Thank you very much for your comments.
Our answers are as below.

General comments:

[G-1]
It makes sense to show the results of the surface fitting in 3D in Figure 5. However, the uncertainty (confidence interval) is not illustrative in 3D. I suggest showing the uncertainty estimates for the fitted surface in 2D instead of 3D, like the visualization in panels (g) and (h).

[About the first general comment.]
To display more understandable and convincing pictures, we created figures in which the estimated uncertainty of TEC values in Figure 5 is projected onto 2-dimension as below.

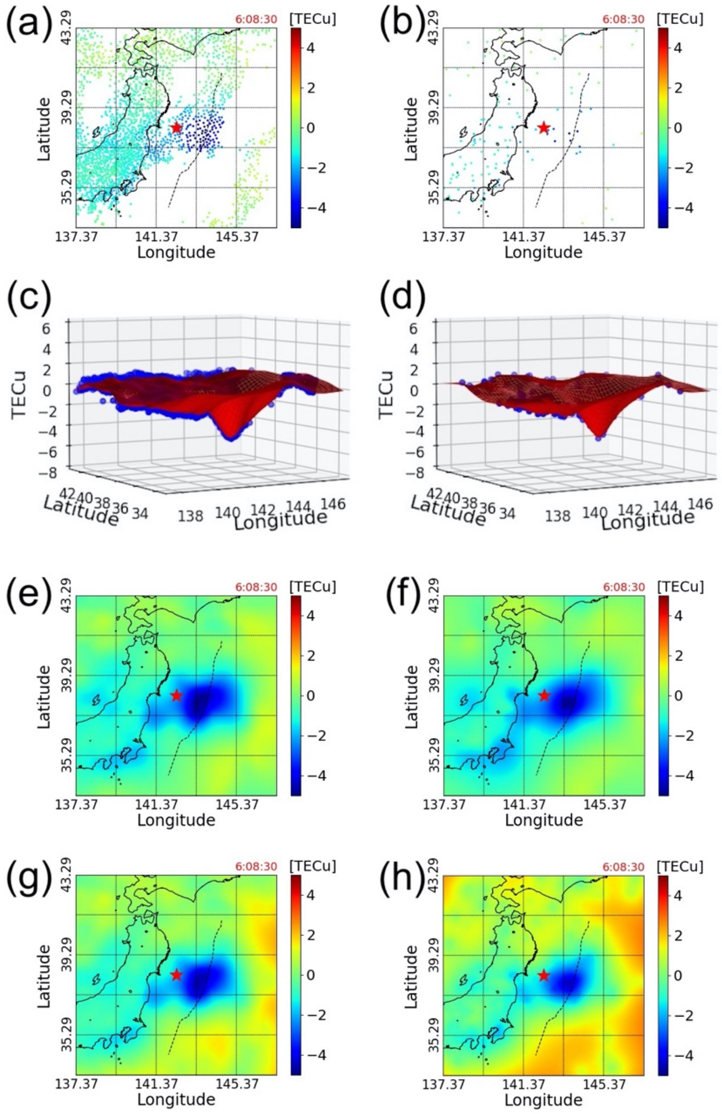

Figure 1：Left-hand side is for the full data and right-hand side is for the sparse data using only 5% of the GEONET receivers. (a) and (b) are measured TEC data. (c) and (d) are measured TEC data (blue dots) and the fitting surface (red surface). (e) and (f) are 2D projection of the fitting surface. (g) and (h) are 2D projection of the 99% one-sided confidence interval of the fitting surface. The fitting surface is computed using the INLA-SPDE method. The black dashed line indicates the Japan Trench.

[G-2]

The authors claim that their surface fitting method reproduces the shape of the TIH in the same region, almost like the initial tsunami source by Saito et al. (2011b). The authors should better specify to what extent (area, wave height) the shape is reproduced. Do uplift or subsidence areas of the initial tsunami match the TIH? In how far are TECu and vertical displacement in meters comparable? Does the TIH mimic the initial tsunami shape? Here the authors could visualize better by comparing the initial tsunami source by Saito et al. (2011b) directly with their results. How does it compare to the initial tsunami source presented by other authors?

[About the second general comment.]

Tatsuhiko SAITO, the first author of Saito et al. (2011b), shared the initial tsunami height data used to depict the figure in their paper.

Using the data, we compared the TEC depression with the initial tsunami shown in the below figures.

Firstly, we created the contours of the initial tsunami. From the figures in the first row, the region where the TEC deepest reaches its minimum is almost the same with the region in which the initial tsunami shows its peak value.

Next, we show the initial tsunami and the TEC depression at three different times at the fixed latitude and longitude where the initial tsunami reaches its peak value,

From the figures in the second and third rows, it is clear that the TIH deepest location and the initial tsunami highest points are the same at the fixed latitude and longitude.

In the next figure, the time series (from 5:46:30 to 6:16:30 (UTC)) data of the TIH is shown with the initial tsunami shape at the fixed latitude and longitude.

In the figure, the time series of the observed data in the atmosphere shows a strong correlation with the initial tsunami in its shape for the first time.

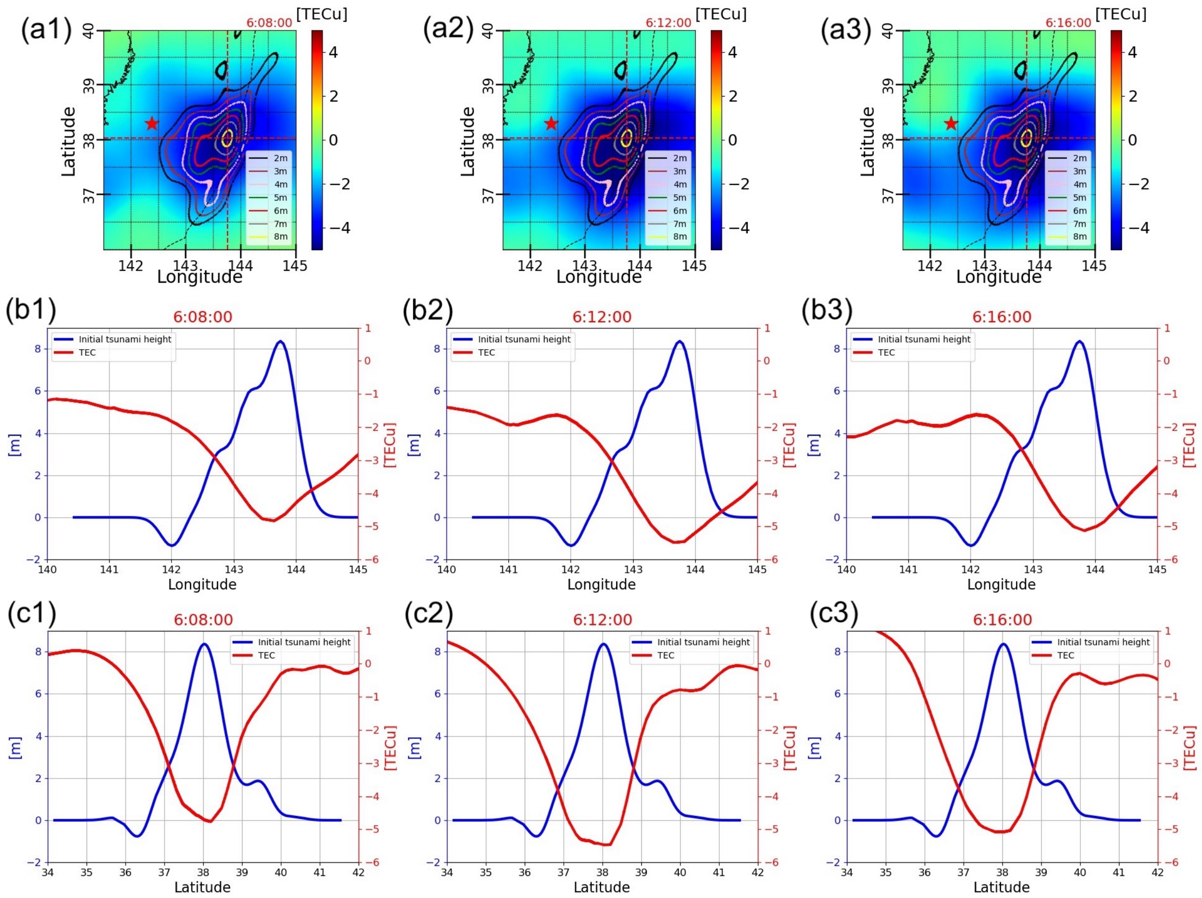

Figure2: (a1) – (a3) are TEC values and the initial tsunami contours. (b1) – (b3) are the TIH shape and the initial tsunami shape at the fixed latitude where the initial tsunami reaches its peak value. (c1) – (c3) are the TIH shape and the initial tsunami shape at the fixed longitude where the initial tsunami reaches its peak value.

The initial tsunami source presented by other researchers, shown in Ohta (2012) and Takagawa (2012), also shows that the initial tsunami region overlaps the TIH region as in Figure 3. In Ohta (2011), they used the algorithm developed by Okada [1985] to compute the initial sea-surface displacement based on their fault-determination procedure. On the other hand, in Takagawa (2012), they investigated the effect of the rupture process on a tsunami source inversion. Estimated sea-surface elevation of tsunami source by their inversion method based on the assumption of finite rupture velocity of 2 km/sec is shown.

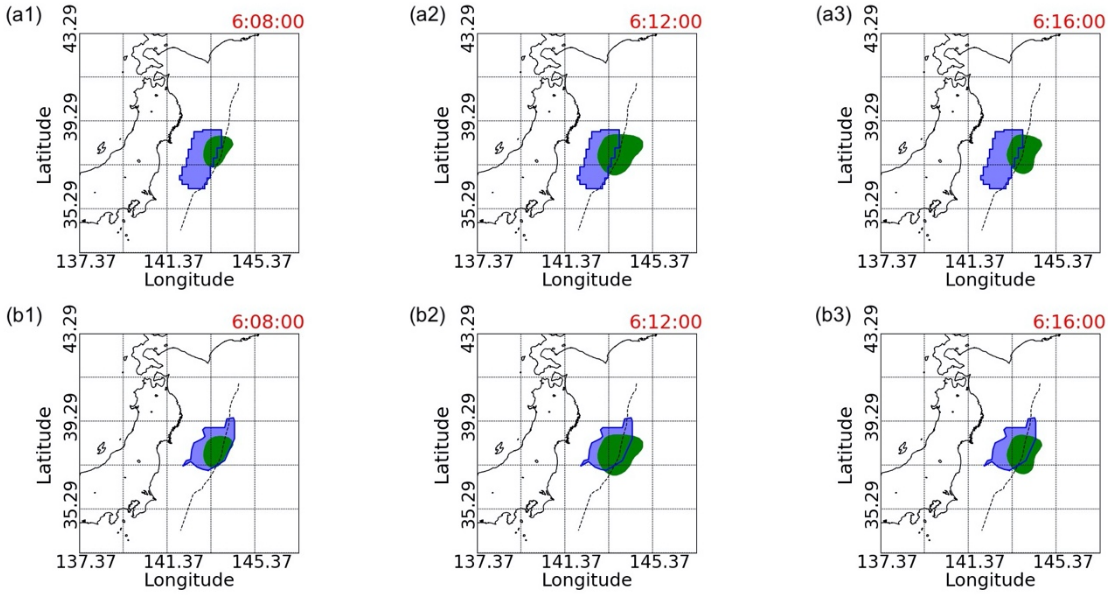

Figure3: TIH and the initial tsunami comparison. In the first row, the initial tsunami estimated by Ohta (2012) is shown. In the second row, the initial tsunami estimated by Takagawa (2012) is shown. In both cases, the TIHs with TEC values less than -4 are described.

[Reference]
Ohta (2012):
Ohta, Yusaku, et al. "Quasi real‐time fault model estimation for near‐field tsunami forecasting based on RTK‐GPS analysis: Application to the 2011 Tohoku‐Oki earthquake (Mw 9.0)." Journal of Geophysical Research: Solid Earth 117.B2 (2012).

Takagawa (2012):
Takagawa, Tomohiro, and Takashi Tomita. "Effects of rupture processes in an inverse analysis on the tsunami source of the 2011 Off the Pacific Coast of Tohoku earthquake." The Twenty-second International Offshore and Polar Engineering Conference. OnePetro, 2012.

[G-3]
The authors should show a couple of other random cases with sparse data for the surface fitting. It will be visually beneficial for their work to demonstrate that their method could enhance future warning systems. Since other randomly chosen datasets should deliver similar results, the authors should demonstrate it in a figure.

[About the third general comment.]
We showed the averaged results of the sparse data experiments in our manuscript.

To show the fitting method can detect the surface in each sparse data experiment in our analysis, several sparse data experiment results are described as below.

If there is no data point in the area where TIH is formed, it is impossible to capture the decrease in TEC, but if there are several observation points in the TEC decrease region, the fitting method works properly.

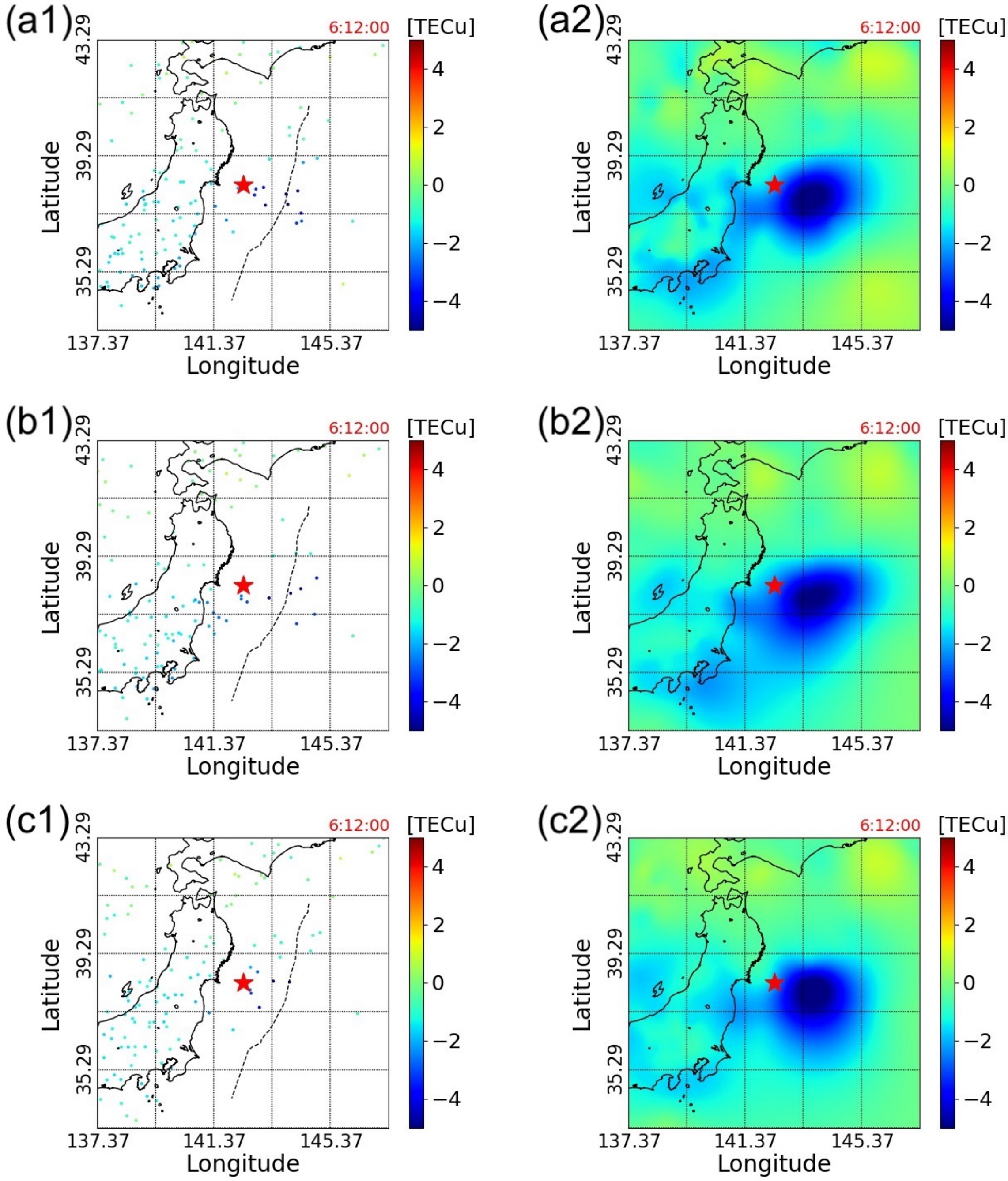

Figure4: Three different sparse data and its fitting surface mapped onto 2 dimension.

TIH overlapping the initial tsunami:

[T-1]
The representation of the initial tsunami is problematic. It is not clear how the authors have chosen the sea-level threshold value to define the area of the initial tsunami. The authors should represent the initial tsunami wave in m concerning the sea level of the event. Tsunamis may contain depression and elevation features in the wave field, which must be shown in the figure. There are many published source inversions for the 2011 Tohoku-Oki tsunami source, and the authors should compare the TIH to other published source inversions (e.g. Ammon et al. 2011, Wei et al. 2012, Yue and Lay 2013); otherwise, it seems they have chosen Saito et al. (2011b) that fits best to their results.

[About the first comment related to TIH overlapping the initial tsunami]
In our manuscript, we used the initial tsunami presented in Saito 2011 to show the overlap with TIH.
The region where the wave height of the initial tsunami is higher than 2 m is shown in blue as the initial tsunami range.
In order to compare the tsunami with TIH, including the shape of the tsunami, the data was provided by Saito and the comparison is shown in Figure2 and Figure5.
Here is a comparison of them using contour lines.
In addition, the shape of the initial tsunami and the shape of the TIH were compared when fixed at the latitude and longitude where the initial tsunami height is the highest.

Moreover, the time series of TIH for fixed latitudes and longitudes are also presented.
From these results, it is inferred that the TIH has information about the initial tsunami.

The papers that the reviewer gave in the comment are not about the initial tsunami height but about the distribution of slips, so it cannot be used for comparison with TIH as Saito 2011 did.

On the other hand, Ohta (2012) and Takagawa (2012) are papers that mention the distribution of initial tsunami heights.

By referring to the figures in these two papers, we can compare the initial tsunami with TIH, and we can confirm that there is an overlap between TIH and the initial tsunami as shown in Figure3.

[T-2]
Moreover, it is essential to explore how the TIH overlaps the initial tsunami. The initial tsunami wavefield values should be compared directly to the TEC values.

[About the second comment related to TIH overlapping the initial tsunami]
In order to make a detailed comparison between the TIH and the initial tsunami, the time series of the shape of the TIH and the shape of the initial tsunami are compared.
The comparison is made by fixing the latitude and longitude at the position where the initial tsunami is the highest.

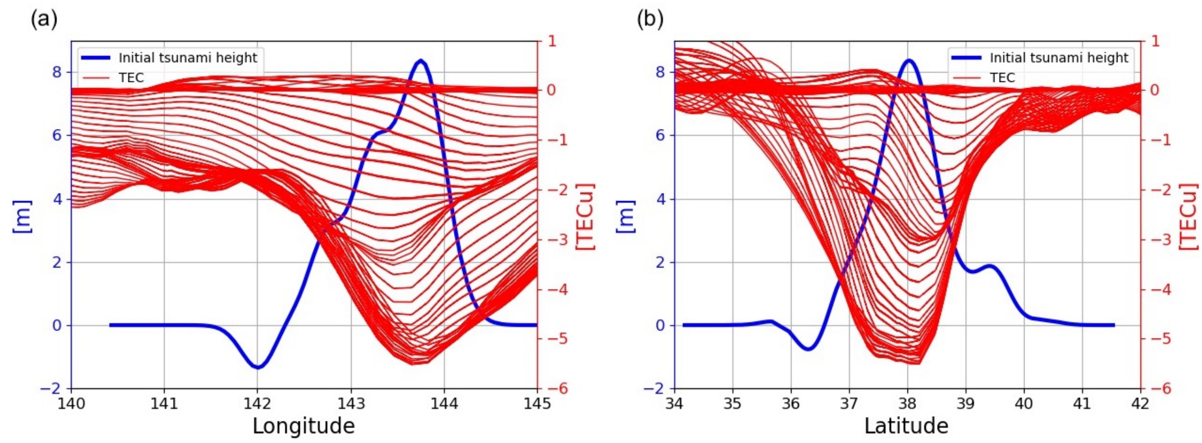

Figure5: The red curves are the time series from 5:46:30 to 6:16:30 of TIH at fixed latitude and longitude where the initial tsunami estimated by Saito (2011) reaches its peak. The blue curve is the initial tsunami.

[T-3]
On page 18, line 359, the authors claim that their method can estimate the tsunami region but the authors only use the Tohoku-Oki event. Before they can draw this conclusion, their method needs verification with other real cases. Moreover, it is not clear to the reader which TECu value (-2, -3) should be used to define the area of the initial tsunami. Is the same TECu value applicable for other tsunami cases?

[About the third comment related to TIH overlapping the initial tsunami]
Our manuscript shows for the first time that our method can properly fit the TIH and obtain the information of the initial tsunami from the fitted data.
The Tohoku-Oki earthquake, which occurred in Japan with a dense GNSS network, is the best case study for the proper application of our method.
The Tohoku-Oki earthquake is the only case in which the tsunami-induced ionospheric changes were observed in such a dense GNSS network.
In addition, this earthquake is the only case where such a huge TIH was observed.

These facts are the reason why we applied our method to the Tohoku-Oki earthquake.
As commented by the reviewer, we consider the application of our method to other cases as a future task.
Since our method has shown its applicability to sparse GNSS networks, we would like to apply it to other cases when the scientific validity of this manuscript is recognized after this peer review.

Minor comments:

[M-1]
Page 3, line 56: Please define TECu the first time it appears in the text.

[About the first minor comment]
We will add the definition of TECu when the word appears the first time on page 3.
Also, we will delete the definition described on page 4.

[M-2]
Page 5, line 132: Please define the variable $O(n^3)$.

[About the second minor comment]
We will add the definition of $O(n^3)$, that is, the computational cost is the cubic of the number of data points.

[M-3]
Page 5, line 136: What is the difference between 'more accurate, less uncertain and more robust'?

[About the third minor comment]
The accuracy is the closeness to the true value, the uncertainties are the dispersion of values in the spatial interpolation, and the robustness is against the absence of measurements.

[M-4]
Page 6, caption figure 2: There is a space missing between 'elementsto'

[About the fourth minor comment]
We will put a space between elements and to.

[M-5]

Page 6, line 148: The text could be deleted since the information is given in figure 3 'The red star is the location of the epicenter of the 2011 off the Pacific coast of Tohoku Earthquake and the two large black circles with slanting lines are outliers.'

[About the fifth minor comment]
We will delete the sentence.

[M-6]
Page 7, line 177: 'which is shown using a red star mark,' could be deleted since the information is in the figure caption.

[About the sixth minor comment]
We will delete the words.

[M-7]
Page 10, line 217: It is sufficient if the triangles' colour coding is in the figure 6 caption.

[About the seventh minor comment]
We will delete the sentence.

[M-8]
Page 10, line 219 – page 11, line 221: Can the authors explain why the tsunami source (Kamogawa et al., 2016) is relevant if they analyze the TIH expansion of their study? If they relate to the source in Kamogawa et al. 2016, they must show it in the figure.

[About the eighth minor comment]
According to Shinagawa (2013), a study that runs a physics-based simulation model, the initial tsunami reproduces TIH. Then, TIH is spread out over the initial tsunami.
Kamogawa (2016) uses the TEC values around the tsunami source area to derive the relationship between TIH and the initial tsunami.
The initial tsunami is shown as a yellow circle in the figure.

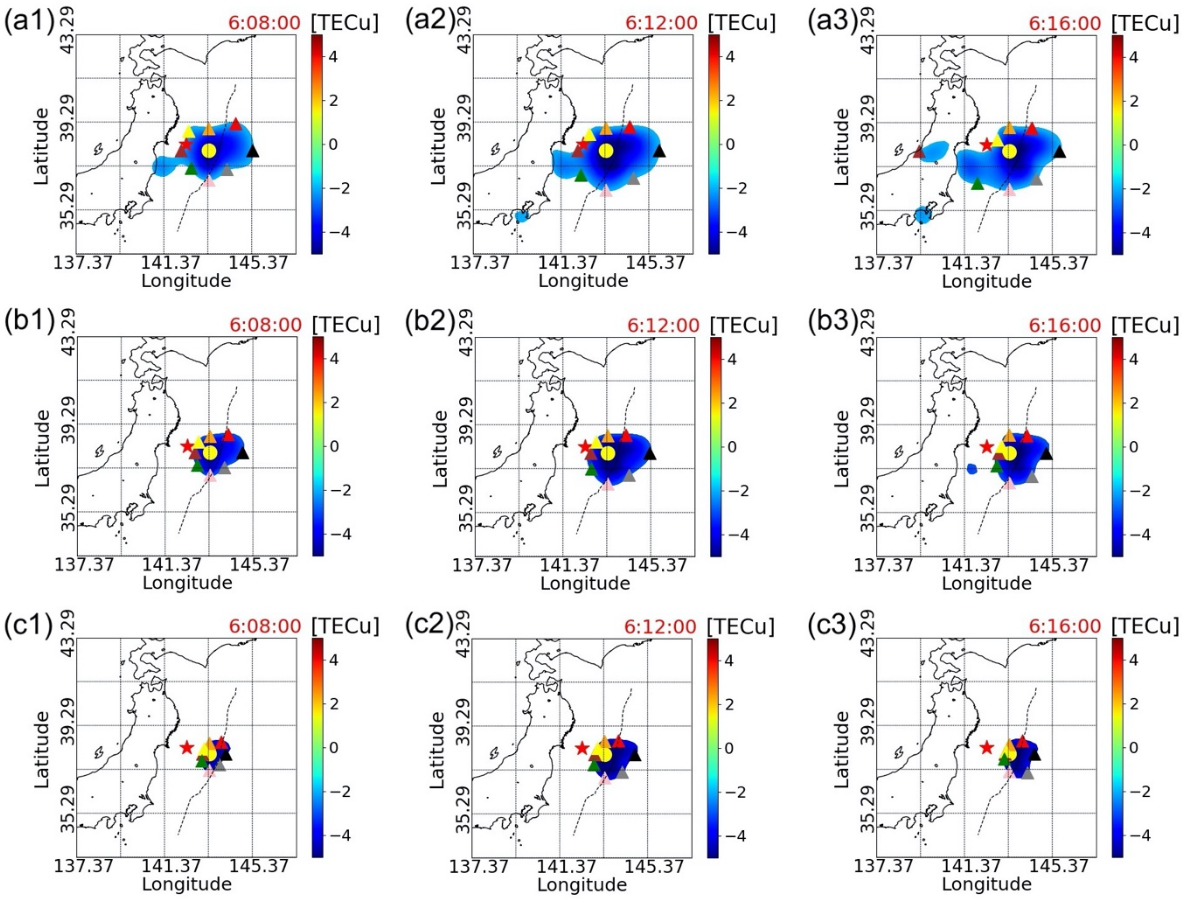

Figure 6: TIH expansion with the tsunami source which is expressed as the yellow circle.

[M-9]
Page 11, line 240: Any reference for the Hubeny's distance formula?

[About the ninth minor comment]
For example, an article, Sato, F., Tanabe, T., Murase, H., Tominari, M., & Kawai, M. (2017). Application of a wearable GPS unit for examining interindividual distances in a herd of Thoroughbred dams and their foals. Journal of equine science, 28(1), 13-17., shows the Hubeny's distance formula in the section Materials and Methods, the paragraph Calculation of the distance between GPS units.
DOI https://doi.org/10.1294/jes.28.13

[M-10]
Figure 7, Panel (a): Why does the TIH withdraw (05:59:30 - 06:01:30) in the southward direction before it starts to expand again?

[About the tenth minor comment]
Due to the influence of the geomagnetic field, the plasma tends to move more to the south.

In addition, the backward moving average frequency filter applied in this study does not completely exclude the high frequency component.
For these reasons, we detected a decrease in the electron density as a result of the recombination caused by the oscillation of the plasma due to the high-frequency component on the south side. This temporary decrease in electron density is separate from the TIH formation caused by the low frequency component.

[M-11]
Page 13, line 273: What do the authors mean? 'if the TEC reduction is larger'. Larger than -2. Please give a more detailed and analytical comparison between the tsunami wavefield and the TEC field.

[About the eleventh minor comment]
In Figure 8, TEC values less than -3 and -4 are depicted. In this context, the larger means TEC values less than -3. However, there is no explicit definition about large TEC reduction.
Based on the atmospheric physics, the background TEC value is around 20 TECu, then 5% of its value, for example, 1 TECu can be mentioned as a large TEC change.
In this case, the magnitude of earthquakes has a big impact on TEC change, so defining the large TEC change is difficult.
About this matter, we created Figure 5 shown above.
The TEC dip corresponds to the initial tsunami shape is described in the figure.

[M-12]
Page 13, line 277: What is meant by the 'TIH almost overlaps the initial tsunami areas'

[About the twelfth minor comment]
In figure 8, panels (a1), (a2), and (a3), the TIH with TEC values less than -3 is located on the region which is almost the same with the initial tsunami region. In other words, the TIH is formed above the initial tsunami region.

[M-13]
Page 15, line 295: Why does the volume of the TIH continue to increase until 28 minutes after the earthquake?

[About the thirteenth minor comment]
TIH is formed by the extremely low frequency component of acoustic waves that travel from the sea surface to the ionosphere.
The recombination of plasma and electrons caused by this low-frequency component is the direct cause of TIH, and this recombination takes time, so the volume continues to increase over a long period of time.
Simulations of TIH formation using a physical model presented by Shinagawa (2013) also show that TIH forms over a similar time period.

[M-14]
Page 15, line 297: Please quantify 'huge simulated tsunami' and 'smaller simulated tsunami'

[About the fourteenth minor comment]
Huge simulated tsunami in this context refers to tsunamis with a simulated initial tsunami height of around 5 m in Kamogawa (2016). On the other hand, smaller simulated tsunami in this context means tsunamis with simulated initial tsunami heights of 1 m or less.

[M-15]
Page 16, Figure 9 caption: Please define acronym CI the first time used in the manuscript.

[About the fifteenth minor comment]
We will add that The CI means the confidence interval, which shows the degree of uncertainty.

[M-16]
Page 16, line 302: Please define what is considered 'a huge' tsunami.

[About the sixteenth minor comment]
In this context, a huge tsunami means a tsunami with the same magnitude with Tohoku-Oki earthquake tsunami.
However, since there is no general definition of a huge tsunami, it is impossible to describe it in a strict and explicit manner.
By convention, tsunamis triggered by magnitude about 9 earthquakes are considered to be huge tsunamis.

[M-17]
Page 16, line 317 to page 17, line 319: This information is redundant. The authors could delete the last phrase of this paragraph.

[About the seventeenth minor comment]
We will delete the phrase.

[M-18]
Page 17, line 327: 'Also, the estimated TIH almost overlaps with the estimated initial tsunami area.' As mentioned earlier, that statement needs better visual representation in figures. I also suggest further exploration, analyzes and discussion.

[About the eighteenth minor comment]
For the visual representation, we created Figure 2 and Figure 5 above based on the initial tsunami estimated by Saito (2011).
In Figure 2, the initial tsunami contours and the fixed latitude and longitude tsunami shape are compared with the TIH. In Figure 5, the time series of TIH shape at the fixed latitude and longitude are compared with the initial tsunami.

In Figure 3 shown above, the comparisons between TIH and the initial tsunami estimated by other research groups (Ohta (2012) and Takagawa (2012)) are shown.

[M-19]
Page 17, line 342: Change the brackets from ‘Heki and Ping (2005)’ to ‘(Heki and Ping 2005)’. Please also correct the brackets in the conclusions or change the text accordingly for the references Zettergren et al. (2017), Zettergren and Snively (2019) & Shinagawa et al. (2013) on page 17, lines 345,347 and 348.

[About the nineteenth minor comment]
We will correct the brackets.

[M-20]
Page 18, line 365: The authors' comment that larger initial tsunamis cause larger decreases in TEC, according to Astafyeva et al. (2013) and Kamogawa et al. (2016): What is the relation between the size of the tsunami and the decreases in TEC. The authors use the volume of the TEC decrease as a measure for the TIH produced by the Tohoku-Oki tsunami. They should relate their measure to a corresponding measure of the tsunami size.

[About the twentieth minor comment]]
In Kamogawa (2016) Figure 3, we can see the positive correlation between the percentage of TEC depression and simulated initial tsunami height.
In our manuscript, we use the volume of TIH as a measure for TIH, but we applied our new method for the Tohoku-Oki tsunami to show our method can detect the TIH volume.
To relate our measure to a corresponding measure of the tsunami size, it is necessary to apply the method to other tsunami cases.
We would like to apply it to other cases when the scientific validity of this manuscript is recognized after this peer review.