## Particle filter algorithm process in this paper.

Step 1: Draw initial particles $\{\boldsymbol{x}_0^i\}_{i=1}^N$ from prior PDF $p(\boldsymbol{x}_0)$ . $\boldsymbol{x}$ is the state vector. N is the number of particles.

Step 2: Compute the mean and variance of posterior particles at step $k$-1:

$$\bar{\boldsymbol{x}}_{k\text{-}1/k-1} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_{k\text{-}1/k-1}^i$$

$$\mathbf{D}_{k\text{-}1/k-1} = \frac{1}{N-1}\sum_{i=1}^{N}(\boldsymbol{x}_{k\text{-}1/k-1}^i - \bar{\boldsymbol{x}}_{k\text{-}1/k-1})(\boldsymbol{x}_{k\text{-}1/k-1}^i - \bar{\boldsymbol{x}}_{k/k-1})^{\mathrm{T}}$$

Step 3: Using the new method in this section, compute the “gains” of particles:

$$\mathbf{D}_{k/k-1} = \left[\frac{\partial f_k}{\partial \boldsymbol{x}_k}(\hat{\boldsymbol{x}}_k)\right]\mathbf{D}_{k-1/k-1}\left[\frac{\partial f_k}{\partial \boldsymbol{x}_k}(\hat{\boldsymbol{x}}_k)\right]^{\mathrm{T}} + \mathbf{G}_{k-1}(\hat{\boldsymbol{x}}_{k-1})\mathbf{Q}_{k-1}\mathbf{G}_{k-1}^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k-1})$$

$$\mathbf{J}_k = \mathbf{D}_{k/k-1}\left[\frac{\partial f_k}{\partial \boldsymbol{x}_k}(\hat{\boldsymbol{x}}_{k/k-1})\right]\left\{\left[\frac{\partial f_k}{\partial \boldsymbol{x}_k}(\hat{\boldsymbol{x}}_{k/k-1})\right]\mathbf{D}_{k/k-1}\left[\frac{\partial f_k}{\partial \boldsymbol{x}_k}(\hat{\boldsymbol{x}}_{k/k-1})\right]^{\mathrm{T}} + \mathbf{R}_k\right\}^{-1}$$

Step 4: Transfer the particles close to the observation:

$$\boldsymbol{x}_k^i = f(\boldsymbol{x}_{k-1}^i) + \hat{\boldsymbol{\varepsilon}}_{k-1} + \mathbf{J}_k[\boldsymbol{z}_k - h(\hat{\boldsymbol{x}}_{k-1})]$$

Step 5: Residual resampling. Each particle generates a set of normally-distributed progeny particles, and all progeny sets make up the resampled particle set:

$$\left\{\boldsymbol{x}_k^{i1}, \boldsymbol{x}_k^{i2}, ..., \boldsymbol{x}_k^{iACT_i}\right\} = \boldsymbol{X}_k^{iACT_i} \sim N\left(\boldsymbol{x}_k^i, \mathbf{G}_k\left(\boldsymbol{x}_k^i\right)\right)$$

$$\left\{\boldsymbol{X}_k^{1ACT_1}, \boldsymbol{X}_k^{2ACT_2}, ..., \boldsymbol{X}_k^{N\ ACT_N}\right\} = \left\{\boldsymbol{x}_k^{*i}\right\}_{i=1,2,...,N}$$

When $ACT_i = 0$ , $\boldsymbol{X}_k^{iACT_i}$ is an empty set.

Step 6: Compute and normalize weights:

$$\tilde{w}_k^i = w_{k-1}^i \cdot p(\boldsymbol{z}_k \mid \boldsymbol{x}_k^i)$$

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^{N}\tilde{w}_k^j}$$

Step 7: Compute the state estimation:

$$\hat{\boldsymbol{x}}_{k/k} = \sum_{i=1}^{N}\boldsymbol{x}_k^{*i} \cdot w_k^i$$

A measure to assess the accuracy of calculation is the root mean square difference (RMSD), which is defined as

$$RMSD = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\boldsymbol{X}}_t - \boldsymbol{X}_t^{obs})^2}$$

where T is the period of assimilation, $\hat{\boldsymbol{X}}_t$ and $\boldsymbol{X}_t^{obs}$ are the assimilated value and the observation of state at time *t*, respectively.