Multilayer-HySEA model validation for landslide generated tsunamis. Part I Rigid slides

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6 Abstract

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This paper is devoted to benchmarking the Multilayer-HySEA model using laboratory experimental data for landslide-generated tsunamis. This article deals with rigid slides and the second part, in a companion paper, addresses granular slides. The US National Tsunami Hazard and Mitigation Program (NTHMP) 10 has proposed the experimental data used and established for the NTHMP Land-11 slide Benchmark Workshop, held in January 2017 at Galveston (Texas). The 12 first three benchmark problems proposed in this workshop deal with rigid slides. 13 Rigid slides must be simulated as a moving bottom topography and, therefore, 14 they must be modelled as a prescribed boundary condition. These three bench-15 marks are used here to validate the Multilayer-HySEA model. This new HySEA 16 model consists of an efficient hybrid finite-volume/finite-difference implementa-17 tion on GPU architectures of a non-hydrostatic multilayer model. A brief de-18 scription of model equations, dispersive properties, and the numerical scheme 19 is included. The benchmarks are described and the numerical results compared 20 against the lab-measured data for each of them. The specific aim is to validate 21 this new code for tsunamis generated by rigid slides. Nevertheless, the over-22 all objective of the current benchmarking effort is to produce a ready-to-use 23 numerical tool for real-world landslide generated tsunami hazard assessment. 24 This tool has already been used to reproduce the Port Valdez Alaska 1964 and 25 Stromboli Italy 2002 events. 26

27 Keywords: Multilayer-HySEA model, tsunamis, rigid slides, model

²⁸ benchmarking, landslide-generated tsunamis, GPU implementation

December 31, 2020

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30 1. Introduction

Model development and benchmarking for earthquake-induced tsunamis is a 31 task addressed in the past and to which much effort and time has been dedicated. 32 In particular, just to mention a couple of NTHMP efforts, the 2011 Galveston 33 benchmarking workshop (Horrillo et al., 2015) and the 2015 Portland workshop 34 for tsunami currents (Lynett et al., 2017) both focused on these topics. However, 35 both model development and benchmarking efforts have advanced at a slower 36 pace for landslide-generated tsunamis. As examples we might mention the 2003 37 NSF-sponsored landslide tsunami workshop organized in Hawaii and a similar 38 follow-up workshop on Catalina Island in 2006 (Liu et al., 2008). Since then, no 39 similar large comprehensive benchmarking workshop has been organized (Kirby 40 et al., 2018). 41

In its 2019 Strategic Plan, the NTHMP required that all numerical tsu-42 nami inundation models to be used in hazard assessment studies in the US 43 be verified as accurate and consistent through a model benchmarking process. 44 This mandate was fulfilled in 2011, but only for seismic tsunami sources and to 45 a limited extent for idealized solid underwater landslides. However, recent work 46 by various NTHMP states has shown that landslide tsunami hazard may in fact 47 be greater than seismically-induced hazard and may be also the dominant risk 48 along significant parts of the US coastline (ten Brink et al., 2014). 49

As a result of this demonstrated gap, a set of candidate benchmarks was pro-50 posed to perform the required validation process. The selected benchmarks are 51 based on a subset of available laboratory data sets for solid slide experiments 52 and deformable slide experiments and include both submarine and subaerial 53 slides. In order to complete this list of laboratory data, a benchmark based 54 on a historic field event (Valdez, AK, 1964) was also chosen. The EDANYA 55 group (www.uma.es/edanya) from the University of Málaga participated in 56 the workshop organized at Texas A&M University, Galveston (January 9-11, 57

2017) presenting results for the benchmarking tests with two numerical codes: 58 Landslide-HySEA and Multilayer-HySEA models. At Galveston, we presented 59 numerical results for six out of the seven benchmark problems proposed, in-60 cluding the field case. The current work presents the numerical results obtained 61 for the Multilayer-HySEA model in the framework of the validation effort de-62 scribed above for the case of rigid slide-generated tsunamis, whereas the bench-63 mark problems dealing with granular slides are presented in a companion paper 64 Macías et al. (2020a). A summary of the results for the field case at Port Valdez 65 can be found at Macías et al. (2017). 66

Twenty years ago, at the beginning of the century, the challenge of solid 67 block landslide modelling was taken by a number of researchers (Grilli and 68 Watts, 1999, 2005; Grilli et al., 2002; Lynett and Liu, 2002; Watts et al., 2003; 69 Wu, 2004; Watts et al., 2005; Liu et al., 2005) and laboratory experiments were 70 developed for those cases and for tsunami model benchmarking (Enet and Grilli, 71 2007) (see also Ataie-Ashtiani and Najafi-Jilani (2008)). The benchmark prob-72 lems performed in the current work are based on the laboratory experiments 73 of Grilli and Watts (2005) for BP1, Enet and Grilli (2007) for BP2, and Wu 74 (2004); Liu et al. (2005) for BP3. The basic reference for these three bench-75 marks, as well as for the three benchmarks related to granular slides and the 76 Alaska field case (all of them proposed by the NTHMP) is Kirby et al. (2018). 77 We highly recommend checking this reference for further details on benchmark 78 descriptions, data provided for performing them, required benchmark items, and 79 inter-model comparison. Finally, we would like to stress that the ultimate goal 80 of our current benchmarking effort is to provide the tsunami community with 81 a NTHMP-approved model for landslide-generated tsunami hazard assessment, 82 similarly to what we have done with the Tsunami-HySEA model for the case of 83 earthquake-generated tsunamis (Macías et al., 2017; Macías et al., 2020c,d). 84

85 2. HySEA models for landslide generated tsunamis

The HySEA (Hyperbolic Systems and Efficient Algorithms) software consists 86 of a family of geophysical codes based on either single-layer, two-layer stratified 87 systems or multilayer shallow-water models. HySEA codes¹ have been under 88 development by the EDANYA Group from UMA (the University of Málaga) 89 for more than a decade. These codes are in continuous evolution and upgrad-٩N ing and they are serving to a wider scientific community every day. The first 91 model we developed dealing with landslide-generated tsunamis consisted of a 92 stratified two-layer Savage-Hutter shallow-water model -the Landslide-HySEA 93 model-. It was implemented based on the model described in Fernández et al. 94 (2008) and was incorporated to the HySEA family. An initial validation of this 95 code, comparing numerical results with the laboratory experiments of Heller and 96 Hager (2011) and Fritz et al. (2001) can be found at Sánchez-Linares (2011). 97 The 2018 numerical simulation of the Lituya Bay 1958 mega-tsunami with real 98 topo-bathymetric data and encouraging results (González-Vida et al., 2019), 99 represented a milestone in the verification process of this code. This validation 100 effort was accomplished under a research contract with PMEL/NOAA. The re-101 sult of this project led the NCTR (NOAA Center for Tsunami Research) to 102 adopt Landslide-HySEA as the numerical code of choice to generate the initial 103 conditions for the MOST model to be initialized in the case of a landslide-104 generated tsunami scenario to be simulated. Further applications of Landslide-105 HySEA can be found at de la Asunción et al. (2013), Macías et al. (2015), and 106 Iglesias (2015). 107

The waves generated in the laboratory tests proposed in the NTHMP selected benchmarks are high frequency and dispersive, and the generated flows have a complex vertical structure. Therefore, the numerical model used must be able to reproduce such effects. This makes the two-layer Landslide-HySEA model unsuitable for reproducing these experimental results as non-hydrostatic

¹https://edanya.uma.es/hysea

effects play an important role and a richer vertical structure is required. To 113 address these requirements, the Multilayer-HySEA model was very recently im-114 plemented, considering a stratified structure in the simulated fluid and including 115 non-hydrostatic terms. A multilayer model is able to better approximate the 116 vertical structure of a complex flow than a standard one-layer depth-averaged 117 model. In particular, increasing the number of layers the linear dispersion re-118 lation of the model converges towards the exact dispersion relation from the 119 Stokes linear theory (see Fernández-Nieto et al. (2018)). 120

121 **3. Model Equations**

The Multilayer-HySEA model implements one of the multilayer non-hydro-122 static models of the family introduced and described in Fernández-Nieto et al. 123 (2018) (model $LDNH_0$). The governing equations, that are obtained by a 124 process of depth-averaging, correspond to a semi-discretization for the vertical 125 variable of the Euler equations following a standard Galerkin approach. The 126 total pressure is decomposed into a sum of hydrostatic and non-hydrostatic 127 pressures and is assumed to have a linear vertical profile. The horizontal and 128 vertical velocities are assumed to have a constant vertical profile. At the discrete 129 level on z, the total pressure matches at the interfases and velocities satisfy a 130 discrete jump condition (see Fernández et al. (2008) or Escalante et al. (2018a)). 131 132

An alternative deduction for this system is performed in Escalante et al. (2018a) assuming linear vertical profiles for pressure and vertical velocity and a constant vertical profile for the horizontal velocity, as well as some extra hypothesis for the case of two layers. The proposed model admits an exact energy balance and, when the number of layers increases, the linear dispersion relation of the linear model converges to the same of Airy's theory (Fernández-Nieto et al., 2018). The model proposed in Fernández-Nieto et al. (2018) can



Figure 1: Schematic diagram describing the multilayer system

¹⁴⁰ be written in a compact form as:

$$\begin{cases} \partial_t h + \partial_x (h\bar{u}) = 0, \\ \partial_t (hu_\alpha) + \partial_x \left(hu_\alpha^2 + \frac{1}{2}gh^2 \right) - gh\partial_x H + u_{\alpha+1/2}\Gamma_{\alpha+1/2} - u_{\alpha-1/2}\Gamma_{\alpha-1/2} = \\ -h \left(\partial_x p_\alpha + \sigma_\alpha \partial_z p_\alpha \right) - \tau, \\ \partial_t (hw_\alpha) + \partial_x (hu_\alpha w_\alpha) + w_{\alpha+1/2}\Gamma_{\alpha+1/2} - w_{\alpha-1/2}\Gamma_{\alpha-1/2} = -h\partial_z p_\alpha, \\ \partial_x u_{\alpha-1/2} + \sigma_{\alpha-1/2}\partial_z u_{\alpha-1/2} + \partial_z w_{\alpha-1/2} = 0, \end{cases}$$

$$(1)$$

where, for $\alpha \in \{1, 2, \dots, L\}$, the following notation is used:

$$f_{\alpha+1/2} = \frac{1}{2} \left(f_{\alpha+1} + f_{\alpha} \right), \ \partial_z f_{\alpha+1/2} = \frac{1}{h\Delta s} \left(f_{\alpha+1} - f_{\alpha} \right), \tag{2}$$

where f denotes one of the generic variables of the system, i.e., u, w and p, and, finally,

$$\sigma_{\alpha} = \partial_x \left(H - h\Delta s(\alpha - 1/2) \right), \ \sigma_{\alpha - 1/2} = \partial_x \left(H - h\Delta s(\alpha - 1) \right).$$
(3)

Total depth, h, is split along the vertical axis into $L \ge 1$ layers and $\Delta s = 1/L$ (see Figure 1). The variables u_{α} and w_{α} are the depth-averaged velocities in the x and z directions, respectively, t is time and g is gravitational acceleration. The non-hydrostatic pressure at the interface $z_{\alpha+1/2}$ is denoted by $p_{\alpha+1/2}$. The water surface elevation measured from the still-water level is $\eta = h - H$, where H is the water depth when the water is at rest. Finally, τ is a friction law term, and the terms $\Gamma_{\alpha+1/2}$ account for the mass transfer across interfaces and are defined by

$$\Gamma_{\alpha+1/2} = \sum_{\beta=\alpha+1}^{L} \partial_x \left(h \Delta s \left(u_\beta - \bar{u} \right) \right), \ \bar{u} = \sum_{\alpha=1}^{L} \Delta s u_\alpha \tag{4}$$

In order to close the system of equations, the following boundary conditions areconsidered

$$p_{L+1/2} = 0, \ u_0 = 0, \ w_0 = -\partial_t H.$$
 (5)

Note that the motion of the bottom surface can be taken into account as a boundary condition, imposing $w_0 \neq 0$. Therefore, this model can simulate the interaction with a slide in the case that the motion of the bottom is prescribed by a function, given by a set of data, or simulated by a numerical model. In the present study, we are going to consider tests where the motion of the seafloor is given by a known function (the solid moving block).

160 3.1. Linear dispersion relation

Some dispersive properties of the system (1) are presented in this subsection, in particular, the phase and group velocities, and the linear shoaling. The first two properties are related to the propagation of dispersive wave trains and the last one to shoaling processes.

To obtain such properties, the system (1) is linearised around the water at rest steady-state solution. After that, a Stokes-type Fourier analysis is carried out looking for first-order planar wave solutions. This method constitutes a standard procedure to study systems that model dispersive water waves (see Escalante et al. (2018a); Lynett and Liu (2004); Madsen and Sorensen (1992); Schäffer and Madsen (1995) and references therein). The phase and group velocities as well as the linear shoaling gradient are, respectively, defined as:

$$C = \omega/k, \quad G = C + k\partial_k C, \quad \frac{\partial_x \eta}{\eta} = -\gamma \frac{\partial_x H}{H},$$
 (6)

where ω denotes the angular frequency, k the local wave-number and H the typical depth.

The measured quantities C, G and γ are solely functions of the local wavenumber and the typical depth H. Thus, one can obtain the so-called linear dispersion relation of the three measured quantities. From the Airy wave theory, one can also obtain the corresponding linear dispersion relations that state the linear theory for the considered quantities (see Schäffer and Madsen (1995) for the Airy reference formulae). For example, the expression for the phase velocity from the Airy's theory is

$$C_{Airy} = gH \frac{\tanh(kH)}{kH}.$$
(7)

181

The expressions of the phase velocity for the system (1) are given in Table 1 for the non-linear hydrostatic shallow water system (SWE) and the Multilayer-HySEA (non-hydrostatic) system with $j \ge 1$ layers (NH-jL). The last two columns contain $Er_C(s)$ for s = 5 and s = 15, where $Er_C(s)$ represents the maximum relative error of the phase velocity with respect to the Airy in a range $kH \in [0, s]$ in percent, i.e.:

$$Er_{C}(s) = 100 \cdot \max_{kH \in [0,s]} \left(\frac{|C(kH) - C(kH)_{Airy}|}{|C(kH)_{Airy}|} \right).$$
(8)

The main goal when deriving dispersive shallow water systems is to get the 188 most accurate dispersive relations as possible, compared with the Airy wave 189 theory, without highly increasing the complexity of the system. See Schäffer 190 and Madsen (1995) for a review on state-of-the-art or a two-layer with improved 191 dispersive relations in Lynett and Liu (2004), and an enhanced two-layer non-192 hydrostatic pressure system in Escalante et al. (2018a). It has been shown 193 (Fernández-Nieto et al., 2018) that increasing the number of layers leads to the 194 convergence of the linear dispersion relation of the linear model to the same of 195

Multilayer System – Phase velocity – Errors for kH up to 5 and 15						
Model	Phase velocity	$Er_C(5)$	$Er_C(15)$			
(SWE)	gH	73.63~%	123.61 %			
(NH-1L)	$gH\frac{1}{1+\frac{1}{4}(kH)^2}$	3.02~%	16.95~%			
(NH-2L)	$gH\frac{1+\frac{(kH)^2}{16}}{1+\frac{3(kH)^2}{8}+\frac{(kH)^4}{256}}$	0.71 %	10.67~%			
(NH-3L)	$\frac{1 + \frac{5(kH)^2}{54} + \frac{(kH)^4}{1296}}{1 + \frac{5(kH)^2}{12} + \frac{5(kH)^4}{432} + \frac{1(kH)^6}{46656}}$	0.31 %	0.62~%			
(NH-5L)	$\frac{1 + \frac{3(kH)^2}{25} + \frac{63(kH)^4}{2510^3} + \frac{3(kH)^6}{2510^4} + \frac{(kH)^8}{10107}}{1 + \frac{9(kH)^2}{20} + \frac{21(kH)^4}{1010^2} + \frac{21(kH)^6}{1010^4} + \frac{9(kH)^8}{2010^6} + \frac{(kH)^{10}}{1010^9}}$	0.11 %	0.11 %			

Table 1: Phase velocity expressions and maximum of the relative error $Er_C(s)$ compared with the Airy's theory for different ranges of $kH \in [0, s]$ for the non-linear hydrostatic shallow water system (SWE) and the Multilayer-HySEA (non-hydrostatic) system with $j \ge 1$ layers (NH-jL).

Airy's theory. Figure 2 shows this behavior and highlights the huge discrepancies between the Airy's theory and the systems (SWE) and (NH-1L). It is well known that waves generated by landslides, might present high characteristic values for kH. For the (SWE) system, it is well known that it has an accurate phase velocity in a small range of kH, and that this system is appropriate for long waves as tsunami waves, but not for dispersive waves with higher values of kH. In the same vein, the one layer non-hydrostatic pressure system (NH-1L) can ²⁰³ improve these results, but again, poor linear dispersive results are achieved in ²⁰⁴ a range of kH between 5 and 15. However, when the number of layers, L, is set ²⁰⁵ to 3 (still a small value) the system (1) is in an excellent agreement with the ²⁰⁶ Airy theory for kH up to 15. For the phase celerity, the percentage error is less ²⁰⁷ than 0.62%, and for the group velocity is less than 1% for kH smaller than 10 ²⁰⁸ (see Figure 2). Linear shoaling is also well reproduced in this same range.

The Multilayer-HySEA model presents enhanced dispersive properties. In 209 order to have similar dispersive results as the ones obtained here using a three-210 layer system, at least five layers are required for other similar multilayer models 211 as the one presented in Bai and Cheung (2018). Furthermore, the results pre-212 sented for the phase velocity with two layers in Table 1 show that the system 213 proposed here produces smaller relative error for kH up to 15 compared with the 214 two-layer system in Cui et al. (2014). That means that the Multilayer-HySEA 215 model can achieve better dispersive properties than models having similar or 216 even more computational complexity. 217

218 3.2. Modeling of breaking waves and wetting and drying treatment

219 3.2.1. Modeling of breaking waves

In shallow areas the breaking of waves can be observed near the coast. As 220 pointed out in Escalante et al. (2018a,b, 2019); Roeber et al. (2010) among 221 others, non-hydrostatic PDE systems such as the one considered in this paper 222 cannot describe this process without the inclusion of an additional term that 223 accounts for the dissipation of the amount of energy required when breaking 224 phenomena occur. In this work, we have implemented a simplified generalization 225 of the breaking mechanism that was introduced in Escalante et al. (2018a) for 226 the case of two layers. To do so, the vertical component of the stress-tensor is 227 depth-averaged on the vertical variable. Thus, adding the proposed integrated 228 viscosity term to system (1), only the vertical momentum equation changes and 229 reads for each $\alpha \in \{1, 2, \dots, L\}$ as: 230

$$\partial_t (hw_\alpha) + \partial_x (hu_\alpha w_\alpha) + w_{\alpha+1/2} \Gamma_{\alpha+1/2} - w_{\alpha-1/2} \Gamma_{\alpha-1/2} = -h \partial_z p_\alpha + 2\varsigma w_\alpha, \quad (9)$$



Figure 2: Relative error for the phase velocities (A), the group velocities (B), and comparison with the reference shoaling gradient (C), with respect to the Airy theory for the described multilayer systems (2L, 3L, and 5L), the one-layer non-hydrostatic, and the shallow-water system.

where $\varsigma = \int_{z_{\alpha-1/2}}^{z_{\alpha-1/2}} \partial_z \nu$ is the eddy viscosity. In this work, as in Escalante et al. (2018a); Roeber et al. (2010) we choose ς to be

$$\varsigma = -KBh \left| \partial_x (h\bar{u}) \right|,\tag{10}$$

where B is an empirical parameter related to a breaking criteria to switch on and off this extra dissipation term. The definition of this empirical parameter is based on a quasi-heuristic strategy to determine when the breaking occurs (see Escalante et al. (2018b); Roeber et al. (2010) and references therein).

²³⁷ Finally, a natural and simple extension of the criterion proposed by Roeber

 $_{238}$ et al. (2010) is adopted

$$B = \max\left(1 - \frac{\partial_x(h\bar{u})}{U_1}, 0)\right) \quad \text{for} \quad |\partial_x(h\bar{u})| \ge U_2, \tag{11}$$

239 where

$$U_1 = B_1 \sqrt{gh}, \quad U_2 = B_2 \sqrt{gh} \tag{12}$$

denote the flow speed at the onset and termination of the wave-breaking process and B_1 , B_2 are calibration coefficients. In this work, we use $B_1 = 0.6$ and $B_2 = 0.15$ for all the test cases studied. Finally, depending on the benchmark problem, we use $K \in \{2, 10\}$.

244 3.2.2. Wetting and drying treatment

For the computation of variables in areas of small water depth, a wet-dry treatment adapting the ideas described in Castro et al. (2005) is applied. The key elements for the numerical treatment of wet-dry fronts with emerging bottom topographies are based on:

• The hydrostatic pressure terms $\partial_x \left(\frac{1}{2}gh^2\right) - gh\partial_x H = gh\partial_x \eta$ at the horizontal velocity equations are modified for emerging bottoms to avoid spurious pressure forces (see Castro et al. (2005)).

• To overcome the difficulties due to large round-off errors in computing the velocities u_{α}, w_{α} from discharges for small values of h, we define the velocities analogously as in Kurganov and Petrova (2007) applying the desingularization formula

$$u_{\alpha} = \frac{\sqrt{2}hhu_{\alpha}}{\sqrt{h^4 + \max(h^4, \epsilon^4)}}, \ w_{\alpha} = \frac{\sqrt{2}hhw_{\alpha}}{\sqrt{h^4 + \max(h^4, \epsilon^4)}}, \ \alpha \in \{1, 2, \dots, L\}$$
(13)

which gives the exact value of u_{α} and w_{α} for $h \ge \epsilon$, and gives a smooth transition of u_{α} and w_{α} to zero when h tends to zero without truncation. In this work we set $\epsilon = 10^{-3}$ for the numerical tests. A more detailed discussion about the desingularization formula can be seen in Kurganov and Petrova (2007).

²⁶¹ 4. Numerical Solution Method

We describe now the discretization of system (1) that follows the natural ex-262 tension of the procedure described in Escalante et al. (2018a,b) for the one and 263 two layer non-hydrostatic system. The numerical scheme employed is based on 264 a two-step projection-correction method, similar to the standard Chorin's pro-265 jection method for Navier-Stokes equations (Chorin (1968)). This is a standard 266 procedure when dealing with dispersive systems (see Escalante et al. (2018b,a); 267 Ma et al. (2012); Kazolea and Delis (2013); Ricchiuto and Filippini (2014) and 268 references therein). 269

First, we shall solve the non-conservative hyperbolic underlying system (1) 271 given by the compact equation

$$\partial_t \mathbf{U} + \partial_x \mathbf{F}_{SW}(\mathbf{U}) + \mathbf{B}_{SW}(\mathbf{U})\partial_x \mathbf{U} = \mathbf{G}_{SW}(\mathbf{U})\partial_x H, \tag{14}$$

²⁷² where the following compact notation has been used:

$$\mathbf{U} = \begin{pmatrix} h \\ hu_1 \\ \vdots \\ hu_L \\ hw_1 \\ \vdots \\ hw_L \end{pmatrix}, \ \mathbf{F}_{SW}(\mathbf{U}) = \begin{pmatrix} hu \\ \frac{hu_1^2}{h} + \frac{1}{2}gh^2 \\ \vdots \\ \frac{hu_L^2}{h} + \frac{1}{2}gh^2 \\ hu_1w_1 \\ \vdots \\ hu_Lw_L \end{pmatrix}, \ \mathbf{G}_{SW}(\mathbf{U}) = \begin{pmatrix} 0 \\ gh \\ \vdots \\ gh \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad (15)$$

and \mathbf{B}_{SW} is a matrix such $\mathbf{B}_{SW}\partial_x \mathbf{U}$ contains the non-conservative products related to the mass transfer across interfaces appearing at the momentum equations.

Then, in a second step, non-hydrostatic terms given by the pressure vector

277 correction term

$$\mathcal{T}_{\mathcal{NH}}(h,\partial_{x}h,H,\partial_{x}H,\mathbf{P},\partial_{x}\mathbf{P}) = -\begin{pmatrix} 0\\h(\partial_{x}p_{1}+\sigma_{1}\partial_{z}p_{1})\\\vdots\\h(\partial_{x}p_{L}+\sigma_{L}\partial_{z}p_{L})\\h\partial_{z}p_{1}\\\vdots\\h\partial_{z}p_{L} \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} p_{1}\\p_{2}\\\vdots\\p_{L} \end{pmatrix},$$
(16)

278 as well as the divergence constraints at each layer

$$\mathcal{B}(\mathbf{U}, \partial_x \mathbf{U}, H, \partial_x H) = \begin{pmatrix} \partial_x u_{1/2} + \sigma_{1/2} \partial_z u_{1/2} + \partial_z w_{1/2} \\ \vdots \\ \partial_x u_{L-1/2} + \sigma_{L-1/2} \partial_z u_{L-1/2} + \partial_z w_{L-1/2} \end{pmatrix}, \quad (17)$$

279 will be taken into account.

System (14) is discretized using a second order finite volume PVM positivepreserving well-balanced path-conservative method (Castro and Fernández-Nieto, 2012). As usual, we consider a set of N finite volume cells $I_i = [x_{i-1/2}, x_{i+1/2}]$ with constant lengths Δx and define

$$\mathbf{U}_{i}(t) = \frac{1}{\Delta x} \int_{I_{i}} \mathbf{U}(x,t) \, dx, \qquad (18)$$

the cell average of the function $\mathbf{U}(x,t)$ on cell I_i at time t. Concerning nonhydrostatic terms, we consider mid-points x_i of each cell I_i and denote the point values of the function \mathbf{P} at time t by

$$\mathbf{P}_{i}(t) = \begin{pmatrix} p_{1}(x_{i}, t) \\ p_{2}(x_{i}, t) \\ \vdots \\ p_{L}(x_{i}, t). \end{pmatrix}$$
(19)

Non-hydrostatic terms will be approximated by second order compact finite differences.

Let us detail de time stepping procedure followed. Assume given time steps Δt^n , and denote $t^n = \sum_{k \leq n} \Delta t^k$. To obtain second order accuracy in time, the two-stage second-order TVD Runge-Kutta scheme is adopted. At the *k*th stage, $k \in \{1, 2\}$, the two-step projection-correction method is given by

$$\begin{cases} \frac{\mathbf{U}^{(\widetilde{k})} - \mathbf{U}^{(k-1)}}{\Delta t} + \partial_x \mathbf{F}(\mathbf{U}^{(k-1)}) + \mathbf{B}(\mathbf{U}^{(k-1)}) \partial_x \mathbf{U}^{(k-1)} & (20a) \\ & = \mathbf{G}(\mathbf{U}^{(k-1)}) \partial_x H, \\ \frac{\mathbf{U}^{(k)} - \mathbf{U}^{(\widetilde{k})}}{\Delta t} = \mathcal{T}(h^{(k)}, \partial_x h^{(k)}, H, \partial_x H, \mathbf{P}^{(k)}, \partial_x \mathbf{P}^{(k)}) & (20b) \end{cases}$$

$$\left(\mathcal{B}(\mathbf{U}^{(k)},\partial_x\mathbf{U}^{(k)},H,\partial_xH)=\mathbf{0}\right)$$
(20c)

where $\mathbf{U}^{(0)}$ is \mathbf{U} at the time level t^n , $\mathbf{U}^{(\tilde{k})}$ is an intermediate value in the twostep projection-correction method that contains the numerical solution of the hyperbolic system (14) at the corresponding kth stage of the Runge-Kutta, and $\mathbf{U}^{(k)}$ is the kth stage estimate. After that, a final value of the solution at the t^{n+1} time level is obtained:

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$$\mathbf{U}^{n+1} = \frac{1}{2}\mathbf{U}^n + \frac{1}{2}\mathbf{U}^{(2)}.$$
 (21)

Observe that, equations (20b-20c) requires, at each stage of the calculation respectively, to solve a Poisson-like equation for each one of the variables contained in $\mathbf{P}^{(k)}$. The resulting linear system is solved using an iterative Jacobi method combined with a scheduled relaxation (see Adsuara et al. (2016); Escalante et al. (2018a,b)). Note that the usual CFL restriction must be imposed for the computation of the time step Δt .

With respect to the breaking mechanism introduced in Subsection 3.2, these terms are semi-implicitly discretized at the end of the second step of the proposed numerical scheme, at each Runge-Kutta stage. The resulting numerical scheme is well-balanced for the water at rest solution and is linearly L^{∞} -stable under the usual CFL condition related to the hydrostatic system. It is also worth mentioning that the numerical scheme is positive preserving and can deal with emerging topographies. Finally, its extension to 2D is straightforward. In

this case, the computational domain is decomposed into subsets with a simple 307 geometry, called cells or finite volumes. The numerical algorithm is well suited 308 for its implementation in GPU architectures, as is shown in Castro et al. (2011). 309 Furthermore, the compactness of the numerical stencil and the natural and the 310 massively parallelization of the Jacobi method makes it possible that the second 311 step can also be implemented in GPUs (see Escalante et al. (2018b,a)). That re-312 sults in a high efficiency of the numerical code and much shorter computational 313 times. 314

5. Benchmark Problem Comparisons

In this Section, the numerical results obtained with the Multilayer-HySEA 316 model and the comparison with the measured lab data for waves generated by 317 the movement of a rigid bottom surface or of a solid block are presented. In 318 particular, BP1 deals with a 2D submarine solid slide, BP2 with a 3D subma-319 rine slide and, finally, BP3 consists of two 3D slides, one partially submerged 320 and a second one representing a completely submarine slide. In all these cases, 321 a moving bottom condition has been used to model the solid block movement. 322 Regarding the wave breaking model, the breaking mechanism described in Sub-323 section 3.2 was implemented, adopting $B_1 = 0.6$, $B_2 = 0.15$ for all the bench-324 mark problems, and K = 10 for the third benchmark and K = 2 for the rest. 325 The description of all these benchmarks can be found at LTMBW (2017) and 326 Kirby et al. (2018). 327

328 5.1. Benchmark Problem 1: Two-dimensional submarine solid block

This benchmark problem is based on the 2D laboratory experiments of Grilli and Watts (2005) which were performed at the University of Rhode Island. Refer to the above-mentioned work to get a detailed description of the present benchmark. Figure 3 depicts the sketch of the laboratory experiment design. The 2D slide model is semi-elliptical, lead-loaded, and rolling down a smooth slope with a slope angle $\theta = 15^{\circ}$ (2 mm above the slope), in between two vertical



Figure 3: BP1. Sketch of main parameters and variables for wave generation by 2D rigid slide. [Modified from Grilli and Watts, 2005].

side walls, 20 cm apart. The water depth is $h_0 = 1.05$ m over the flat bottom part. The slide dimensions were, length B = 1 m, maximum thickness $T = T_{ref}$ = 0.052 m, and width w = 0.2 m. The model initial submergence d was varied in experiments and the free surface elevation recorded at 4 capacitance wave gauges installed at locations: x' = 1.175, 1.475, 1.775, and 2.075 m, the first location being nearly identical to $x'_g = 1.168$ m (where de tilde variables, as x', mean than non-dimensional units are used -see Table 3-).

x'_g	T'	d'	θ	В	$b(\epsilon)$
1.168	0.052	0.259	15	1	1.225

Table 2: Values for variables defining setup configuration.

	g_0	g_1	g_2	g_3
x	1.234	1.549	1.864	2.179
$x' = x/h_0$	1.175	1.475	1.775	2.075

Table 3: Gauge positions in dimensional and non-dimensional units.

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In this benchmark, two items remained not completely determined in the original description provided: the first one is related with the initialization of the numerical experiment, the second one is related with how and where the solid moving block must stop. Other small issues related to the description of the benchmark were put forward in Macías et al. (2017) at our NTHMP report. The motion of the rigid slide was prescribed as a function of time as

$$S(t) = S_0 \log(\cosh(t/t_0)), \qquad (22)$$

where $S_0 = u_t^2/a_0 = 2.110 \text{ m}$, $t_0 = u_t/a_0 = 1.677 \text{ s}$, $a_0 = 0.75 \text{ m/s}^2$ and $u_t = 1.258 \text{ m/s}$ is the terminal velocity. Figure 4 shows the prescribed acceleration, velocity and rigid slide displacement. In the laboratory experiment, the block is stopped at time $t = t_0 = 1.667 s$. and we replicate this behaviour in the numerical model. We also performed numerical experiments (not presented here) where the block continued moving at constant speed.



Figure 4: BP1. Prescribed acceleration, velocity and displacement of the solid slide.

The benchmark here consists of using the above information on slide shape, submergence, and kinematics, together with reproducing the experimental setup to simulate surface elevations measured at the four wave gauges (average of 2 replicates of experiments provided).

Then, in order to reproduce the lab experiment, the interval [-1, 10] discretized with $\Delta x = 0.02 \ m$, is the computational domain considered. In the vertical, taking three layers seems to produce optimal results. Increasing the number of layers gives similar results increasing the computational cost. The stability *CFL* number was set to 0.9 and g = 9.81. The numerical simulation performed was 4 s long in real time. As boundary conditions, outflow conditions were imposed at x = -1, x = 10.

In Figure 5 the comparison of the numerical results with the filtered lab measured data is presented. A good overall agreement between them can be observed. Some discrepancies can be seen after draw-down in all the gauges.

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This behavior could also be observed, except for the last gauge, at Grilli and Watts (2005) results. These authors explained that this behavior could be due to unwanted surface tension effects. Given this comparison, and considering the experimental variations and errors inherent to laboratory work and data processing, it can be concluded that the Multilayer-HySEA model performs optimally the present benchmark test.



Figure 5: BP1. Filtered data (in red) and numerically simulated (in blue) time series at wave gauges (A) g_0 , (B) g_1 , (C) g_2 , and (D) g_3 .

374 5.2. Benchmark Problem 2: Three-dimensional submarine solid block

This second benchmark consists of a 3D extension of BP1. The longitudinal 375 sketch of the experiment is the same as in Figure 3. In the horizontal plane, 376 cross-sections are elliptic, the plan view of the rigid slide, for the case d = 61 mm, 377 is presented in Figure 6. It is based on the 3D laboratory experiments of Enet 378 and Grilli (2007). The experiments were also performed at the University of 379 Rhode Island in a water wave tank of width 3.6 m and length 30 m, with a still 380 water depth of 1.5 m over the flat bottom portion. As in the previous benchmark, 381 the angle of the plane slope where the slide slid down is $\theta = 15^{\circ}$. The submarine 382 slide model was built as a streamline Gaussian-shaped aluminum body with 383 elliptical footprint (see Figure 6), with down-slope length $b = 0.395 \,\mathrm{m}$, cross-384 slope width $w = 0.680 \,\mathrm{m}$, and maximum thickness $T = 0.082 \,\mathrm{m}$. Complete 385 details about the analytic definition of the slide shape and the experimental 386 setting can be found at Kirby et al. (2018) and at LTMBW (2017). 387



Figure 6: BP2. Sketch of the plan view (case 61 mm). [From Kirby et al. (2018)].

For the numerical simulations, the two-dimensional computational domain $[-1, 10] \times [-1.8, 1.8]$ is considered and discretized with $\Delta x = \Delta y = 0.02 m$. The number of layers was set to 3. Numerical tests were performed using more layers and similar results were obtained. The CFL number was set to 0.9 and g = 9.81. The simulated time was 6 s. As boundary conditions, rigid wall conditions were imposed at y = -1.8, y = 1.8 and outflow conditions at x = -1, x = 10.

The benchmark test proposed consists in reproducing the slide shape and complete experimental set-up in and using the information about submergence and kinematics to replicate numerically Enet and Grilli's experiments for d =61 and d = 120 mm. It is required to simulate surface elevations measured at the four wave gauges (average of 2 replicates of experiments) and present comparisons of the model with the experimental results.

Enet and Grilli (2007) performed experiments for 7 initial submergence depths d. They are listed in Table 4, together with values of related slide parameters and some measured tsunami wave characteristics. Here, the numerical results corresponding to the two NTHMP required experiments (for d = 61and d = 120 mm) will be presented first, then, as data for the seven experiments were provided, the comparison for the remaining five cases will also be presented.

d (mm)	61	80	100	120	140	149	189
$x_g \text{ (mm) (measured)}$	551	617	696	763	846	877	1017
$x_g \text{ (mm) (theoretical)}$	560	630	705	780	854	888	1037
$\eta_0 \ (\mathrm{mm})$	13.0	9.2	7.8	5.1	4.4	4.2	3.1
$a_0 (m/s)$	1.20	1.21	1.19	1.17	1.14	1.20	1.21
$u_t (m/s)$	1.70	1.64	1.93	2.03	2.13	1.94	1.97
t_0 (s)	1.42	1.36	1.62	1.74	1.87	1.62	1.63
S_0 (m)	2.408	2.223	3.130	3.522	3.980	3.136	3.207

Table 4: Measured and curve-fitted slide and wave parameters for the 7 experiments performed by Enet and Grilli (2007). Nomenclature: Measured characteristic amplitude η_0 (at $x = x_0$). Slide kinematics parameters a_0, u_t and t_0 .

In Figure 7 the comparison of the Multilayer-HySEA model numerical results with measured data for the first case, $d = 61 \, mm$, in the four gauges, is

g_1	g_2	g_3	g_4
$(x_0,0)$	(1469, 350)	(1929,0)	(1929, 500)

Table 5: Wave gauge locations (x, y) in mm, as shown in Figure 6.

⁴⁰⁹ presented. An excellent agreement can be observed between these time series. ⁴¹⁰ The comparisons for the second required case (d = 120 mm) in the 3 gauges ⁴¹¹ with data provided (gauge g_3 was not available) are shown in Figure 8. Good ⁴¹² agreement can also be observed in this case. Finally, Figure 9 shows the com-⁴¹³ parison for the five remaining cases provided by Enet and Grilli. In all cases (for ⁴¹⁴ all submergences), a good agreement between simulated results and measured ⁴¹⁵ lab data can be observed.



Figure 7: Test case d = 61 mm. Numerically computed (in blue) time time series at wave gauges (A) g_1 , (B) g_2 , (C) g_3 , and (D) g_4 compared with the lab measured data (in red).



Figure 8: Test case $d = 120 \ mm$. Numerically computed (in blue) time time series at wave gauges (A) g_1 , (B) g_2 , and (C) g_4 for compared with the lab measured data (in red).



Figure 9: Comparison of data time series and numerical at wave gauges (dashed) for the cases (A) d = 80 mm, (B) d = 100 mm, (C) d = 140 mm, (D) d = 149 mm, and (E) d = 189 mm.

In Table 6, the execution times for simulations performed on a NVIDIA Tesla P100 GPU are presented. It can be observed that including non-hydrostatic terms in the NLSW equations results in an increase of the computational time in 2.65 times. If a richer vertical structure is considered, then larger computational times are required. As examples for the two and three-layer systems, 3.3 and 4.45 times increase in the computational effort.

	Runtime (s)	Ratio
SWE	23.08	1
1L-NH	61.20	2.65
2L-NH	76.35	3.30
3L-NH	102.93	4.45

Table 6: Execution times in seconds for SWE and non-hydrostatic GPU implementations. Ratios compared with SWE.

Figure 10 shows the comparison, for the four models considered, of the numerical results obtained with the measured data at gauge g_4 for the case $d = 189 \ mm$. It can be observed that a model vertical structure considering only one layer is not enough to reproduce the observed data, and that considering 2 and 3 layers in the model produce much better numerical results.

Moreover, Table 7 shows the period times, T, of the time series data in Figures 7, 8, and 9 for all the wave gauges. To obtain the vales for the periods, we have computed the elapsed time between the first two wave troughs in each time series. We have omitted the measurement for wave gauge g_1 because it was not clear how to measure the period in this case. Once the period from each time series has been measured, we have computed the wave number from the dispersion relation given from the Airy theory:

$$\frac{2\pi}{T} = \sqrt{gk \tanh(kH)}.$$
(23)

Table 8 shows the computed values kH by solving the dispersion relation (23). On the view of the computed kH values it can be stated that $kH \in [2.815, 4.528]$. Since multilayer models have good dispersion relation errors within this range $_{437}$ of kH (see Table 1 and Figure 2), this explains the aforementioned excellent $_{438}$ agreement between the computed time series and the measured lab data.

Finally, although the phase velocity for the one-layer system shows an error bounded by only 3.02% for $kH \in [0, 5]$ (see Table 1), it can be seen in Figure 10 that the one-layer non-hydrostatic pressure system cannot represent the waves correctly. In contrast, the one-layer system tends to amplify waves. This behaviour can be explained by observing the shoaling gradient for this model (see Figure 2). The shoaling gradient verifies the ODE

$$A' = -A \cdot \gamma(kH) \cdot \frac{\partial_x H}{H},\tag{24}$$

where A denotes the amplitude. Then, it can be stated by inspecting the solutions of the ODE (24) that if the shoaling gradient of the model γ(kH) is
underestimated with respect to the Airy theory (γ < γ_{Airy}), then the solutions
of the system tend to amplify waves, in this case, for offshore wave propagation.
The poor behavior shown by the one-layer system in some cases justifies the need to incorporate the improved multilayer model considered here.

$d \ (mm)$	61	80	100	120	140	149	189
g_2	0.69	0.686	0.704	0.69	0.676	0.692	0.8
g_3	0.66	0.716	_	_	0.702	_	0.694
g_4	0.84	0.8	0.784	0.75	0.794	0.784	0.751

Table 7: Measured wave period T (s) for test cases d = 61, 120 (Figures 7 and 8) and d = 80, 100, 140, 149, 189 (Figure 9).

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451 5.3. Benchmark Problem 3: Three-dimensional submarine/subaerial triangular 452 solid block

This benchmark problem is based on the 3D laboratory experiment of Wu (2004) and Liu et al. (2005), for a series of triangular blocks of several aspect ratios moving down a plane slope into the water from a dry (subaerial) or wet (submarine) location. Figure 11 shows the schematic description of the set-up for this benchmark in the case of a partially submerged block. Further details

d (mm)	61	80	100	120	140	149	189
g_2	3.114	3.15	2.995	3.114	3.242	3.097	2.35
g_3	4.528	3.85	—	_	4.004	—	4.097
g_4	2.815	3.093	3.218	3.512	3.14	3.218	3.512

Table 8: Computed kH values from the measured wave period (see Table 7) and the Airy dispersion relation $2\pi/T = \sqrt{gk \tanh(kH)}$ for test cases d = 61, 80, 100, 120, 140, 149 and 189.



Figure 10: Test case $d = 189 \ mm$. Lab measured data (red) and numerically computed time series at wave gauge g_4 using different numerical models.

can be found at Kirby et al. (2018) and at LTMBW (2017). The laboratory
experiments were conducted in a wave tank at Oregon State University of length
104 m, width 3.7 m, and depth 4.6 m.

⁴⁶¹ A plane slope 1:2 (as the one shown in Figure 11 upper panel) with $\theta = 26.6^{\circ}$ ⁴⁶² was located near one end of the tank and a dissipating beach in the other. In all ⁴⁶³ the experiments, the water depth was $h_0 = 2.44$ m. The experiments retained ⁴⁶⁴ for the present benchmark were all performed with a triangular block of length ⁴⁶⁵ b = 0.91 m, width w = 0.61 m, and vertical front face a = b/2 = 0.455 m.

The block movement was provided by means of a polynomial fitting to measured data, giving the horizontal distance as:

$$x_{0,t} = x_{(0,t=0)} + (a t^3 + b t^2 + c t) \cos \beta, \qquad (25)$$

with $\beta = \arctan(1/2)$ and $x_{(0,t=0)} = -2\Delta$. The polynomial coefficients for the two cases proposed are given in Table 9.

Δ	a	b	с
0.10 m	-0.097588	0.759361	0.078776
-0.25 m	-0.085808	0.734798	-0.034346

Table 9: Polynomial coefficients defining slide motion.

For each case, measured free surface elevations for two wave gauges placed at (x, y) = (1.83, 0) (in m) and (x, y) = (1.2446, 0.635), where x is the distance to the initial coastline and y is the distance to the central cross-section (see location at Fig. 11 lower panel). Also measured runup for each case is given at runup gauges 2 and 3 in Figure 11 lower panel, lying on the slope at a distance 0.305 m and 0.611 m from the central cross-section, respectively.

The two-dimensional computational domain $[-2, 6] \times [-2, 2]$ is discretised with $\Delta x = \Delta y = 0.025 m$ and the number of layers was set up to 3. Numerical experiments using more number of layers were performed, obtaining similar results. The stability CFL number was set to 0.9 and g = 9.81. The simulated time was 4 s. The same boundary conditions, as in the previous case, were imposed.

The numerical results obtained for the subaerial test case are presented in 482 Figures 12 and 13. Figure 12 depicts the comparison for the time series at the 483 wave gauges and Figure 13 at the runup gauges. The same comparison has been 484 performed for the submerged test case, and it is presented in Figures 14 and 485 15. The agreement for the wave gauges is quite good for WG1 in both cases. 486 For WG2, just in front of the block, an overshoot after the first depression wave 487 is observed in both cases related to the turbulent nature of the experiment. 488 Note that although a turbulent model is not considered here, we have noted 489 that the breaking criteria helps to dissipate energy associated to this turbulent 490 process. For the run-up, the qualitative agreement is quite good, with the larger 491 discrepancies in RG3 for the submarine test case. 492



Figure 11: Definition sketch for BP3 laboratory experiments. Here for a submerged ($\Delta < 0$) slide. Upper panel vertical cross section, lower panel plan view. All units are in meters.



Figure 12: Subaerial test case. Lab measured water height (red) and numerical time series (blue) at wave gauges (A) WG1 and (B) WG2 .



Figure 13: Subaerial test case. Lab measured runup (red) and numerical time series (blue) at runup gauges (A) RG2 and (B) RG3.



Figure 14: Submerged test case. Lab measured water height (red) and numerical time series (blue) at wave gauges (A) WG1 and (B) WG2.



Figure 15: Submerged test case. Lab measured runup (red) and numerical time series (blue) at runup gauges (A) RG2 and (B) RG3.

493 6. Concluding Remarks

Validation of numerical models is a first unavoidable step before their use 494 as predictive tools. This requirement is even more necessary when the devel-495 oped models are going to be used for risk assessment in natural events where 496 human lives are involved. The present work is the first step in this task for the 497 Multilayer-HySEA model, a novel dispersive multilayer model of the HySEA 498 suite developed at the University of Malaga. This model considers a stratified 499 vertical structure and includes non-hydrostatic terms, this is done in order to 500 include the dispersive effects in the propagation of the waves in a homogeneous, 501 inviscid, and incompressible fluid. The numerical scheme implemented, com-502 bines a highly robust and efficient finite volume path-conservative scheme for 503 the underlying hyperbolic system and finite differences for the discretization of 504 the non-hydrostatic terms. In order to increase numerical efficiency, the numeri-505 cal model is implemented to run in GPU architectures. In particular in NVIDIA 506 graphics cards and using CUDA language. In the case of the traditional SW 507 non-dispersive model, this kind of implementations produces an extremely ef-508 ficient and fast code (Macías et al., 2020d). Increasing the number of layers 509 in SW models provides an enhanced vertical resolution and, at the same time, 510 increases the computational cost. Despite this, from a computational point of 511 view, the two-layer non-hydrostatic code presents a good computational effi-512 ciency, and computing times with respect to the one layer SWE GPU code are 513 absolutely reasonable, being only from 2 to 2.5 larger that for the one layer case. 514 In the numerical simulations performed in the present work, the non-hydrostatic 515 wall-clock times are always below 4.45 times those for the traditional SWE Hy-516 SEA model, for a number of vertical layers up to three. The numerical scheme 517 presented here and the corresponding multilayer SW water model proposed, is 518 highly efficient and is able to model dispersive effects with a low computational 519 cost. 520

Regarding model results, they show a good agreement with the experimental data for the three benchmark problems studied in the present work. In partic-

ular, for BP2, but this also occurs for the other two benchmark problems, we 523 have shown that a one layer, hydrostatic or non-hydrostatic, model is not able 524 to reproduce the complexity in the observed lab data considered in the pro-525 posed benchmarks. The waves to be modeled in the test cases proposed here 526 are high-frequency and dispersive. Hence, it is at least necessary a two-layer 527 structure and non-hydrostatic terms in the model to be used in order to capture 528 the dynamics of the generated waves. As noted in Kirby et al. (2018) and in 529 view of the results presented, the non-hydrostatic multilayer model discussed 530 here can adequately represent the physics and behavior of the waves generated 531 with a reasonable low computational cost. 532

533 7. Code and data availability

The numerical code used to perform the numerical simulations in this paper is available at HySEA codes web page at https://edanya.uma.es/hysea/index.php/ download.

All the data used in the present work and necessary to reproduce the experiments set-up of the numerical experiments and the laboratory measured data to compared with, can be downloaded from LTMBW (2017) at the web site http://www1.udel.edu/kirby/landslide/. Finally, the NetCDF files containing the numerical results obtained with the Multilayer-HySEA code can be found and download from Macías et al. (2020b).

543 8. Authors' contributions

JM is leading the HySEA codes benchmarking effort undertaken by the EDANYA group, he wrote most of the paper, reviewed and edited it, assisted in the numerical experiments and in their set up. CE implemented the numerical code and performed all the numerical experiments, he also contributed to writing the manuscript. JM and CE did all the figures. MC strongly contributed to the design and implementation of the numerical code.

550 9. Competing interest

⁵⁵¹ The authors declare that they have no conflict of interest.

552 10. Acknowledgements

This research has been partially supported by the Spanish Government-FEDER funded project MEGAFLOW (RTI2018-096064-B-C21), the Junta de Andalucía-FEDER funded project UMA18-Federja-161 and the University of Málaga, Campus de Excelencia Internacional Andalucía Tech.

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