Report#1 - accepted subject to minor revisions.

We wish to thank the Referee for his/her time and effort reviewing the manuscript. We are grateful for the helpful comments and suggestions, which we have carefully addressed in the following responses and in the revised manuscript. Please note that all references to line numbers in this response refer to the marked-up version of the manuscript.

-Overall, much improved from the previous version, more clear, and I liked the figures and table- really helped. The only revision I would request is to be really careful conceptually. A lot of people using these terms in their own disciplines! So its not totally new to do holistic risk assessment-but perhaps new to the authors. Maybe sure to cite and recognize other disciplines/folks working on the topic (some of which are not Italian!).

-Line 43- I still don't think there is even consensus among disciplines! Geography and the social sciences approaches vulnerability really differently and measures it differently to this day. Rephrase.

We acknowledge that the calculation of risk is in continuous discussion in the scientific community, particularly among disciplines. We have rephrased this sentence accordingly:

> L49: "The research community concerned with Disaster Risk Reduction (DRR), particularly in the fields of physical risk~~and environmental sciences~~, has generally agreed on a common approach for the calculation of risk (*R*) as a function of hazard (*H*), exposure (*E*), and vulnerability (*V*): $R = f (H, E, V)$ (e.g. ~~Balbi *et al.*, 2010)~~ Balbi et al., 2010; David, 1999; IPCC, 2012; Schneiderbauer and Ehrlich, 2004)."

We have also included additional references to support the provided definition:

- David, C. (1999). *The Risk Triangle*. London, UK: Jon Ingleton.
- IPCC, 2012: *Managing the Risks of Extreme Events and Disasters to Advance Climate Change Adaptation*. A Special Report of Working Groups I and II of the Intergovernmental Panel on Climate Change [Field, C.B., V. Barros, T.F. Stocker, D. Qin, D.J. Dokken, K.L. Ebi, M.D. Mastrandrea, K.J. Mach, G.-K. Plattner, S.K. Allen, M. Tignor, and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, UK, and New York, NY, USA, 582 pp.
- Schneiderbauer, S. and Ehrlich, D.: *Risk, hazard and people's vulnerability to natural hazards: A review of definitions, concepts and data*, Eur. Comm. Jt. Res. Centre. EUR, 21410, 40(January), doi:10.1007/978-3-540-75162-5_7, 2004.

-Line 64… could add Eakin et al 2017 as a reference (to include resilience cities approaches, ub habitat etc also demanding holistic approaches) Eakin, H., Bojórquez-Tapia, L. A., Janssen, M. A., Georgescu, M., Manuel-Navarrete, D., Vivoni, E. R., … Lerner, A. M. (2017). Opinion: Urban resilience efforts must consider social and political forces. Proceedings of the National Academy of Sciences, 114(2), 186–189. http://doi.org/10.1073/pnas.1620081114 Also there is the SETS literature e.g.: Markolf, S. A., Chester, M. V, Eisenberg, D. A., Iwaniec, D. M., Davidson, C. I., Zimmerman, R., … Chang, H. (2018). Interdependent Infrastructure as Linked Social, Ecological, and Technological Systems (SETSs) to Address Lock-in and Enhance Resilience. Earth's Future, 6(12), 1638–1659.

We agree on the relevance of both references and have added them to the revised manuscript at L72.

-LINE 121. Two periods? Remove one?

We propose the followings modifications:

> L132: "Tsuruta and Kataoka (2008) use matrices ~~for determining~~to determine damage propagation within infrastructure networks (e.g. electric power, waterworks, sewerage, telecommunication, road, and social functions like finance, medical treatment and administration) due to interdependency based on earthquake data and expert judgment considering infrastructure networks ~~(e.g. electric power, waterworks, sewerage, telecommunication, road, and social functions like finance, medical treatment and administration)~~."

-Line 270…more thorough assessments? More thorough than what? I am still not totally clear as to what graphs and graph concepts tell us about risk in a systems that we could not get through another method. What is its unique contribution?

This manuscript contributes to explore some fundamental aspects of complex systems exposed to natural hazard risk leveraging well known graph properties. Furthermore, the graph is proposed as tool to model the propagation of impacts of a natural hazard in order to analyse second-order consequences and quantitatively estimate risk. Both aspects analysed by the graph, properties of the systems and cascading effects, allow a more thorough assessment of risk compared to traditional approaches. We agree that this may not have been fully clear in the previous versions. To address this, we have completely rewritten sub-section "*1.3 Positioning and aims*", modified Figure 2 in order to show the two major advantages of the proposed approach, and considerably expanded sections 3 and 4.

-Line 329. I agree that resilience does include more time dimensions but I don't think the analogy of cinematography is right. Resilience is about the ability to preserve system structure and function with a perturbation. Vulnerability is not a subcomponent of resilience. In fact, some disciplines use the work resilience in opposition to vulnerability. (See Anderies's work on robustness-vulnerability tradeoffs) or this article by Turner Turner, B. L. (2010). Vulnerability and resilience: Coalescing or paralleling approaches for sustainability science? Global Environmental Change, 20(4), 570–576. http://doi.org/10.1016/j.gloenvcha.2010.07.003 I do agree however that the percolation threshold is a new and interesting way to measure resilience.

We propose rephrasing this sentence as described below, removing the cinematography analogy as suggested:

> L394: "These two features, dynamic aspect and whole system, make vulnerability different from resilience and further clarify the need to develop an approach that it is able to consider the dynamic of the system as a whole. ~~this difference can be expressed by a cinematography analogy: vulnerability is a single frame of the resilience video~~."

-Line 455. Consider a stronger conclusion, that really gets us excited about why using graph theory can help us unlock large theoretical questions in vulnerability/resilience science (and then state what those questions are!)

We have restructured the original sections "*4. Discussion*" and "*5. Conclusion*" into a new section "*5. Discussion and final considerations*". Following the Reviewer's much appreciated comment, we have extensively rewritten this new section.

Report#2 - reconsidered after major revisions: I am willing to review the revised paper.

We wish to thank the Referee for his/her time and effort reviewing the manuscript. We are grateful for the helpful comments and suggestions, which we have carefully addressed in the following responses and in the revised manuscript. Please note that all references to line numbers in this response refer to the marked-up version of the manuscript.

- The authors have improved this article by describing how their model is situated in the literature. Their review of the state of the art (section 1.2) is an improvement from their previous draft, and now touches on some of the relevant issues in holistic approaches to risk and the application of network analysis. However, the specific contribution of this model can be further clarified. The authors make a convincing argument that most approaches to risk assessment utilize reductionist approaches, and note that their model is designed to contribute to the holistic approaches to assessing risk.

- They argue that they provide a new way to model holistic risk, specifically of interconnected infrastructures, and provide a short review of the interconnected infrastructure/SoS literature. Their treatment of the interconnected infrastructure/SoS literature could be improved. In their critique they state that "this well developed branch of research is mostly focused on the analysis of a single infrastructure typology and the aim is usually to assess the efficiency of the infrastructure rather than the impact that its failure may have on society." However, in the preceding paragraph they provide several references (such as Lewis, Rinaldi) that focus specifically on interconnected infrastructure and failures therein. These authors represent part of a very large literature that has been developing since the early 2000s (see for instance some of the references to Rinaldi (link) including network - centric approaches (e.g. work of Lewis). The authors

should clarify specifically how their model relates to this work and what specifically it provides. What is the current state of literature, what are the gaps, and what does this model provide that is different?

We have expanded the introduction section regarding the current state of the literature ("*1.2 Modelling natural hazard risk in complex systems: state of the art and limitations*"), particularly from L115. A recent key reference (Pant et al., 2018) that helps clarify what the gaps area has been added, as described below. We believe that this addition, together with the extensive changes and new content added to sub-section 1.3 and section 3, now allow for a much better understanding of what this approach can bring to the field of natural hazard risk.

> L113: "Trucco et al. (2012) propose a functional model aimed at i) propagating impacts, within and between infrastructures, in terms of disservice due to a wide set of threats and ii) applying it to a pilot study in the metropolitan area of Milan. Pant et al. (2018) proposed a spatial network model~~s~~ to quantify ~~infrastructure~~ flood impacts on infrastructures in terms of disrupted customer services both directly~~s~~ ~~linked directly to flood assets and customers disrupted~~ and indirectly linked to flooded assets ~~due to network effects.~~ These analyses could inform flood risk management practitioners to identify and compare critical infrastructure~~s~~ risks on flooded and non-flooded land, for prioritising flood protection investments and improve resilience of cities.
>
> However, this well-developed branch of research is mostly focused on the analysis of a single infrastructure typology, and the aim is usually to assess the efficiency of the infrastructure itself rather than the impact that its failure may have on society. In particular, "*representations of infrastructure network interdependencies in existing flood risk assessment frameworks are mostly non-existent*" (Pant et al., 2018). These interdependencies are crucial for understanding how the impacts of natural hazards~~flood risks, as well as other natural hazards,~~ propagate across infrastructures and towards society."

- On page 5 they state that "there is still a gap in current practice when it comes to modelling the complexity of interconnections between individual elements that do not explicitly constitute a network". It seems like this could be one potential contribution. Further articulating this contribution (or others that are identified following a more engaged literature review) will help understand the added value of this model.

We agree, and have modified sub-section 1.3 accordingly (see below). We have also considerably expanded Section 3 and made various other changes throughout the document which should help clarify the added value of the approach.

> L145: "In fact, although several authors have shown how to model risk in systems which are already networks by construction (Havlin et al., 2010; Reed, Kapur, & Christie, 2009; Rinaldi, 2004; Zio, 2016), fewer have addressed the topic of risk modelling in systems where that is not the case, i.e. systems are not immediately and manifestly depicted as a network (Hammond et al., 2013; Zimmerman et al., 2019). These include cities, regions or countries, which are complex systems made of different elements (e.g. people, services, factories) connected in different ways among each other in order to carry out their own activities. Therefore, in this manuscript we ~~propose an~~would like to promote an approach, which has previously deserved the attention of other authors, to model ~~such~~ the interconnections between the elements that constitute those systems and assess collective risk in a holistic manner. The approach involves the translation of the complex system into a graph, i.e. a mathematical structure used to model relations between elements."

- The discussion on the contributions, once articulated in the literature review and positioning and aims, should also be used to improve the discussion. What are the specific implications that this model provides that are beyond what is already known in the literature? Based on the model, where should the field of practice be going? As with the literature review it would be important to reference relevant articles in describing where the field is and what your findings suggest.

We agree with the Reviewer that the discussion should be improved in accordance with the new clarifications of literature review and positioning and aims. For this reason, we have made considerable modifications to sub-sections "*1.2. Modelling natural hazard risk in complex systems: state of the art and limitations*" and "*1.3. Positioning and aims*". Section 3, which describes the methodology, has also been expanded, which should now better highlight the novelty of the model within the field of natural hazard risk. Finally, we have merged the original sections "*4. Discussion*" and "*5. Conclusion*" into a new section "*5. Discussion and final considerations"*, which has been rewritten extensively.

This article still does not seem substantial enough to stand on its own. It offers a short literature review and a detailed methodology, which is then used in the second article on the case study. Perhaps the value of this article will become more apparent once the contribution is better articulated. Better linking the model to the various bodies of literature (risk assessment, infrastructure interdependencies, and network analysis) will also help identify which elements of both articles are critical and which should be streamlined and reduced. This could help to determine whether this should be one article or two. Without such clarity I still suggest integrating both articles into a single article. Again, these articles could be a nice addition to network analysis approaches of interdependent infrastructure, but it is not yet clear what they add to the literature.

We believe that the extensive modifications introduced to the article have resulted in a much-improved articulation between what its contribution and the existing body of literature. In particular, we have expanded the literature review and integrated additional relevant references, significantly improved the "*Positioning and aims*" sub-section, and expanded the workflow in Section 2.2 (including Figure 2) to highlight the main contributions of the methodology, among other changes aimed at improving overall clarity. We are convinced that this new version of the article has sufficient substance to stand its own, which should now be more easily evaluable.

Report#3

We wish to thank the Referee for his/her time and effort reviewing the manuscript. We are grateful for the helpful comments and suggestions, which we have carefully addressed in the following responses and in the revised manuscript. Please note that all references to line numbers in this response refer to the marked-up version of the manuscript.

General summary

I carefully read the second version of both the manuscripts which is organized in two parts: part I describes the proposed "graph theory" approach, and part II illustrates an application of the graph theory to a pilot study in Mexico City. The author(s) argues that from a more practical perspective, these two companion papers may address different audiences: part I is targeted to a more general audience who may be interested in understanding the foundations of the approach, while part II points to technical experts and researchers who may want to implement this approach for their own practical applications. I identified various loopholes in the written manuscripts which needs to be sincerely addressed before this manuscript could proceed with publications, which are as follows:

- 1. Unjustified strong claim

I do not agree with the author(s) claim that this manuscript has proposed a new approach. Graph theory is an existing approach which has invaded almost all branches of science (as clearly mentioned by both the reviewers in their first round of review). This manuscript only extends its application to "Natural hazard risk of a complex system". I would suggest moderating the tone of the paper mentioning that they have used/applied graph theory for X purpose properly giving credit to all seminal papers.

For instance, see Part I: page 6, second aim, "to propose a new approach…….".

We agree, and have moderated the tone throughout the document. In the revised manuscript, the approach is not presented as "new" anymore. We have also completely rewritten the "*1.3 Positioning and aims*" sub-section, including the main aims:

L164: "The aims of this paper ~~are~~ can be summarized as follows:

- to call for a paradigm shift from a reductionist to a holistic approach to assess natural hazard risk, supported by the construction of a graph;
- to show the potential advantages of the use of a graph: (1) understanding fundamental aspects of complex systems which may have relevant implications to natural hazard risk, leveraging well known graph properties, (2) using

the graph as a tool to model the propagation of impacts of a natural hazard and, eventually, assess risk in complex systems;

- to discuss the limitations, potentialities and future developments of this approach compared to other more traditional approaches.
- ~~to present a new perspective to promote a paradigm shift from a reduction to holistic approach;~~
- ~~to propose a new approach to analyse the risk of complex systems based on Graph Theory;~~
- ~~to link traditional risk variables (exposure, vulnerability, and resilience) to certain properties of a graph;~~
- ~~to introduce a debate on the new perspective and approach, and directions for future developments.~~”

- 2. Confusing between Graph and Network theory, terms and concepts

Reading both the manuscripts it seems that the author(s) did not do a detailed (breadth and depth) literature review. For instance,

(a) throughout both the manuscripts authors used particular terms associated with network and graph interchangeably which is not scientifically correct and created confusion;

We recognize that the use of these terms interchangeably throughout the document is not scientifically correct and might create confusion among readers. Where appropriate, we have replaced the “network” term with “graph”: L182, L186, L190, L191, L213, L214, L220, L221, L227, L280, L309 and L380.

(b) many seminal papers, on which graph theory is based, are omitted from the citation list,

We have added references to several seminal papers regarding graph theory: Barabasi, 2016; Biggs, Lloyd, & Wilson, 1976; Börner, Soma, & Vespignani, 2007; Euler, 1736; Luce & Perry, 1949; Wilson, 1996. We have also included additional references to support the provided definition:

- Barabasi, A. L. (2016). Network Science (Cambridge University Press, ed.). Retrieved from http://barabasi.com/networksciencebook/%3E
- Biggs, N. L., Lloyd, E. K., & Wilson, R. J. (1976). Graph Theory 1736-1936 (Clarendon Press, ed.).
- Börner, K., Soma, S., & Vespignani, A. (2007). Network Science. In Medford (Ed.), Annual Review of Information & Technology (Vol. 41, pp. 537–607). New Jersey.
- Euler, L. (1736). Solutio problematis ad geometrian situs pertinentis. Comentarii Academiae Scientarum Petropolitanae, 8(1741), 128–140. https://doi.org/002433.d/232323
- Luce, R. D., & Perry, A. D. (1949). A method of matrix analysis of group structure. Psychometrika, 14(2), 95–116. https://doi.org/10.1007/BF02289146
- Wilson, R. J. (1996). Introduct to graph theory. Oliver & Boyd.

(c) vertices and edges are particularly associated with graph whereas node and links with network. Either stick to one terminology or highlight the difference between graph and network and make a statement that all terminologies (Vertices and node, edge and link) are same and they are using it without any distinction.

We have addressed this by adding the following clarification when the definition of graph is presented Section 2.1:

> L199: “Formally, a complex network can be represented by a graph *G* which consists of a finite set of elements *V(G)* called vertices (or nodes, in network terminology), and a set *E(G)* of pairs of elements of *V(G)* called edges (or links, in network terminology (Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006).”

- Further, in a single statement, the study is using Network and graph simultaneously, in my opinion, which is not acceptable. For instance, “Part I: Section 2.1, In the scientific community, the mathematical properties of a network are studied using Graph theory,”

We agree, and have modified the sentence accordingly:

> L181: “~~As discussed above, a network graph could allow portraying the complexity of a risk system.~~ T~~In the scientific community, t~~he mathematical properties of a ~~network~~ graph ~~are~~ can be studied using Graph Theory (Biggs et al.,

> 1976), which as mentioned above ~~could also~~could provide a framework~~ provide a better angle~~ to assess risk from a holistic and systemic viewpoint. ~~The following paragraphs review~~ This section summarizes some of the main ~~aspects~~ concepts of Graph Theory~~,~~ on which ~~our approach~~the proposed methodology, presented in Section 3, is based."

- Also, I am not fine with the definition of Graph theory. In part I, the author write that Graph Theory is the branch of mathematics that studies the properties of networks (P6/L143) whereas in the part II author states that Graph Theory is the branch of mathematics that studies the properties of graphs (P2/L37). Does the author think that Graph and Network are exactly the same? Well, in my understanding they are different indeed there is overlapping.

We are aware of the difference between Graph and Network and agree with the Reviewer's comment. We recognize that clearer explanations were needed regarding this aspect and for this reason we added new references and modified the network and graph terminology in section 2.1 as illustrated in the previous point. Regarding to the definition of Graph Theory, we have kept the definition of Part II and modified the following sentence in Part I accordingly:

> L190: "Graph Theory is the branch of mathematics that studies the properties of ~~networks~~ graphs (Barabasi, 2016). ~~Network~~ Graphs can represent ~~comprise~~ networks of physical elements in the Euclidean space (e.g. electric power grids~~, the Internet,~~ and highways~~, neural networks~~) or of entities defined in an intangible space (e.g. collaborations between individuals) (Wilson, 1996).

- I am struggling to understand why authors decided to call the approach based on Graph theory and not Network theory. What makes it really a graph theory? Does the author agree that "a network is a diagrammatical representation of some physical system or structure whereas a graph, on the other hand, is a mathematical notion that represents only the structure of a network without physical meanings?" If yes, I would prefer to call approach based on Network theory.

As underlined above, we are aware of the difference between Graph and Network. However, we are also aware that these two terms are often synonyms, as underlined by Barabasi (2016)[1]. "*In the scientific literature the terms network and graph are used interchangeably […] Yet, there is a subtle distinction between the two terminologies […] Yet, this distinction is rarely made, so these two terminologies are often synonyms of each other.*" (Barabasi, 2016). Nevertheless, we recognize that this could generate confusion, and for this reason we have adjusted the name of proposed approach to "graph-based" instead using "network". Furthermore, we prefer to use the term graph because we are suggesting to use it not only for networks with physical meaning (e.g. electric or sewage networks) but also interconnections between individual elements that do not explicitly constitute a network (e.g. population, houses, schools, industry). Finally, since in the proposed approach we analysed the analogy between the risk variable and graph properties, with the adoption of "graph-based" terminology we would like to put more emphasis on the study of mathematical properties ("*we use the terms {graph, vertex, edge} when we discuss the mathematical representation of these networks*" (Barabasi, 2016)).

- Further, I also have a notion that Graph theory largely has its root in Mathematics where it has been used to conceptualize the problems into a graph whereas Network theory provides a set of techniques for analyzing such graphs. Further, many concepts such as multi-layer network, dynamic network, coarse-graining our flourishing with network theory only. At last, I would say that I am not strict with terminology since every researcher has its own notion but indeed I am more inclined to use Network theory instead of graph theory. Further, if the author continues to go with graph theory makes sure manuscripts clearly deal with the terms and concepts of only graph theory.

As illustrated above, we have adjusted the name of the proposed approach to "graph-based" instead using "network", and we have made modifications accordingly throughout the manuscripts. We still mention the main mathematical properties of Graph Theory in sub-section 2.1, but we have removed Graph Theory from the designation of proposed approach, as can be recognized also from the proposed change to the title:

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[1] Barabasi has H-index 139

L1: “Natural hazard risk of complex systems – the whole is more than the sum of its parts: I. A holistic graph-based assessment approach ~~modelling approach based on Graph Theory~~”

3. Structuring and content of part I
The author(s) argues that part I is targeted to a more general audience who may be interested in understanding the graph theory. Being a general audience, I sincerely have difficulties to understand many terms theoretically as well as mathematically, presented examples, terminologies, mathematical concepts and more importantly the aim. As both the reviewer mentioned (reviewer 2, specifically) I feel the paper is disorganized and at last, it does not convince me. My very specific observations are as follows:

- Abstract: L14, this paper proposes a new holistic approach to assess the risk in a complex system based on Graph theory. What is the approach? To identify vertices of a graph, setup edges and analyze the resultant graph? I feel it is a standard way, isn't it?
As requested, we have reduced the tone also in this specific part of the text. We clarify that the aim of the paper is to call for a paradigm shift from a reduction to holistic approach supported by the construction of a graph. This is supported by several changes throughout the document, for example in Section 2.2. and Figure 2.

- Then, how this manuscript justify the approach? Only based on a hypothetical city example, which is very subjective. I am still struggling to understand what this manuscript contributes to the existing knowledge.

The manuscript shows that adopting an approach based on a graph could contribute to solve some of the challenges that existing reductionist risk assessment approaches have. We restructured and amply the section “3 Methodology, the new workflow in Figure 2 shows the two major advantages of the proposed approach. We improved the section “5 Discussion and final considerations” of the manuscript in accordance with the new clarification applied in sub-section “*1.2 Modelling natural hazard risk in complex systems: state of the art and limitations*” and the new restructured sub-section “*1.3 Positioning and aims*”. In particular, we clarify that this manuscript proposes a graph-based approach to assess the risk of a system that are not immediately depicted as a network: we have rewritten this at L136:

L145: “In fact, although several authors have shown how to model risk in systems which are already networks by construction (Havlin et al., 2010; Reed et al., 2009; Rinaldi, 2004; Zio, 2016), fewer have addressed the topic of risk modelling in systems where that is not the case, i.e. systems are not immediately and manifestly depicted as a network (Hammond et al., 2013; Zimmerman et al., 2019). These include cities, regions or countries, which are complex systems made of different elements (e.g. people, services, factories) connected in different ways among each other in order to carry out their own activities. Therefore, in this manuscript we ~~propose an~~would like to promote an approach, which has previously deserved the attention of other authors, to model ~~such~~ the interconnections between the elements that constitute those systems and assess collective risk in a holistic manner.”

We are aware that this paper is only the first attempt in this direction, the illustrative example of this manuscript is then expanded in real case study application in the second part of the companion paper.

My biggest concern is that the study deals with graph theory but does not provide any mathematical details. Which is unacceptable in such kind of study.

We agree, and have included all relevant mathematical formulas in Table 1 (L211).

Table 1: ~~**Graph properties description**~~**Properties of a graph G with N nodes defined by its adjacency matrix *A(G)* with *N x N* elements a_{ij}, whose value is $a_{ij}>0$ if nodes *i* and *j* are connected, and 0 otherwise**

Property	**Description**	**Formula**
Degree (k)	The number of edges incident with the node	$k_i = \sum_j a_{ij}$
Diameter (D)	The maximum value of all path lengths d_{ij}	$D = \max_{i,j} d_{ij},$ where d_{ij} is the geodesic length from node i to node j *(i.e. path length)*:
Characteristic path length (d)	The average shortest path length	$d = \frac{1}{N*(N-1)} * \sum_{i,j(i\neq j)} d_{ij}$
Closeness (c)	Shortest path length from the node to every other node in the network	$c_i = \frac{1}{l_i}, \quad where\ l_i = \frac{1}{n-1} * \sum_j d_{i,j}$
Betweenness (b)	Number of shortest paths between pairs of nodes that pass through a given node	$b_i = \sum_{j,k} \frac{n.\ of\ shortest\ paths\ connecting\ j,k\ via\ i}{n.\ of\ shortest\ paths\ connecting\ j,k} = \sum_{j,k} \frac{n_{jk}(i)}{n_{jk}}$
Authority (x)	The value proportional to the sum of the node hub values pointing to it	$x_i = \alpha * \sum_j a_{ji} y_j$ -> $A * A^T$, where α is a proportional constant
Hub (y)	The value proportional to the sum of authority of nodes pointing to it	$y_i = \beta * \sum_j a_{ij} x_j$ -> $A^T * A$, where β is a proportional constant
Percolation threshold (pc)	The minimum value of fraction of remaining nodes (p) that leads to the connectivity phase of the graph	For random graph $p_C = \frac{1}{\bar{k}}$, $\bar{k}$ is the average of degree

Page 6/Line 158, section 2.1: electrical power grid, the internet, highway and neural network, being general audience I am not able to visualize what is vertices and edges in above-mentioned graphs and importantly directions (if directed network).

We agree that it might not be obvious what vertices and edges are, particularly on the internet and on a neural network, and that these examples are unnecessary for the purpose at hand in any case. For this reason, we have made the following modification:

> L191: ~~Network~~ Graphs can represent ~~comprise~~ networks of physical elements in the Euclidean space (e.g. electric power grids~~, the Internet,~~ ~~and~~ highways~~, neural networks~~) or of entities defined in an intangible space (e.g. collaborations between individuals) (Wilson, 1996).

- *Coming back to the aim of the paper, the entire study deals with the directed network and hence since the beginning the author (s) need to put more stress on the directed network. For example, section 2.1 need more words, more examples and mathematical notions about a directed network.*

After careful consideration, we consider that the tone taken throughout the article to describe networks is suitable for the purpose at hand. We believe that all the concepts of directed networks that are necessary to grasp the approach are presented in a clear manner. We agree with the Reviewer that section 2.1 lacked mathematical formulations, and have expanded Table 1 in order to address this.

- The author(s) take many different examples to explain the network concept whenever they need, without any coherence structure, For instance, P13/L351: As an example, in a road graph, a bridge node has a higher value of betweenness because all the nodes of a sub-graph (e.g. one side of the river) need to pass through the bridge node in order to connect to the nodes of the other sub-graph (the other side of the river). In the case of bridge failure, the two sides of the river are isolated and the original road graph splits into two sub-graphs. What is the road graph? What are the edges and vertices? What is the bridge node? How to decide? Do we have a river in the road graph? It is all theoretical and very subjective. Not acceptable and convincing.

We agree. We propose to improve this by introducing the following modifications:

> L379: “As an example, consider~~in~~ a road network ~~(~~where road segments are represented by links, whereas crossroads and bridges are represented by nodes.~~i.e. nodes are the crossroad and bridges, instead the road are the links) graph,~~ In this case, a bridge would likely be the node ~~has~~ with a higher value of betweenness, because all the nodes of a sub-~~graph~~ network (e.g. all the nodes that are in one side of the river) need to pass through the bridge node in order to connect to the nodes of the other sub~~-graph~~ network (all the nodes on~~in~~ the other side of the river). In the case of a bridge failure, the two sides of the network, separated geographically by~~of~~ the river, are isolated and the original road network ~~graph~~ splits into two sub~~-graphs~~networks.”

Another example is section 3.1.3 of the earthquake.

We have removed this example from the manuscript.

4. Mathematical details and graph theory measures

Page 7/line 194: A node with high hub value points to many other nodes, while a node with high authority value is linked by many different hubs. Mathematically, the authority value of a node is proportional to the sum of the node hubs pointing to it and the hub value of a node is proportional to the sum of the authority of nodes pointing to it. Is this definition provided by the authors exclusively? If not, I couldn’t find any citation to valid above-written definition.

This definition is not provided by the authors. As suggested, we have added the following citations to support the definition; (Newman, 2010) and (Nepusz & Csard, 2018).

I am wondering whether or not this problem has a unique solution. Author claim that this article is suitable to a general audience, being an expert in network theory I am unable to understand it. I request to take a dummy graph (very simple) and explain how the author(s) setup direct graph, decide hubs, authority values and hub value of a node.

The graph is setup following the workflow presented in section 2.2, which has been improved in the revised manuscript. We do not decide a-priori which nodes are hubs, but we use the equations presented in Table 1 that are proposed in (Newman, 2010) and implemented in R software (Nepusz & Csard, 2018).

Page 8/line 201: Depending on the statistical properties of the degree distribution, there are two broad classes of networks: homogeneous and heterogeneous……. Again, is this definition provided by the authors exclusively? If not, I couldn’t find any citations. Is the author sure about the above-mentioned statement? I am struggling to validate this definition. What do authors mean by the homogeneous network? What are the properties of the homogeneous network? I assume the author might be pointing to a regular network because, by definition, each node in a regular network has the same number of links. If so, this is a very absurd statement. So as per the claim, all the network having Poisson *distributions are homogeneous? This again makes my conviction strong that authors did not check the literature appropriately.*

The definition is not provided by the authors but by Boccaletti et al. (2006). This reference, which has received thousands of citations (e.g. 6006 citations on Scopus; 9221 citations on Google Scholar), at paragraph 2.2.2 "Scale-free degree distributions" reports: "*The usual case in Science until a few years ago was that of homogeneous networks. Homogeneity in the interaction structure means that almost all nodes are topologically equivalent, like in regular lattices or in random graphs. In these distribution is binomial or Poisson in the limit of large graph size. It is not startling then that, when the scientists approached the study of real networks from the available databases, it was considered reasonable to find degree distributions localized around an average value, with a well-defined average of quadratic fluctuations. In contrast with all the expectancies, it was found that most of the real networks display power law shaped degree distribution. […] Such networks have been named scale-free networks [2,93], because power-laws have the property of having the same functional form at all scales. […] These networks, having a highly inhomogeneous degree distribution, result in the simultaneous presence of a few nodes (the hubs) linked to many other nodes, and a large number*".

What a random network is, as mentioned in (P8/L224)? Further, $Pc=1/\bar{K}$*, I do not understand this mathematical expression?*

We propose to add the definition of random network:

> L257: "In a random network (i.e. network with N nodes where each node pair is connected with probability p), for example, pc=1/$\bar{k}$, where $\bar{k}$ is the mean of degree *k* (Bunde & Havlin, 1991)."

Section 3.1.2: P12/L343: closeness: a shorter path between a node and the network? Do you mean shorted path between a node and all other nodes? Difficult to understand, please Rewrite it.

We propose to modify as requested:

> L370: "A lower value of closeness, i.e. a shorte~~r~~st path length from ~~between~~ a node ~~and~~ to every other nodes in the network, means a higher probability of a node ~~to remain isolated because of a~~of being impacted by a hazard event. On the other hand, high value of closeness, i.e. a longer path length from ~~between~~ a node ~~and~~ to every other nodes in the network, means a low probability ~~that the node *i* will be isolated.~~of being impacted."

Table 1: closeness: Which is closeness author talking about? I did not understand the definition? Is it closeness centrality? If yes, please rewrite it to "shortest path length from the node to every other node in the network". Again mathematical formulas of all the terms are unavoidable.

Yes, we refer to closeness centrality and we propose to modify it as requested and provided the mathematical formulas in the table.

Hub? Does the author mean two different hubs in the network? Hub value of a node and hub node itself? Confusing. Make it clear.

We agree that it might not be clear and we modify accordingly the definition of authority in the table:

> "The value proportional to the sum of the node hub values pointing to it"

I just did not understand the concept of percolation and others in the absence of proper mathematical definitions. Mathematical details are indeed important.

We have inserted the relevant mathematical formulas in Table 1.

5. Section 2.2

"We proposed an approach based on the following two major phases". I appreciate the *author's creativity in designing the text and section however, this is more general and clubbed into one section called graph construction to reduce the redundancy. In an entire section of based on topology, I couldn't see any seminal paper cited. Not appreciated. Did the*

author has heard about coarse-graining of network or topological scale in the network? It goes in the same direction what the author has explained (entire network, community and a single node).

Section 2 is divided in two sub-sections: the first illustrates the theoretical background and now cites a number of seminal papers (e.g Barabasi, 2016; Biggs et al., 1976; Boccaletti et al., 2006; Luce & Perry, 1949), while the second sub-section illustrates the proposed workflow. As suggested, we clubbed into one section all the original "*2.2 Proposed workflow*" and restructure the section to emphasise the two main contributes of this manuscripts: to understand the fundamental aspects of complex systems leveraging well known graph properties and invite to use the graph as a tool to model the propagation of impacts of a natural hazard.

P10/L279: it is necessary to define rules? I would be happy to see the rules since the author claim that they are proposing an approach and hence very consolidate approach with the full explanation is needed. Section 2.2 from part II should have been here.

At L314 we presented the example of "student go to school" where we adopt the geographical proximity rule, the same rule that is adopted in the second part of the companion paper. In the first part we provide the conceptual phases of the approach and an illustrative example, instead in the second part we apply the approach to a real data in Mexico City.

6. Section 3.2

I am sorry to the author but I couldn't find anything new here.

The aim of this paper is to introduce a new perspective in the traditional risk assessment based on a graph and not add new discovery in the network theory. Sub-section 3.2, Section 4 in the revised manuscript, shows the feasibility of the proposed approach and illustrates with a simple example the two main advantages of using the proposed graph-based approach: "*(1) understanding fundamental aspects of complex systems which may have relevant implications to natural hazard risk leveraging well known graph properties, (2) using the graph as a tool to model the propagation of impacts of a natural hazard and, eventually, assess risk in complex system*". Regarding to the second point, we had a new section in the methodology "*3.3. Hazard impact propagation via the graph*" and the Section 3.2 has been also expanded with an explanation of the potential impact propagation into the graph of an external hazard event, as describe below:

> L373: "We assume that these elements are located in a flood-prone area and Bridge 3 and Block 6 are directly flooded (Figure 3.d). Since those elements are directly damaged it is possible to follow the cascading effect following the direction of the service into the graph from providers to receivers. In this artificial example, the service of transportation provided from the Bridge is lost and this has an indirect consequence to the Hospital 16 which is not directly damaged but cannot provide humanitarian services since people cannot reach the hospital any more. The graph allows to extend the impact not only to the elements directly hit by the hazard but also to all elements that receive service from element directly or indirectly affected by the hazard."

7. Discussion

Too much repetition, again and again, that we have proposed an approach which can solve all the problems

Page 16/line 443: *"This new approach…."* Is it really a new approach?

As mentioned previously, we have moderated the tone throughout the manuscript. The approach is no longer presented as new.

Line 443 to line 454 should go to introduction.

As requested by the reviewers, we propose to restructure the original sections "*4 Discussion*" and "*5 Conclusion*" into a new section "*5 Discussion and final consideration*", as described below. We kept the suggested sentences (L443 and L 454) because we think are useful to critically discuss and underline the limits of the adopted approach.

Line 455 and line 342: "*The proposed approach is suitable for multi-hazard assessment" many times repeated.*
As previous comments, we completely modify the conclusion and discussion and the new version is more synthetic and avoid repetition.

8. Other minor comments
Fig. 1c could be improved by weighting the links.
We modified the Figure as requested.

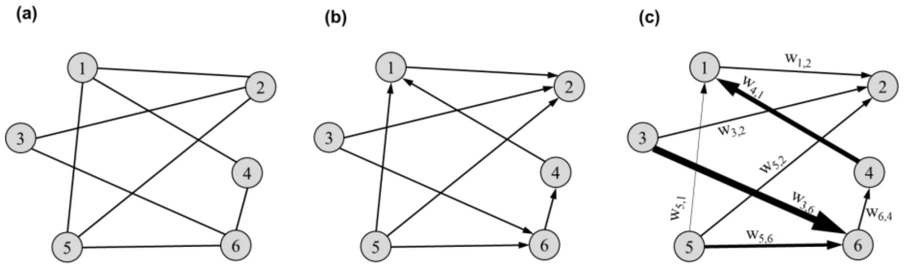

Fig.3: there are 9 blocks, not 8. Correct in the text (Section 3.2, line 403).
We correct the error.

L449: "In specific, our example includes 20 elements: ~~8~~ 9 Blocks of residential buildings, 1 Hospital, 2 Fire Stations, 3 Schools, 3 Fuel Stations and 2 Bridges."

Summary
I am not convinced with the author's claim that this study is suitable for General Audience. The author (s) has tried to oversell the content without giving proper justification and showing any results. Indeed it fails in giving credit to previous studies. Many seminal papers are missing and it clearly seems that vast and in-depth literature review is missing. I found many terse and absurd statements. The study clearly lacks in terms of mathematical justifications. Too many over claim statement such as approach "Could be used for risk mitigation strategies", "Similar analysis could be carried out for betweenness to obtain more insights into the risk assessment". Therefore, I recommend rejection of part I.
Nevertheless, it was a great attempt and I motivate the team to work and explore more to fill the gaps. Due to this, I am afraid that the entire manuscript should be rejected, and the authors should given an opportunity to merge both the manuscripts and resubmit. I am willing to review the contribution, if the authors want to submit a revised work. I hope I am not unduly discouraging, but the problems detailed above are sufficiently severe for the work not to be considered for publication in the current form. However, I do think that the authors have applied an interesting methodology that should be investigated thoroughly, and which may lead to important breakthroughs in the area of interest. Therefore, I encourage the authors to further pursue this method, and I hope my suggestions are useful for this endeavor.

We appreciate the Reviewer's candid comments. We recognize that the previous version of the article had a considerable number of flaws, many of which are well summarized in this comment. In the revised version that we are now submitting, we believe to have addressed most of the issues raised, as described in the responses above. This includes a moderation of the overall tone regarding the novelty of a graph-based approach. Having said this, we must note that we found the Reviewer's criticism on the overall scientific contribution of the manuscripts excessive. We remain convinced that the novelty and potential of the application of graphs to model complex systems of elements within the field of natural hazard risk is in itself a significant contribution (as confirmed also by our expansion of the literature review in the new version), and the reviewer may not be able to fully appreciate being an expert in the field of network theory. Nevertheless, we fully recognize that the shortcomings present in the previous version made this evaluation more difficult. We believe that the extensive modifications introduced to both articles represent a vast improvement, and are thankful to the Reviewer for his/her willingness to review this new version, despite his/her concerns regarding the previous version.

Natural hazard risk of complex systems – the whole is more than the sum of its parts: I. A holistic graph-based assessment approach ~~modelling approach based on Graph Theory~~

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Abstract: Assessing the risk of complex systems to natural hazards is an important but challenging problem. In today's intricate socio-technological world, characterized by strong urbanization and technological trends, the connections and interdependencies between exposed elements are crucial. These complex relationships call for a paradigm shift in collective risk assessments, from a reductionist approach to a holistic one. Most commonly, the risk of a system is estimated through a reductionist approach, based on the sum of the risk evaluated individually at each of its elements. In contrast, a holistic approach considers the whole system as a unique entity of interconnected elements, where those connections are taken into account in order to assess risk more thoroughly. To support this paradigm shift, this paper proposes a holistic approach to analyse risk in complex systems based on the construction and study of a graph, the mathematical structure to model connections between elements. We demonstrate that representing a complex system such as an urban settlement by means of a graph, and using the techniques made available by the branch of mathematics called Graph Theory, will have at least two advantages. First, it is possible to establish analogies between certain graph metrics (e.g. authority, degree, hub values) and the risk variables (exposure, vulnerability and resilience) and leveraging these analogies to obtain a deeper knowledge of the exposed system to an hazard (structure, weaknesses, etc.). Second, it is possible to use the graph as a tool to propagate the damage in the system, not only direct but also indirect and cascading effects and, ultimately, to better understand the risk mechanisms of natural hazards in complex systems. ~~Assessing the risk of complex systems to natural hazards is an important and challenging problem. In today's intricate socio-technological world, characterized by strong urbanization and technological trends, the connections and interdependencies between exposed elements are crucial. These complex relationships call for a paradigm shift in collective risk assessments, from a reductionist approach to a holistic one. Most commonly, the risk of a system is estimated through a reductionist approach, based on the sum of the risk evaluated individually at each element. In contrast, a holistic approach considers the whole system as a unique entity of interconnected elements, where those connections are taken into account in order to more thoroughly assess the risk. To support this paradigm shift, this paper proposes a~~

~~new holistic approach to assess the risk in complex systems based on Graph Theory. The paper is organized in two parts: part I, presented here, describes the proposed approach, and part II illustrates an application to a pilot study in Mexico City. Here we demonstrate that by representing a complex system such as an urban settlement by means of a network (i.e. a graph), it is possible to take advantage of the techniques made available by the branch of mathematics called Graph Theory to analyse its properties. Moreover, it is possible to establish analogies between certain graph metrics (e.g. authority, degree, hub values) and the risk variables (exposure, vulnerability and resilience). Leveraging these analogies, one can not only obtain a deeper knowledge of the system (structure, weaknesses, etc.), but also understand its risk mechanisms (how the impacts of a single or multiple natural hazards are propagated, where they are exacerbated), and therefore assess the disaster risk of the system as a whole, including second-order impacts and cascade effects.~~

1. Introduction

We live in an increasingly complex world. Today's societies are interconnected in complex and dynamic social-technological networks, and have become more dependent on the services provided by critical facilities. Population and assets in natural hazard-prone areas are increasing, which translates into higher economic losses (Bouwer et al., 2009). In coming years, climate change is expected to exacerbate these trends (Alfieri et al., 2017). In this context, ~~disaster~~ natural hazard risk is a worldwide challenge that institutions and private individuals must face at both global and local scales. Today, there is growing attention to the management and reduction of natural ~~disaster~~ hazard risk, as illustrated for example by the wide adoption of the Sendai Framework for Disaster Risk Reduction (SFDRR, 2015).

1.1. Collective disaster risk assessment: traditional approaches

The effective implementation of strategies to manage and reduce collective risk, i.e. the risk assembled by a collection of elements at risk, requires support from Risk Assessment (RA) studies that quantify the impacts that hazardous events may have on the built environment, economy and society (Grossi and Kunreuther, 2005). The research community concerned with Disaster Risk Reduction (DRR), particularly in the fields of physical risk~~and environmental sciences~~, has generally agreed on a common approach for the calculation of risk (*R*) as a function of hazard (*H*), exposure (*E*), and vulnerability (*V*): $R = f(H, E, V)$ (e.g. ~~Balbi *et al.*, 2010)~~ Balbi et al., 2010; David, 1999; IPCC, 2012; Schneiderbauer and Ehrlich, 2004). Hazard defines the potentially damaging events and their probabilities of occurrence, exposure represents the population or assets located in hazard zones that are therefore subject to potential loss, and vulnerability links the intensity of a hazard to potential losses to exposed elements. This framework has been in use by researchers and practitioners in the field of seismic risk assessment for some time (Bazzurro and Luco, 2005; Crowley and Bommer, 2006), and has more recently also become standard practice for other types of hazards, such as floods (Arrighi et al., 2013; Falter et al., 2015).

Despite consensus on the conceptual definition of risk, different stakeholders tend to have their own specific perspectives. For example, while insurance and reinsurance companies may focus on physical vulnerability and potential economic losses, international institutions and national governments may be more interested in the social behaviour of society or individuals in coping with or adapting to hazardous events (Balbi et al., 2010). As such, even though this risk formulation can be a powerful tool for RA, it has its limits. For instance, it does not consider social conditions, community adaptation or resilience (i.e. a system's capacity to cope with stress and failures and to return to its previous state). In fact, resilience is still being debated and there is not a common and consolidated approach to assess it (Bosetti et al., 2016; Bruneau et al., 2004; Cutter et al., 2008, 2010).

To overcome some of these limits, different approaches have been put forward in recent research. For example, Carreño et al. (2007b, 2007a, 2012) have proposed to include an aggravating coefficient in the risk equation in order to reflect socio-economic and resilience features. Another example can be found in the Global Earthquake Model, which aims to assess so-called integrated risk by combining hazard (seismic), exposure and vulnerability of structures with metrics of socio-economic vulnerability and resilience to seismic risk (Burton and Silva, 2015). Multi-risk assessment studies resulting from a combination of multiple hazards and vulnerabilities are also receiving growing scientific attention (Gallina et al., 2016; Karagiorgos et al., 2016; Liu et al., 2016; Wahl et al., 2015; Zscheischler et al., 2018,~~)~~ (Eakin et al., 2017; Markolf et al., 2018). These new approaches are seen with increasing international interest, particularly with regard to climate change adaptation (Balbi et al., 2010; Terzi et al., 2019).

While some research has explored the potential of an integrated approach to risk and multi-risk assessment of natural hazards, quantitative collective RA still requires further development to consider the connections and interactions between exposed elements. Although holistic approaches are in strong demand (Cardona, 2003; Carreño et al., 2007b; IPCC, 2012), the majority of methods and especially models developed so far are based on a reductionist paradigm, which estimates the collective risk of an area as the ~~combination~~ sum of the risk of its exposed elements individually, neglecting the links between them.

1.2. Modelling natural hazard risk in complex systems: state of the art and limitations

~~In a changing~~Modern society ~~which~~ increasingly relies on interconnections.~~,~~ ~~t~~The links between elements are now crucial, especially considering ~~the~~ current urbanization and technological trends ~~that modern-day society is strongly promoting~~. Urban population growth means that people are depending more and more on critical facilities, and there is an increasing interdependency between infrastructures. Complex socio-technological networks, which increase the impact of local events on broader crises, characterize the modern technology of present-day urban society (Pescaroli and Alexander, 2016). Lhomme et al. (2013) showed that the “*city has to be considered as an entity composed by different elements and not merely as a set of concrete buildings*”.

Such aspects support the perception that collective risk assessment requires a more comprehensive approach than the traditional reductionist one, as it needs to involve “whole systems” and “whole life” thinking (Albano et al., 2014). The reductionist approach, in which the "*risks are an additive product of their constituent parts*” (Clark-Ginsberg et al., 2018), contrasts with the complex nature of disasters. In fact, these tend to be strongly non-linear i.e. the ultimate outcomes (losses) are not proportional to the initial event (hazard intensity and extensions) and are expressed by emergent behaviour (i.e. macroscopic properties of the complex system) that appear when the number of single entities (agents) operate in an environment, giving rise to more complex behaviours as a collective (Bergström, Uhr and Frykmer, 2016). In the last decade, many disasters have shown high levels of complexity and the presence of nonlinear paths and emergent behaviour that have led towards secondary events. Examples of such large-scale extreme events are the eruption of the Eyjafjallajokull volcano in Iceland in 2010, which affected Europe’s entire aviation system, the flooding in Thailand in 2011, which caused a worldwide shortage of computer components, and the energy distribution crisis triggered by hurricane Sandy in New York in 2012.

Secondary events (or indirect losses) due to dependency and interdependency have been thoroughly analysed in the field of critical infrastructures such as telecommunications, electric power systems, natural gas and oil, banking and finance, transportation, water supply systems, government services and emergency services (Buldyrev et al., 2010). Rinaldi et al. (2001), in one of the most quoted papers on this topic, proposed a comprehensive framework to identify, understand and analyse the challenges and complexities of interdependency. Since then, numerous works have focused on the issue of systemic vulnerability, due to the increase in interdependencies in modern society (e.g. Lewis, 2014; Menoni et al., 2002; Setola et al., 2016). Menoni (2001) defines systemic risk as *“the risk of having not just statistically independent failures, but interdependent, so-called ‘cascading’ failures in a network of N interconnected system components.”* The article also highlights that *“In such cases, a localized initial failure (‘perturbation’) could have disastrous effects and cause, in principle, unbounded damage as N goes to infinity.* Ouyang (2014) reviews existing modelling approaches of interdependent critical infrastructure systems and categorizes them into six groups: empirical, agent-based, system dynamics-based, economic theory-based, network-based, and others. This wide range of models reflects the different levels of analysis of critical infrastructures (physical, functional or socio-economic). Trucco et al. (2012) propose a functional model aimed at i) propagating impacts, within and between infrastructures, in terms of disservice due to a wide set of threats and ii) applying it to a pilot study in the metropolitan area of Milan. Pant et al. (2018) proposed a spatial network model~~s~~ to quantify ~~infrastructure~~ flood impacts on infrastructures in terms of disrupted customer services both directly~~s~~ ~~linked directly to flood assets and customers disrupted~~ and indirectly linked to flooded assets ~~due to network effects.~~ These analyses could inform flood risk management practitioners to identify and compare critical infrastructure~~s~~ risks on flooded and non-flooded land, for prioritising flood protection investments and improve resilience of cities.

However, this well-developed branch of research is mostly focused on the analysis of a single infrastructure typology, and the aim is usually to assess the efficiency of the infrastructure itself rather than the impact that its failure may have on society. In particular, "*representations of infrastructure network interdependencies in existing flood risk assessment frameworks are mostly non-existent*" (Pant et al., 2018). These interdependencies are crucial for understanding how the impacts of natural hazards~~flood risks, as well as other natural hazards,~~ propagate across infrastructures and towards society.

A full research branch analyses the complex socio-physical-technological relationships of society considering a System-of-System (SoS) perspective, whereby systems are merged into one interdependent system of systems. In a SoS, people belong to and interact within many groups, such as households, schools, workplaces, transport, health care systems, corporations and governments. In a SoS, the dependencies are therefore distinguished between links within~~ (intra)~~ the same system~~,~~ or between ~~(inter)~~ different systems (Alexoudi et al., 2011). The relation between different systems are modelled in the literature using qualitative graphs or flow diagrams (Kakderi et al., 2011) and by matrices (Abele and Dunn, 2006). Tsuruta and Kataoka (2008) use matrices ~~for determining~~to determine damage propagation within infrastructure networks (e.g. electric power, waterworks, ~~sewerage,~~ telecommunication, road)~~, and social functions like finance, medical treatment and administration)~~ due to interdependency, based on past earthquake data and expert judgment~~ considering infrastructure networks (e.g. electric power, waterworks, sewerage, telecommunication, road, and social functions like finance, medical treatment and administration)~~. Menoni (2001) proposes a framework showing major systems interacting in a metropolitan environment based on observations on the Kobe earthquake. Lane and Valerdi (2010) provide a comparison of various SoS definitions and concepts, while Kakderi et al. (2011) have delivered a comprehensive literature review of methodologies to assess the vulnerability of a SoS.

1.3. Positioning and aims

The aspects of complexity and interdependency have been investigated by various models of critical infrastructure as a single system, or as systems of systems, which are networks by construction (e.g. drainage system or electric power network, Holmgren, 2006; Navin, 2016). However, ~~although a consistent literature,~~ there is still a gap in current practice when it comes to modelling the complexity of interconnections between individual elements that do not explicitly constitute a network, which tend to be neglected by traditional reductionist risk assessments. In fact, although several authors have shown how to model risk in systems which are already networks by construction (Havlin et al., 2010; Reed et al., 2009; Rinaldi, 2004; Zio, 2016), fewer have addressed the topic of risk modelling in systems where that is not the case, i.e. systems are not immediately and manifestly depicted as a network (Hammond et al., 2013; Zimmerman et al., 2019). These include cities, regions or countries, which are complex systems made of different elements (e.g. people, services, factories) connected in different ways among each other in order to carry out their own activities. Therefore, in this manuscript we ~~propose an~~would like to promote an approach, which has previously deserved the attention of other authors, to model ~~such~~ the interconnections between the elements that constitute those systems and assess collective risk in a holistic manner. ~~In particular,~~ The approach involves the translation of the complex system into a graph, i.e. a mathematical structure used to model relations between elements.

~~Furthermore, t~~The ~~proposed approach~~is allows modelling and assessing interconnected risk (due to the complex interaction between human, environment and technological systems) and cascading risk (which results from escalation processes). The interactions between elements at risk and their influence on~~f~~ indirect impacts are assessed within the framework of Graph Theory, the branch of mathematics concerned with graphs~~ of vertices connected by edges~~. The results can ~~then~~ be used to support more informed DRR decision making (Pescaroli and Alexander, 2018).

~~The analyses of the interactions between elements at risk and their influence of indirect impacts are assessed in this work by adopting the framework of Graph Theory, the branch of mathematics for the treatment of networks, which has been used to address a wide range of practical questions in many sectors (Boccaletti *et al.*, 2006). Given this context, this paper proposes an insight into collective risk assessment from an innovative holistic perspective.~~ The aims of this paper ~~are~~ can be summarized as follows:

- to call for a paradigm shift from a reductionist to a holistic approach to assess natural hazard risk, supported by the construction of a graph;
- to show the potential advantages of the use of a graph: (1) understanding fundamental aspects of complex systems which may have relevant implications to natural hazard risk, leveraging well known graph properties, (2) using the graph as a tool to model the propagation of impacts of a natural hazard and, eventually, assess risk in complex systems;
- to discuss the limitations, potentialities and future developments of this approach compared to other more traditional approaches.

- ~~to present a new perspective to promote a paradigm shift from a reduction to holistic approach;~~
- ~~to propose a new approach to analyse the risk of complex systems based on Graph Theory;~~
- ~~to link traditional risk variables (exposure, vulnerability, and resilience) to certain properties of a graph;~~
- ~~to introduce a debate on the new perspective and approach, and directions for future developments.~~

~~2. A Graph Theory approach to modelling complex systems~~

~~3.~~2. ~~Background and~~Summary of relevant graph properties

~~It is probably necessary before going into the methodology, to provide a short summary on the general concepts and the main properties of the graph that will be used in the proposed graph-based approach.~~

~~As discussed above, a network graph could allow portraying the complexity of a risk system.~~ T~~In the scientific community, t~~he mathematical properties of a ~~network~~ graph ~~are~~ can be studied using Graph Theory (Biggs et al., 1976), which as mentioned above ~~could also~~could provide a framework ~~provide a better angle~~ to assess risk from a holistic and systemic viewpoint. ~~The following paragraphs review~~ This section summarizes some of the main ~~aspects~~ concepts of Graph Theory~~,~~ on which ~~our approach~~the proposed methodology, presented in Section 3, is based.

Over recent decades, studies of ~~network~~ graph concepts, connections and relationships have strongly accelerated in every area of knowledge and research (from physics to information technology, from genetics to mathematics, to building and urban design), showing the image of a strongly interconnected world in which relationships between individual objects are often more important than the objects themselves (Mingers and White, 2009).

Graph Theory is the branch of mathematics that studies the properties of ~~networks~~ graphs (Barabasi, 2016). ~~Network~~Graphs can represent ~~comprise~~ networks of physical elements in the Euclidean space (e.g. electric power grids~~, the Internet,~~ and highways~~, neural networks~~) or of entities defined in an intangible space (e.g. collaborations between individuals) (Wilson, 1996).

Since its inception in the eighth century (Euler, 1736), Graph Theory has provided answers to questions in different sectors, such as pipe networks, roads, and the spread of epidemics. During the last decade, there has been an increase in interest in the study of complex networks (e.g. irregular structures, dynamically evolving in time), paying renewed attention to the dynamic properties of networks (Börner et al., 2007; Newman, 2003).

Formally, a complex network can be represented by a graph *G* which consists of a finite set of elements *V(G)* called vertices (or nodes, in network terminology), and a set *E(G)* of pairs of elements of *V(G)* called edges (or links, in network terminology) (Boccaletti et al., 2006). The graph can be undirected or directed (Figure 1a and b). In an undirected graph, each of the links is defined by a pair of nodes *i* and *j*, and is denoted as l_{ij}. The link is said to be incident in nodes *i* and *j*, or to join the two nodes; the two nodes *i* and *j* are referred to as the end-nodes of link l_{ij}. In a directed graph, the order of the two nodes is important: l_{ij} stands for a link from *i* to *j*, node *i* points to node *j*, and $l_{ij} \neq l_{ji}$. Two nodes joined by a link are referred to as adjacent (Börner et al., 2007; Luce and Perry, 1949).

In addition, a graph could have edges of different weights representing their relative importance, capacity or intensity. In this case, a real number representing the weight of the link is associated to it, and the graph is said to be weighted (Figure 1c) (Börner et al., 2007).

A short list of the most common set of node, edge and graph measures used in Graph Theory is presented here and summarized in Table 1 (Nepusz and Csard, 2018; Newman, 2010). There are measures that analyse the properties of nodes or edges, local measures that describe the neighbourhood of a node (single part of the system), and global measures that analyse the entire ~~network~~ graph (whole system). From a holistic point of view, it is important to note that since some node/edge measures require the examination of the complete ~~network~~graph, this allows looking the studied area as a unique entity that results from the connections and interactions between its parts and characterizing the whole system.

The degree (or connectivity, *k*) of a node is the number of edges incident with the node. If the graph is directed, the degree of the node has two components: the number of outgoing links (referred to as the degree-out of the node), and the number of ingoing links (referred to as the degree-in of the node). The distribution of the degree of a graph is its most basic topological characterization, while the node degree is a local measure that does not take into account the global properties of the ~~network~~graph. On the contrary, path lengths, closeness and betweenness centrality are properties that consider the complete ~~network~~graph. The path length is the geodesic length from node *i* to node *j*: in a given graph, the maximum value of all path lengths is called diameter and the average shortest path length is named characteristic path length. Closeness is the shortest path length from a node to every other nodes in the network ~~computes the distance (number of links) of a node to all others~~, and betweenness is defined as the number of shortest paths between pairs of nodes that pass through a given node.

Other relevant characteristics that are commonly analysed in directed graphs to assess the relative importance of a node, in terms of the global structure of the ~~network~~graph, are the hub and authority properties. A node with high hub value points to many other nodes, while a node with high authority value is linked by many different hubs. Mathematically, the authority value of a node is proportional to the sum of the node hubs pointing to it and the hub value of a node is proportional to the sum of authority of nodes pointing to it (Nepusz and Csard, 2018; Newman, 2010). In the World Wide Web, for example, websites (nodes) with higher authorities contain the relevant information on a given topic (e.g. wikipedia.com) while websites with higher hubs point to such information (e.g. google.com).

The mathematical properties presented above are useful metrics to analyse the structural (i.e. network topology, arrangement of a network) and functional (i.e. network dynamics, how the network status changes after perturbation) properties of complex networks. Depending on the statistical properties of the degree distributions, there are two broad classes of networks: homogeneous, and heterogeneous (Boccaletti et al., 2006). Homogeneous networks show a distribution of the degree with a typically exponential and fast decaying tail, such as Poissonian distribution, while heterogeneous networks have a heavy-tailed distribution of the degree, well-approximated by a power-law distribution. Many real-world complex networks show power-law distribution of the degree and these are also known as scale-free networks, because power-laws have the same functional form on all scales (Boccaletti et al., 2006). Networks with highly heterogeneous degree distribution have few nodes linked to many other nodes (i.e. few hubs), and a large number of poorly connected elements.

The properties of the static network structure are not always appropriate to fully characterize real-world networks that also display dynamic aspects. There are examples of networks that evolve with time or according to external environment perturbations (e.g. removal of nodes/links). Two important properties to explore the dynamic response to a perturbation are percolation thresholds and fragmentation modes.

Percolation was born as the model of a porous medium, but soon became a paradigm model of statistical physics. Water can percolate in a medium if a large number of links exists (i.e. the presence of links means the possibility of water flowing through the medium), and this depends largely on the fraction of links that are maintained. When the graph has is characterized by many links, there is a higher probability that connection between two nodes may exist and, in this case, the system percolates. Vice versa, if most links are removed, the network becomes fragmented (Van Der Hofstad, 2009). The percolation threshold is an important network feature resulting from the percolation concept which is obtained by removing vertices or edges from a graph. When a perturbation is simulated as a removal of nodes/links, the fraction of nodes removed is defined as $f = \frac{Nodes_{removed}}{Nodes_{Total}}$, and the probability of nodes/links present in a percolation problem is $p = 1 - f = \frac{Nodes_{remainig}}{Nodes_{Total}}$. Consequently, it is possible to define the percolation threshold (p_c) as the minimum value of p that leads to the connectivity phase of the graph (Gao et al., 2015). In practical terms, the percolation threshold discriminates between the connected and fragmented phases of the network. In a random network (i.e. network with N nodes where each node pair is connected with probability p), for example, $pc=1/\bar{k}$, where $\bar{k}$ is the mean of degree k (Bunde and Havlin, 1991).

The second property that investigates dynamic evolution is the fragmentation (i.e. number and size of the portions of the network that become disconnected). The number and the size of the sub-networks obtained after removing the vertices/edges provide useful information. In the case of a so-called giant component fragmentation, the network retains a high level of global connectivity even after a large amount of nodes have been removed, while in the case of total fragmentation, the network collapses into small isolated portions. For this reason, "*keeping track of the fragmentation evolution permits the determination of critical fractions of removed components (i.e., fraction of component deletion at which the network becomes disconnected), as well as the determination of the effect that each removed component has on network response*" (Dueñas-Osorio et al., 2004).

~~4.~~3. ~~Proposed workflowologys~~Methodology

~~Within the framework of Graph Theory, we propose an approach based on the following two major phases:~~

- ~~Network conceptualization;~~
- ~~Graph construction.~~

In this section, which presents the methodology, we aim to answer the three following questions:

1) How to "translate" a complex system into a graph?
2) Which properties of the graph could give us insights on the risk related properties of the system?
3) How to propagate the impacts of a natural hazard by means of the graph?

The answers to these questions are formulated proposing the workflow of the graph-based approach, which is divided in three main steps described in the following sub-sections: 3.1 Construction of the graph; 3.2 Analogy between graph properties and risk variables; and 3.3 Hazard impact propagation via the graph. The workflow is presented in Figure 2~~Figure 2~~.

~~4.1.~~3.1. ~~Network conceptualization~~Construction of the graph

According to the objects of each specific context, the ~~network~~ graph ~~conceptualization~~ construction phase starts by definging~~es~~ the hypothesis of the analysis and the system boundaries according to the objects of each specific context. In particular, it establishes the two main objects of the ~~network~~graph: vertices (nodes) ~~(vertices)~~ and ~~links (edges)~~ edges (links) and their characteristics.

The nodes can theoretically represent all the entities that the analysis wants to consider: physical elements like a single building, bridge and electric tower, suppliers of services such as schools, hospitals and fire brigades, or beneficiaries such as population, students or specific vulnerable groups such as elderly people. Due to the very wide variety of elements that can be chosen, it is necessary to select the category of nodes most relevant to the specific context of analysis. It is also necessary to define, for each node, the operational state that can be characterized, from the simplistic Boolean (functional/non-functional), to discreet states (30/60/100% of service/functionality), or even a complete continuous function (similarly to vulnerability functions). In a graph, the states of each node depend both on the states of the adjacent nodes and on the hazard. In this paper, we use the term *node* to refer to its graph characteristics and term *element* to refer to the entity that it represents in the real world. The links between the nodes that create the graph can range from physical, geographical, cyber or logical connections (Rinaldi et al., 2001). According to the different typologies of connections and nodes selected, it is necessary to define direction and weight of the links. The graph will be directed when the direction of the connection between elements is relevant and it will be weighted if the links have different importance, intensity or difference capacity.

In defining the topology, it is crucial to define the level of analysis details coherently with the scope and scale, both for the selection of elements and for the relationship between elements that need to be considered. In the case of a very high detail for example, a node of the graph could represent a single person within a population, and in the case of lower resolution, it could represent a large group of people with a specific common characteristic, such as living in the same block or having the same hobby. In the case of analyses at a coarser level, an entire network (e.g. electric power system) can be modelled as a single node of another larger network (e.g. national power system). The definition of the topology structure of the graph also identifies immediately the system boundaries (e.g. which hospitals to be considered in the analysis: only the potential flood area, the ones in the district or in the region?). Up to which extent it is necessary to consider elements as nodes of the graph? The topology definition is a necessary step to perform the computational analysis and introduces approximations of the open systems that need to be acknowledged.

~~2.2.1. Graph construction~~

Once the ~~network~~ graph is conceptually defined, in order to actually build the graph, it is then necessary to establish the connection between all the selected elements. The ~~conceptual network~~ relations ~~defined~~described above determine~~s~~ the existence of connections between categories of elements, but it does not define how a single node of one category is linked to a node of another category. Therefore, it is necessary to define rules that establish the connections between each single node. For the sake of clarity, an example could be the following: ~~in the conceptual network,~~ the conceptual relationship is defined between students and school (“students go to school”); subsequently, it is necessary to make the link between each student and a school in the area, applying a rule such as ~~the following rule –~~"students go to the closest school". This is an example of geographical connection with nodes that are linked by their spatial proximity.

The connections between the single elements can be represented either by a list of pairs of nodes or, more frequently, by the adjacency matrix. Any graph G with N nodes can be represented in fact by its adjacency matrix $A(G)$ with $N \times N$ elements A_{ij}, whose value is $A_{ij} = A_{ij} = 1$ if nodes i and j are connected, and 0 otherwise. If the graph is weighted, $A_{ij}=A_{ji}$ can have a value between 0 and 1 expressing the weight of the connection between the nodes. The properties of the nodes are represented in both cases by another matrix, with a column for each property associated with the node (e.g. name, category, type).

In practical terms, the list of all connections or the adjacency matrix can be automatically obtained via GIS analysis, in the case of geographical connections, or by database analysis, in the case of other categories of connections. The list of nodes, together with either the list of links or the adjacency matrix, are the inputs to build the mathematical graph~~ using dedicated tools. For example, igraph (http://igraph.org/r/), the package for network analysis of the R environment provides a set of data types and functions for the implementation of graph algorithms, and is able to handle large graphs with millions of vertices and edges.~~.

Once a graph has been setup and a constructed, it is then possible to compute and analyse its properties by means of Graph Theory and propagate the hazard impact into the graph as illustrated in the following sub-sections.

~~5. Graph concepts in the context of collective risk assessment~~

~~Once a network graph has been setup and a graph has been constructed, it is then possible to compute and analyse its properties by means of Graph Theory. These analytical tools can be very useful for understanding risk mechanisms: How is it generated? How is it propagated? Which are the weaknesses of the system? Information such as this is key to perform more thorough risk assessments, where second-order and cascade effects are considered, and to support the implementation of more effective risk mitigation actions.~~

~~The traditional conceptual skeleton to describe risk as a function of hazard, exposure, vulnerability and resilience can still be adopted within the framework of the proposed graph-based approach. In fact, the properties calculated from a graph consist in a new layer of information for some of those risk variables that go beyond their traditional interpretations within the reductionist paradigm. In particular, they provide a more comprehensive characterization of the single nodes (deriving from their relationships with other nodes), as well as of the system as a whole. In the following paragraphs, we present~~

~~the analogies that can be established between the graph properties described in Section 2.1 and certain risk variables (Table 2), and then provide a simple illustrative example.~~

3.2. Analogy between graph properties and risk variables

The proposed graph properties can be used to more thoroughly characterize systems of exposed elements. In fact, the traditional conceptual skeleton to describe risk can still be adopted within the framework of the proposed graph-based approach. The properties calculated from a graph consist in a new layer of information for some of those risk variables that go beyond their traditional interpretations within the reductionist paradigm. In particular, they provide a more comprehensive characterization of the single nodes (deriving from their relationships with other nodes), as well as of the system as a whole. As such, from the risk variables presented in Section 1,~~the introduction,~~ the hazard preserves its traditional definition as an event that can impact such systems, or part(s) of it, with certain intensities and associated probabilities of occurrence. For the three other variables, namely, exposure, vulnerability, and resilience, below we propose and provide a brief discussion on their analogies with the graph properties presented in Section 2.~~previously.~~

Exposure

In analogy with the traditional approach but at the same time extending its concept, the value of each exposed ~~node~~ elements can be ~~assessed~~ estimated as the relative importance that is given to it by the graph, which is measured by the network itself by means of the connections that point to each node. In Graph Theory, this relative importance among elements, based on standardized values, can be investigated through the authority analysis. A high authority value of a node indicates that there are many other nodes (or otherwise some hubs) that provide services (i.e. providers or suppliers) to that node. In other words, the system privileges it compared with others according to their connections with the provider nodes. For example, a factory settled in an industrial district may receive more services (e.g. electric power, roads for heavy vehicles, logistic systems) than a factory located in the old quarter of a city; in this case, the former is structurally privileged by the system compared with the latter.

Vulnerability

In the reductionist approach, vulnerability is the propensity of an asset to be damaged because of a hazardous event. By adopting a graph perspective, the vulnerability can be ~~defined~~ estimated both for the single node as well as for the system as a whole.

In the first case, the vulnerability depends on the relationship that the node has with the others. In particular, the closeness represents the likelihood of a node to be affected indirectly by a hazard event due to the lack of services provided by other nodes. A lower value of closeness, i.e. a shorte~~r~~st path length from ~~between~~ a node ~~and~~ to every other nodes in the network, means a higher probability of a node ~~to remain isolated because of a~~of being impacted by a hazard event. On the other hand, high value of closeness, i.e. a longer path length from ~~between~~ a node ~~and~~ to every other nodes in the network, means a low probability ~~that the node *i* will be isolated.~~of being impacted.

In the second case, the vulnerability can be defined as the propensity of the network to be split into isolated parts due to a hazardous event. In that condition, an isolated part is unable to provide and receive services, which can translate into indirect losses. The system vulnerability, therefore, can be evaluated by means of the following graph properties: hubs, betweenness and degree out distribution. The presence of nodes with high hub values– indicates a propensity of the network to be indirectly affected more extensively by a hazard event, since a large number of nodes are connected with the hubs. Betweenness manifests the tendency to create isolated sub-networks. As an example, in a road network where road segments are represented by links, whereas crossroads and bridges are represented by nodes. ~~i.e. n graph~~In this case, a bridge would likely be the node ~~has~~ with a higher value of betweenness because all the nodes of a sub~~-graph~~ network (e.g. all the nodes that are in one side of the river) need to pass through the bridge node in order to connect to the nodes of the other sub~~-graph~~ network (all the nodes on the other side of the river). In the case of a bridge failure, the two sides network, separated geographically by~~of~~ the river are isolated and the original road network ~~graph~~ splits into two sub-~~graphs~~networks. A network that has nodes with high betweenness values has a higher tendency to be fragmented, because it has a strong aptitude to generate isolated sub-networks. Finally, the degree distribution, which expresses network connectivity of the whole system (i.e. the existence of paths leading to pairs of vertices), has a strong influence on network vulnerability after a perturbation. The shape of the degree distribution determines the class of a network: heterogeneous graphs (power-law distribution and scale-free network) are more resistant to random failure, but they are also more vulnerable to intentional attack (Schwarte et al., 2002). As emphasised above, scale-free networks have few nodes linked to many nodes (i.e. few hubs), and a large number of poorly connected elements. In the case of random failure, there is a low probability of removing a hub, but if an intentional attack hits the hub, the consequences for the network could be catastrophic.

Resilience

Resilience differentiates from vulnerability in terms of dynamic features of the system as a whole. The properties and functions used to model vulnerability are static characteristics that do not consider any time evolution, or using the words of Sapountzaki (2007), “*vulnerability is a state, while resilience is a process*”; in fact the definition of resilience implies a time evolution of the characteristics of the whole system. In addition, Lhomme et al. (2013) underline “*the need to move beyond reductionist approaches, trying, instead, to understand the behaviour of a system as a whole*”. These two features, dynamic aspect and whole system, make vulnerability different from resilience and further clarify the need to develop an approach that it is able to consider the dynamic of the system as a whole.~~ this difference can be expressed by a cinematography analogy: vulnerability is a single frame of the resilience video~~.

In this context, the study of the percolation threshold (p_c) can be used to explain the resilience of the network after a perturbation. The p_c value distinguishes between the connectivity phase (above p_c) and the fragmented phase (below p_c). In the connectivity phase, the network can lose nodes without losing the capacity to cope with the perturbation as a network, while in the fragmented phase, the network does not actually exist anymore and the remaining nodes are unable to cope with the disruption alone.

This critical behaviour is a common feature also observed in natural disasters. In some cases, the exposed elements withstand some damage and loss, but the overall system maintains its structure. However, there are events in which the amount of loss (affected nodes) is so relevant that the system loses the overall network structure. In the first case, the system has the capacity to cope independently and tackle the event, while in the second case, the system is unable to cope. ~~In the case of an earthquake, for example, if a large number of exposed elements are damaged, it is common for affected regions to be unable to cope with the situation and require help from outside their borders (i.e. the graph needs new nodes). Asking for support from outside would increase the extension of the area (graph) and offers new resources (nodes), therefore decreasing the percolation threshold value increasing connectivity. In this perspective, the percolation threshold should define when a specific area is not self-sufficient to overcome that level of loss and requires help from outside.~~

The dynamic responses are characterized by the network fragmentation property, which describes the performance of a network when its components are removed (Dueñas-Osorio and Vemuru, 2009). For instance, the so-called giant component fragmentation (the largest connected sub-network) and the total fragmentation describe network connectivity and determine the failure mechanism (Dueñas-Osorio et al., 2004). Keeping track of fragmentation evolution makes it possible to determine both the critical fraction of components removed (i.e. the smallest component deletion that disconnects the network), and the effect that each component removed has on the network response.

For these reasons, we consider percolation threshold and network fragmentation a good indicators of resilience, also because it is able to show the emergent behaviour of the whole system beyond just considering the single parts of the network (e.g. node).

3.3. Hazard impact propagation via the graph

Besides the considerable amount of information that can be obtained by analysing graph properties from the viewpoint of natural hazard risk, the graph itself also provides an optimal structure to propagate the impacts of a hazard throughout an affected system. Indeed, the use of a graph allows estimating, besides direct losses to elements directly affected (such as elements within a flooded area), also indirect losses to elements outside the affected area that rely on services provided by directly hit elements, which may have lost some capacity to provide those services as a result. This propagation and quantification of impacts through a graph allows understanding the risk mechanisms of the system, and identifying weaknesses that can translate into larger indirect consequences. It also enables the possibility of quantitatively estimating risk considering those indirect consequences.

In order to propagate the impacts by means of the graph and quantify indirect losses resulting from second-order and cascading effects, the modelled graph must first be integrated with hazard data. This data should include a hazard footprint that allows establishing the hazard intensity (e.g. water depth) at the location of each element. After the impact to directly hit elements is estimated through traditional vulnerability functions (which estimate a level of impact for an asset type as a function of the hazard intensity), the impact can then be propagated through the graph based on the modelled connections between elements. This process should account for the vulnerability of the service itself (relating the damage sustained by a directly hit node to a level of lost capacity to provide a service), and finally the vulnerability of the receiver node(s) (relating the level of lost service to an estimate of indirect loss). By computing impacts for hazard scenarios with different probabilities of occurrence, a quantitative estimate of risk can be obtained. A preliminary example of propagation of impacts is presented in Section 4, and more detailed information on the propagation of impacts through the graph and the estimation of risk are presented in part II of the manuscript.

~~6.~~4. Illustrative example

In order to illustrate the application of the graph-based approach ~~Graph Theory~~ in the characterization of a system exposed to natural hazards, in Figure 3~~Figure 3~~ we present an example of a hypothetical city comprising various elements of different types which provide services among them. In specific, our example includes 20 elements: ~~8~~ 9 Blocks of residential buildings, 1 Hospital, 2 Fire Stations, 3 Schools, 3 Fuel Stations and 2 Bridges. ~~We assume that these elements are located in a flood-prone area.~~ Blocks are intended to represent the population, which receives services from the other nodes. Bridges provide a transportation service, Fire Stations provide a recovery service, Hospitals provide a healthcare service, Schools provide an education service, and Fuel Stations provide a power service. Figure 3~~Figure 3~~(a) shows how the elements are connected into a graph. The authority and hub values adopted in this illustrative example have been computed using the R igraph package. The full library of functions adopted are available in Nepusz and Csard (2018).

In Figure 3~~Figure 3~~(b), the size of the elements is proportional to their authority values. Blocks 6, 18, 19 and 20 have higher authority values than the other elements of this typology because they receive a service from the Hospital (node

16), which is an important hub. Fire Station 5 and School 9 have high values of authority because they are serviced by Bridge 3, which is also an important hub. The importance of a node in Graph Theory is closely connected with the concept of topological centrality. Referring to the illustrative example, Block 6 has the highest authority value; if a flood hit it, it would therefore affect the most central node of the network, or in other words, the node which is implicitly more privileged by the system.

In Figure 3~~Figure 3~~(c), the major hubs are the elements with largest diameters: Hospital 16, Bridge 3, School 7 and Fuel Station 15. Bridge 3 is an important hub since it provides its service to Block 6, which has the highest authority value, and to Fire Station 5 and School 9. Fuel Station 15 and School 7 are also important hubs because they provide services to Block 6. The elements in the south-east part of the network inherited a relative importance (i.e. authority) from the most important hub in that area (i.e. Hospital 16). Bridge 3 is an exception to this aspect; in fact, this Bridge connects the south part (i.e. Block 6) with the north part of the city (i.e. Fire Station 5 and School 9). A flood event in the south-east part of the network would likely generate a major indirect impact on the whole system compared to other parts of the network.

This illustrative example shows how the single elements can be considered as part of the whole network and not as single separate entities. This holistic approach adds information to the traditional approach: it considers the exposed asset as whole system and it exploits the properties of single elements in order to make decisions for risk mitigation strategies.

We assume that these elements are located in a flood-prone area and Bridge 3 and Block 6 are directly flooded (Figure 3d). Since those elements are directly damaged, it is possible to follow the cascading effects following the direction of the service within the graph from providers to receivers. In this artificial example, the transportation service provided from by Bridge is lost and this has an indirect consequence to the Hospital 16, which is not directly damaged but cannot provide healthcare services since people cannot reach the hospital any more. The graph allows to extend the impact not only to the elements directly hit by the hazard, but also to all elements that receive service from element directly or indirectly affected by the hazard.

Note that similar analyses could be carried out for other properties of the graph (e.g. betweenness) in order to obtain additional insight into the properties of the system, which could be useful for the purpose of a risk assessment. For the sake of brevity, such analyses have not been included here. A complete study of all relevant graph properties discussed above and a more realistic hazard scenario~~s~~ ~~for a selected case study~~ are presented in Part II of this manuscript for a selected case study.

~~7.~~5. Discussion and final considerations

In this paper we have tried to look at the problem of risk assessment of natural hazards in a holistic perspective, focussing on the “system” as a whole. We used “system” as a general term to identify the set of the different entities, assets, parts of a mechanism connected among each other in order to function a, for instance, an organism, an organization, a city. Most of such systems are complex because of the high number of elements and the large variety of connections linking them. Nevertheless, our society is structured in these complex systems which are widespread everywhere. How can we assess the risk of such complex systems? We believe that a reductionist approach that separates the parts of the system from each other, compute the risk (losses, impacts, effects, etc.) for each of them and then sums them up to come out with a total estimate of risk is not adequate. Most of the research on natural hazards and their risk adopted implicitly the reductionist approach (i.e. “split the problem in small parts and solve it”). However, we mentioned a large and emerging literature which adopts a different approach (“keep the system as whole”), a holistic approach. Despite the improvements in risk assessment within this systems perspective, Clark-Ginsberg *et al.* (2018) highlights that there are “*questions about the validity of such assessment*” regarding the ontological foundations of networked risk, the non-linearity and emergent phenomena that characterize system phenomena. The emergence of the risk system demonstrates that the risk will never be completely knowable, and for this reason the “*unknown unknowns are an inseparable part of a risk networks*"; in fact, the boundary definition of open systems is by nature artificial.

How can the system be represented as a single, intact, entire entity? And how can all the connections of its parts be represented? We believe, as other authors do, that the best approximation to represent a complex system is the graph. Many authors have already used the graph to model systems already organized as networks by construction (e.g. electric power network) and assess the risk of natural hazards in such a manner. Fewer authors have used the graph to model systems not immediately and manifestly depicted as physical networks and proceed in this manner to model the risk. Once the effort to “translate” a system with all its components into a graph is made, there are many advantages and benefits.

First of all, there is a mature theory of mathematics, the Graph Theory, that already studies the properties of a graph. Are these graph properties telling us something useful to assess the risk of natural hazards affecting these complex systems? We showed that some of the graph properties can disclose some relevant characteristics of the system related to the risk assessment. What is the vulnerability and exposure of the system? There are some interesting analogies between graph properties such as hub, betweenness and degree-–out values and the “systemic” vulnerability. The adoption of these analogies is supported also by the recent work published by (Clark-Ginsberg et al., 2018). Despite having a different scope, they also use certain graph properties to assess the hazards of the companies operating in the case study and promote a network representation of the risk as well.

A second advantage is that we can use the graph as a tool to propagate the impacts of a natural hazard all over the system from wherever the hazard hit it, including indirect or cascading effects. The links between nodes allow passing from the direct physical damage to broader economic and social indirect impacts. It is worth noting that the concept of indirect impact needs to be expanded and more explored. The indirect impact suffered by a certain node may be defined as a function of two factors: 1) the direct damage sustained by one or more of its parent nodes (i.e. traditional impact); 2) the loss of service the latter provide to the former (i.e. vulnerability function). The integration of indirect impact quantification within the graph-based framework it is addressed in part II using a simplified binary vulnerability function.

Furthermore, the proposed approach could introduce a common base for both multi-hazard and integrated risk assessment. Being the graph properties hazard independent, it is possible to integrate these properties with the characteristics of the single node, such as the physical vulnerability of a building with respect to earthquake or flooding (adopted by reductionist approaches) and analyse multi-hazard using the same graph. Besides, the use of this approach can be applied to physical as well as social or integrated risk. In the former case, the graph has only physical elements (e.g. buildings), the latter case the graph has nodes that reflect also social aspects (e.g. population, age, education, etc.).

Several open questions arise, and a number of unresolved issues remain for future developments. Nonetheless, we tried to show that the proposed approach to assess the risk of natural hazards in complex systems is profitable and promising; Part II of this paper presents an application to the case of urban flooding in Mexico City to support this direction.

~~Many open questions raise, and many unresolved issues remains for future developments. Nonetheless, we believe we showed that the proposed approach to assess the risk of natural hazards in complex systems is profitable and promising.~~

~~This paper proposes a new approach to model the risk of complex systems based on Graph Theory to represent them. By leveraging certain analogies that can be established between graph properties and risk concepts, this approach allows obtaining a more thorough knowledge of a system compared to traditional approaches, in terms, for example, of its structure and vulnerabilities. It also allows understanding certain risk mechanisms, such as how the impacts of a hazard are propagated or where they are exacerbated, and therefore assessing the disaster risk of the system as a whole, including second-order impacts and cascade effects.~~

~~The proposed approach can bring important advantages to collective risk assessment: it provides a systemic and holistic perspective, it is suitable for multi-hazard assessment, it introduces a common base for an integrated risk assessment, and it promotes the study of second-order impacts and cascade effects.~~

~~The new holistic perspective introduces an important paradigm shift in the risk conceptualization: the most widely accepted risk concepts of hazard, vulnerability, exposure and resilience do not lose their validity but are integrated into a systemic perspective rather than considered separately. The properties of the whole graph show the studied area as a unique entity, and how the whole system together is vulnerable to an external perturbation, such as a hazardous event that can affect part of it. Beside this whole system perspective, it is also possible to assess the properties of the single parts of the graph (e.g. nodes) and detect which element, or set of elements, are more critical for the entire system.~~

~~This new approach, through which a system can be modelled as a graph, and the analysis of its properties within the RA framework, provides a systemic and holistic perspective that is missing in the traditional RA. The adoption of analogies as proposed in this methodology is supported by the recent work published by (Clark-Ginsberg et al., 2018). Despite having a different scope, it also uses certain graph properties to analyse the 15 main hazards of the companies operating in Khorasan Razavi Province, promoting a network representation of the risk. This perspective, innovative in the context of collective risk assessment, uses the information contained both in the vertices and in the whole network.~~

~~Despite the improvements in risk assessment within this systems perspective, Clark-Ginsberg *et al.* (2018) highlights that there are “*questions about the validity of such assessment*” regarding the ontological foundations of networked risk, the non-linearity and emergent phenomena that characterize system phenomena. The emergence of the risk system demonstrates that the risk will never be completely knowable, and for this reason the “*unknown unknowns are an inseparable part of a risk networks*"; in fact, the boundary definition of open systems are by nature artificial.~~

~~The proposed approach is suitable for multi-hazard assessment, For the first, bBas the graph properties of the system are independent of the type of hazard to be analysed. Moreover, these properties can be easily integrated with the properties of the single node estimated by reductionist approaches, such as the physical vulnerability of a building with respect to earthquake or flooding.~~

~~The graph-based approach also introduces a common base for integrated risk assessment in terms of different features, not only in relation to the physical component of the elements but also with regard to social aspects that express the capacity of the system to respond to perturbations. Therefores, the use of Graph Theory in this field can be applied to physical, as well as social or integrated risk. In the first case, the analysis can focus on physical aspects if the graph has physical elements (e.g. buildings). The second case focuses on social aspects if the nodes represent the population with characteristics that reflect different types of vulnerability (e.g. age, education).~~

~~Lastly, he intrinsic network perspective can be applied to model cascade effects and the dynamic consequences of disruptive events. The links between nodes allow passing from the direct physical damage to broader economic and social indirect impacts. Based on the type of disruptive event and how the network is setup in terms of nodes and links, the~~

~~spread of the impacts throughout the network can be assessed. Furthermore, during the evolution of such cascade effects, it is possible to analyse the structural evolution of the network, the main properties of which are emphasised above, and study how those measurements change during the propagation of loss. If a cascade process needs few or high number of propagation steps to reach most of the nodes, this shows a lower or higher capacity to cope and adapt to the perturbation, and therefore to be less or more resilient.~~

~~**8. Final Considerations**~~

~~This paper proposes a new approach to model the risk of complex systems based on Graph Theory. By leveraging certain analogies that can be established between graph properties and risk concepts, this approach allows obtaining a more thorough knowledge of a system compared to traditional approaches, in terms, for example, of its structure and vulnerabilities. It also allows understanding certain risk mechanisms, such as how the impacts of a hazard are propagated or where they are exacerbated, and therefore assessing the disaster risk of the system as a whole, including second-order impacts and cascade effects.~~

~~The natural continuation of this study, which focuses mainly on theoretical aspects, is to implement and test the approach in case studies, verifying its feasibility. ManySeveral open questions raisearise, and manya number of unresolved issues remains for future developments. Nonetheless, we believe we showedtried to show that the proposed approach to assess the risk of natural hazards in complex systems is profitable and promising.Therefore, part II of this paper presents an application to the case of urban flooding in Mexico City.~~ Further research will aim to fully implement and integrate the graph-based approach in quantitative risk assessments, both at scenario and probabilistic level.

A possible future extension of this framework is to model the physical hazard as one or more nodes linked to the elements at risk, rather than through a traditional approach where elements are overlapped with hazard footprints. This approach may be advantageous, as it would allow including all the factors of risk directly into the topology structure of the graph~~, and will be explored in further research~~.

Moving forward, one of the challenges that will need to be addressed is related with data requirements and availability. Currently, most exposure and vulnerability databases focus on the properties of single elements, and tend to contain little to no information on the connections between them. As we have discussed, this information is key for more thoroughly understanding and assessing the risk of a system. For this reason, developing and collecting data with information related to the connections between the elements is paramount. To promote this perspective, it is necessary consider shifting the RA from using traditional relational databases to so-called graph databases. In such databases, each node contains, further to the traditional characteristics, also a list of relationship records which represent its connections with other nodes. The information on these links is organized by type and direction, and may hold additional attributes.

Finally, ~~T~~the introduction of the ~~network perspective of Graph Theory~~ graph-based approach into the RA for collective disaster risk aims, in the long term, to be a first step for future developments of Agent Based Models and Complex Adaptive Systems in collective risk assessment. In this perspective, the nodes of the network are agents, with defined state (e.g. level of damage), and the interaction between the other agents is controlled by specific rules (e.g. vulnerability and functional functions) inside the environment within they live (e.g. natural hazard phenomena).

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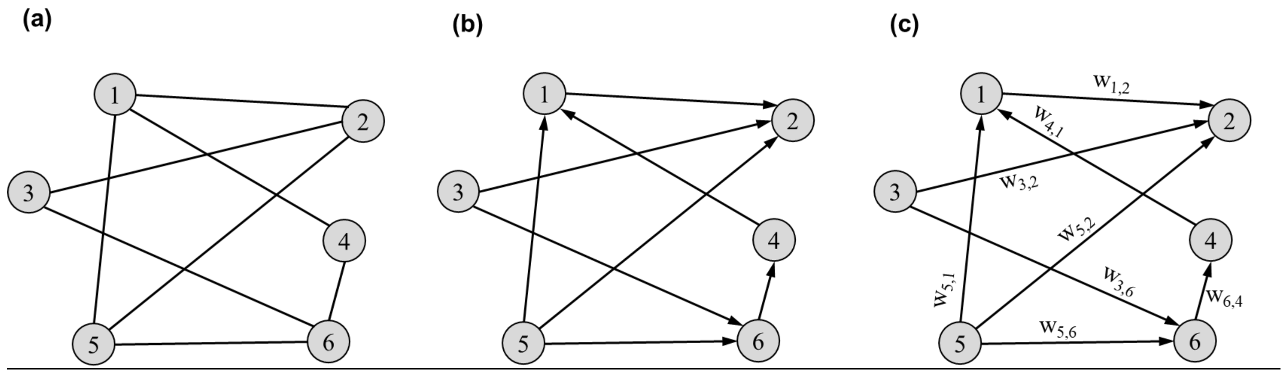


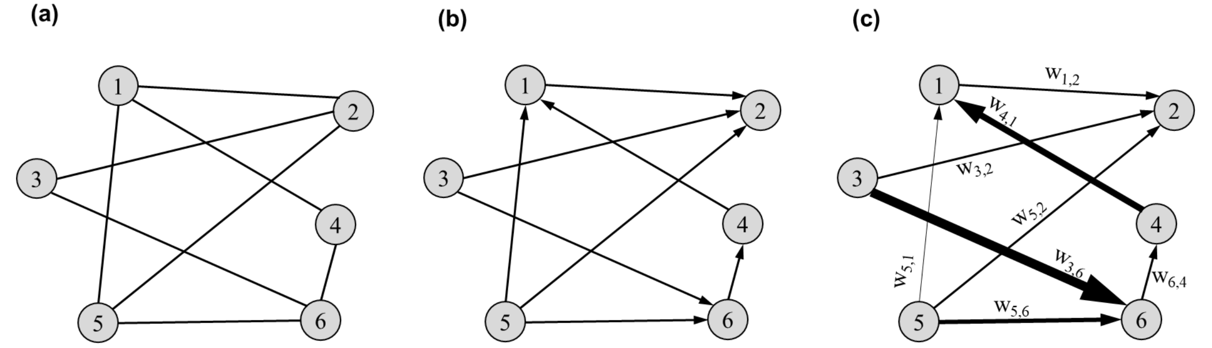
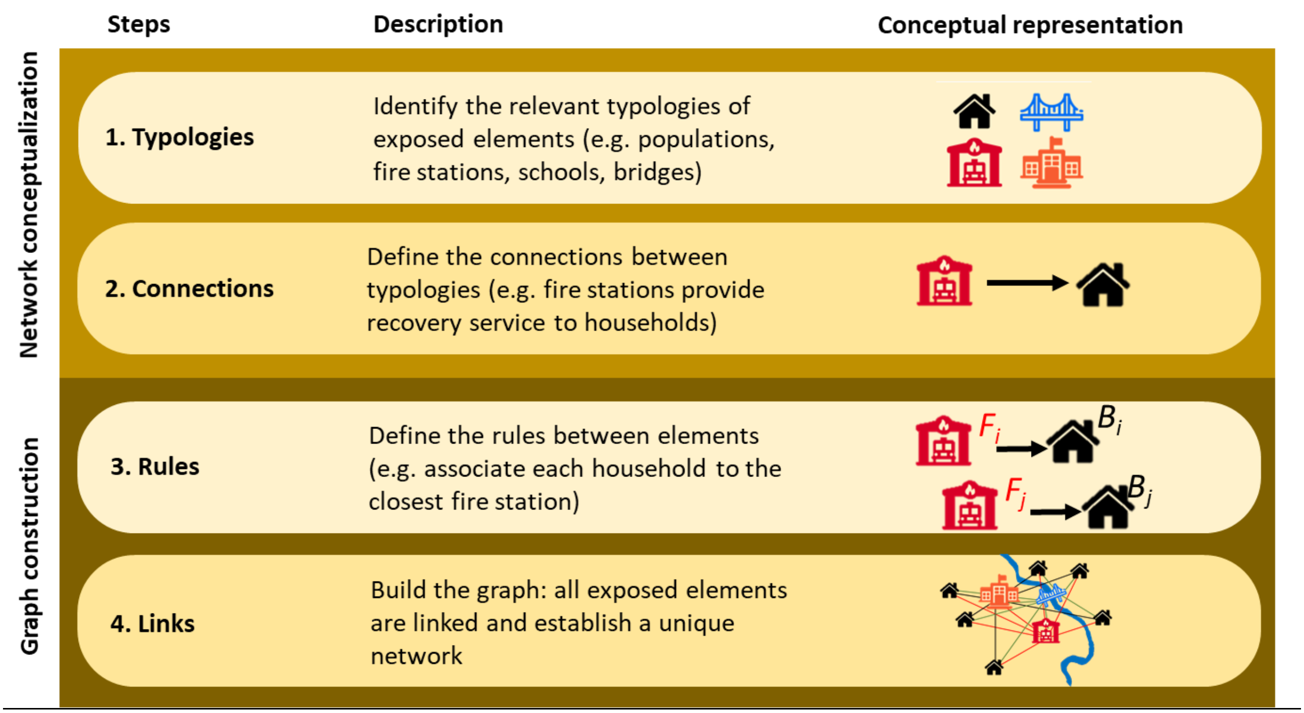


Figure 1: Graph representation of a network. (a) Undirected. (b) Directed. (c) Weighted directed.



	Steps	Description	Conceptual representation
Network conceptualization	**1. Typologies**	Identify the relevant typologies of exposed elements (e.g. populations, fire stations, schools, bridges)	
	2. Connections	Define the connections between typologies (e.g. fire stations provide recovery service to households)	
Graph construction	**3. Rules**	Define the rules between elements (e.g. associate each household to the closest fire station)	$F_i \rightarrow B_i$ $F_j \rightarrow B_j$
	4. Links	Build the graph: all exposed elements are linked and establish a unique network	

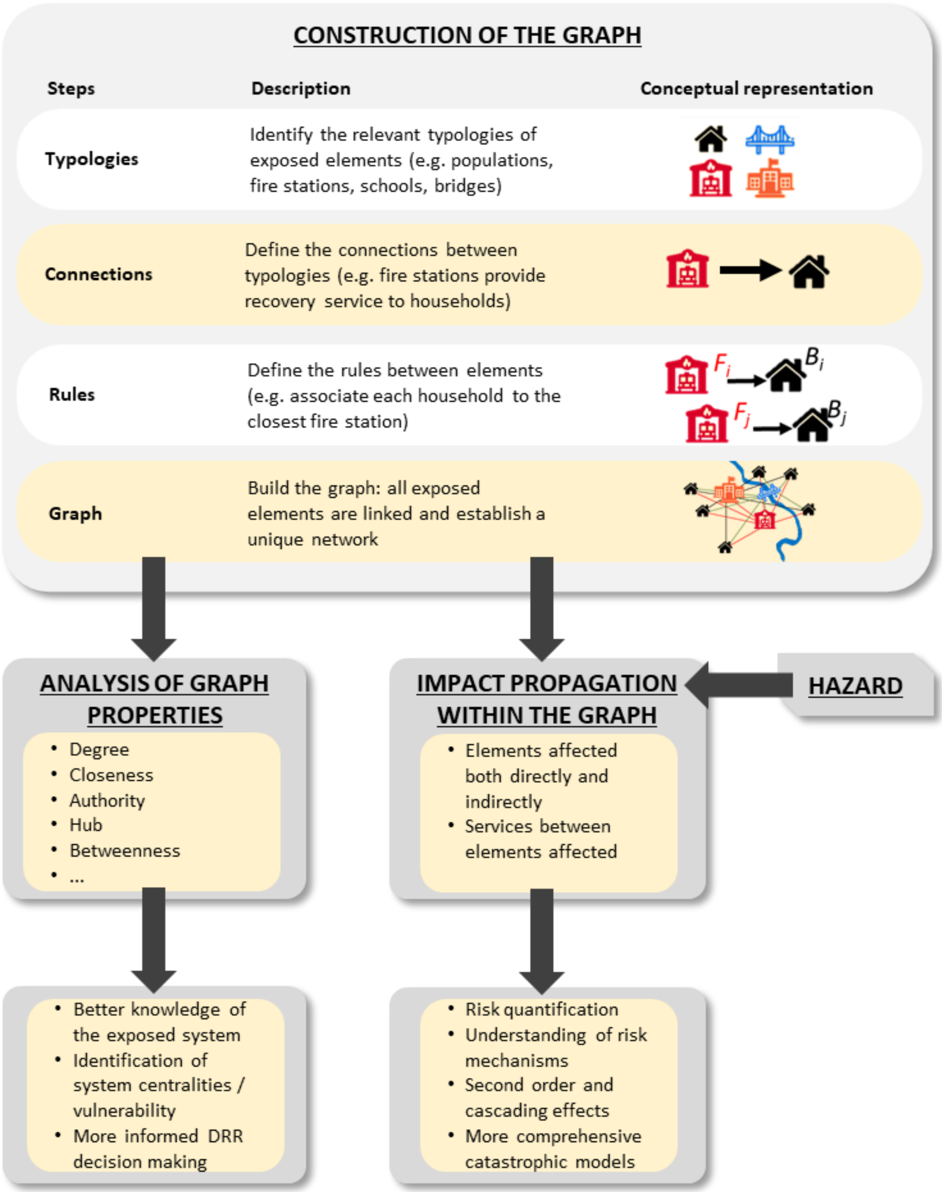
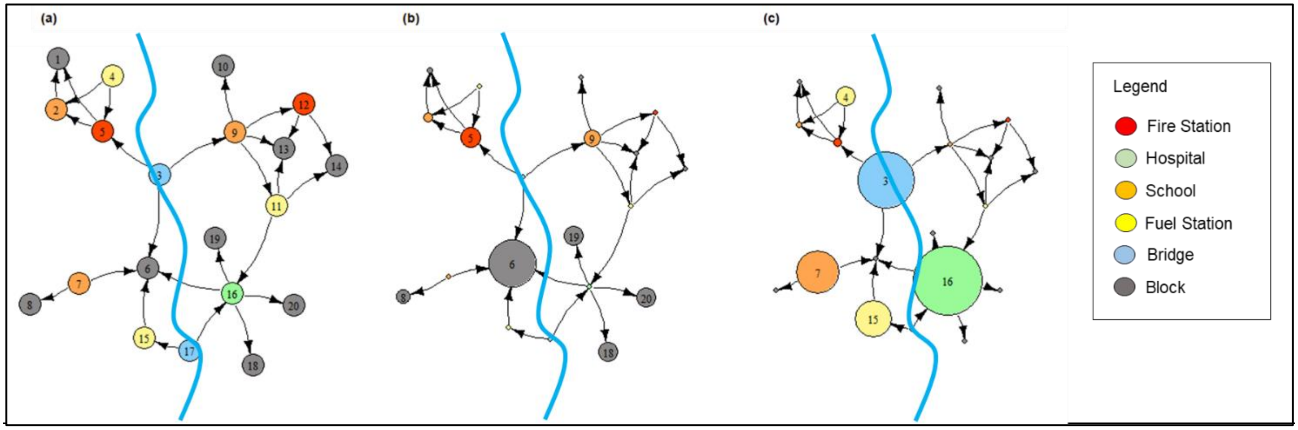


Figure 2: Workflow.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🟢 Hospital
🟠 School
🟡 Fuel Station
🔵 Bridge
⚫ Block

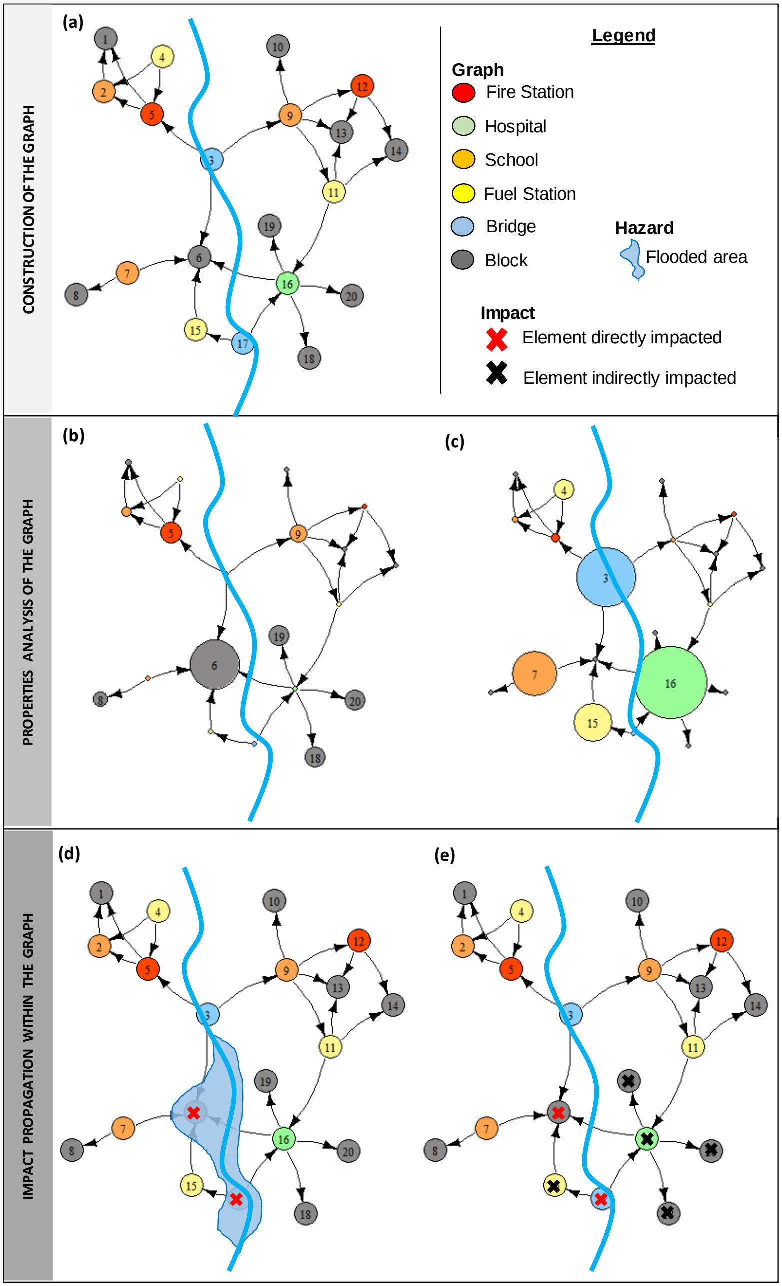

Figure 3: (a) Map of the various elements of a hypothetical municipality in a flood-prone area; (b) Same, with node sizes proportional to authority values; (c) Same, with node sizes proportional to hub values; d) Same, with flood area and nodes directly impacted highlighted with in red cross; (e) Same, with also the nodes indirectly impacted highlighted with black cross.

Table 1: ~~**Graph properties description**~~**Properties of a graph G with N nodes defined by its adjacency matrix *A(G)* with *N x N* elements a_{ij}, whose value is $a_{ij}>0$ if nodes *i* and *j* are connected, and 0 otherwise**

~~**Graph properties**~~	~~**Description**~~
~~Degree (k)~~	~~The number of edges incident with the node~~
~~Path length~~	~~The geodesic length from node i to node j~~
~~Closeness~~	~~The distance (number of links) of a node to all others~~
~~Betweenness~~	~~The shortest paths between pairs of nodes that pass through a given node~~
~~Authority~~	~~Value of a node proportional to the sum of the node hubs pointing to it~~
~~Hub~~	~~Value of a node proportional to the sum of authority of nodes pointing to it~~
~~Percolation threshold (pc)~~	~~The minimum value of fraction of remaining nodes (p) that leads to the connectivity phase of the graph~~

Property	**Description**	**Formula**
Degree (k)	The number of edges incident with the node	$k_i = \sum_j a_{ij}$
Diameter (D)	The maximum value of all path lengths d_{ij}	$D = \max_{i,j} d_{ij}$, where d_{ij} is the geodesic length from node *i* to node *j (i.e. path length)*:
Characteristic path length (d)	The average shortest path length	$d = \frac{1}{N*(N-1)} * \sum_{i,j(i\neq j)} d_{ij}$
Closeness (c)	Shortest path length from a node to every other nodes in the network	$c_i = \frac{1}{l_i}, \quad where\ l_i = \frac{1}{n-1} * \sum_j d_{i,j}$
Betweenness (b)	Number of shortest paths between pairs of nodes that pass through a given node	$b_i = \sum_{j,k} \frac{n.\ of\ shortest\ paths\ connecting\ j,k\ via\ i}{n.\ of\ shortest\ paths\ connecting\ j,k} = \sum_{j,k} \frac{n_{jk}(i)}{n_{jk}}$
Authority (x)	The value proportional to the sum of the node hub values pointing to it	$x_i = \alpha * \sum_j a_{ji} y_j$ -> $A * A^T$, where α is a proportional constant
Hub (y)	The value proportional to the sum of authority of nodes pointing to it	$y_i = \beta * \sum_j a_{ij} x_j$ -> $A^T * A$, where β is a proportional constant
Percolation threshold (pc)	The minimum value of fraction of remaining nodes (p) that leads to the connectivity phase of the graph	For random graph $p_c = \frac{1}{\bar{k}}$, $\bar{k}$ is the average of degree

Table 2: Analogy of risk variables with graph properties.

Risk variables	Analogy with graph properties
Exposure	The authority represents how the system privileges the nodes, conferring them more or less importance compared with others, according to the connections established in the system.
Vulnerability	The propensity of parts of the network to be isolated because of hazard events. The closeness of a node is a measure of the single node vulnerability within the system, while degree distribution, hub, and betweenness are measures of vulnerability of the system as a whole.
Resilience	The percolation threshold, together with the network fragmentation analysis, explain the resilience of the network after a perturbation.