THE UNIVERSITY OF HONG KONG

January 12, 2019

The Editorial Board
Natural Hazards and Earth System Sciences

Dear Editors of Natural Hazards and Earth System Sciences,

We are pleased to submit the revised version of the manuscript, **nhess-2018-238**, by Kwok Wing Chow, Hiu Ning Chan and Roger H. J. Grimshaw. We have changed the title of the paper as part of the response to the requests of one referee. All modifications implemented are highlighted in BLUE and underlined in the revised manuscript. A detailed point-by-point response to the concerns raised by the referees is documented in the following pages. We wish to thank the referees for their valuable opinions and also the Editorial Office for handling the manuscript.

Please feel free to contact us if you need further information. Thank you.

Yours sincerely,

KWC

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nhess-2018-238
Modified title: Modulation instability as a generation mechanism for internal oceanic rogue waves: A modelling and computational study
by Kwok Wing Chow, Hiu Ning Chan and Roger H. J. Grimshaw.

Reply to Referee 1:

We wish to thank Referee 1 for his insightful and supportive comments. We respond in detail as follows.

(1) '*…it would be reasonable to give an example of rogue wave characteristics in numbers using formula (1), for instance, characteristic lengths of carrier and envelope waves in 100m-depth basin…*'

Response: Thank you for pointing out the need to verify the actual numerical orders of magnitude. For a basin depth (h) of say 500m, the critical wavelength (λ_c), wavenumber (k_c) and internal wave mode (n), the formulation in the text gives

$$\lambda_c = \frac{2\,\pi}{k_c} = \frac{2h}{n(4^{1/3}-1)^{1/2}}$$

and with h = 500m, we can construct the following table:

n (internal mode number)	Rogue waves and instability can occur for wavelengths longer than λ_c given by (in meters)
1	1305
2	652
3	435
4	326
5	261

Hence ranges of 'shallow' and 'intermediate' depths are covered. This information has been added in page 2 (after Equation (1)) of the revised text. (Note: we change the suggested depth to 500m, to get a better approximation for the oceanic situation.)

(2) '*…rogue waves now occur for the shallow water regime…, but this conclusion has been made earlier in the paper*…2010, *and previous papers*…2011…'

Response: Thank you for reminding us of these relevant works on the long wave (shallow water) regime. However, there is a subtle difference between the two approaches. In the previous work by one of the authors (RHJG) in 2010, the starting point was a long wave model, the extended Korteweg-de Vries equation. There is thus an assumption of long waves in the basic carrier wave envelope. In contrast, the Taylor-Goldstein equation for linear modes is utilized in the present approach, and hence the fast oscillations inside the carrier envelope need not be in the long wave regime. We have enhanced the connection to this body of works in the literature by incorporating these new references:

Didenkulova, I. and Pelinovsky, E.: Rogue wave in nonlinear hyperbolic systems (shallow-water framework), Nonlinearity, 24, R1-R18, 2011.

Pelinovsky, E., Talipova, T., and Kharif, Ch.: Nonlinear-dispersive mechanism of the freak wave formation in shallow water, Physica D, 147, 83-94, 2000.

Talipova, T. G., Pelinovsky, E. N., and Kharif, Ch.: Modulation instability of long internal waves with moderate amplitudes in a stratified horizontally inhomogeneous ocean, JETP Letters, 94, 182-186, 2011.

'*…the author's criterion should include the positivity of cubic nonlinear term in (the) Gardner (equation) as a particular case. Is it correct?*'

Yes, if we take the small wavenumber regime for the Taylor-Goldstein equation, then we can recover the Korteweg-de Vries and Gardner equations. However, such an asymptotic calculation will take us way beyond the 4-page limit of a 'brief communication' paper, and thus only a brief remark is made at the end of Section 2.

(3) '*…important result is that the modulation instability can occur not only in shallow water,…but also in the intermediate depth basin*.' (underline = our re-phrasing)

Response: Yes, that is exactly one of our messages in writing this paper and we will emphasize this point, both in the Abstract (line 4 of that paragraph) as well as Section 4 Discussions and Conclusions (line 4 of that section).

(4) '*…the following papers…should be cited*.'

Response: Thank you. The Physica D 2000 and Nonlinearity 2011 papers have been included in the References.

nhess-2018-238
Modified title: Modulation instability as a generation mechanism for internal oceanic rogue waves: A modelling and computational study
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Reply to Referee 2:

We thank the Referee for the constructive comments, and also for the assertion that the present work can be potentially an important contribution to ocean science.

(1) '*Title is misleading… occurrence of rogue waves, which makes me think that formation of internal rogue waves is discussed within a proper statistical framework…*'

Response: There are several widely cited review articles on rogue waves where various approaches of investigations are presented, including both deterministic and stochastic models. An example is 'Oceanic rogue waves' by K. Dysthe, H. E. Krogstad, P. Müller, *Annual Reviews of Fluid Mechanics* **40**, 287 (2008). From the various mechanisms discussed, and to avoid the possible confusion associated with the word 'occurrence', the terminologies of 'nonlinear focusing' and 'modulation instability' are perhaps more appropriate for our paper. Hence we suggest a possible change to a new title 'Modulation instability as a generation mechanism for internal oceanic rogue waves: A modelling and computational study'.

(2) '*…There is an extensive literature discussing generation of internal rogue waves, but this is not discussed in details in the present manuscript. I am thinking, for example, to Grimshaw, R., Pelinovsky, E., Stepanyants, Y. and Talipova, T., 2006. Modelling internal solitary waves on the Australian North West Shelf. Marine and Freshwater Research, 57(3), pp.265-272; and Chapter 25 of Osborne, A.R., 2002. Nonlinear Ocean Wave and the Inverse Scattering Transform. In Scattering (pp. 637-666), and reference therein. To justify a rapid communication, more effort should be put to highlight the original contribution of the present manuscript…*'

Response: There is indeed an extensive literature on large amplitude oceanic internal waves. In particular, the two references quoted and many other related works are mainly on the topic of 'internal solitary waves'. These are spatially localized pulses propagating essentially without change of form, but they are **not** localized in time. In this paper we consider simple analytical description of a wave pulse localized in ***both*** space and time. In widely used phrase in this field, rogue waves are 'waves that appear from nowhere and disappear without a trace'. We emphasize on this difference in page 2 of the revised manuscript (7 lines above Equation (1)), and nevertheless have included some relevant references.

(3) '…*The authors mention that "classical" modulation instability would cease at kh<1.36. However, there is evidence that instability can survive for shallower relative depth if the wave field is sufficiently directional (Toffoli, A., Fernandez, L., Monbaliu, J., Benoit, M., Gagnaire-Renou, E., Lefevre, J.M., Cavaleri, L., Proment, D., Pakozdi, C., Stansberg, C.T., Waseda, T., Onorato, M., 2013. Experimental evidence of the modulation of a plane wave to oblique perturbations and generation of rogue waves in finite water depth. Phys. Fluids, 25, 09170). Also, effects of current have been discussed in detail in Onorato M., Proment D., Toffoli A., 2011. Triggering rogue waves in opposing currents. Phys. Rev. Lett.,107, 184502, doi: 10.1103/PhysRevLett.107.184502; and Toffoli, A., Waseda, T., Houtani, H., Kinoshita, T., Collins, K., Proment, D., Onorato, M., 2013. Excitation of rogue waves in a variable medium: An experimental study on the interaction of water waves and currents. Phys. Rev. E, 87, 051201(R), before Liao et al 2017…*'

Response: Thank you for these references, where the well-known constraint of $kh > 1.363$ was extended to lower numerical values. However, it is not clear (at least to us) how far can these numerical values go. In contrast,

► we are studying internal waves as opposed to surface waves, and

► our proposed constraint is very well defined, i.e. $kh < 0.766n\pi$. The limit of k or h tending to zero is explicitly included.

The effects of ocean / shear currents will be taken up in future studies. Experimental verification will be beyond the scope of the present study. Nevertheless we have made relevants remarks, mentioned all three papers in Section 4 and included them in the References section.

(4) '…*The theoretical framework, especially the NLS equation, seems to be already published. Nevertheless, the title mentions modelling study. What is the novel model the authors are proposing?…*'

Response: The word 'modelling' is used here as opposed to numerical simulations or field data comparison. When the paper by Liu and Benney (*Studies in Applied Mathematics* 1981) was published, the focus then was internal solitary wave. Our proposed contributions are:

(a) This formulation as applied to the setting of internal rogue waves will provide a nonlinear focusing mechanism in the long internal waves (shallow water) regime, as opposed to the usual deep water scenario for surface waves.

(b) Numerical simulations from random and specially prescribed initial conditions, a practice frequently implemented only in the past ten years, is pertinent for internal wave investigations.

(5) '…Section 3, *Computational Simulations, is may major concern. It should be the core of the manuscript and yet it is reduced to 7 lines. This section does not convene a message at all and needs to be re-written and expanded…*' (our guess: *my major concern*?)

Response: Please see point (6) below for a full explanation.

(6) We provide a response to each query individually. As an overview, the primary intention of this 'brief communication' is to demonstrate that unexpectedly large displacements (rogue waves) may occur in internal waves too. Indeed they can occur in the shallow water regime, in sharp contrast to the surface wave scenarios. Numerical presentations were condensed in the initial submission due to the 4-page limit. We have substantiated the contents in this revised version, and we can expand this part further if necessary, subject to editorial advice.

'…*What simulations did the author carried out?...*'
Response: We conduct simulations with specially selected initial conditions to determine how rogue-wave-like structures can emerge. More precisely, we choose a mode with an optimal modulation instability growth rate.

'…*What are the initial conditions? Are regular or irregular waves considered?...*'
Response: Specially selected conditions mean choosing a modulation instability mode with the optimized (or maximum) growth rate. Hence we can roughly classify them as 'regular waves'. Numerical simulations for the nonlinear Schrödinger equation with random initial conditions had been conducted earlier in the literature (Akhmediev et al., *Physical Review E* 2009, cited in the manuscript).

'…*What are the values of key parameters? etc…*'
Response: For surface rogue waves described by the nonlinear Schrödinger equation, the key parameters are k, the wave number of the carrier wave envelope and h, the water depth. For the present wave packet dynamics in a stratified flow model, two

additional parameters are N_0, the constant buoyancy frequency of the background stratification and n, the mode number of the internal wave.

'…*It also seems that no sensitivity analysis has been done and only one specific "lucky"case is discussed*…'
Response: Standard quality control processes were routinely performed for similar simulations in our papers in the past. Our present results, analogous to those of other research groups (e.g. Baronio et al, *Physical Review A* 2015), are that rogue-wave-like structures will emerge, and this is ***not*** a 'lucky' result. The goal of this portion of the paper is to convince the reader that such dynamics also holds true for internal wave scenarios too. We have expanded Section 3 by providing highlights of the computational schemes and can substantiate with further details, depending on editorial advice on the classification as a 'brief communication' versus 'full paper'.

'…*What is the effect of wave steepness? What is the threshold of relative water depth below which internal rogue waves do not occur? what is the effect of density gradient?...*'
Response: The wave steepness must scale with the small parameter describing the long modulation scale as given in any standard derivation of the nonlinear Schrödinger equation (e.g. the paper by Liu and Benney, *Studies in Applied Mathematics*, 1981, amongst many others). The threshold of relative water depth for ***internal rogue waves to occur*** is $kh < 0.766n\pi$, four lines below Equation (7) of the text (strong contrast with $kh > 1.363$ of surface waves – this constitutes the theme of the paper). This new constraint means that internal rogue waves can thus occur for small h (or shallow water regime). The density gradient, or more precisely, the buoyancy frequency parameter N_0, will affect the horizontal length scale of the rogue wave and a precise description will constitute one of the long term objectives of this study.

'…*None of these points are discussed, leaving the reader completely unaware of the number computations. In addition, I am not sure to understand Figure 1. Or better, I can guess what it is and its meaning, but the authors did not put any effort to describe it*…'
Response: Again we wish to emphasize that we are constrained by the 4-page limit of a 'brief communication' in the initial submission. To address a relatively broad audience, we have described the dynamics of the nonlinear Schrödinger equation in the first half of the paper. We have included descriptions of numerical schemes in Section 3 now, and can elucidate the numerical details in a full paper if necessary. The caption of Figure 1 has been expanded to 6 lines, hopefully the science is more comprehensible now.

(7) ‘…*Throughout the paper and in the title, it is mentioned that likelihood of occurrence of rogue waves is assessed. However, I do not see any discussion of a proper statistical framework that can justify new results on the probability of occurrence of internal rogue waves*…’

Response: As discussed earlier, it is beyond the scope of this paper to carry out a statistical assessment. To avoid possible confusion with phrases like ‘likelihood’ or ‘occurrence’, we shall adopt the words ‘modulation instability’ in the modified title and also discussions in Section 4.

(8) Final paragraph:
‘…*section 3 has to be significantly redeveloped and more details provided to support results*…’

Response: Again the motivation of writing this ‘brief communication’ is to show this rather unexpected parameter regime for the modulation instability of internal rogue waves. Due to the 4-page limit on a ‘brief communication’ in the initial submission, we have of necessity condensed the numerical treatment. We beef up the simulation portions already and can further expand on those treatments, subject to editorial approval.

‘…*If this is done properly, this manuscript has the potential to become a significant contribution to ocean science*…’

Response: Thank you for providing a very positive opinion on our work.

Brief communication: Modulation instability as a generation mechanism for internal oceanic rogue waves: A modelling and computational study

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Abstract. Unexpectedly large displacements in the interior of the oceans are studied through the dynamics of packets of internal waves, where the evolution is governed by the nonlinear Schrödinger equation. The case of constant buoyancy frequency permits analytical treatment. While modulation instability for surface wave packets only arises for sufficiently deep water, 'rogue' internal waves may occur for the shallow water and intermediate depth regimes. The dependence on the stratification parameter and choice of internal modes can be demonstrated explicitly. The spontaneous generation of rogue waves is tested by numerical simulations.

1 Introduction

Rogue waves are unexpectedly large displacements from equilibrium positions or otherwise tranquil configurations. Oceanic rogue waves obviously pose immense risk to marine vessels and offshore structures (Dysthe, et al., 2008). After these waves were observed in optical waveguides, studies of such extreme and rare events have been actively pursued in many fields of science and engineering (Onorato et al., 2013). Within the realm of oceanic hydrodynamics, observation of rogue waves in coastal regions has been recorded (Nikolkina and Didenkulova, 2011; O'Brien et al., 2018).

Theoretically the propagation of weakly nonlinear, weakly dispersive narrow-band wave packets is governed by the nonlinear Schrödinger equation, where the dynamics is dictated by the competing effects of second order dispersion and cubic nonlinearity (Zakharov, 1968; Ablowitz and Segur, 1979). Modulation instability of plane waves and rogue waves can then occur only if dispersion and cubic nonlinearity are of the same sign. For surface wave packets on a fluid of finite depth, rogue modes can emerge for $kh > 1.363$ where k is the wavenumber of the carrier wave packet and h is the water depth. Hence conventional understanding is that such rogue waves can only occur if the water depth is sufficiently large.

Other fluid physics phenomena have also been considered, such as the effects of rotation (Whitfield and Johnson, 2015) or the presence of shear current or an opposing current (Onorato et al., 2011; Toffoli et al., 2013a; Liao et al., 2017) or oblique perturbations (Toffoli et al., 2013b). While such considerations may change the numerical value of the threshold (1.363) and extend the instability region, the requirement of water of sufficiently large depth is probably unaffected. For wave packets of large wavelengths, dynamical models associated with the shallow water regime have been employed (Didenkulova and Pelinovsky, 2011, 2016), such as the well-known Korteweg-de Vries and Kadomtsev-Petviashvili types of equations (Grimshaw et al., 2010, 2015; Pelinovsky et al., 2000; Talipova et al., 2011), which may also lead to modulation instability under several special circumstances.

The goal here is to establish another class of rogue wave occurrence through the effects of density stratification, namely, internal waves in the interior of the oceans. There is an extensive literature on large amplitude internal solitary waves which are spatially localized pulses propagating essentially without change of form (Grimshaw et al., 2004; Osborne, 2010). Our focus here is on internal rogue wave which is modelled as a wave pulse localized in ***both*** space and time. The asymptotic multiple scale expansions for internal wave packets under the Boussinesq approximation also yield the nonlinear Schrödinger equation (Grimshaw, 1977, 1981; Liu and Benney, 1981). When the buoyancy frequency is constant, modulation instability in one horizontal space dimension will only occur for $kh < k_c h = 0.766 n\pi$ where the fluid is confined between rigid walls distance h apart, n is the vertical mode number of the internal wave, and the critical wave number k_c given by (Liu et al., 2018):

$$k_c = \frac{n\pi}{h}\left(4^{1/3} - 1\right)^{1/2} . \tag{1}$$

For a basin depth (h) of say 500m, the critical wavelength (λ_c) is

$$\lambda_c = \frac{2\,\pi}{k_c} = \frac{2h}{n(4^{1/3} - 1)^{1/2}}$$

and ranges of 'shallow' and 'intermediate' depths are covered (Table 1):

n (internal mode number)	Rogue waves and instability can occur for wavelengths longer than λ_c given by (in meters)
1	1305
2	652
3	435
4	326
5	261

Table 1: Critical wavelength λ_c as a function of the internal mode number n (with h = 500 m).

The important point is not just a difference in the numerical value of the cutoff, but rogue waves now occur for water depth ***less*** than a certain threshold. Our contribution is to extend this result. The nonlinear focusing mechanism of internal rogue waves is: (i) determined by estimation of the growth rate of modulation instability, and (ii) elucidated by a numerical simulation of emergence of rogue modes with the optimal modulation instability growth rate as the initial condition.

2 Formulation

2.1 Nonlinear Schrödinger theory for stratified shear flows

The dynamics of small amplitude (linear) waves in a stratified shear flow with the Boussinesq approximation is governed by the Taylor-Goldstein equation ($\phi(y)$ = vertical structure, k = wavenumber, c = phase speed, $U(y)$ = shear current):

$$\phi_{yy} - \left(k^2 + \frac{U_{yy}}{U-c}\right)\phi + \frac{N^2\phi}{(U-c)^2} = 0\ , \tag{2}$$

where N is the Brunt-Väisälä frequency or more simply 'buoyancy frequency' ($\bar{\rho}$ is the background density profile):

$$N^2 = -\frac{g}{\bar{\rho}}\frac{d\bar{\rho}}{dy}\ . \tag{3}$$

The evolution of weakly nonlinear, weakly dispersive wave packets is described by the nonlinear Schrödinger equation for the complex-valued wave envelope S, obtained through a multi-scale asymptotic expansion, which involves calculating the induced mean flow and second harmonic (β, γ being parameters determined from the density and current profiles):

$$iS_\tau - \beta S_{\xi\xi} - \gamma|S|^2 S = 0\ ,\ \ \tau = \varepsilon^2 t\ ,\ \ \ \xi = \varepsilon\left(x - c_g t\right) \tag{4}$$

where τ is the slow time scale, ξ is the group velocity (c_g) coordinate and ε is a small amplitude parameter.

2.2 Constant buoyancy frequency

For the simple case of **constant** buoyancy frequency N_0, the formulations simplify considerably in the absence of shear flow ($U(y) = 0$). The linear theory Eq. (2) yields simple solutions for the mode number n:

$$N = N_0\ ,\ \ \ \phi = \sin\left(\frac{n\pi y}{h}\right), \tag{5}$$

with the dispersion relation, phase velocity (c) and group velocity (c_g) given by

$$\omega^2 = \frac{k^2 N_0^2}{\dfrac{n^2\pi^2}{h^2} + k^2}\ , \qquad c = \frac{\omega}{k}, \qquad c_g = \frac{d\omega}{dk}\ , \qquad c_g = \frac{c}{1 + \dfrac{k^2 h^2}{n^2\pi^2}}\ . \tag{6}$$

The subsequent nonlinear analysis yields the coefficients of the nonlinear Schrödinger equation in explicit forms:

$$\beta = \frac{3n^2\pi^2 c^2}{2h^2 k N_0^2}\left(c - c_g\right), \qquad \gamma = -\frac{6N_0^2 k c_g^3\left(c - c_g\right)}{c^4\left(c^3 - 4c_g^3\right)}\ . \tag{7}$$

A plane wave solution for Eq. (4) (or physically a continuous wave background of amplitude A_0) is

$$S = A_0 \exp[-i\gamma A_0^2 \tau]\ . \tag{8}$$

Small disturbances with modal dependence $\exp[i(r\xi - \Omega\tau)]$ will exhibit modulation instability if

(a) $\Omega^2 = \beta r^2(\beta r^2 - 2\gamma A_0^2)$ is negative, i.e. for $\beta\gamma > 0$; calculations using Eqs. (6, 7) lead to $kh < k_c h = 0.766n\pi$ (Eq. (1));

(b) the maximum growth rate is (imaginary part of Ω) $= \Omega_i = |\gamma|A_0^2$ for a special wavenumber given by $\beta^{1/2}r = \gamma^{1/2}A_0$;

(c) the growth rate for long wavelength disturbance is $|\Omega_i/r| = (2\beta\gamma)^{1/2}A_0$ for $r \to 0$.

In terms of significance in oceanography, the constraint $kh < k_c h = 0.766n\pi$ does *not* depend on the constant buoyancy frequency N_0. However, it *does* depend on the mode number (n) of the internal wave, with the higher order modes permitting a large range of carrier envelope wavenumber and fluid depth for rogue waves to occur. An analysis in the long wave regime of this Taylor-Goldstein formulation would in principle recover the previous results related to the Korteweg-de Vries and Gardner equations, and details will be reported in the future.

3. Computational Simulations

An intensively debated issue in the studies of rogue waves through a deterministic approach is the proper initial condition which may generate or favour the occurrence of such large amplitude disturbances. One suggestion is the role played by long wavelength modes associated with modulation instability, or 'baseband instability' (Baronio et al., 2015). To highlight this effect and to clarify the role of stratification as well as the choice of internal wave modes, numerical simulations are performed where modes with the proper modulation instability growth rate on a plane wave background and say 5% amplitude are selected as the initial condition (Chan and Chow, 2017; Chan et al., 2018):

$S(\xi,0) = [1 + 0.05\exp(ir\xi)]A_0$,

(A_0 = the amplitude parameter defined by Eq. (8) and r = wavenumber of the optimal mode).

A pseudospectral method with a fourth-order Runge-Kutta scheme for marching forward in time is applied. When the wavenumber r of the disturbance is small, corresponding to a baseband mode, rogue wave can be generated from the plane wave background (Figure 1). Physically this spontaneous growth of disturbance due to modulation instability is closely associated with the 'focusing' of energy and thus the formation of rogue waves.

The growth rate of the baseband mode is a crucial factor of rogue wave generation. A stronger baseband growth rate will trigger a rogue wave within a shorter period of time. From Eqs. (6) and (7), the baseband growth rate $(2\beta\gamma)^{1/2}A_0$ increases as the depth h or wavenumber k increases (Figure 2), but this growth rate decreases as the mode number n increases. However, this baseband rate is independent of the buoyancy frequency N_0.

Figure 1 shows that rogue waves can emerge sooner when the fluid is deeper. Remarkably, this implies that baseband instability is stronger when the system is closer to the singular limit where the cubic nonlinearity changes sign. On the other hand, the degree of the background density stratification posts only a minor effect to the baseband mode. Apart from choosing a preferred baseband mode, another perspective taken in the literature is to select a random field as the initial condition. For the present nonlinear Schrödinger equation, 'rogue wave like' entities would then emerge too (Akhmediev et al., 2009).

4. Discussions and Conclusions

An analytically tractable model for packets of internal waves is studied through four input parameters, h (fluid depth), k (wavenumber of the carrier envelope packet), N_0 (buoyancy frequency), and n (mode number of the internal wave), with only h and k relevant for surface waves. For internal waves, modulation instabilities and rogue waves may now arise for the shallow water and intermediate depth regimes if N_0 is constant. With knowledge of baseband instability and supplemented by computer simulations, the nonlinear focusing mechanism of rogue waves is assessed. Remarkably the constant buoyancy frequency may not play a critical role in the existence condition in terms of focusing, but the mode number of the internal wave does. For breathers or other pulsating modes, this buoyancy frequency parameter will enter the focusing mechanism consideration and further analytical and computational studies will be valuable (Sergeeva et al., 2014). In the next phase of this research effort, contrasts and similarities with surface waves should also be pursued, where a directional field or opposing currents can provide rogue waves generation mechanisms beyond the well established criterion of kh>1.363. Such effects of shear currents and comparisons with experimental / field data will be taken up in future studies (Onorato et al. 2011; Toffoli et al. 2013a; Toffoli et al. 2013b).

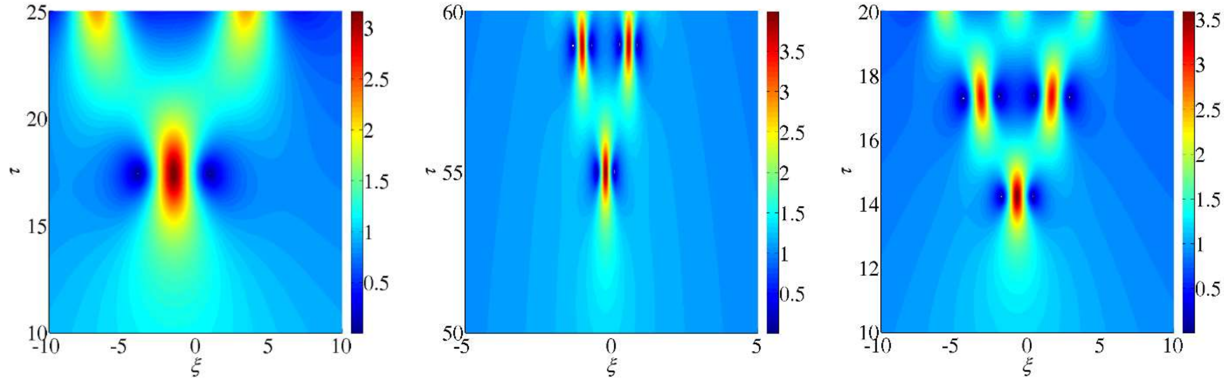

Figure 1: The emergence of rogue wave modes from a background continuous wave perturbed by a long wavelength unstable mode. Larger baseband gain implies a smaller time is required for the rogue wave modes to emerge. Left: For $N_0 = 2$, $h = 4$, $k = 0.5$, $n = 1$, $r = 0.2$, baseband instability growth rate = 0.868, rogue wave emerges at $\tau \approx 17$; Middle: For $N_0 = 2$, $h = 1$, $k = 0.5$, $n = 1$, $r = 0.2$, baseband instability growth rate = 0.193, longer time is required for the emergence of rogue wave in a shallower fluid ($\tau \approx 55$); Right: For $N_0 = 1$, $h = 4$, $k = 0.5$, $n = 1$, $r = 0.2$, baseband instability growth rate = 0.868, rogue wave emerges at about the same time ($\tau \approx 14$) as compared to the case with a higher buoyancy frequency $N_0 = 2$.

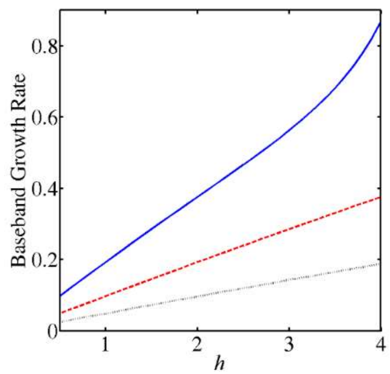

Figure 2: The baseband growth rate increases as the fluid depth h increases: $N_0 = 2$, $k = 0.5$, $n = 1$ (blue solid line); $N_0 = 2$, $k = 0.5$, $n = 2$ (red dashed line) ; $N_0 = 2$, $k = 0.25$, $n = 1$ (black dotted line).

Acknowledgements

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