Dear Editor, dear other referees, dear Perry and co-authors, dear interested readers,

As I'm still not convinced by the mass conservation (Eq. (4)) proposed by Perry Bartelt and co-workers, please allow me the following comments:

1) Mass conservation given by the jump condition is the correct way to write mass conservation when a flow impacts a rigid wall that spans the entire width of the incoming flow. As a result of the impact of the flow with the wall, a discontinuity in both velocity and density (if $h_\Omega = h_\Phi$), and in height (if $h_\Omega > h_\Phi$), forms and propagates at speed $\dot{S}_\Phi$ upstream of the wall. I maintain that the mass conservation across the MOVING discontinuity reads as follows (see details in item 2 below):

$$\rho_\Phi h_\Phi V_\Phi = -\rho_\Omega h_\Omega \dot{S}_\Phi + \rho_\Phi h_\Phi \dot{S}_\Phi. \tag{1}$$

(I'm using the notation of the authors.)

2) The sketch provided by Perry Bartelt and co-workers to interpret such a jump in velocity (from u_1 to $u_2 = 0$), density (from ρ_1 to $\rho_2 > \rho_1$) and height (from h_1 to h_2), as drawn in figure 1 –mid panel– of Perry Bartelt and co-workers' response published on 30 October 2018, is misplaced. The correct sketch rather corresponds to the right panel of figure 1 provided by Perry Bartelt and co-workers' in their response published on 30 October 2018, with the (more classical) following notation: $u_1 = V_\Phi$, $u_2 = V_\Omega = 0$, $h_1 = h_\Phi$, $h_2 = h_\Omega$, $\rho_1 = \rho_\Phi$, $\rho_2 = \rho_\Omega$, and $U = \dot{S}_\Phi$ ($= -u_n$ in my initial referee report on the paper).

To write mass conservation in such a situation, it is important to note that there exist discontinuities in variables (velocity, density, height) and the singular surface (where the discontinuities appear) is MOVING. This produces extra terms in the Reynolds transport theorem (see for instance the valuable references provided by Peter Gauer in his last comment). The Reynolds transport theorem reads as follows:

$$\oint_S \rho \, u \, \mathrm{d}S + \frac{\partial}{\partial t} \iint_V \rho \, V \mathrm{d}t = 0. \tag{2}$$

In the case when $h_2 = h_1$, the variation of mass in a control volume V (surrounding the discontinuity) during dt is:

$$\frac{\partial}{\partial t} \iint_V \rho \, V \mathrm{d}t = \rho_2 U - \rho_1 U, \tag{3}$$

(note that the terms above are $\rho_i h_1 U$ in the case $h_2 > h_1$.),

and the balance of mass fluxes across the control surface S is:

$$\oint_S \rho \; u \; \mathrm{d}S = \rho_1 u_1 - \rho_2 u_2. \tag{4}$$

(Note that the terms above are $\rho_i h_i u_i$ in the case $h_2 > h_1$, and because $h_2 \neq h_1$ you have to add the extra mass flux term $-\rho_2(h_2 - h_1)U$ to the sum above.)

The Reynolds transport theorem then gives:

$$\rho_1 u_1 - \rho_2 u_2 = -\rho_2 U + \rho_1 U. \tag{5}$$

Using $u_2 = 0$ (mass stopped against the wall), it becomes:

$$\rho_1 u_1 = -\rho_2 U + \rho_1 U. \tag{6}$$

(for the case $h_2 > h_1$, we have: $\rho_1 h_1 u_1 = -\rho_2 h_2 U + \rho_1 h_1 U$. The term $\rho_2 h_1 U$ appears twice: in the variation of mass during dt but also in the sum of mass fluxes. Thus, that term $\rho_2 h_1 U$ does not appear in the mass balance at the end. But the term $\rho_1 h_1 U$ is still here.)

With the notation of the authors, this reads:

$$\rho_\Phi V_\Phi = -\rho_\Omega \dot{S}_\Phi + \rho_\Phi \dot{S}_\Phi. \tag{7}$$

3) Perry Bartelt and co-authors are pointing out the analogy to kinematic waves formed during traffic jam. Please allow me to note that the Rankine-Hugoniot relation for mass continuity given by Eq. (5) is largely used in studies about traffic jam to extract the speed U at which the wave propagates: $U = \frac{\Delta(\rho h)}{\Delta \rho}$. See for instance:

P. I. Richards. Shock Waves on the Highway. Operations research, 4(1): 42–51, 1956).

M. Lighthill and G. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads. Proceedings of the Royal Society of London: Series A, 229 (1955), 317–345.

Regards,
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