

Quotation of the general comment: “It is suggested that the abstract need to be revised to be as concise and understandable as possible”

Reply: Thank you very much for your suggestion. We revised the manuscript to make it as concise and understandable as possible.

Quotation of the general comment: “IIR method is seldom introduced in this article, and the method is the key to this article. What work has the author done on IIR method?”

Reply: Thank you very much for your suggestion. Yes, the method is important in this article, we explain it in the article simple. We will explain it in this supplement document.

First, the frequency-dependent relative site amplification factors are assumed to be modelled by a following linear system of first- and second order filters,

$$F(s) = G_0 \prod_{n=1}^N \left(\frac{\omega_{2n}}{\omega_{1n}} \right) \cdot \frac{s + \omega_{1n}}{s + \omega_{2n}} \cdot \prod_{m=1}^M \left(\frac{\omega_{2m}}{\omega_{1m}} \right)^2 \cdot \frac{s^2 + 2h_{1m}\omega_{1m} + \omega_{1m}^2}{s^2 + 2h_{1m}\omega_{1m} + \omega_{2m}^2} \quad (1)$$

where N and M stand for the numbers of the first and second-order filters, respectively, and $s = i\omega$. Here ω are angular frequencies and h are damping factors that characterize the frequency dependence,

respectively. $s^2 + 2h\omega + \omega_m^2$ represents a damping oscillation.

Then, we need to use the following formula (2) to transfer the continuous-time system representations to discrete-time system

The bilinear transform is introduced as:

$$s = \frac{2}{\Delta T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \quad (2)$$

Then the pre-warping equation

$$\omega \rightarrow \frac{2}{\Delta T} \tan\left(\frac{\omega \Delta T}{2}\right) \quad (3)$$

where ΔT is the sampling interval of the digital waveforms and $z = \exp(s\Delta T)$.

Formula (3) is applied to ω_{1n} , ω_{2n} , ω_{1m} , ω_{2m} ,

By the above two steps, the transfer function $F(z)$ is obtained,

Eq. (2) and Eq. (3) are the necessary procedures to obtain the coefficients of a causal recursive filter for real time processing.

Now we will show the detailed process for the IIR method:

In this method, the filter is modelled as the formula (1), $F(s)$ is written as combined formula of first order and second order s ,

$$F_{1n}(s) = \left(\frac{\omega_{1n}}{\omega_{2n}}\right) \cdot \frac{s + \omega_{1n}}{s + \omega_{2n}},$$

$$F_{2m}(s) = \left(\frac{\omega_{2m}}{\omega_{1m}}\right)^2 \cdot \frac{s^2 + 2h_1\omega_{1m} + \omega_{1m}^2}{s^2 + 2h_2\omega_{2m} + \omega_{2m}^2} \quad (4)$$

Then we will explain how to transfer formula (4) to IIR filter

$$F_{1n}(z) = g_0 \cdot \frac{a_0 + a_1 z^{-1}}{1 + b_1 z^{-1}},$$

$$F_{2m}(z) = g_0 \cdot \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} \quad (5)$$

For $F_{1n}(z)$, the recursive filter is given as

$$y_k = g_0(a_0 x_k + a_1 x_{k-1}) - b_1 y_{k-1} \quad (6)$$

For $F_{2m}(z)$, the recursive filter is given as

$$y_k = g_0(a_0 x_k + a_1 x_{k-1} + a_2 x_{k-2}) - (b_1 y_{k-1} + b_2 y_{k-2}) \quad (7)$$

Where x_k and y_k stands for the input discrete waveform time history and the output discrete waveform time history.

Through the bilinear transform formula (2) and the pre-warping formula (3) ,

For $F_{1n}(z)$, a_0 , g_0 , a_1 and b_1 is given as:

$$\begin{aligned}
 g_0 &= \frac{\tan\left(\frac{\omega_{2n}\Delta T}{2}\right)}{\tan\left(\frac{\omega_{1n}\Delta T}{2}\right)} \cdot \frac{1}{1 + \tan\left(\frac{\omega_{2n}\Delta T}{2}\right)} \\
 a_0 &= 1 + \tan\left(\frac{\omega_{2n}\Delta T}{2}\right), \\
 a_1 &= \tan\left(\frac{\omega_{2n}\Delta T}{2}\right) - 1, \\
 b_1 &= \frac{\tan\left(\frac{\omega_{2n}\Delta T}{2}\right) - 1}{1 + \tan\left(\frac{\omega_{2n}\Delta T}{2}\right)}, \tag{8}
 \end{aligned}$$

For $F_{2m}(z)$, a_0 , g_0 , a_1 , a_2 , b_1 and b_2 is given as,

$$\begin{aligned}
 g_0 &= \left\{ \frac{\tan\left(\frac{\omega_{2n}\Delta T}{2}\right)}{\tan\left(\frac{\omega_{1n}\Delta T}{2}\right)} \right\}^2 \cdot \frac{1}{1 + 2h_2 \tan\left(\frac{\omega_{2n}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{2n}\Delta T}{2}\right)}, \\
 a_0 &= 1 + 2h_1 \tan\left(\frac{\omega_{1m}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{1m}\Delta T}{2}\right), \\
 a_1 &= 2 \tan^2\left(\frac{\omega_{1m}\Delta T}{2}\right) - 2, \\
 a_2 &= 1 - 2h_1 \tan\left(\frac{\omega_{1m}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{1m}\Delta T}{2}\right),
 \end{aligned}$$

$$\begin{aligned}
 b_1 &= \frac{2 \tan^2\left(\frac{\omega_{2m}\Delta T}{2}\right) - 2}{1 + 2h_2 \tan\left(\frac{\omega_{2m}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{2m}\Delta T}{2}\right)}, \\
 b_2 &= \frac{1 - 2h_2 \tan\left(\frac{\omega_{2m}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{2m}\Delta T}{2}\right)}{1 + 2h_2 \tan\left(\frac{\omega_{2m}\Delta T}{2}\right) + \tan^2\left(\frac{\omega_{2m}\Delta T}{2}\right)}
 \end{aligned} \tag{9}$$

The above formulas gives detailed explain about how to model site amplification factor as the IIR filter.

We searched the possible combination of N and M trough iteration method. As the give N and M, We use the least-square method to determine parameters such as $G_0, \omega_{1n}, \omega_{2n}, \omega_{1m}, \omega_{2m}$. If we get the similar residual, we should choose the smaller N and M, because it could be more easily implemented for hardware filter design.

Quotation of the general comment: “Somethings are wrong in Table 1 and Table 2. For example, in Table 1 Surface in PGA includes Obs. and Sim., but it only includes Obs. Now. And value 8.3 in the Sim. in Table 1 is calculation error. Please check it carefully”.

Reply: Thank you very much for your carefully checking this manuscript, Yes, it does make a mistake in the table, the he data shows in the fig.6 is 5.6255 and 0.67548, so the value 8.3 is calculated correctly, we carefully check the data and redesign the Table 1 and Table 2. In the revised manuscript, it has been fixed.

Quotation of the general comment: “English expression needs improvement, such as, (1) Pay attention to the comparative level in English, Line 25, It also shows good performance: ::, should be “It also shows better performance ::::” (2) There is something wrong with the presentation of the month, Line 8, Septemper.1, 2000, should be “Sept. 1, 2000” or “September 1, 2000” (3) It’s better to use the third person in the manuscript unless you have done so already, such as Line 9 and Line 11, I selected 673, I selected data, :::: (4) Attention should be paid to the singular and plural forms in English expressions, for example in Line 14, There exists 20 meter :::: it should be “There exists 20 meters ::::” (5) What does “the he M5.2” mean in Line 2 and Line 3 below the table 4? (6) ::::::”

Reply: Thank you very much for your suggestion. We will check the grammar error and type error in the manuscript and improve the English expression in the manuscript following your detailed suggestion.