Supporting Information

1 Markov model for weather pattern (WP) prediction

Let $W(t) = i$ represent a particular WPi, with $i = 1, \ldots, 30$, on day t. We construct a first-order, nonhomogeneous Markov chain to predict WPs using the following procedure:

1. Calculate the 12 monthly transition matrices, $\mathbf{X}^{\boldsymbol{m}}, m = 1, \ldots, 12$, of MO30:
$$x_{i,j}^{(m)} = \Pr(W_t = i | W_{t-1} = j), \text{for } i, j = 1, \ldots, 30.$$
Each matrix is calculated using transition probabilities from three consecutive months, centred on the month of interest. For example, the transition matrix for June is calculated using data from May, June and July.

2. Set $W(t)$ as the observed WP at time $t = 0$, the forecast initialisation day.

3. Generate a random number x^* from the standard uniform distribution $U(0,1)$.

4. Find the index y such that:
$$\sum_{j=1}^{y-1} x_{i,j}^{(m)} < x^* < \sum_{j=1}^{y} x_{i,j}^{(m)}$$
where $i = W(t)$.

5. Set $W(t+1) = y$ and $t = t + 1$.

6. Repeat steps 3 to 5 until $t = T$, the final day to be forecast.

To provide probabilistic output, for each forecast we run 1000 Markov models in parallel.

2 Estimating precipitation forecasts from weather patterns

We process the daily HadUKP precipitation data by discretising into v bins with historical probabilities p_b for $b = 1, \ldots, v$. Dry days form one bin and bin intervals increase for higher precipitation values, see Table 1 in the main document. This gives a discrete distribution of precipitation interval relative frequencies, $D(z)$, with conditional distributions for each WP given by $D(z|W = i)$, for $i = 1, \ldots, 30$. We also define w summed precipitation intervals s_c for $c = 1, \ldots, w$. Forecast probabilities of these summed intervals are derived from the WP forecast models as follows:

1. Set the ensemble member $e \in (e_1, \ldots, e_N)$, where N_e is the number of ensemble members; time $t = 0$, the first day of the forecast, and then the predicted WP by ensemble member e at time t is $W_e(t) = i$ for $i = 1, \ldots 30$.

2. Set $p_0 = 0$, calculate the probabilities $p_1, \ldots p_m$ of each of the m daily precipitation bins from the discrete precipitation distribution that is conditional on $W_e(t)$ and on the 91-day windows centred on t (i.e. $t - 45, \ldots, t + 45$) from every year except the current year. This last condition is equivalent to a leave-one-year-out cross-validation procedure.

3. Define the maximum value of each bin as $l_{p_b}, b = 1, \dots v$, with $l_{p_0} = 0$. Note that $l_{p_0} = l_{p_1} = 0$, ensuring zero precipitation days can be simulated.

4. Generate u random variables $p_k^* \sim U(0,1)$ for $k = 1, \dots, u$.

5. For each p_k^*, find the index q such that

$$\sum_{j=0}^{q} p_j < p_k^* < \sum_{j=0}^{q+1} p_j \, .$$

Set $P_q = \sum_{j=0}^{q-1} p_j$ and $P_{q+1} = \sum_{j=0}^{q} p_j$, the cumulative probabilities of the bins adjacent to p_k^*.

6. Define the difference between the adjacent bins as $\alpha = P_{q+1} - P_q$ and the difference between the random number and the lower cumulative probability as $\beta = p_k^* - P_q$.

7. Estimate the precipitation value for each p_k^* as $r_k(t) = l_{p_q} + \frac{\beta}{\alpha}\left(l_{p_{q+1}} - l_{p_q}\right)$. We now have u predicted daily precipitation values at time t, $\boldsymbol{r}(t) = (r_1(t), \dots, r_n(t))$.

8. Set $t = t + 1$ and repeat steps 3 to 6 until the final day of the forecast, $t_{\max}$, is processed.

9. Sum the daily precipitation vectors and divide by the random-sample size $(\sum_\tau \boldsymbol{r}(t))/u$ for $\tau = 0, \dots, t_{\max}$.

10. Discretise according to the w summed precipitation bins $s_1, \dots s_w$ to obtain a distribution of relative frequencies for this ensemble member $\boldsymbol{f_e} = (f_1, \dots, f_w)$.

11. Set a new ensemble member $e^* \in \left(e_1, \dots e_{N_e}\right), e^* \neq e$ and repeat steps 2 to 10 until every ensemble member has been processed.

12. Sum each ensemble member's distribution of summed precipitation relative frequencies and divide by the number of ensemble members to obtain a final forecast probability distribution:

$$\boldsymbol{F} = \left(\sum_e \boldsymbol{f_e}\right) \Big/ N_e.$$

The number of ensemble members depends on the model. For EPS-WP, $N_e = 11$, i.e. the number of ensemble members of the ECMWF dynamical model. For the Markov model $N_e = 1000$. We set the number of samples drawn from each WP-precipitation conditional distribution as $u = 10{,}000$.