

## ***Interactive comment on “Trivariate copula to design coastal structures” by Olivier Orcel et al.***

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### **Trivariate copula to design coastal structures**

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5 **Abstract.** Some coastal structures must be redesigned in the future due to rising sea levels caused by global warming. The design of structures subjected to the actions of waves requires an accurate estimate of the long return period of such parameters as wave height, wave period, storm surge and more specifically their joint exceedance probabilities. The simplified Defra method that is currently used in particular for European coastal structures makes it possible to directly connect the joint exceedance probabilities to the product of the univariate probabilities by means of a single factor. These schematic correlations do not, however, represent all the complexity of the reality because of the use of this single factor. That may lead to damaging errors in coastal structure design. The aim of this paper is therefore to remedy the lack of robustness of these current approaches. To this end, we use copula theory with a copula function that aggregates joint distribution function to its univariate margins. We select a bivariate copula that is adapted to our application by the likelihood method with a copula parameter that is obtained by the error method. In order to integrate extreme events, we also resort to the notion of tail dependence. We select the copulas with the same tail dependence as data. In the event of an opposite tail dependence structure, we resort to the survival copula. The tail dependence parameter makes it possible to estimate the optimal copula parameter. The most robust copulas for our practical case with applications in Saint-Malo and Le Havre (in Northern France) are the Clayton normal copula and the Gumbel survival copula. The originality of this paper is the creation of a new and robust trivariate copula with an analysis of the sensitivity to the method of construction and to the choice of the copula. Firstly, we select the best fitting of the bivariate copula with its parameter for the two most correlated univariate margins. Secondly, we build a trivariate function. For this purpose, we aggregate the bivariate function with the remaining univariate margin with its parameter. We show that this trivariate function satisfies the mathematical properties of the copula. We finally represent joint trivariate exceedance probabilities for a return period of 10, 100 and 1000 years. We finally conclude that the choice of the bivariate copula is more important for the accuracy of the trivariate copula than its own construction.

### 25 **1 Introduction**

The design of coastal structures requires the multiplicity of variables and their degree of correlation to be taken into account. We must therefore address the lack of robustness in the modelling procedure of the dependencies between the different variables characterizing the sea state (Sergent *et al.*, 2014; Hawkes, 2005) such as wave height  $H$ , wave period  $T$  and storm surge  $S$ . The design of coastal structures is based in particular on the return periods of wave overtopping or of armour damage (Ciria *et al.*, 2007). Since the applications on wave overtopping and armour damage depend on the parameters of the coastal structure, we will not deal with the return periods of these quantities. The aim of this paper is however to improve the methods of estimating them in order to avoid costly and inappropriate decisions (Li *et al.*, 2008). To this end, we provide accurate estimates of the correlations between the variables  $H$ ,  $T$  and  $S$  and obtain reliable return periods. Currently, in reference manuals such as the Rock Manual (Ciria *et al.*, 2007), it is recommended that a factor be applied to the product of univariate survival functions in order to determine the joint period. Copulas are mathematical tools for modelling the dependence structure of several random variables. The theory of copulas was developed by the mathematician Sklar (1959). The copula is a written form of the joint distribution function that provides all the information on the dependency structure. The recent interest in copulas started in financial risk management and insurance. Its use in environmental science especially concerns hydrology

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**Fig. 1.**

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#### Reply to Referee #1

Several references are added in the paper. Colloquial language is removed. Sections 2.2 to 2.4 are moved to the appendix. Figure 1 appears now immediately after the first introduction of Le Havre and Saint-Malo in the main body of the manuscript. The columns of the tables are grouped by site. More detailed explanations of the results are added.

The limitations of the Defra method is commented in section 3.2. They are due to the use of the simplified version of the Defra method but also to the choice of a Gauss copula without tail dependence.

We mention now Aas and Berg (2009) who propose copula construction with conditional sets: the pair copula construction (PCC). As the bivariate copulas that are selected as the most promising in our application are Archimedean copulas, simpler methods of construction are available. We find that it is useless to use more complex techniques with compatibility problem and that have been less robust than fully nested method in some applications.

#### Reply to Referee #2

The Defra method is now presented as a current practice. The section 3.2 explains in which context the Defra method is a current practice. We have recalled that FD=25 is a weak dependence and the FD=20 is lower than the value that is recommended by Kergadallan.

In section 2.1, we acknowledge that the choice of the values at high tide is not the only choice. We had omitted to mention that we used the same data as Kergadallan and his own method as we selected the maximum  $H_s$  value within a time window centred on the time of high water. That is now mentioned in the paper.

We nevertheless consider that a threshold on  $H_s$  is inappropriate in regards of the distribution function [this threshold is applied for copula but not for the distribution function]. Since we have two wave populations, we have indeed used a threshold and excluded wave height values less than one meter (see figure 2). We acknowledge that the independence assumption is not completely valid when two tuples per day are selected but this is a common assumption. Full compliance with independence would lead to ignore some relevant pairs of wave height and surge values.

We focus on the lower tail dependence of the survival copula. That is now better explained in section 3.3. We choose the survival copula instead of the standard copula because it simplifies the equations (22), (26), (30), (34).

We acknowledge that we do not address the return period of wave overtopping and of armour damage. We also recall that the definition of the return period is not unique.

The mixture model is similar to Chakak and Koehler (1995) method that is explained line 50. Its compatibility problem is explained line 51. The title of section 3.1 is changed. The caption of figure 7 is completed in order to improve the understanding of the figure. Five proposed references are added to the text.

**Fig. 2.**