definition of compound hazards risk, we take time dimension into consideration to introduce the concept of dynamic compound hazards risk. (ii) Considering that compound hazards have different measurement indicators for the comprehensive evaluation, a hybrid model of Variable Fuzzy Sets and the Information Entropy Method has been proposed to improve the accuracy of compound hazards level evaluation. (iii) According to the concept of dynamic compound hazards risk, we apply Information Diffusion Method to estimate the conditional probability distribution and the vulnerability distribution using the comprehensive hazard levels, hazards occurrence time and the losses of compound hazards. The proposed model VFS-IEM-IDM can be used to deal with the problem of incomplete and limited information in dynamic risk assessment. (iv) By evaluating the expected value of the conditional probability distribution and the vulnerability distribution, we quantify the Typhoon-Rainstorm dynamic risk which shows that the occurrence of Typhoon-Rainstorm hazards brings Shenzhen 114 million RMB and 167 million RMB losses in August and September respectively. These risk assessment results obtained by VFS-IEM-IDM model are in line with the actual situation and ~~may~~ can be used to guide the emergency management in Shenzhen ~~, which also shows the potential of VFS-IEM-IDM being applied to~~

Dynamic risk assessment is a relatively new topic and we have proposed a hybrid model to assess the compound hazards. risk, but there are somewhere need to be improved. On one hand, the weight calculation of different types of hazards indicators is subjective and the results of vulnerability curve have not taken the changes in the internal attributes of the affected area into consideration. On the other hand, there are also some subjective issues regarding how to process the data sets, so maybe we can consider adopting a more scientific method to process the original data to obtain more scientific conclusions.

Author contributions. GWW and YLL conceived the research framework and developed the methodology. GWW was responsible for the code compilation, data analysis, graphic visualization. GWW and JJ had done the first draft writing. YLL managed the implementation of research activities and revised the manuscript. All authors discussed the results and contributed to the final version of the paper.

Competing interests. The authors declare that they have no conflict of interest.

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Appendix A: Data Source

For the Typhoon-Rainstorm dynamic compound hazards risk assessment, the useful data sets, collected from Meteorological Bureau of ShenZhen Municipality (http://weather.sz.gov.cn/qixiangfuwu/qihoufuwu/qihouguanceyupinggu/nianduqihougongbao/) and TYPHOON ONLINE (http://typhoon.nmc.cn/web.html), have been sorted out in Table A1. In this table, ~~MP~~ MDP denotes as Maximum ~~Precipitation, SWI denotes as Strong~~ Daily Precipitation, EWI denotes as Extreme Wind Intensity, DEL denotes as Direct Economic ~~. The~~ Loss, and the Transformed Location Number (TLN) ~~is also denoted as the landing location by~~ denotes as the Typhoon Landing Location which is determined by radio distance transform using expertise knowledge.

Table A1: Data sets of Typhoon-Rainstorm hazards in Shenzhen.

Hazards Number	Impact Time	~~MP~~ MDP (mm)	~~SWI~~ EWI (m/s)	Landing Location	~~Transformed Location~~ TLN	DEL (l
~~20090627~~ 0904	0627	67.3	16.8	Huizhou	8.5	0.3
0906	0719	80	27.3	Shenzhen	10	1.1
0915	0912	127.9	28	Taibei	6	1.0
~~20100724~~ 1003	0724	31.3	16.2	Zhanjiang	6.5	0.2
1010	0912	62.4	13.7	Quanzhou	3	0.3
1011	0922	51.9	15.8	Heyuan	7	0.2
~~20110624~~ 1105	0624	41.7	14	Yangjiang	4.5	0.0
1006	0930	~~117.3~~ 53.0	15.2	Wenchang	2.5	0.8
~~20120630~~ 1206	0630	33.6	16.8	Zhuhai	6.5	0.6
1208	0724	152.3	~~24.9~~ 23.9	Taishan	7	2.2
1213	0817	~~66.1~~ 46.1	13.5	Zhanjiang	3	0.9
~~20130615~~	0615	36.5	~~12.4~~ 8.4	Wenchang	4	0.3
1306	0702	38.6	~~13.9~~ 10.9	Zhanjiang	3	0.2
1309	0802	40.7	~~14.7~~ 10.7	Wenchang	3	0.0
1311	0814	47.8	14.2	Yangxi	3	0.6
1319	0922	72.4	21.6	Shanwei	8.5	1.1
~~20140718~~ 201409	0718	31.6	14.7	Wenchang	2.5	0.0
201415	0916	73.5	18.9	Xuwen	2.5	0.9
201517	0823	69.4	13.6	Shanwei	10	1.0
201522	1004	108.5	13.5	Zhanjiang	5.5	0.9
~~20160802~~ 201604	0802	166	19.2	Shenzhen	10	2.
201608	0818	45.5	9.1	Zhanjiang	5.5	0.0
201621	1018	117.6	12.3	Wanning	1.5	0.4

201622	1021	83.7	18.8	Shanwei	7.5	0.8
~~20170612~~ 1702	0612	161.8	~~23~~ 16.9	Shenzhen	10	2.1
1707	0723	33.4	~~18~~ 10.6	Xianggang	9	0.5
1713	0823	56.3	~~23.9~~ 23.4	Zhuhai	8.5	1.3
1714	0827	114.5	17.5	Jiangmen	8.5	1.7
1716	0903	82.4	14.4	Shanwei	7.5	0.9
1720	1016	40	20.3	Zhanjiang	7.5	0.7
~~20180606~~ 1804	0606	97.2	~~10.8~~ 8.8	Xuwen	8.5	0.9
1809	0718	50.7	11.1	Wanning	1.5	0.0
1816	0811	45.3	10.8	Yangjiang	7	0.5
1822	0916	173.5	30	Taishan	7.5	2.3
~~20190703~~ 1904	0703	48.8	11	Wanning	1.5	0.0
1907	0811	~~178.5~~ 99.1	14.1	Wenchang	5.5	0.9
1911	0824	~~97.6~~ 49.4	12.7	Zhangzhou	6	0.5
1914	0902	~~86.9~~ 52.2	11.3	Wanning	1	0.0

Appendix B: Comprehensive Compound Hazards Level

Based on the Dimension Reduction Model VFS-IEM, this paper takes the average of $\alpha = \beta = 1$ and $\alpha = \beta = 2$ to denote the final Typhoon-Rainstorm hazards level. The following Table B1 has shown that the whole results of comprehensive degree value.

Table B1: Comprehensive compound hazards level in ShenZhen

Time	$\alpha = \beta = 1$	$\alpha = \beta = 2$	Average Level (D)	Comprehensive Level
20090627	3.07	2.36	2.72	III
0719	3.34	2.65	3.00	III
0915	3.93	3.55	3.74	IV
20100724	2.67	1.96	2.32	II
0912	2.68	2.29	2.49	III
0922	3.02	2.45	2.74	III
20110624	2.12	1.73	1.93	I
0930	2.87	2.57	2.72	III
20120630	2.66	1.95	2.31	II
0724	3.97	3.93	3.95	IV
0817	2.8	2.32	2.56	II
20130615	2.08	1.79	1.94	I
0702	2.28	1.7	1.99	I
0802	1.65	1.4	1.53	I
0814	2.22	2.03	2.13	II
0922	3.44	2.67	3.06	III
20140718	1.93	1.73	1.83	I
0916	2.65	2.3	2.48	II
0823	3.19	2.64	2.92	III
1004	3	2.91	2.96	III
20160802	3.66	3.69	3.68	IV
0818	1.96	1.8	1.88	I
1018	2.52	2.03	2.28	II
1021	33.1	2.91	3.11	III
20170612	3.69	3.83	3.76	IV
0723	2.52	1.7	2.11	II

0823	2.89	2.03	2.46	II
0827	3.35	3.04	3.2	III
0903	3.22	2.83	3.03	III
1016	2.95	2	2.48	II
20180606	2.75	2.18	2.17	II
0718	1.57	1.45	1.51	I
0811	2.72	2.17	2.45	II
0916	3.87	3.98	3.93	IV
20190703	1.52	1.48	1.5	I
0811	3.25	2.79	3.02	III
0824	2.96	2.83	2.9	III
0902	1.93	1.67	1.8	I