Interactive comment on “Evaluation of global seismicity along Northern and Southern hemispheres” by Olaide Sakiru Hammed et al.

Yavor Kamer (Referee)
yaver.kamer@gmail.com

Received and published: 26 June 2020

Dear Anonymous Reviewer,

The paper that you recommend uses Bender’s 1983 formula for grouped data. In 2003 Marzocchi and Sandri [1] showed that Tinti and Mulargia’s 1987 formula (equation 3.9 and 3.10) is more accurate.

It is unfortunate that a paper investigating systematic biases uses a biased estimator, while being recommended here as a reference to guide others.

Kind regards, Y.Kamer

the b-value and its uncertainty. Annals of geophysics, 46(6).

The validity of eqs. (3.6) and (3.7) deserves further
explanations. In particular, these equations assume that
\( E(x) = b \) and \( E(b) = b \), respectively. If we take the expected value of Taylor's
expansion around the true value \( \mu \) of eqs. (2.3)
and (3.1), we see that these assumptions hold only for small deviations of \( \mu \), i.e. for large
datasets. Numerical investigations have shown that
the biases are negligible for datasets with
50 or more earthquakes.

By comparing eqs. (3.6) and (3.7) we obtain
\[
\sigma_\lambda = \left( 1 + \frac{\theta}{b} \right)^3 \sigma_\lambda
\]
(3.8)
therefore, \( \sigma_\lambda \gtrsim \sigma_\lambda \). From eqs. (3.5) and (3.8),
we can conclude that the true dispersion of the
RV \( E(\sigma_\lambda) \) increases more than the increase in
the estimation of the uncertainty \( \sigma_\lambda \). In other
words, eq. (2.4) provides an underestimation of
the true dispersion.

3.2. Binned formulas

After the correction suggested by Utsu (1966), Bender (1983), Tinti and
Mulargia (1987) provided formulas to estimate the \( b \)
value, by properly taking into account the
biasing of the magnitudes. Remarkably, be-
sides very few exceptions (e.g., Frohlich and
Davis, 1983), these formulas were almost ig-
nored in subsequent applications. We argue
that the reasons are mainly of a technical
nature. Bender's (1983) formulas, for example,
can be solved only numerically. Moreover, in
her analysis she gave more emphasis to the bias \( \theta \) introduced by the use of the con-
tinuous approximation (eq. (2.3)), concluding that
the latter provides almost unbiased estima-
tions of the \( b \)-value if the magnitude interval
for the grouping is \( \Delta M < 0.1 \).
A definite improvement to the estima-
tion of the \( b \)-value was provided by Tinti and
Mulargia (1987). Their formula reads
\[
b_{\text{bw}} = \frac{1}{\ln(10) \Delta M \ln(p)}
\]
(3.9)
where
\[
p = \frac{1}{\mu - M_{\text{min}}} \frac{\Delta M}{(b - M_{\text{min}})}
\]
(3.10)
and the associated asymptotic error is
\[
\sigma_{bw} = \frac{1}{\ln(10) \Delta M \sqrt{np}}
\]
(3.11)
where \( N \) is the number of earthquakes. In this
case, we think the very scarce use of these
formulas was probably due to some kind of cri-
pticism of the paper.

3.3. Numerical check

In order to check the reliability of the formulas
described above, we simulate 1000 seismic cata-
logs, for different catalog sizes. The magnitudes \( M \)
are obtained by binning, with \( \Delta M = 0.1 \) (as for the
instrumental magnitudes), a continuous RV dis-
tributed with a pdf given by eq. (2.2); in other
words, \( M \) is the magnitude attached to all the syn-
thetic seismic events with real continuous magni-
dude in the range \( M - 0.05 \leq M < M + 0.05 \).

In fig. 1a,b we report the medians of \( b \); \( b \) and
\( b_{\text{bw}} \) calculated in 1000 synthetic catalogs as a func-
tion of the number of data, for the case \( b = 1 \)
and \( b = 2 \). To each median is attached the 95\% confi-
dence interval, given by the interval between the
2.5 and 97.5 percentile. From fig. 1a,b, we can see
that the estimation \( b_{\text{bw}} \) (Tinti and Mulargia, 1987)
is bias free, also for a small dataset. As regards the
continuous formulas, with and without correction
(respectively eqs. (3.1) and (2.3)), we can see that
the bias \( \theta \) reported in fig. 1a,b is comparable to
the theoretical expectation given by eq. (3.3). The
corrected estimation \( b \) is undoubtedly much closer
to the real \( b \)-value. The slight underestimation of \( b \)
(much less than 1\% of the real \( b \)-value) is due to the
bias \( \theta \) previously discussed (Bender, 1983).
Therefore, at least for \( \Delta M = 0.1 \), \( \theta \) can be neglect-
ated (e.g., Bender, 1983), but \( \theta \) is certainly relevant.
In order to evaluate the reliability of the estima-
tions of the uncertainty, it is necessary to compare each estimation with the true dis-