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Natural Hazards and Earth System Sciences
Discussions

Interactive comment on "Evaluation of global seismicity along Northern and Southern hemispheres" *by* Olaide Sakiru Hammed et al.

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Dear Anonymous Reviewer,

The paper that you recommend uses Bender's 1983 formula for grouped data. In 2003 Marzocchi and Sandri [1] showed that Tinti and Mulargia's 1987 formula (equation 3.9 and 3.10) is more accurate.

It is unfortunate that a paper investigating systematic biases uses a biased estimator, while being recommended here as a reference to guide others.

Kind regards, Y.Kamer

[1] Marzocchi, W. and Sandri, L., 2009. A review and new insights on the estimation of

the b-value and its uncertainty. Annals of geophysics, 46(6).

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The validity of eqs. (3.6) and (3.7) deserves further explanations. In particular, these equations assume that $E(\hat{b}^*) = b^*$ and $E(\hat{b}) = b$, respectively. If we take the expected value of Taylor's expansion around the true value μ of eqs. (2.3) and (3.1), we see that these assumptions hold only for small deviations of $\hat{\mu}$, *i.e.* for large datasets. Numerical investigations have shown that the biases are negligible for datasets with 50 or more earthquakes.

By comparing eqs. (3.6) and (3.7) we obtain

$$\sigma_{\hat{b}^*} = \left(1 + \frac{\theta_2}{\hat{b}}\right)^2 \sigma_{\hat{b}} \qquad (3.8)$$

therefore, $\sigma_{\hat{b}^*} > \sigma_{\hat{b}}$. From eqs. (3.5) and (3.8), we can conclude that the true dispersion of the RV $\hat{b}^*(\sigma_{\hat{b}^*})$ increases more than the increase in the estimation of the uncertainty $\hat{\sigma}_{\hat{b}^*}$. In other words, eq. (2.4) provides an underestimation of the true dispersion.

3.2. *Binned formulas*

After the correction suggested by Utsu (1966), Bender (1983), Tinti and Mulargia (1987) provided formulas to estimate the *b*-value, by properly taking into account the grouping of the magnitudes. Remarkably, besides very few exceptions (*e.g.*, Frohlich and Davis, 1983), these formulas were almost ignored in subsequent applications. We argue that the reasons are mainly of a technical nature. Bender's (1983) formula, for example, can be solved only numerically. Moreover, in her analysis she gave more emphasis to the bias θ_1 introduced by the use of the continuous approximation (eq. (2.3)), concluding that the latter provides almost unbiased estimations of the *b*-value if the magnitude interval for the grouping is $\Delta M = 0.1$.

A definite improvement to the estimation of the *b*-value was provided by Tinti and Mulargia (1987). Their formula reads

$$\hat{b}_{TM} = \frac{1}{\ln(10)\,\Delta M} \ln(p) \qquad (3.9)$$

where

$$p = \left(1 + \frac{\Delta M}{\hat{\mu} - M_{thresh}}\right) \qquad (3.10)$$

and the associated asymptotic error is

$$\hat{\sigma}_{\hat{b}_{TM}} = \frac{1 - p}{\ln(10)\,\Delta M \sqrt{Np}} \qquad (3.11)$$

where N is the number of earthquakes. In this case, we think the very scarce use of these formulas was probably due to some kind of crypticism of the paper.

3.3. *Numerical check*

In order to check the reliability of the formulas described above, we simulate 1000 seismic catalogs, for different catalog sizes. The magnitudes M are obtained by binning, with $\Delta M = 0.1$ (as for the instrumental magnitudes), a continuous RV distributed with a pdf given by eq. (2.2); in other words, M_i is the magnitude attached to all the synthetic seismic events with real continuous magnitude in the range $M_i - 0.05 \le M < M_i + 0.05$.

In fig. 1a,b we report the medians of $\hat{b}^*$, $\hat{b}$ and $\hat{b}_{TM}$ calculated in 1000 synthetic catalogs as a function of the number of data, for the case $b = 1$ and $b = 2$. To each median is attached the 95% confidence interval, given by the interval between the 2.5 and 97.5 percentile. From fig. 1a,b, we can see that the estimation $\hat{b}_{TM}$ (Tinti and Mulargia, 1987) is bias free, also for a small dataset. As regards the continuous formulas, with and without correction (respectively eqs. (3.1) and (2.3)), we can see that the bias θ_2 reported in fig. 1a,b is comparable to the theoretical expectation given by eq. (3.3). The corrected estimation $\hat{b}$ is undoubtedly much closer to the real *b*-value. The slight underestimation of $\hat{b}$ (much less than 1% of the real *b*-value) is due to the bias θ_1 previously discussed (Bender, 1983). Therefore, at least for $\Delta M = 0.1$, θ_1 can be neglected (*e.g.*, Bender, 1983), but θ_2 is certainly relevant.

In order to evaluate the reliability of the estimations of the uncertainty, it is necessary to compare each estimation with the true dis-

Fig. 1.