Response to Referee #1

We wish to thank the Referee for his/her time and effort reviewing the manuscript. We greatly appreciate the constructive comments and suggestions, which we have carefully addressed in this response. Where applicable, changes are proposed to the manuscript accordingly (and marked up for clarity). Following the guidelines of the NHESS Editorial Board, the revised manuscript was not prepared at this point.

Natural hazard risk of complex systems part I introduces graph theory into risk analysis to promote a paradigm shift from reductive to holistic approaches to risk assessment and assess the risk of complex systems. Through a review of graph theory as it relates to risk, including issues of exposure, vulnerability, and resilience, and the development of an illustrative case, the authors show how network analysis can be employed to assess complex interdependent systems. The authors' main argument is that current risk assessment approaches fail to capture complex interactions between systems as a whole, and that network analysis techniques can be used to capture that complexity.

The authors are correct that current risk assessments are often reductionist and fail to account for interconnections and the properties of the system as a whole. Readers will also benefit from this topic given the prevalence of risk analysis that take a reductionist perspective.

- However, there is a significant body of work using graph theory for risk analysis. A large literature builds on Rinaldi et al. (2001) to use graph theory to assess critical infrastructure risks, interdependencies, and cascades (Lewis 2014; Setola et al. 2016), and another focuses on the systemic risks in financial systems (Summer 2013). Instead of focusing primarily on the connections between physical structures of infrastructure, another body of work focuses on the interconnections between hazards or hazards and vulnerabilities showing how risks can propagate and cascade (Clark- Ginsberg 2017; Gill and Malamud 2014).

Graph Theory is a well-established branch of mathematics. As such, it has been used to address a wide number of problems in many different fields, where risk analysis is included. However, risk analysis is, in itself, a very large field. Natural hazard risk, despite falling under the 'risk analysis' umbrella, requires its own specific modelling approaches, which are necessarily different from other types of risk, such as financial contagion in banking systems (as covered by Summer (2013) mentioned by the Referee), or others like car accidents, disease, conflict, to name a few. As this paper focuses of natural hazard risk, we have engaged with literature primarily from this field, where the application of Graph Theory is much sparser. We agree that some of the references mentioned here by the Referee are relevant in this context, and were missing from the original manuscript. Accordingly, in the revised manuscript we will add Clark-Ginsberg *et al.* (2018) at L84,L446,L450, Lewis (2014) and Setola *et al.* (2016) at L100.

- This literature (and the broader qualitative literature on networks of risk) identifies several challenges with using network analysis for risk. Chief among them is how to account for the multi-level, open-ended nature of systems in graph based approaches. For instance, Schulman and Roe (2016) and other high reliability theorists point out that infrastructure systems are vastly more complex than modelers make them out to be, with substantial coupling across components that is difficult to discern. Clark- Ginsberg et al. (2018) applies these insights to argue that network bases approaches of open-ended systems can never be complete and require careful decisions on how to delimit boundaries and describe networks. The authors allude to the idea of system incompleteness when discussing the nestled nature of power infrastructure, but then purport to offer a complete network (p6), which is not possible given the open-ended nature of risk.

We agree that the issue of modelling open-ended systems is central in any study of networks, and that this aspect was not sufficiently discussed in the original manuscript. For this reason, following the suggested reference, we propose adding the following paragraph to the Discussion section (L450) in the revised manuscript:

> “Despite the improvements in risk assessment within this systems perspective, Clark-Ginsberg *et al.* (2018) highlights that there are “questions about the validity of such assessment” regarding the ontological foundations of networked risk, the non-linearity and emergent phenomena that characterize system phenomena. The emergence of the risk system demonstrates that the risk will never be completely knowable, and for this reason the “unknown unknowns” are an inseparable part of a risk networks; in fact, the boundary definition of open systems are by nature artificial.”

We believe this issue should also be discussed in Section 2.2.1 (Network Conceptualization), and therefore propose to add the following sentence (L270):

> “In defining the topology, it is crucial to define the level of analysis details coherently with the scope and scale, both for the selection of elements and for the relationship between elements that need to be considered. In the case of a very high detail for example, a node of the graph could represent a single person within a population, and in the case of a lower resolution, it could represent a large group of people with a specific common characteristic, such as living in the same block or having the same hobby. In the case of analyses at a coarser level, an entire network (e.g. electric power system) can be modelled as a single node of another larger network (e.g. national power system). The definition of the topology structure of the graph also identifies immediately the system boundaries (e.g. which hospitals to be considered in the analysis: only the potential flood area, the ones in the district or in the region?). Up to which extent it is necessary to consider elements as nodes of the graph? The topology definition is a necessary step to perform the computational analysis and introduces approximations of the open systems that need to be acknowledged.”

- *This literature shows how graph theory can be used for representing complex issues of risk in a holistic way and also provides a grounding in some of the challenges associated with the topic. The authors need to clearly state how their work contributes to this literature.*

As stated in the first comment, it is true that Graph Theory has been used to model risk in different fields, and it is also true that some literature proposes the use of Graph Theory specifically to model natural hazard risk of specific types of systems (most often infrastructures, such as Dueñas-Osorio et al. (2004). However, to the best of our knowledge, the application of Graph Theory for broader disaster risk reduction and collective risk assessment purposes, as proposed in the article, is new.

As we aimed to describe in the Introduction, the common practice in the field of natural hazard risk is to adopt reductionist methods, which focus on exposed elements individually and therefore neglect a very significant parcel of the actual impacts. This is very clearly an under-explored area in catastrophe risk modelling, and one where more research work is warranted. As such, we are firmly convinced that this work makes a relevant contribution to the field.

However, we agree that the original manuscript fails to unequivocally identify this gap through a well-structured literature review, and therefore also fails to clearly position itself among that literature. While the introduction of the original manuscript aimed to achieve this, it probably did so in an insufficiently organized and incomplete manner. We therefore propose to restructure it by splitting it into three subsections and expanding certain parts, as described below:

1. Introduction

1.1 Collective disaster risk assessment: traditional approaches

This subsection provides a brief contextualization of current practice and limitations in disaster risk assessments, making use of key references. Here we propose to add a relevant reference related to multi-hazard risk Zscheischler *et al.* (2018), as suggested by Referee #2 (L64).

1.2 Modelling natural hazard risk in complex systems: state of the art and limitations

This subsection introduces the need for holistic approaches that are able to handle the complexity of contemporary society. This is in contrast with the reductionist approaches presented in subsection 1.1, which only partially contribute to the assessment of the total impact, because they do not consider the connections between the exposed elements. The literature in this subsection aims to give an overview of the state of art and limitations of existing models to study complex systems. We propose to improve it by adding the suggested references listed here:

L79: Lhomme et al. (2013) showed that the "*city has to be considered as an entity composed by different elements and not merely as a set of concrete buildings.*"

L84: "The reductionist approach, in which the "risks are an additive product of their constituent parts" (Clark-Ginsberg *et al.*, 2018), contrasts with the complex nature of disasters."

We then show how the networks are treated in the infrastructure sectors, one of the sectors that traditionally address the complexity of interdependency. This brings to the concept of systemic vulnerability typical of cascading failures in the network, for which we also propose to add two suggested references Lewis (2014) and Setola *et al.* (2016) at L100. Subsection 1.2 ends with the presentation of the system of systems perspective.

1.3 Positioning and aims

~~Although t~~The aspects of complexity and interdependency have been investigated by various models of critical infrastructure as a single system, or as systems of ~~systems,~~ systems, which are networks by construction (e.g. drainage system or electric power network, Åke J. Holmgren, 2006; Navin, 2016). However, there is still a gap in current practice when it comes to modelling~~we would like to further explore~~ the complexity of interconnections between individual ~~the exposed~~ elements that do not explicitly constitute a network, which tend to be neglected by traditional reductionist risk assessments. ~~and their interconnections, and propose an approach to develop more~~ Therefore, in this manuscript we propose an approach to model such interconnections and assess collective risk in a holistic manner~~collective risk assessment~~. . In particular, the proposed approach allows modelling and assessing interconnected risk (due to the complex interaction between human, environment and technological systems) and cascading risk (which results from escalation processes). The results can then support more informed DRR decision making (Pescaroli and Alexander, 2018).

The analyses of ~~it is necessary to better analyse~~ the interactions between elements at risk and their influence of indirect impacts are assessed in this work by adopting the framework of Graph Theory, the branch of mathematics for the treatment of networks, which has been used to address a wide range of practical questions in many sectors (Boccaletti *et al.*, 2006) ~~assessment~~. Given this context, this paper proposes an insight into collective risk assessment from an innovative holistic perspective. The aims of this paper are"

- *Because they do not engage with this literature, I do not believe there is enough for a standalone theoretical paper on their topic. Rather than publishing this as a separate piece, I recommend using this article as a basis the literature review/methodology of the empirical paper, which provides a useful contribution to the literature.*

This article proposes the theoretical framework for a new approach to model collective risk of natural hazards in complex systems, such as urban environments. In order to do so, the introduction aims to engage with literature that is representative of the state of the art in this field, following the logical sequence described in the previous comment. Ultimately, the goal of the introduction is to provide a concise overview (i.e. brief but comprehensive), and then position this work among the existing body of literature.

We fully recognize that certain key references suggested by both Referees were missing, and following the very useful suggestions provided, we have reorganized and expanded the introduction and added them. We would like to highlight, nevertheless, that the main goal of this article is *not* to provide an exhaustive literature review of the application of Graph

Theory to risk analysis, or of collective risk assessments – these topics would likely require extensive review articles by themselves. As such, we believe that this does not justify the insufficiency of the article as a standalone theoretical paper, as its contents go incontrovertibly beyond the literature review (note that sections 2 to 5 account for over 75% of the article). Moreover, we believe that merging the two papers, even after a significant hypothetical reduction of both of them, would be harmful for their quality, and would still result in an excessively long manuscript. Finally, it is worth noting that from a more practical perspective, the two papers may address different audiences: part I is targeted to a more general audience who may be interested in understanding the foundations of the approach, while part II points to technical experts and researchers who may want to implement this approach for their own practical applications. For these reasons, we believe that keeping the current structure with two companion papers is the optimal solution. We would like to kindly ask the Referee to reconsider his/her position on this matter, taking into account the major revision that we have carried out.

Response to Referee #2

We wish to thank the Referee for his/her time and effort reviewing the manuscript. We greatly appreciate the constructive comments and suggestions, which we have carefully addressed in this response. Where applicable, changes are proposed to the manuscript accordingly (and marked up for clarity). Following the guidelines of the NHESS Editorial Board, the revised manuscript was not prepared at this point.

- This paper only takes us about 60% of the way there. While I do think you have a novel idea of using graph theory to model risk transfer in a way that has not been done, you don't fully show us how to do it conceptually. e.g. you explain how graphs work. and give some discussion of how these graph properties link to vulnerability, resilience, and exposure. But you need to go much further.

This article is organized as two companion papers, and our understanding is that this may have been overlooked by the Referee. The article is organized such that part I provides the theoretical framework, and part II demonstrates how it can be applied using a pilot study. We believe that part II (https://www.nat-hazards-earth-syst-sci-discuss.net/nhess-2018-278/) fully covers the issue raised by the Referee.

- What metrics do you propose we use from graph theory that link to which metrics in risk assessment? Maybe this is what you are trying to do with percolation but it is still very unclear. How are you going to get us towards measure cascading risks with your new approach.

These two companion papers, from a theoretical point of view in part I and a practical application in part II, propose a list of selected graph properties and discuss how these can be used in the assessment of the traditional components of risk. In particular, authority, closeness and percolation threshold are proposed respectively as metrics for the risk variables: exposure, vulnerability and resilience. Furthermore, these analogies are summarized in Table 2, and finally most of proposed metrics are applied in a case study in part II.

- Part of the problem is the disorganized literature review and background. You are missing a lot of the resilience literature on this topic, and it feels like you are describing papers selectively. Please organize this into topics, themes, that lead to the demonstrating the gap in the lit that your new graph theory approach will allow us to fill.

We fully agree with the Reviewer that the structure of the literature review of the manuscript can be improved. For this reason, we propose to restructure the Introduction, introduce the reference relevant for to the aim of the introduction, and write a new subsection (1.3) that help positioning the paper among the existing body of literature. These changes are described in detail below:

1. Introduction

1.1 Collective Disaster Risk Assessment: traditional approaches

This subsection provides a brief contextualization of current practice and limitations in disaster risk assessments, making use of key references. Here we propose to add a relevant reference related to multi-hazard risk Zscheischler *et al.* (2018), as suggested by Referee #2 (L64).

1.2 Modelling natural hazard risk in complex systems: state of the art and limitations

This subsection introduces the need for holistic approaches that are able to handle the complexity of contemporary society. This is in contrast with the reductionist approaches presented in subsection 1.1, which only partially contribute to the assessment of the total impact, because they do not consider the connections between the exposed elements. The literature in this subsection aims to give an overview of the state of art and limitations of existing models to study complex systems. We propose to improve it by adding the suggested references listed here:

> L79: Lhomme et al. (2013) showed that the "*city has to be considered as an entity composed by different elements and not merely as a set of concrete buildings.*"
>
> L84: "The reductionist approach, in which the "risks are an additive product of their constituent parts" (Clark-Ginsberg *et al.*, 2018), contrasts with the complex nature of disasters."

We then show how the networks are treated in the infrastructure sectors, one of the sectors that traditionally address the complexity of interdependency. This brings to the concept of systemic vulnerability typical of cascading failures in the network, for which we also propose to add two suggested references Lewis (2014) and Setola *et al.* (2016) at L100. Subsection 1.2 ends with the presentation of the system of systems perspective.

1.3 Positioning and aims

~~Although t~~The aspects of complexity and interdependency have been investigated by various models of critical infrastructure as a single system, or as systems of ~~systems,~~ systems, which are networks by construction (e.g. drainage system or electric power network, Åke J. Holmgren, 2006; Navin, 2016). However, there is still a gap in current practice when it comes to modelling~~we would like to further explore~~ the complexity of interconnections between individual ~~the exposed~~ elements that do not explicitly constitute a network, which tend to be neglected by traditional reductionist risk assessments. ~~and their interconnections, and propose an approach to develop more~~ Therefore, in this manuscript we propose an approach to model such interconnections and assess collective risk in a holistic manner~~collective risk assessment~~. . In particular, the proposed approach allows modelling and assessing interconnected risk (due to the complex interaction between human, environment and technological systems) and cascading risk (which results from escalation processes). The results can then support more informed DRR decision making (Pescaroli and Alexander, 2018).

> The analyses of ~~it is necessary to better analyse~~ the interactions between elements at risk and their influence of indirect impacts are assessed in this work by adopting the framework of Graph Theory, the branch of mathematics for the treatment of networks, which has been used to address a wide range of practical questions in many sectors (Boccaletti *et al.*, 2006) ~~assessment~~. Given this context, this paper proposes an insight into collective risk assessment from an innovative holistic perspective. The aims of this paper are: (…)"

We believe that this new structure of the Introduction and its proposed improvements address the Referee's comment and provide a much more logical sequence and organization for these topics. It is worth noting that the purpose of this section is not to provide an exhaustive literature review of a specific sector (e.g. resilience or critical infrastructure), but to engage with literature that is representative of the state of the art in this field, providing a concise overview (i.e. brief but comprehensive) to readers, and then position this work among the existing body of literature.

- This idea has a lot of promise, but needs work. The conclusion should make me feel like I have a new tool and idea to measure risk. But I am left feeling confused.

Presumably this comment applies to part I of the article. We believe that the Referee's idea, with which we agree, is covered by the full article (i.e. both companion papers together). Nevertheless, note that this topic warrants further research, which is duly acknowledged in the manuscript.

- Line 45. Vulnerability does consider social conditions. That is a wrong statement

We agree and do not state otherwise. The contrast presented here is between social and *physical* vulnerability, as explicitly written in the previous line of the manuscript.

- Line 59. See the work on compound flood risk. Eg. Wahl, Thomas, Shaleen Jain, Jens Bender, Steven D. Meyers, and Mark E. Luther. "Increasing risk of compound flooding from storm surge and rainfall for major US cities." Nature Climate Change 5, no. 12 (2015): 1093. Zscheischler, J., Westra, S., Hurk, B.J., Seneviratne, S.I., Ward, P.J., Pitman, A., AghaKouchak, A., Bresch, D.N., Leonard, M., Wahl, T. and Zhang, X., 2018. Future climate risk from compound events. Nature Climate Change, p.1.

We agree on the relevance of both references and propose adding them to the article at L64

- Line 80. Great examples. Surprise to see lack of citatations for the large literature on compo unding risk and cascading failtures from the resilience field. E.g. Buldyrev, S. V., Parshani, R., Paul, G., Stanley, H. E., & Havlin, S. (2010). Catastrophic cascade of failures in interdependent networks. Nature, 464(7291), 1025. Chicago

We add the suggested reference in the Introduction at L96

- Line 84. I have never heard of this rinaldi paper. I doubt it is the most quoted.

Rinaldi et al. (2001) is an essential reference in the assessment of critical infrastructure risks, with currently over 2000 citations on Google Scholar for example, as also underlined by Referee #1 "*A large literature builds on Rinaldi et al. (2001) to use graph theory to assess critical infrastructure risks*". We recognize that this article may be not as well known in other neighbouring fields of science (e.g. other sectors of risk, or applications of graph theory), but the validity of this statement is indisputable and we therefore propose to keep it.

- In general this literature review feels selective and disorganized. Use subheadings. What is the gap you are filling? Are you really the only/first people to use graph theory to assess risk. I somehow doubt it. A simple google scholar search revealed many articles:

Heckmann, T., Schwanghart, W., & Phillips, J. D. (2015). Graph theoryâATRecent developments of its application in geomorphology. Geomorphology, 243, 130-146. Holmgren, Åke J. "Using graph models to analyze the vulnerability of electric power networks." Risk analysis 26, no. 4 (2006): 955-969. Lhomme, S., Serre, D., Diab, Y., & Laganier, R. (2013). Analyzing resilience of urban networks: a preliminary step towards more flood resilient cities. Natural hazards and earth system sciences, 13(2), 221-230.

Also see risk transfer analysis 1. Sapountzaki, K. Social resilience to environmental risks: A mechanism of vulnerability transfer? Manag. Environ. Qual. An Int. J. 18, 274–297 (2007).

We agree with the Reviewer that the structure of the literature review of the manuscript can be improved, and above we proposed a significant number of improvements to address this issue. In the revised manuscript we also propose adding the following references suggested by the Referee: Åke J. Holmgren (2006); Lhomme et al. (2013) and Sapountzaki (2007).

Note that the manuscript does not claim in any way to "*be the only/first people to use Graph Theory to assess risk*": As we underline above, the literature on the use Graph Theory in risk assessment is large, but also (and more importantly here) extremely diverse. Many (if not most) of the articles that show up on a "*simple google scholar search"* actually have little relevance given the scope of our manuscript. The manuscript includes the references that we believe are most relevant and representative of the state of the art in this field (i.e. natural hazard risk modelling of systems that are not explicitly arranged as a network but whose underlying connections can significantly magnify impacts and risk), and could help the reader to understand the purpose of the proposed approach.

- Page 165. Its hard to read all your definitions in prose. Made a table or a diagram that shows in a depiction each term. Add more to figure 1.

We agree, and propose adding the following table at L177 to address this issue:

Table 1: Graph properties description

Graph properties	**Description**
Degree (k)	The number of edges incident with the node
Path length	The geodesic length from node i to node j
Closeness	The distance (number of links) of a node to all others
Betweenness	The shortest paths between pairs of nodes that pass through a given node
Authority	Value of a node proportional to the sum of the node hubs pointing to it
Hub	Value of a node proportional to the sum of authority of nodes pointing to it
Percolation threshold (pc)	The minimum value of fraction of remaining nodes (p) that leads to the connectivity phase of the graph

- Line 196. What is pc. What is k.

The definitions of pc and k are presented at L221 and L224, respectively.

- Line 265. It is not until here that you tell me what graph theory contributes to vulnerability analysis.

This follows the logic behind the structure of the manuscript, where we first provide context for the research (Introduction), then present some relevant aspects of graph theory, followed by the workflow that we propose in our approach, and finally show the analogy between graph properties and exposure, vulnerability and resilience. We believe this aids overall clarity and organization.

- WHY is current risk analysis lacking and WHAT does graphs uniquely help us understand.

We believe that the main shortcoming of current reductionist approaches is the impossibility to consider the connections between exposed elements, as also underlined by the suggested reference Lhomme et al. (2013). Our manuscript proposes an approach based on Graph Theory that aims to take into consideration these connections, and treat the exposed elements as part of a whole system. The analogy proposed in part I and the application in part II show how the properties of a graph can provide information on the risk variables.

- *330. I have never heard of this definition of resilience. This need to be motivated by the enourmous literature on the topic to some degree.*

The definition of resilience is proposed at line 51 ("system's capacity to cope with stress and failures and to return to its previous state"). Instead, at L369 we specifically underline the dynamic features of resilience compared to vulnerability. Since this was not clear enough, we are now suggesting a modification that also incorporates two references proposed by the Reviewer L371-L372:

> Resilience differentiates from vulnerability in terms of dynamic features of the system as a whole. The properties and functions used to model vulnerability are static characteristics that do not consider any time evolution, or using the words of Sapountzaki (2007), "*vulnerability is a state, while resilience is a process*"; ~~instead,~~ in fact the definition of resilience implies a time evolution of the characteristics of the whole system. In addition, Lhomme et al. (2013) underline "*the need to move beyond reductionist approaches, trying, instead, to understand the behaviour of a system as a whole*". These two features, dynamic aspect and whole system, make vulnerability different from resilience and ~~This~~ this difference can be expressed by a cinematography analogy: vulnerability is a single frame of the resilience video.

Natural hazard risk of complex systems – the whole is more than the sum of its parts: I. A holistic modelling approach based on Graph Theory

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Abstract: Assessing the risk of complex systems to natural hazards is an important and challenging problem. In today's intricate socio-technological world, characterized by strong urbanization and technological trends, the connections~~,~~ and interdependencies ~~and interactions~~ between exposed elements are crucial. These complex relationships call for a paradigm shift in collective risk assessments, from a reductionist approach to a holistic one. Most commonly, the risk of a system is estimated through a reductionist approach, based on the sum of the risk ~~of its elements~~evaluated individually at each element. In contrast, a holistic approach considers the whole system as a unique entity of interconnected elements, where those connections are taken into account in order to more thoroughly assess the risk. To support this paradigm shift, this paper proposes a new holistic approach to assess the risk in complex systems based on Graph Theory. The paper is organized in two parts: part I, presented here, describes the proposed approach, and part II ~~presents~~ illustrates an application to a pilot study in Mexico City. ~~In part I,~~Here we demonstrate that by representing a complex system such as an urban settlement by means of a network (i.e. a graph), it is possible to take advantage of the techniques made available by the branch of mathematics called Graph Theory to analyse its properties. Moreover, it is possible to establish analogies between certain graph metrics (e.g. authority, degree, hub values) and the risk variables (exposure, vulnerability and resilience). ~~-~~Leveraging these analogies, one can not only obtain a deeper knowledge of the system (structure, weaknesses, etc.), but also understand its risk mechanisms (how the impacts of a single or multiple natural hazards are propagated, where they are exacerbated), and therefore assess the disaster risk of the system as a whole, including second-order impacts and cascade effects.

1. Introduction

We live in an increasingly complex world. Today's societies are interconnected in complex and dynamic social-technological networks, and have become more dependent on the services provided by critical facilities. Population and assets in natural hazard-prone areas are increasing, which translates into higher economic losses (Bouwer et al., 2009). In coming years, climate change is expected to exacerbate these trends (Alfieri et al., 2017). In this context, disaster risk is a worldwide challenge that institutions and private individuals ~~need to~~must face at both global and local scales. Today, there is growing attention to the management and reduction of disaster risk, as illustrated for example by the wide adoption of the Sendai Framework for Disaster Risk Reduction (SFDRR, 2015).

1.1. Collective disaster risk assessment: traditional approaches

The effective implementation of strategies to manage and reduce collective risk, i.e. the risk assembled by ~~the~~ a collection of ~~risk assets/~~elements at risk, requires support from Risk Assessment (RA) studies that quantify the impacts that hazardous events may have on the built environment, economy and society (Grossi and Kunreuther, 2005). ~~As such, RA is a fundamental step in the Disaster Risk Management cycle, particularly within a risk management framework where Disaster Risk Reduction (DRR) is directly integrated in national development strategies.~~ The research community concerned with Disaster Risk Reduction (DRR), particularly in the fields of physical and environmental sciences, has generally agreed on a common approach for the calculation of risk (R) as a function of hazard (H), exposure (E), and vulnerability (V): $R = f(H, E, V)$ (e.g. Balbi *et al.*, 2010). Hazard defines the potentially damaging events and their probabilities of occurrence, exposure represents the population or assets located in hazard zones that are therefore subject to potential loss, and vulnerability links the intensity of a hazard to potential losses to exposed elements. This framework has been in use by researchers and practitioners in the field of seismic risk assessment for some time (Bazzurro and Luco, 2005; Crowley and Bommer, 2006), and has more recently also become standard practice for other types of hazards, such as floods (Arrighi et al., 2013; Falter et al., 2015).

Despite consensus on the conceptual definition of risk, different stakeholders tend to have their own specific perspectives. For example, while insurance and reinsurance companies may focus on physical vulnerability and potential economic losses, international institutions and national governments may be more interested in the social behaviour of society or individuals in coping with or adapting to hazardous events (Balbi et al., 2010). As such, even though this risk formulation can be a powerful tool for RA, it has its limits. For instance, it does not consider social conditions, community adaptation or resilience (i.e. a system's capacity to cope with stress and failures and to return to its previous state). In fact, resilience is still being debated and there is not a~~n~~ common and consolidated approach to assess it (Bosetti et al., 2016; Bruneau et al., 2004; Cutter et al., 2008, 2010).

To overcome some of these limits, different approaches have been ~~proposed~~ put forward in recent research. For example, ~~(~~Carreño et al.~~,~~ (2007b, 2007a, 2012) have proposed to include an aggravating coefficient in the risk equation in order to reflect socio-economic and resilience features. Another example can be found in the Global Earthquake Model, which aims to assess so-called integrated risk by combining hazard (seismic), exposure and vulnerability of structures with metrics of socio-economic vulnerability and resilience to seismic risk (Burton and Silva, 2015). ~~These new integrated approaches are seen with increasing interest, particularly with regard to Climate Change Adaptation (CCA). In fact, CCA-related research activities are strongly focusing on how local communities are able to adapt to and cope with a disaster (Balbi et al., 2010). In addition to integrated risk, there is also an increasing international interest in m~~Multi-risk assessment studies resulting from ~~as~~ a combination of multiple ~~-~~hazards ~~(natural and manmade) with~~ and ~~multi-~~vulnerabilities are also receiving growing scientific attention~~y (physical and social) assessments~~ (Gallina et al., 2016; Karagiorgos et al., 2016; Liu et al., 2016;~~)~~ Wahl et al., 2015; Zscheischler et al., 2018). These new approaches are seen with increasing international interest, particularly with regard to climate change adaptation (Balbi et al., 2010; Terzi et al., 2019)~~ (Balbi et al., 2010)~~.

While ~~S~~some research has explored the potential of an integrated approach to risk and multi-risk assessment of natural ~~disasters~~ hazards~~in various fields. However~~, quantitative collective RA ~~(both scenario- and event-based)~~ still requires further development to consider the connections and interactions between exposed elements. Although holistic approaches are in strong demand (Cardona, 2003; Carreño et al., 2007b; IPCC, 2012), the majority of methods and especially models developed so far are based on a reductionist paradigm, which estimates the collective risk of an area as the combination of the risk of its exposed elements individually, neglecting the links between them.

1.2. Modelling natural hazard risk in complex systems: state of the art and limitations

In a changing society which increasingly relies on interconnections, the links between elements are crucial, especially considering the urbanization and technological trends ~~which~~ that modern-day society is strongly promoting. Urban population growth means that people are depending more and more on critical facilities, and there is an increasing interdependency between infrastructures. Complex socio-technological networks, which increase the impact of local events on broader crises, characterize the modern technology of present-day urban society (Pescaroli and Alexander, 2016). Lhomme et al. (2013) showed that the “*city has to be considered as an entity composed by different elements and not merely as a set of concrete buildings*”.

~~This partly~~Such aspects support~~s~~ the perception that ~~the~~ collective risk assessment~~ paradigm, where the total risk is the combination of the risk of elements individually,~~ requires a more comprehensive approach than the traditional reductionist one, as it needs to involve~~s~~ “whole systems” and “whole life” thinking (Albano et al., 2014). The reductionist approach, in which the "*risks are an additive product of their constituent parts*” (Clark-Ginsberg et al., 2018), contrasts with the complex nature of disasters. In fact, ~~T~~these tend to be strongly non-linear i.e. the ultimate outcomes (losses) are not proportional to the initial event (hazard intensity and extensions) and are expressed by emergent behaviour (i.e. macroscopic properties of the complex system) that appear when the number of single entities (agents) operate in an environment, giving rise to more complex behaviours as a collective (Bergström, Uhr and Frykmer, 2016). In the last decade, many disasters have shown high levels of complexity and the presence of nonlinear paths and emergent behaviour that have led towards secondary events. Examples of such large-scale extreme events are the eruption of the Eyjafjallajokull volcano in Iceland in 2010, which affected Europe’s entire aviation system, the flooding in Thailand in 2011, which caused a worldwide shortage of computer components, and the energy distribution crisis triggered by hurricane Sandy in New York in 2012.

Secondary events (or indirect losses) due to dependency and interdependency have been thoroughly analysed in the field of critical infrastructures~~:~~ such as telecommunications, electric power systems, natural gas and oil, banking and finance, transportation, water supply systems, government services and emergency services (Buldyrev et al., 2010). Rinaldi et al. (2001), in one of the most quoted papers on this topic, proposed a comprehensive framework to identify, understand and analyse the challenges and complexities of interdependency. Since then, numerous works have ~~tried to study the increase in~~focused on the issue of systemic vulnerability~~ (Menoni et al., 2002)~~, ~~-~~due to the increase in interdependencies in ~~our~~ modern society (e.g. Lewis, 2014; Menoni et al., 2002; Setola et al., 2016)~~: ~~. Menoni (2001) defines systemic risk~~s~~ as “~~systemic risk is ~~*the risk of having not just statistically independent failures, but interdependent, so-called ‘cascading’ failures in a network of N interconnected system components*~~. ~~.” The ~~paper~~article also highlights that “~~*That is, systemic risks result from connections between risks (‘networked risks’).* ~~*In such cases, a localized initial failure (‘perturbation’) could have disastrous effects and cause, in principle, unbounded damage as N goes to infinity*~~ (Menoni, 2001)~~. Ouyang (2014) reviews ~~the ~~existing modelling approaches of interdependent critical infrastructure systems and categorizes them into six groups: empirical, agent-based, system dynamics-based, economic theory-based, network-based, and others. This wide range of models reflects the different levels of analysis of critical infrastructures (physical, functional or socio-economic). Trucco et al. (2012) propose a functional model aimed at i) propagating impacts, within and between infrastructures, in terms of disservice due to a wide set of threats and ii) applying it to a pilot study in the metropolitan area of Milan. However, this well-developed branch of research is mostly focused on the analysis of a single infrastructure typology, and the aim is usually to assess the efficiency of the infrastructure itself rather than the impact that its failure may have on society.

A full research branch analyses the complex socio-physical-technological relationships of ~~our ~~society considering a System-of-System (SoS) perspective, whereby systems are merged into one interdependent system of systems. In a SoS, people belong to and interact within many groups, such as households, schools, workplaces, transport, health care systems, corporations and governments. In a SoS, the dependencies are therefore distinguished between links within (intra) the same system, or between (inter) different systems (Alexoudi et al., 2011). The relation between different systems are modelled in literature using qualitative graphs or flow diagrams (Kakderi et al., 2011) and by matrices (Abele and Dunn, 2006). Tsuruta and Kataoka (2008) use matrices for determining damage propagation due to interdependency based on earthquake data and expert judgment considering infrastructure networks (e.g. electric power, waterworks, sewerage, telecommunication, road, and social functions like finance, medical treatment and administration). Menoni (2001) proposes a framework showing major systems interacting in a metropolitan environment based on observations on the Kobe earthquake. Lane and Valerdi (2010) provide a comparison of various SoS definitions and concepts, while Kakderi et al. (2011) have delivered a comprehensive literature review of methodologies to assess the vulnerability of a SoS.

1.3. Positioning and aims

~~Although t~~The aspects of complexity and interdependency have been investigated by various models of critical infrastructure as a single system, or as systems of ~~systems,~~ systems, which are networks by construction (e.g. drainage system or electric power network, Åke J. Holmgren, 2006; Navin, 2016). However, there is still a gap in current practice when it comes to modelling~~we would like to further explore~~ the complexity of interconnections between individual ~~the exposed~~ elements that do not explicitly constitute a network, which tend to be neglected by traditional reductionist risk assessments.~~ and their interconnections, and propose an approach to develop more~~ Therefore, in this manuscript we propose an approach to model such interconnections and assess collective risk in a holistic manner~~collective risk assessment~~. . In particular, the proposed approach allows modelling and assessing interconnected risk (due to the complex interaction between human, environment and technological systems) and cascading risk (which results from escalation processes). The results can then support more informed DRR decision making (Pescaroli and Alexander, 2018).

The analyses of ~~it is necessary to better analyse~~ the interactions between elements at risk and their influence of indirect impacts are assessed in this work by adopting the framework of Graph Theory, the branch of mathematics for the treatment of networks, which has been used to address a wide range of practical questions in many sectors (Boccaletti *et al.*, 2006) ~~assessment~~. Given this context, this paper proposes an insight into collective risk assessment from an innovative holistic perspective.

The aims of this paper are:

- to present a new perspective to promote a paradigm shift from a reduction to holistic approach;
- to propose a new approach to analyse the risk of complex systems based on Graph Theory~~, the branch of mathematics for the treatment of networks~~;
- to link traditional risk variables (exposure, vulnerability, and resilience) to certain properties of a graph;
- to introduce a debate on the new perspective and approach, and directions for future developments.

2. A Graph Theory approach to modelling complex systems

2.1. Background and relevant graph properties

As discussed ~~in detail~~ above, a network could allow ~~to portray~~portraying the complexity of a risk system. In the scientific community, the mathematical properties of a network are studied using Graph Theory, which could also provide a better angle ~~also~~ to assess risk from a holistic and systemic viewpoint. The following paragraphs review the main aspects of Graph Theory, on which our approach is based.

Over recent decades, studies of network concepts, connections and relationships have strongly accelerated in every area of knowledge and research (from physics to information technology, from genetics to mathematics, to building and urban design), showing the image of a strongly interconnected world in which relationships between individual objects are often more important than the objects themselves (Mingers and White, 2009).

Graph Theory is the branch of mathematics that studies the properties of networks. Networks can comprise physical elements in the Euclidean space (e.g. electric power grids, the Internet, highways, neural networks) or entities defined in an intangible space (e.g. collaborations between individuals).

Since its inception in the eighth century, Graph Theory has provided answers to questions in different sectors, such as pipe networks, roads, and the spread of epidemics. During the last decade, there has been an increase in interest in the study of complex networks (e.g. irregular structures, dynamically evolving in time), paying renewed attention to the dynamic properties of networks (Börner et al., 2007; Newman, 2003).

Formally, a complex network can be represented by a graph G which consists of a finite set of elements $V(G)$ called vertices (or nodes), and a set $E(G)$ of pairs of elements of $V(G)$ called edges (or links) (Boccaletti et al., 2006). The graph can be undirected or directed (Figure 1a and b). In an undirected graph, each of the links is defined by a pair of nodes i and j, and is denoted as l_{ij}. The link is said to be incident in nodes i and j, or to join the two nodes; the two nodes i and j are referred to as the end-nodes of link l_{ij}. In a directed graph, the order of the two nodes is important: l_{ij} stands for a link from i to j, node i points to node j, and $l_{ij} \neq l_{ji}$. Two nodes joined by a link are referred to as adjacent.

In addition, ~~A~~a graph could have edges of different weights representing their relative importance, capacity or intensity. In this case, a real number representing the weight ~~(i.e. the relative importance/capacity/intensity)~~ of the link is associated to it, and the graph is said to be weighted ~~. Also tThe weighted graph can also be either directed or undirected. A graph that shows variability in the importance, capacity and intensity values of edges is called weighted, and a real number representing the strength or value of the connection is associated to each link of the network~~ (Figure 1c) (Börner et al., 2007).

A short list of the most common set of node, edge and graph measures~~ments~~ used in Graph Theory is presented here and summarized in Table 1~~1~~. There are measures~~ments~~ that analyse the properties of nodes or edges, local measures~~ments~~ that describe the neighbourhood of a node (single part of the system), and global measures~~ments~~ that analyse the entire network (whole system). From a holistic point of view, it is important to note that since some node/edge measures~~ments~~ require the examination of the complete network, this allows looking the studied area as a unique entity that results from the connections and interactions between its parts and characterizing the whole system.

The degree (or connectivity, *k*) of a node is the number of edges incident with the node. If the graph is directed, the degree of the node has two components: the number of outgoing links (referred to as the degree-out of the node), and the number of ingoing links (referred to as the degree-in of the node). The distribution of the degree of a graph is its most basic topological characterization, while the node degree is a local measure~~ment~~ that does not take into account the global properties of the network. On the contrary, path lengths, closeness and betweenness centrality are properties that consider the complete network. The path length is the geodesic length from node *i* to node *j*: in a given graph, the maximum value of all path lengths is called diameter and the average shortest path length is named characteristic path length. Closeness computes the distance (number of links) of a node to all others, and betweenness is defined as the number of shortest paths between pairs of nodes that pass through a given node.

Other relevant characteristics that are commonly analysed in directed graphs to ~~find~~ assess the relative importance of a node, in terms of the global structure of the network, are the hub and authority properties. A node with high hub value points to many other nodes, while a ~~vertex~~ node with high authority value is linked by many different hubs. Mathematically, the authority value of a node is proportional to the sum of the node hubs pointing to it and the hub value of a node is proportional to the sum of authority of nodes pointing to it. In the World Wide Web, for example, websites (nodes) with higher authorities contain the relevant information on a given topic (e.g. wikipedia.com) while websites with higher hubs point to such information (e.g. google.com).

The mathematical properties presented above are useful metrics to analyse the structural (i.e. network topology, arrangement of a network) and functional (i.e. network dynamics, how the network status changes after perturbation) properties of ~~large~~ complex networks. Depending on the statistical properties of the degree distributions, there are two broad classes of networks: homogeneous, and heterogeneous. Homogeneous networks show a distribution of the degree with a typically exponential and fast decaying tail, such as Poissonian distribution, while heterogeneous networks have a heavy-tailed distribution of the degree, well-approximated by a power-law distribution. Many real-world complex networks show power-law distribution of the degree and these are also known as scale-free networks, because power-laws have the same functional form on all scales (Boccaletti et al., 2006). Networks with highly heterogeneous degree distribution have few nodes linked to many other nodes (i.e. few hubs), and a large number of poorly connected elements.

The properties of the static network structure are not always appropriate to fully characterize real-world networks that also display dynamic aspects. There are examples of networks that evolve with time or according to external environment perturbations (e.g. removal of nodes/links). Two important properties to explore the dynamic response to a perturbation are percolation thresholds and fragmentation modes.

Percolation was born as the model of a porous medium, but soon became a paradigm model of statistical physics. Water can percolate in a medium if ~~an infinite number~~a large number of links exists (i.e. the presence of links means the possibility of water flowing through the medium), and this depends largely on the fraction of links that are maintained. When the graph has ~~most of the~~is characterized by many links, there is a higher probability that connection between two nodes may exist and, in this case, the system percolates. Vice versa, if most links are removed, ~~the connected components tend to be small and~~ the network becomes fragmented (Van Der Hofstad, 2009). The percolation threshold is an important network feature resulting from the percolation concept ~~and it~~which is~~:~~ obtained by removing vertices or edges from a graph. When a perturbation is simulated as a removal of nodes/links, the fraction of nodes removed is defined as $f = \frac{Nodes_{removed}}{Nodes_{Total}}$, and the probability of nodes/links present in a percolation problem is $p = 1 - f = \frac{Nodes_{remainig}}{Nodes_{Total}}$. Consequently, it is possible to define the percolation threshold (p_c) as the minimum value of p that leads to the connectivity phase of the graph (Gao et al., 2015). In practical terms,~~, i.e.~~ the percolation threshold ~~that~~ discriminates between the connected and fragmented phases of the network. In a random network, for example, pc=1/$\bar{k}$, where $\bar{k}$ is the mean of degree k (Bunde and Havlin, 1991).

The second property that investigates dynamic evolution is the fragmentation ~~property~~ (i.e. number and size of the portions of the network that become disconnected). The number and the size of the sub-networks obtained after removing the vertices/edges provide useful information. In the case of a so-called giant component fragmentation, the network retains a high level of global connectivity even after a large amount of nodes have been removed, while in the case of total fragmentation, the network collapses into small isolated portions. For this reason, "*keeping track of the fragmentation evolution permits the determination of critical fractions of removed components (i.e., fraction of component deletion at* *which the network becomes disconnected), as well as the determination of the effect that each removed component has on network response*" (Dueñas-Osorio et al., 2004).

2.2. Proposed workflow

Within the framework of Graph Theory, we propose an approach based on the following two major phases:

- Network conceptualization;
- Graph construction.

The workflow is presented in Figure 2~~Figure 2~~.

2.2.1. Network conceptualization

According to the objects of each specific context, the network conceptualization phase defines the hypothesis of the analysis and the system boundaries. In particular, it establishes the two main objects of the network: nodes (vertices) and links (edges) and their characteristics.

The nodes can theoretically represent all the entities that the analysis wants to consider: physical elements like a single building, bridge and electric tower, suppliers of services such as schools, hospitals and fire brigades, or beneficiaries such as population, students or specific vulnerable groups such as elderly people. Due to the very wide variety of elements that can be chosen, it is necessary to select the category of nodes most relevant to the specific context of analysis. It is also necessary to define, for each node, the operational state that can be characterized, from the simplistic Boolean (functional/non-functional), to discreet states (30/60/100% of service/functionality), or even a complete continuous function (similarly to vulnerability functions). In a graph, the states of each node depend both on the states of the adjacent nodes and on the hazard. In this paper, we use the term *node* to refer to its graph characteristics and term *element* to refer to the entity that it represents in the real world.

The links between the nodes that create the graph can range from physical, geographical, cyber or logical connections (Rinaldi et al., 2001). ~~Two nodes are physically connected if one node is dependent on the material output(s) of the other (e.g. a power grid system provides electricity to an industrial area). Two nodes are geographically connected if their close spatial proximity is able to influence the reciprocal state (e.g. building close to a nuclear power plant). If a node depends on the information provided by another node, these two nodes are cyber connected (e.g. supervisory control and data acquisition (SCADA) systems, industrial productivity). When the nodes are connected but the mechanisms are not physical, geographical and cyber, then the connection is logical (e.g. the increasing of petroleum price has an influence on the traffic congestion). Depending on the selected category for the node~~According to the different typologies of connections and nodes selected, it is necessary to define ~~the type of connection between elements, which are essential for the analysis, and its characteristics:~~ direction and weight of the links. The graph will be directed when the direction of the connection between elements is relevant and it will be weighted if the links have different importance, intensity or difference capacity.~~, otherwise it will be undirected (e.g. the world wide web vertices represent webpages and directed edges represent hyperlinks; network based on friendship is an undirected graph). Furthermore, it is possible to assign a value to each link and to weight the different connections in the graph.~~

In defining the topology, it is crucial to define the level of analysis details coherently with the scope and scale, both for the selection of elements and for the relationship between elements that need to be considered. In the case of a very high detail for example, a node of the graph could represent a single person within a population, and in the case of lower resolution, it could represent a large group of people with a specific common characteristic, such as living in the same block or having the same hobby. In the case of analyses at a coarser level, an entire network (e.g. electric power system) can be modelled as a single node of another larger network (e.g. national power system). The definition of the topology structure of the graph also identifies immediately the system boundaries (e.g. which hospitals to be considered in the analysis: only the potential flood area, the ones in the district or in the region?). Up to which extent it is necessary to consider elements as nodes of the graph? The topology definition is a necessary step to perform the computational analysis and introduces approximations of the open systems that need to be acknowledged.

2.2.2. Graph construction

Once the network is conceptually defined, in order to actually build the graph, it is then necessary to establish the connection between all the selected elements. The conceptual network determines the existence of connections between categories of elements, but it does not define how a single node of one category is linked to a node of another category. Therefore, it is necessary to define rules that establish the connections between each single node. For the sake of clarity, an example could be the following: in the conceptual network, the relationship is defined between students and school ("students go to school"); subsequently, it is necessary to make the link between each student and a school in the area, applying the following rule - "students go to the closest school". This is an example of geographical connection with nodes that are linked by their spatial proximity.

The connections between the single elements can be represented either by a list of pairs of nodes or, more frequently, by the adjacency matrix. Any graph G with N nodes can be represented in fact by its adjacency matrix $A(G)$ with $N \, x \, N$ elements A_{ij}, whose value is $A_{ij} = A_{ij} = 1$ if nodes i and j are connected, and 0 otherwise. If the graph is weighted, $A_{ij}=A_{ji}$ can have a value between 0 and 1 expressing the weight of the connection between the nodes. The properties of the nodes are represented in both cases by another matrix, with a column for each property associated with the node (e.g. name, category, type).

In practical terms, the list of all connections or the adjacency can be automatically obtained via GIS analysis, in the case of geographical connections, or by database analysis, in the case of other categories of connections. The list of nodes, together with either the list of links or the adjacency matrix, are the inputs to build the mathematical graph using dedicated tools. For example, ~~the~~ igraph ~~package~~ (http://igraph.org/r/), the ~~specific library and~~ package for network analysis of the R environment~~,~~ provides a set of data types and functions for the implementation of graph algorithms, and is able to handle large graphs with millions of vertices and edges.

3. Graph concepts in the context of collective risk assessment

Once a network has been setup and a graph has been constructed, it is then possible to compute and analyse its properties by means of Graph Theory. ~~Recall that the topologic properties (i.e. degree, path length, degree distribution, hub, authority) represent the structural characteristics of the network, while dynamic properties (i.e. percolation threshold and fragmentation) provide information about the response of the network to a perturbation or disruption.~~ These analytical tools can be very useful for understanding risk mechanisms: How is it generated? How is it propagated? Which are the weaknesses of the system? Information such as this is key to perform more thorough risk assessments, where second-order and cascade effects are considered, and to support the implementation of more effective risk mitigation actions.

The traditional conceptual skeleton to describe risk as a function of hazard, exposure, vulnerability and resilience can still be adopted within the framework of the proposed graph-based approach. In fact, the properties calculated from a graph consist in a new layer of information for some of those risk variables that go beyond their traditional interpretations within the reductionist paradigm. In particular, they provide a more comprehensive characterization of the single nodes (deriving from their relationships with other nodes), as well as of the system as a whole. In the following paragraphs, we present the analogies that can be established between the graph properties described in Section 2.1 and certain risk variables (Table 2~~Table 2~~), and then provide a simple illustrative example.

3.1. Analogy between graph properties and risk variables

The proposed graph properties can be used to more thoroughly characterize systems of exposed elements. As such, from the ~~four~~ risk variables ~~shown above~~presented in the introduction, the hazard preserves its traditional definition as an event that can impact such systems, or part(s) of it, with certain intensities and associated probabilities of occurrence. For the three other variables, namely, exposure, vulnerability, and resilience, below we provide a brief discussion on their analogies with the graph properties presented previously. ~~It is worth noting that the concept of impact that occurs as a result of the above factors can also be expanded within the graph-based approach. For example, the indirect impact due to a hazardous event suffered by a certain node may be defined as a function of the direct damage sustained by one or more of its parent nodes (i.e. traditional impact), and of the type of service the latter provide to the former. This could be given by a vulnerability function defining the consequences of such a cascade effect. The integration of indirect impact quantification within the graph-based framework will be addressed in future research.~~

3.1.1. Exposure

In analogy with the traditional approach but at the same time extending its concept, the value of each exposed node can be assessed as the relative importance that is given to it by the graph, which is measured by the network itself by means of the connections that point to each node. In Graph Theory, this relative importance ~~between~~ among elements, based on standardized values, can be investigated through the authority analysis. ~~The distribution of authority shows which the elementss might havewith relative a lower or higher values within the system. In particular, a node (element) has~~ A ~~Hh~~high authority value of a node ~~if in the whole system there~~indicates that there are many other nodes (or otherwise some hubs) that provide services (i.e. providers or suppliers) to that node~~it in order to make it functional~~.~~;~~ I~~i~~n other words,~~this case, higher authority values imply that~~ the system privileges it ~~some receiver nodes~~ compared with others according to their connections with the provider nodes. For example, a~~A~~ factory settled in an industrial district ~~receives~~ may receive more services (e.g. electric power, ~~special~~ roads for heavy vehicles~~ and~~, logistic systems) ~~compared with~~than a factory ~~historically developedsettled~~located in the old quarter of ~~the~~a city~~ and therefore~~ ; in this case, the ~~first factory~~former is structurally privileged by the system~~.~~ ~~This simple example shows that the factory in the district is privileged by the system connections compared to the industry settled in the old quarter.~~compared with the ~~second one~~latter.

3.1.2. Vulnerability

In the reductionist approach, vulnerability is ~~referred to~~ the propensity of an asset to be damaged because of a hazardous event. ~~In~~ By adopting a ~~the~~ graph perspective, the vulnerability can be defined both for the ~~the whole system and the single node.~~single node as well as for the ~~whole~~ system as a whole.

In the first case, the vulnerability depends on the relationship that the node has with the others. In particular, the closeness represents the likelihood of a node to be affected indirectly by a hazard event due to the lack of services provided by other nodes. A lower value of closeness, i.e. a shorter path between a node and the network, means a higher probability of a node to remain isolated because of a hazard event. On the other hand, high value of closeness, i.e. a longer path between a node and the network, means a low probability that the node *i* will be isolated.

In the ~~first~~ second case, the vulnerability ~~of a system~~ can be defined as the propensity of the network to be split into isolated parts ~~after because of~~due to a hazardous event. In that condition, an isolated part is unable to provide and receive services, which can translate into indirect losses. The system vulnerability can be ~~expressed~~ evaluated by means of the following graph properties: hubs, betweenness and degree out distribution. The presence of nodes with high ~~values of~~ hub values ~~(namely hubs)~~ indicates a propensity of the network to be indirectly affected more extensively by a hazard event, since a large number of nodes are connected with the hubs. Betweenness manifests the tendency to create isolated sub-networks. As an example, in a road graph, ~~the~~ a bridge node has a higher value of betweenness because all the nodes of a sub-graph (e.g. one side of the river) need to pass through the bridge node in order to connect to the nodes of the other sub-graph (the other side of the river). In the case of a bridge failure, the two sides of the river are isolated and the original road graph splits into two sub-graphs. A network that has nodes with high betweenness values has a higher tendency to be fragmented, because it has a strong aptitude to generate isolated sub-networks. Finally, the degree distribution, which expresses network connectivity of the whole system (i.e. the existence of paths leading to pairs of vertices), has a strong influence on network vulnerability after a perturbation. The shape of the degree distribution determines the class of a network: heterogeneous graphs (power-law distribution and scale-free network) are more resistant to random failure, but they are also more vulnerable to intentional attack (Schwarte et al., 2002). As emphasised above, scale-free networks have few nodes linked to many nodes (i.e. few hubs), and a large number of poorly connected elements. In the case of random failure, there is a low probability of removing a hub, but if an intentional attack hits the hub, the consequences for the network could be catastrophic.

~~In the second case, the single node vulnerability depends on the relationship that one node has with the others. In particular, the closeness represents the likelihood of a node to be affected indirectly by a hazard event due to the lack of services provided by other nodes. A lower value of closeness, i.e. a shorter path between a node and the network, means a higher probability of a node to remain isolated because of a hazard event. On the other hand, high value of closeness, i.e. a longer path between a node and the network, means a low probability that the node *i* will be isolated.~~

3.1.3. Resilience

Resilience differentiates from vulnerability in terms of dynamic features of the system as a whole. The properties and functions used to model vulnerability are static characteristics that do not consider any time evolution, or using the words of Sapountzaki (2007), "*vulnerability is a state, while resilience is a process*"; ~~instead,~~ in fact the definition of resilience implies a time evolution of the characteristics of the whole system. In addition, Lhomme et al. (2013) underline "*the need to move beyond reductionist approaches, trying, instead, to understand the behaviour of a system as a whole*". These two features, dynamic aspect and whole system, make vulnerability different from resilience and ~~This~~ this difference can be expressed by a cinematography analogy: vulnerability is a single frame of the resilience video.

In this context, the study of the percolation threshold (p_c) can be used to explain the resilience of the network after a perturbation. The p_c value distinguishes between the connectivity phase (above p_c) and the fragmented phase (below p_c). In the connectivity phase, the network can lose nodes without losing the capacity to cope with the perturbation as a network, while in the fragmented phase, the network does not actually exist anymore and the remaining nodes are unable to cope with the disruption alone.

This critical behaviour is a common feature also observed in natural disasters. In some cases, the exposed elements withstand some damage and loss, but the overall system maintains its structure. However, there are events in which the amount of loss (affected nodes) is so relevant that the system loses the overall network structure. In the first case, the system has the capacity to cope independently and tackle the event, while in the second case, the system is unable to cope. In the case of an earthquake, for example, if a large number of exposed elements are damaged, it is common for affected regions to be unable to cope with the situation and require help from outside their borders (i.e. the graph needs new nodes). Asking for support from outside would increase the extension of the area (graph) and offers new resources (nodes), therefore decreasing the percolation threshold value increasing connectivity. In this perspective, the percolation threshold should define when a specific area is not self-sufficient to overcome that level of loss and requires help from outside.

The dynamic responses are characterized by the network fragmentation property, which describes the performance of a network when its components are removed (Dueñas-Osorio and Vemuru, 2009). For instance, the so-called giant component fragmentation (the largest connected sub-network) and the total fragmentation describe network connectivity and determine the failure mechanism (Dueñas-Osorio et al., 2004). Keeping track of fragmentation evolution makes it possible to determine both the critical fraction of components removed (i.e. the smallest component deletion that disconnects the network), and the effect that each component removed has on the network response.

For these reasons, we consider percolation threshold and network fragmentation a good indicators of resilience, also because it is able to show the emergent behaviour of the whole system beyond just considering the single parts of the network (e.g. node).

3.2. Illustrative example

In order to illustrate the application of Graph Theory in the characterization of a system exposed to natural hazards, in Figure 3~~Figure 3~~ we present an example of a hypothetical city comprising various elements of different types which provide services among them. In specific, our example includes 20 elements: 8 Blocks of residential buildings, 1 Hospital, 2 Fire Stations, 3 Schools, 3 Fuel Stations and 2 Bridges. We assume that these elements are located in a flood-prone area. Blocks are intended to represent the population, which receives services from the other nodes. Bridges provide a transportation service, Fire Stations provide a recovery service, Hospitals provide a healthcare service, Schools provide an education service, and Fuel Stations provide a power service. Figure 3~~Figure 3~~(a) shows how the elements are connected into a graph. The authority and hub values adopted in this illustrative example have been computed using the R igraph package. The full library of functions adopted are available in Nepusz and Csard (2018).

In Figure 3~~Figure 3~~(b), the size of the elements is proportional to their authority values. Blocks 6, 18, 19 and 20 have higher authority values than the other elements of this typology because they receive a service from the Hospital (node 16), which is an important hub. Fire Station 5 and School 9 have high values of authority because they are serviced by Bridge 3, which is also an important hub. The importance of a node in Graph Theory is closely connected with the concept of topological centrality. Referring to the illustrative example, Block 6 has the highest authority value; if a flood hit it, it would therefore affect the most central node of the network, or in other words, the node which is implicitly more privileged by the system.

In Figure 3~~Figure 3~~(c), the major hubs are the elements with largest diameters: Hospital 16, Bridge 3, School 7 and Fuel Station 15. Bridge 3 is an important hub since it provides its service to Block 6, which has the highest authority value, and to Fire Station 5 and School 9. Fuel Station 15 and School 7 are also important hubs because they provide services to Block 6. The elements in the south-east part of the network inherited a relative importance (i.e. authority) from the most important hub in that area (i.e. Hospital 16). Bridge 3 is an exception to this aspect; in fact, this Bridge connects the south part (i.e. Block 6) with the north part of the city (i.e. Fire Station 5 and School 9). A flood event in the south-east part of the network would likely generate a major indirect impact on the whole system compared to other parts of the network.

This illustrative example shows how the single elements can be considered as part of the whole network and not as single separate entities. This holistic approach adds information to the traditional approach: it considers the exposed asset as whole system and it exploits the properties of single elements in order to make decisions for risk mitigation strategies.

Note that similar analyses could be carried out for other properties of the graph (e.g. betweenness) in order to obtain additional insight into the properties of the system, which could be useful for the purpose of a risk assessment. For the sake of brevity, such analyses have not been included here. A complete study of all relevant graph properties discussed above for a selected case study are presented in Part II of this manuscript.

4. Discussion

The proposed approach ~~may~~ can bring ~~some~~ important advantages to collective risk assessment: it provides a systemic and holistic perspective, it is suitable for multi-hazard assessment, it introduces a common base for an integrated risk assessment, and it promotes the study of second-order impacts and cascade effects.

The new holistic perspective introduces an important paradigm shift in the risk conceptualization: the most widely accepted risk concepts of hazard, vulnerability, exposure and resilience do not lose their validity but are integrated into a systemic perspective rather than considered separately. The properties of the whole graph show the studied area as a unique entity, and how the whole system together is vulnerable to an external perturbation, such as a hazardous event that can affect part of it. Beside this whole system perspective, it is also possible to assess the properties of the single parts of the graph (e.g. nodes) and detect which element, or set of elements, are more critical for the entire system.~~The analogy between graph properties and traditional risk variables (exposure, vulnerability, and resilience) provides useful information that can enrich traditional reductionist approaches to RA. These consider collective risk as the combination of each individual element at risk, without accounting for the interaction or interdependencies between them.~~

Thise new approach~~ based on Graph Theory~~, through which a system can be modelled as a graph, and the analysis of its properties within the RA framework, provides a systemic and holistic perspective that is missing in the traditional RA. The adoption of analogies as proposed in this methodology is supported by the recent work published by (Clark-Ginsberg et al., 2018). Despite having a different scope, it also uses certain graph properties to analyse the 15 main hazards of the companies operating in Khorasan Razavi Province, promoting a network representation of the risk. This perspective, innovative in the context of collective risk assessment, uses ~~both~~ the information contained both in the vertices and in the whole network.

Despite the improvements in risk assessment within this systems perspective, Clark-Ginsberg *et al.* (2018) highlights that there are "*questions about the validity of such assessment*" regarding the ontological foundations of networked risk, the non-linearity and emergent phenomena that characterize system phenomena. The emergence of the risk system demonstrates that the risk will never be completely knowable, and for this reason the "*unknown unknowns are an inseparable part of a risk networks*"; in fact, the boundary definition of open systems are by nature artificial.

The proposed approach is suitable for multi-hazard assessment, as the graph properties of the system are independent of the type of hazard to be analysed. Moreover, these properties can be easily integrated with the properties of the single node ~~, already well~~ estimated by ~~the~~ reductionist approaches ~~(e.g.~~, such as the physical vulnerability of a building with respect to earthquake or flooding~~)~~.

The graph-based approach also introduces a common base for integrated risk assessment in terms of different features, not only in relation to the physical component of the elements but also with regard to social aspects that express the capacity of the system to respond to perturbations. Therefore, the use of Graph Theory in this field can be applied to physical, as well as social or integrated risk. In the first case, the analysis can focus on physical aspects if the graph has physical elements (e.g. buildings). The second case focuses on social aspects if the nodes represent the population with characteristics that reflect different types of vulnerability (e.g. age, education).

Lastly, ~~the new holistic perspective introduces an important paradigm shift in the risk conceptualization: the most widely accepted risk concepts of hazard, vulnerability, exposure and resilience do not lose their validity but are integrated into a systemic perspective rather than considered separately. The graph shows properties relevant for all factors together: in fact, in the graph it is possible to study some exposure characteristics, some nodes bring information on potential cascading effects, and others convey vulnerability information of the system as a whole. The properties of the whole graph show the studied area as a unique entity, and how the whole system together is vulnerable to an external perturbation, such as a hazardous event that can affect part of it. Beside this whole system perspective, it is also possible to assess the properties of the single parts of the graph (e.g. nodes) and detect which element, or set of elements, are more critical for the entire system. One of the most important features and consequences promoted by this approach is the possibility to investigate cascade effects. Indeed, t~~The intrinsic network perspective can be applied to model cascade effects~~, showing~~ and the dynamic consequences of disruptive events. The links between nodes allow ~~to pass~~passing from the direct physical damage to ~~a~~ broader economic and social indirect impacts. ~~Depending~~ Based on the type of disruptive event and how the network is setup in terms of nodes and links, the spread of the impacts ~~can spread~~ throughout the network can be assessed. Furthermore, during the evolution of such cascade effects, it is ~~interesting~~ possible to analyse the structural evolution of the network, the main properties of which are emphasised above, and study how those measurements change during the propagation of loss. If a cascade process needs few or high number of propagation steps to reach most of the nodes, this shows a lower or higher capacity to cope and adapt to the perturbation, and therefore to be less or more resilient.

5. Final Considerations

This paper proposes a new approach to model the risk of complex systems based on Graph Theory. By leveraging certain analogies that can be established between graph properties and risk concepts, this approach allows obtaining a more thorough knowledge of a system compared to traditional approaches, in terms, for example, of its structure and vulnerabilities. It also allows understanding certain risk mechanisms, such as how the impacts of a hazard are propagated or where they are exacerbated, and therefore assessing the disaster risk of the system as a whole, including second-order impacts and cascade effects.

The natural continuation of this study, which focuses mainly on theoretical aspects, is to implement and test the approach in case studies, verifying its feasibility. Therefore, part II of this paper presents an application to the case of urban flooding in Mexico City. Further research will aim to fully implement and integrate the graph-based approach in quantitative risk assessments, both at scenario and probabilistic level.

A possible extension of this framework is to model the physical hazard as one or more nodes linked to the elements at risk, rather than through a traditional approach where elements are overlapped with hazard footprints. This approach may be advantageous, as it would allow including all the factors of risk directly into the topology structure of the graph, and will be explored in further research.

Regarding the cascading effects, it is worth noting that the concept of indirect impact needs to be expanded and more explored. For example, the indirect impact due to a hazardous event suffered by a certain node may be defined as a function of the direct damage sustained by one or more of its parent nodes (i.e. traditional impact), and of the type of service the latter provide to the former. This could be given by a vulnerability function defining the consequences of such a cascade effect. The integration of indirect impact quantification within the graph-based framework will be addressed in future research.

Moving forward, one of the challenges that will need to be addressed is related with data requirements and availability. Currently, most exposure and vulnerability databases focus on the properties of single elements, and tend to contain little to no information on the connections between them. As we have discussed, this information is key for more thoroughly understanding and assessing the risk of a system. For this reason, developing and collecting data with information related to the connections between the elements is paramount. To promote this perspective, it is necessary consider shifting the RA from using traditional relational databases to so-called graph databases. In such databases, each node contains, further to the traditional characteristics, also a list of relationship records which represent its connections with other nodes. The information on these links is organized by type and direction, and may hold additional attributes.

The introduction of the network perspective of Graph Theory into the RA for collective disaster risk aims, in the long term, to be a first step for future developments of Agent Based Models and Complex Adaptive Systems in collective risk assessment. In this perspective, the nodes of the network are agents, with defined state (e.g. level of damage), and the interaction between the other agents is controlled by specific rules (e.g. vulnerability and functional functions) inside the environment within they live (e.g. natural hazard phenomena).

Acknowledge

This research was partly funded by Fondazione Cariplo under the project “NEWFRAME: NEtWork-based Flood Risk Assessment and Management of Emergencies"

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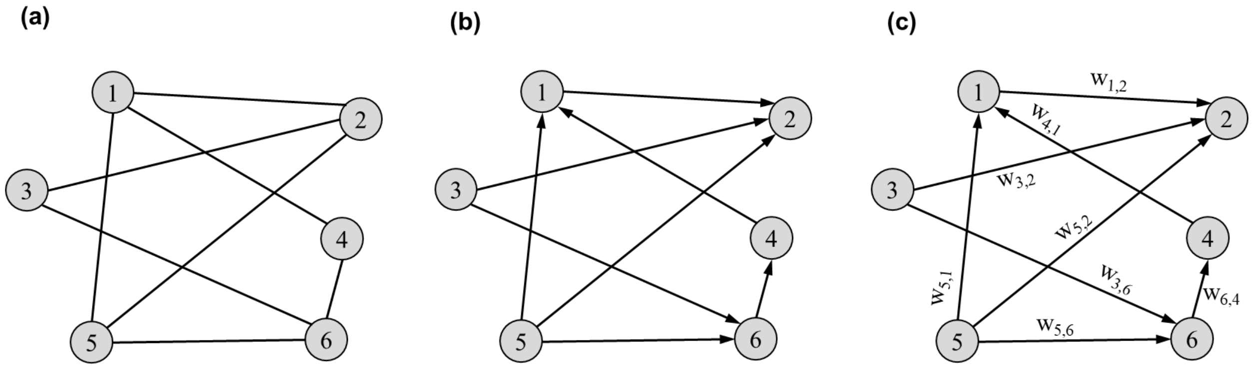
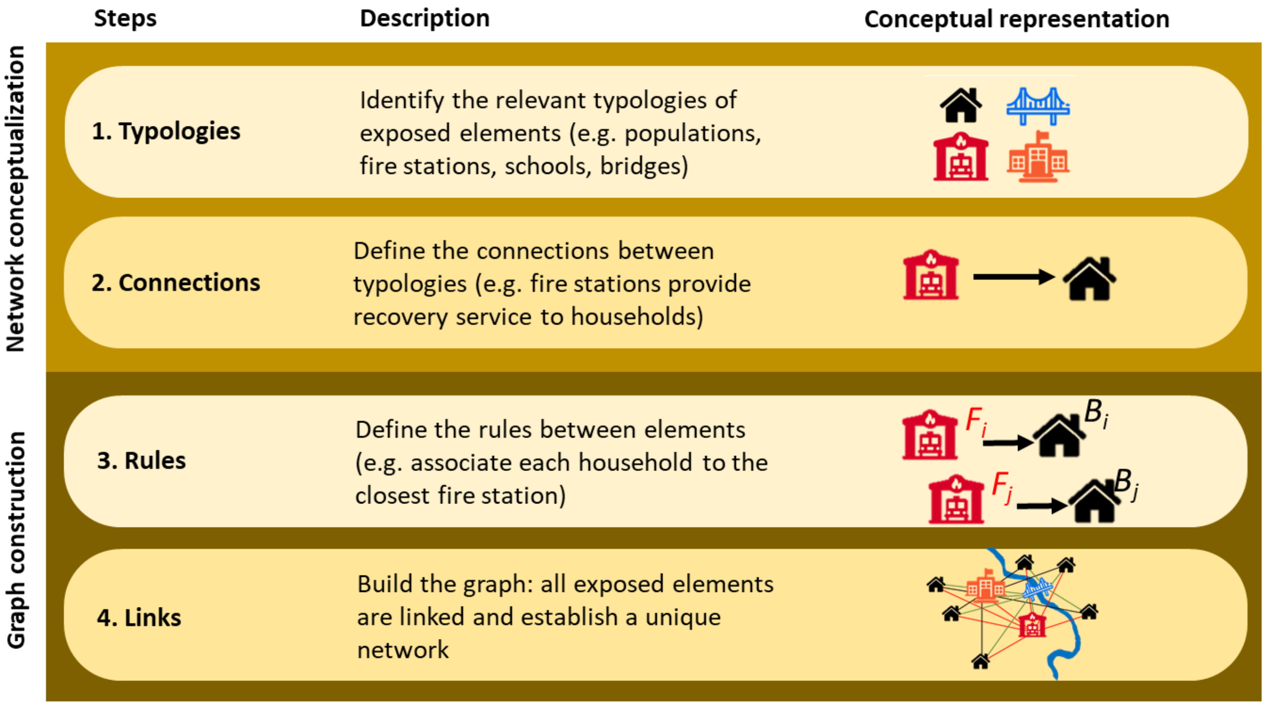


Figure 1: Graph representation of a network. (a) Undirected. (b) Directed. (c) Weighted directed.



	Steps	Description	Conceptual representation
Network conceptualization	**1. Typologies**	Identify the relevant typologies of exposed elements (e.g. populations, fire stations, schools, bridges)	
	2. Connections	Define the connections between typologies (e.g. fire stations provide recovery service to households)	
Graph construction	**3. Rules**	Define the rules between elements (e.g. associate each household to the closest fire station)	F_i B_i F_j B_j
	4. Links	Build the graph: all exposed elements are linked and establish a unique network	

Figure 2: Workflow.

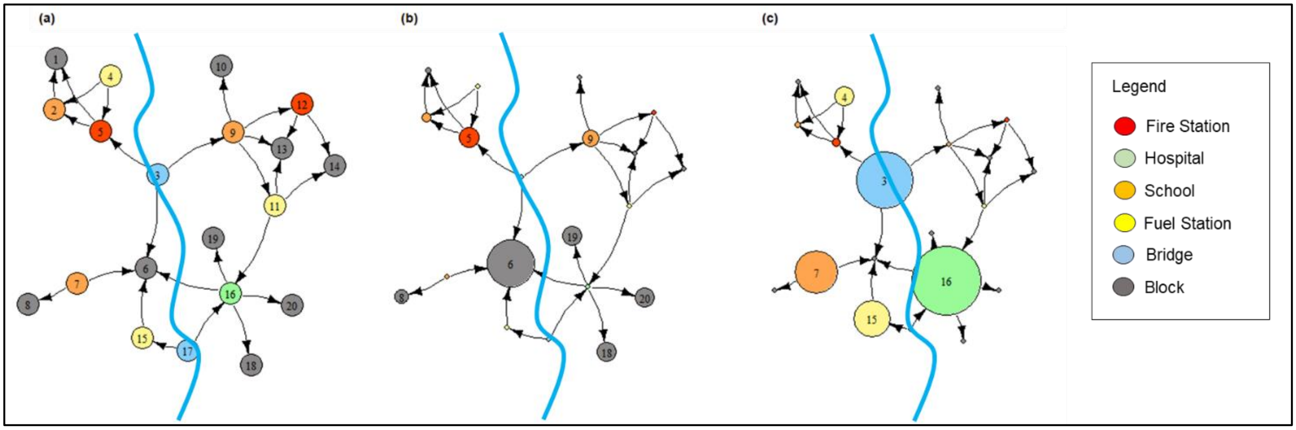


Figure 3: (a) Map of the various elements of a hypothetical municipality in a flood-prone area; (b) Same, with node sizes proportional to authority values; (c) Same, with node sizes proportional to hub values.

Table 1: Graph properties description

Graph properties	Description
Degree (k)	The number of edges incident with the node
Path length	The geodesic length from node i to node j
Closeness	The distance (number of links) of a node to all others
Betweenness	The shortest paths between pairs of nodes that pass through a given node
Authority	Value of a node proportional to the sum of the node hubs pointing to it
Hub	Value of a node proportional to the sum of authority of nodes pointing to it
Percolation threshold (pc)	The minimum value of fraction of remaining nodes (p) that leads to the connectivity phase of the graph

Table 2~~1~~: Analogy of risk variables with graph properties.

Risk variables	Analogy with graph properties
Exposure	The authority represents how the system privileges the nodes, conferring them more or less importance compared with others, according to the connections established in the system.
Vulnerability	The propensity of parts of the network to be isolated because of hazard events. The closeness of a node is a measure of the single node vulnerability within the system, while degree distribution, hub, and betweenness are measures of vulnerability of the system as a whole.
Resilience	The percolation threshold, together with the network fragmentation analysis, explain the resilience of the network after a perturbation.