Where is the mass that has velocity zero?

Dear reviewers, editor and interested readers,

We thank all three reviewers for their positive and critical comments. The suggestions to improve the paper from positive reviewers 1 and 2 will be used to amend the text. The third reviewer, Thierry Faug, has stated bluntly that our mass balance equation is wrong, because it does not agree with the "shockwave" treatment of avalanche impact on a rigid structure. We are sincerely thankful for Thierry Faug's comments, because it forced us to review the "shockwave" approach. After our review of Thierry Faug's comments, we could identify the differences between our approaches and show why our method (application of the work-energy theorem) is correct, but also why we simply do not understand the "shockwave" treatment, since it has no physical relationship to the pile-up process.

The shockwave approach begins by considering the mass balance of an elastic wave travelling at the speed of u_n. Our approach is based on the idea of a kinematic wave[1], travelling with a speed equal to the pile-up rate of the avalanche snow in front of the wall. We denote the pile-up speed as Sdot. In our approach, there is no elasticity and therefore no energy exchange between the kinetic and potential (strain) energy driving the speed of the shockwave. We make no references to the speed of sound. Avalanche snow is considered to be an ideal, compactible, plastic material. The difference in this characterization between a pile-up and a "shockwave" leads to a fundamental change in the definition of the speed of the pile-up or "shock". For us, a "shock" simply cannot exist. It is a compaction of the mass in front of the wall.

Thierry Faug maintains that the correct balance is given by the formula

$$u\_1\ (rho1\ h1) = u\_n\ (rho2\ h2 - rho1\ h1)$$

which is the same as writing

$$u\_n = \frac{u\_1}{X-1}\ with\ X = \frac{rho2\ h2}{rho\ 1\ h1}$$

Here, u_1 is the incoming avalanche speed, u_n is the "shock" wave speed propagating backwards, opposite to u_1. During the impact the snow mass compacts from the incoming density rho1 to the compacted density rho2. The incoming height of the avalanche is h1, the pile-up height is h2. Again, we emphasize, that this approach is correct for an elastic wave propagating with the speed u_n. However, it is incorrect for a kinematic or "pile-up" wave. We have no source for an elastic wave.

Consider the first time interval dt when an avalanche with height h1 hits the wall. Consider further that in this case the pile-up height doubles h2 = 2 h1, but there is no compaction, rho2 = rho1. This is shown in the figure 1 below. For this case, the "shock" wave model predicts that the out-going "wave" speed is equal to the incoming speed of the avalanche u_n = u_1. Thus, for the interval dt,

[1] The term "kinetic" wave comes from the theory of traffic jams, see "Kinetic Theory of Vehicular Traffic", by Prigogine and Herman, Elsevier Publishing, New York, 1971. Indeed, we treat snow pile-up as a "traffic jam".

the distance that the “shock” travels back (u_n dt) is equal to the length the incoming mass travels towards the wall (u_1 dt). Moreover, the avalanche hits the wall, and then is immediately deflected, traveling “on top” of the incoming avalanche. A shear plane must therefore develop in the pile-up zone and the “shock” front breaks down. In fact, there is no pile-up, only a deflection of mass. Moreover, the starting assumption of the shockwave model (a “shockwave” of height h2 propagates backward) is violated. It simply cannot happen with u_n = u_1. With our calculated speed Sdot, the pile-up wave extends over the entire height h2. The entire mass (u_1 h1 rho1) is stopped in front of the wall. We ask a simple question: where is the mass with zero velocity? Where is the pile-up? For us, it is in front of the wall.

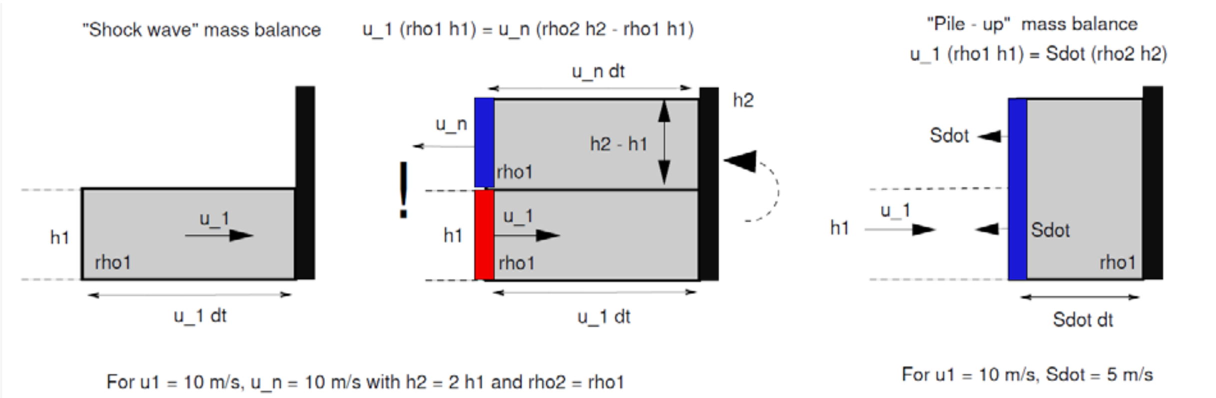

Figure 1: An avalanche with density rho1 and height h1 hits a rigid wall with the speed u_1. At impact, the mass does not compact rho2 = rho1, but grows to twice the height h2 = 2 h1. In the time interval dt, the volume of mass that moves towards the wall is u_1 dt and the mass that moves away from the wall u_n dt. This can only happen if a shear plane develops between the incoming and outgoing mass. Moreover, the “shock” front breaks down. There is no “pile-up”. The avalanche hits the wall and is directed backwards. During a pile-up, the kinematic wave speed Sdot is maintained over the entire height h2.

Another example serves to demonstrate why the shockwave approach is not suitable to model a pile-up and therefore avalanche impact pressures. Consider the case where the height of the incoming and piled-up mass are equal h2 = h1. The avalanche snow, however, compacts to twice the incoming density rho2 = 2 rho1. Again the speed of the “shock” (according to Thierry Faug) is given by u_n = u_1. This case is shown in Figure 2.

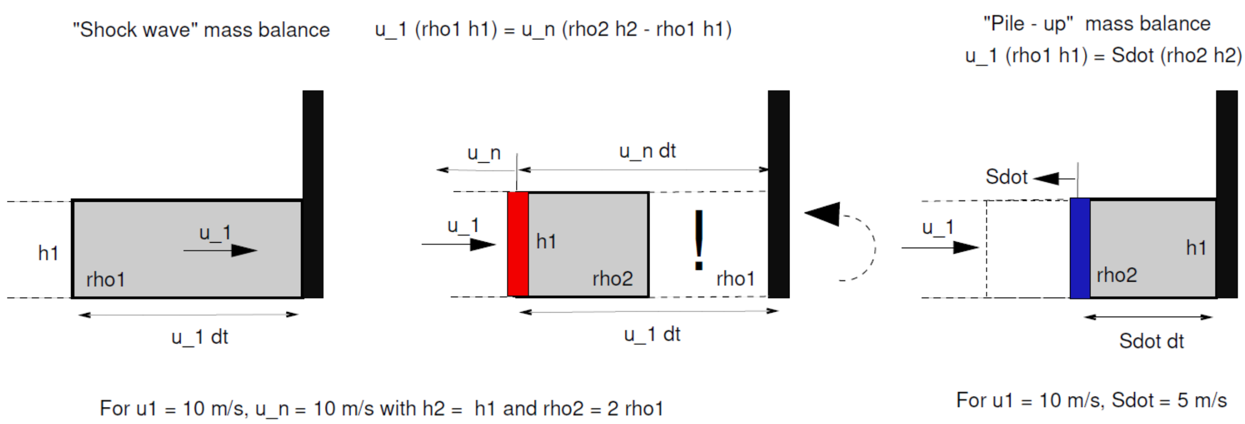

Figure 2: : An avalanche with density rho1 and height h1 hits a rigid wall with the speed u_1. At impact, the mass compacts to rho2 = 2 rho1. The height of the pile-up does not change h2 = h1. In the time interval dt, the volume of mass that moves towards the wall is u_1 dt and the mass that moves away from the wall u_n dt. The shockwave model predicts u_n = u_1. Because the outgoing mass is compacted, this can only happen if an empty space develops between the compacted mass and the wall. Again, there is no “pile-up”. The shockwave mass balance predicts that the empty space is filled with mass of density rho1. How?

The results from the “shockwave” approach are simply bizarre. The avalanche mass hits the wall and propagates backwards with the density rho2 to the location u_1 dt (the end location of the incoming mass). Moreover the speed of the incoming mass is equal to the “shock” speed u_n = u_1. However, the outgoing mass is compacted (the density behind the shock has increased). Therefore, the length of the compacted mass must be reduced because the height remains the same. Thus, it appears that an empty space develops between the outgoing mass and the wall. The empty space, however, is, according to the shockwave model, filled with the the mass, u_n rho1 h1. This is incredible: mass must travel through the outgoing wave (or perhaps it simply jumps over it) and piles-up with the density rho1 in front of the wall! This is simply not our picture (or any suitable mathematical characterization) of snow pile-up. For us, this completely erroneous result is a direct consequence of subtracting the amount u_n rho1 h1 from u_n rho2 h2 in the mass balance equation. This is the term that Thierry Faug wants us to be included in our mass balance. What is the physical reason for this inclusion? Simply because it agrees with a “shockwave” model? Again, we ask the question: where is the mass with zero velocity?

The same exercise can be performed over and over with different compaction ratios or outgoing/incoming height ratios h2/h1. The result is always the same. The shockwave model predicts incoming mass reaches the wall, where it is deflected backwards. This is not the pile-up process as we observe it, or how we model it. The predicted speeds of the pile-up front are too high. The shockwave model predicts that there is no non-moving mass of snow in front of the wall; the shockwave model does not allow a “pile-up” with density rho2 and height h2 (and speed zero, u_2 = 0).

Based on these arguments, we have come to the conclusion that the “shockwave” analogy to mass pile-up in front of walls is wrong. Above all, we dislike the use of an “elastic” theory to explain the deformation of a plastic material and what we consider to be an entirely irreversible process.

The consequences of this disagreement are significant, for it prevents a better understanding of avalanche impact pressures. Our interest in the snow impact problem is motivated by improving the impact pressure calculations in the simulation model **RAMMS**, as well as providing rational explanations to practitioners, especially during user workshops. Our study of the existing literature, both experimental and theoretical, led us to seek better methods to calculate (and explain) impact pressures. From our reading, it appears to us that three areas of confusion (and therefore contention) now exist within the avalanche dynamics community:

1) Lack of a consistent theoretical model describing the impact process. For example, why should the method to calculate impact pressures change when the avalanche impacts a wall or thin pylon? The model should automatically take into account the geometry of the impacted object. Again our model is based on the application of the work-energy theorem to simplify the complex deformation mechanics of snow. It is a step in this direction because different structural geometries induce different pile-up processes and therefore braking distances. The shockwave model cannot resolve the problem of how snow deforms behind a specific geometry. Plastic, irreversible pile-up is central to understanding impact forces on BOTH wide and thin objects.

2) Lack of information on impact times. A terrible mistake that is now being propagated in the avalanche community is to equate the "external" avalanche forces to the "internal" structural forces (see several papers in the recent ISSW 2018 proceedings where measured external impacts forces are used to calculate internal bending stresses of the construction, e.g. VdlS mast). The pile-up force is an external force. The internal forces account for the inertial forces, which depend on the mass of and mass distribution within the impacted structure. Large, short duration external forces may have no effect on heavy (large mass) structures. Key in the future engineering analysis will be then to identify the duration of the pile-up loading, because this will determine how a particular structure reacts to an impact. The answer to the question, "where is the pile-up mass" is central. Because the external pressures are large does not necessarily mean the internal stresses are likewise large. The converse is also true: we can have external forces that excite resonance in the structure, increasing the internal stresses well above the static (i.e. external = internal) loads.
3) Interpretation of impact pressure measurements. To understand impact measurements it is essential to understand the pile-up geometry and duration time. The measured forces depend on the speed of the pile-up and therefore the geometry of the measurement device, which controls the stopping process. Each measurement device will have different pressure factors Cd = ld/d (ld = effective length of incoming mass/d=braking distance). Frankly, the interpretation of measurements is lacking a theoretical model indicating what data and how must be collected in front of the structure (pile-up geometries, densities, etc). Because this additional information is missing, it is impossible to interpret pressure measurements and develop a consistent theory of avalanche impact. Empirical formulas could be replaced.

We conclude that the application of the work energy theorem, and the correct calculation of "shockwave" propagation speeds with the corresponding impact duration times, is a small step to understanding the snow avalanche impact problem. Where is the mass with zero velocity?