Replies to the comments by Editor

Comments to the Author:

Dear authors

We have now received three extensive reviews and three detailed answers by the authors. The authors largely modified the skeleton of their manuscript adding new figures and subsections. This gives rise to some critical enhancements of the quality of the manuscript. However, it is still not clear how the experiments brings new information to either the geophysical community working on the mechanics of free falls, or to the modelling community trying to advance the state of the art of dynamic propagation models. I suggest the authors to continue to rephrase mainly their introduction and discussion section with the goal of providing answers on how their results could be used for these two communities. Limitations of their experiments (a certain type of block geometry) should also be expressed in this view.

I suggest the authors to submit their revised version for further review by the editorial board.

Best regards,

Jean-Philippe Malet

Reply: Thanks you very much for your comments!

1. Based on the present study, we have two main results: (1) the general laws regarding the effect of the impact angle on the coefficients of restitution is not altered when the test scale changes. Increasing the impact angle results in a reduction of R_n, R_v and R_E, and causes an increase in R_t. (2) The rotation plays an important role in the effect of the impact angle. A smaller impact angle is easier to have a higher β/α (the ratio between the rebound angle and the impact angle) for the indentations and macro roughness caused by the impacts. With increasing β/α, the percentage of kinetic energy converted to rotational energy become higher and induces a higher R_n and a lower R_t. The first result confirms the validity of the existing conclusions based on small scale tests. The second result construct the correlation occurs between the rotation and the effect of the impact angle on the coefficient of restitution. They provides some new insights on how to interpret the results of small scale tests, and how to understand the effect of the impact angle on the coefficients of restitution. In the revised paper, the related contents are modified in the introduction, discussion and conclusion section.
2. Up to now, it is still very difficult to introduce the effect of the impact angle on the coefficients of restitution into rockfall trajectory model using a reasonable and comprehensive method. The main barrier comes from the discreteness of the measured data under small impact angle condition and the absence of a uniform and reasonable function describing the effect of the impact angle. We don't try to solve the problem in this study and just suggested that the stochastic model maybe a better choice. The related content is in Section 5.4 in the revised paper.
3. In this study, the effect of block shape and the detailed impact orientations are not involved, which induces some limitation of our test results. As you suggested, the limitations of our experiments are illustrated in the conclusion and abstract section.
4. For the consistency, Fig. 8 in Section 4.1 is redrawn by taking the ratio between the rebound angle and the impact angle β/α as an indicator, and the related contents are modified accordingly.
5. In the revised paper, all changes are marked using red font.

Thank you again for your advices!

Replies to the comments by Anonymous Referee #1

I, on behalf of other co-authors, would like to express our gratitude for the reviewer's attitude towards the reviewing.

General comments: The article presents a laboratory study on the dependence of the coefficient of restitution regarding the impact angle, falling height etc. Based on the results a regression has been formulated to obtain normal and tangential coefficients of restitution. The R2 are not very high. This – in my opinion – has one main reason: the blocks are not spherical but have edges an corners. Their impact on the ground mains defines the rebound angle and velocity. The model itself cannot reflect this effect because it neglects the rotational movement of the block that has a significant influence. Therefore, the model presented should be reported as being valid only for trajectory simulation codes based on point masses used to simulate the blocks. The model would not work for simulation codes that use folly shaped three-dimensional blocks. This should be stated in the introduction, handled in the discussion and be summarized in the conclusions.

Reply: Thanks you very much for your comments!

In this study, the free fall tests are performed and the sample impacts the slope without rotation. When the sample leaves from the slope, the rotation is observable and the angular velocities are also recorded. Just as you indicated, the angular velocities are not involved in the previous manuscript because the assumption of a lumped-mass model is adopted in this study.

Considering the comments by all reviewers, the rotation is involved in evaluating the effect of the impact angle on the coefficients of restitution. Results show that the percentage of kinetic energy converted to rotational energy increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples. And a higher percentage of kinetic energy converted to rotational energy always induces a higher normal coefficient of restitution R_n and a lower tangential coefficient of restitution R_t. Although the impact orientations during impact are not involved in this study, the results may be useful for those codes based on a rigid body model when predicting the trajectory of spherical rocks with rough surface.

The main changes in manuscript:

Considering all comments, the structure of this paper is rearranged. All figures are modified and rearranged.

The purpose of this study is described as: (1) to verify whether the test scale influence the laws regarding the effect of the impact angle on the coefficients of restitution, (2) to determine the role of rotation in the effect of the impact angle on the coefficients of restitution.

Rotation is involved in this study. As a consequence, the kinetic energy coefficient of restitution R_E is recalculated, and results of the kinematic coefficient of restitution R_v is added in this study.

The fitting curve are replaced by mean value lines of data points, and the fitting formula is removed.

In the original manuscript, we considered the impact velocity difference as the main reason for the magnitude difference in the coefficients of restitution between the tests compared. In the revision, we withdraw this deduction.

The role of rotation in the effect of the impact angle on the coefficients of restitution is investigated. As the percentage of the total kinetic energy converted to rotational energy increases, R_n increases but R_t decreases. The percentage increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples.

To special comments

P1L13: Please, add short term on the kind of rock movements with or without rotation, "jumping" or vertically falling.

Reply: As your suggestion, we have rephrased the sentences. "Free fall test are conducted and the velocities before and after the impact are obtained by a 3D motion capture system." The rotation is little before impact, but is observable after impact.

P2L4: Outdated references!

Reply: Thanks you a lot! The simulation codes listed in the previous paper is too outdated and cannot reflect the new progress. We have inspected literatures till 2018 and some representative simulation codes are added in the revised manuscript.

P2L19: The COR is a model only. In reality it is almost zero. Example: take a spherical rock and let it fall –> it barely jumps.

Reply: You are right! The coefficients of restitution is only useful for the bouncing phenomenon. When computer simulation codes are adopted in the trajectory predication, the coefficients of restitution should be input by users. Some typical values has been recommended for normal and tangential COR values according to the slope properties, such as clean hard rock, bedrock outcrops with boulders, and so on. Some summary work was listed in this paper, which may benefit some readers. As the structure of the paper is rearranged, the related sentence is removed to the end of Section 1.1, "So, R_n and R_t attracted most attentions in the previous studies, and some typical values of R_n and R_t had been summarized (Agliardi and Crosta, 2003; Heidenreich, 2004; Scioldo, 2006)."

P4L23, P9L14, P16L8: replace "increases in the impact" by "increasing"

Reply: Thanks you for your suggestion! The expression has been revised as your suggestion.

P5L2: Glover also evaluated coefficients of restitution in http://etheses.dur.ac.uk/10968

Reply: Thanks you very much! James Glover did excellent works on how the shape of rock affect the rockfall dynamics. And we noticed that some results about the effect of the impact angle were also presented in the thesis, which has been added in the Section 1.2.

P5L12: use kg instead of g because it is doubtful that exact this weight is kept.

Reply: Thanks you for your suggestion! The expression has been revised as your suggestion.

P7L5: 60fps might not be enough precise to capture the accelerations (during impact= time of the highest acceleration) there are only very small displacements that are not covered by the resolution of the cameras?

Reply: Thanks you a lot! We inspected the data information and found that the accelerations are not provided by the system. In the revision the "accelerations" has been removed.

P12L8: Of course, if only translational movements are looked at. The hardness of the impact partners involved is not very relevant. The rebound is influenced mainly from the rock's edges and therefore related to its rotational movement.

Reply: You are right! In the revised manuscript, the title of related figure is revised as "Direction transitions of translational velocities induced by impacts" and the related sentences is rephrased. The impact orientations, a corner contact or an edge contact, will affect the rebound motion. In the revised manuscript, the rotation is involved.

P13L7: This is a very precise weight....

Reply: Yes, it is a quite precise value. We inspected Chau's article "Coefficient of restitution and rotational motions of rockfall impacts, Int J Rock Mech Min Sci, 39, 69–77, 2002" again. And the weight is given clearly in the literature. So, we keep the value unchanged.

P16L30: "Assume" –> "Assuming"

Reply: Thanks you for your suggestion! The expression has been revised as your suggestion.

P20L9: The presented concept of COR analysis an experimental/laboratory trajectory regarding the block's center of gravity. The shape of the block does not play any role as well as its rotational movement. The presented model to determine R_n therefore only works if the trajectory model simulates small mass points without rotational movements. As soon as the trajectory code aims to model spatially shaped blocks with edges and corners above data cannot be used. This consequence should be added to discussion and conclusions.

Reply: In the previous paper we didn't point out that the assumption of a lumped-mass model in the study. Considering all comments, the rotation is involved in the revised paper, and its role in the effect of the impact angle is investigated. In this study the surface of samples are constituted by small artificial facets, the impact orientation can't be distinguished during the collision. Although the orientations of the block during impact are not involved in this study, the results maybe useful for those codes based on a rigid body model when simulating the trajectory of spherical rocks with rough surface.

References:

Reply: Thank you very much for your reminding! We have inspected the details and formats of the references. Some new literatures are added according to the revision, and a unify format is used.

Replies to the comments by M. Farin

I, on behalf of other co-authors, would like to express our gratitude for the reviewer's attitude towards the reviewing.

General comments: Most of the results given in the paper, in particular the variation of the coefficients of restitution as a function of the impact angle, were already reported in previous studies. It is not clear what this paper brings new to the research on energy losses during impacts. Please state clearly in the introduction what are the main questions that are posed at the end of the previous studies and needed additional experiments and answer to these specific questions in the conclusions. It is not clear what people doing computer simulations of rockfalls should retain from this work and how they could use the presented results.

I think that one important parameter that could allow us to better understand why kinetic energy losses are larger at high impact angles is the energy lost in rotational modes of the impactor. The more energy is dissipated in rotation after the impact, the less energy is restituted to the block as kinetic energy for rebound (cf Farin et al. (2015) Characterization of rockfalls from seismic signal: Insights from laboratory experiments, JGR:Earth Surface, Figure C1b). The authors could take advantage of the fact that their experimental setup has 8 cameras around the impact to measure precisely the rotation of the impactors before and after the impact and evaluate the rotational energy. This energy could be defined as 1/2* I * omega_rˆ2, where I is the moment of inertia of the block (that could be approximated to a full sphere) and omega_r is its rotation speed. A figure showing the kinetic coefficient of restitution, Re, as a function of the rotational energy after impact could be interesting to show to bring additional contribution with respect to the previous work on the subject. Also, it is important to precise in the paper that the 'energy coefficient of restitution' is the 'kinetic energy coefficient of restitution', which does not represent the whole energy lost by the block but only the kinetic energy Ek lost. If a lot of energy is transmitted in rotation energy Er maybe the total energy of the block Ek + Er does not decreases at large impact angles (?).

The authors suspect at several times in the paper that the impact speed has an influence on the coefficients of restitution. Thus, they should produce a Figure showing the coefficients of restitutions (and the rebound angle) as a function of the impact speed (event if only 3 different impacts speeds are investigated here, they could also use the data from previous work). Such a figure could support their discussion.

I find that the discussion section is a bit difficult to follow. Maybe it could be reworked with subsections, discussing for example 'Interpretation of normal coefficient of restitution larger than 1', 'Relation between kinetic energy losses and normal coefficient of restitution': : :

Reply: Thanks you very much for your suggestion!

The initial purpose of this study is to investigate whether the existing conclusions is valid when the test scale changes. To date, restrained by the measure devices, the existing laboratory test are mainly small scale tests. For the model test on coefficients of restitution, the similarity theory is still absent because the influence factors are much more than the material properties and sizes. It is questionable whether the test scale influence the laws regarding the effect of the impact angle on the coefficients of restitution. So, bigger samples and a new measure technique are adopted to perform a medium-scale test, and the above question is expected to be answered by the result comparisons between our test and the existing small scale tests.

Considering comments by all reviewers, the rotation is involved in the calculation of the energy coefficient of restitution R_E, and the role of rotation in the effect of the impact angle on the coefficients of restitution is investigated. Because the magnitudes of the total kinetic energy before impact varies, the percentage of the total kinetic energy converted to rotational energy is used as a reference. Results show that the percentage increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples. A higher percentage always induces a higher R_n and a lower R_t. While, no clear correlations occurs between the percentage and the other two coefficients, R_v and R_E. In the revised manuscript, this has been listed as another contribution of this study. Thank you again for your suggestion!

In this study, the small scale tests performed by Chau (2002), Cagnoli and Manga (2003), Asteriou (2012) are selected in the comparison. It is our pleasure that Cagnoli has also posted his comments. The magnitude difference in the coefficients of restitution between the tests compared attracted our attention, and we considered the impact velocity difference as the main reason. Actually, this deduction is arbitrary, considering that those tests differ from each other in multiple test conditions listed in Table 2. Cagnoli suggested that "The small R_n values in Cagnoli and Manga (2003) are due to the weak strength of pumice whose damage upon impact dissipates energy" in the comment. In Asteriou's latest paper (Asteriou, P. and Tsiambaos, G.: Effect of impact velocity, block mass and hardness on the coefficients of restitution for rockfall analysis, Int J Rock Mech Min Sci, 106, 41-50, 2018), a free fall test is performed using spherical balls vertically impacting the surface, and results show that R_n reduces when increasing the impact velocity, and increases as the material become harder. Because multiple factors can affect the magnitude, it is unreasonable to appraise the effect of one specific factor on the magnitude of the coefficient of restitution using data from the tests under various conditions together. The mean value of R_n versus the impact velocity in this study are drawn with different slope angles in the new manuscript as Fig. 7, and no determined trend is observable. We cannot make a definitive conclusion which factor is the main reason for the magnitude difference in the coefficients of restitution between the tests compared.

I am very sorry for the poor structure in the previous manuscript. Considering all comments, the structure of the paper and all figures are rearranged. I hope the new manuscript has an easy access to be scanned.

The main changes in manuscript:

Considering all comments, the structure of this paper is rearranged. All figures are modified and rearranged.

The purpose of this study is described as: (1) to verify whether the test scale influence the laws regarding the effect of the impact angle on the coefficients of restitution, (2) to determine the role of rotation in the effect of the impact angle on the coefficients of restitution.

Rotation is involved in this study. As a consequence, the kinetic energy coefficient of restitution R_E is recalculated, and results of the kinematic coefficient of restitution R_v is added in this study.

The fitting curve are replaced by mean value lines of data points, and the fitting formula is removed.

In the original manuscript, we considered the impact velocity difference as the main reason for the magnitude difference in the coefficients of restitution between the tests compared. In the revision, we withdraw this deduction.

The role of rotation in the effect of the impact angle on the coefficients of restitution is investigated. As the percentage of the total kinetic energy converted to rotational energy increases, R_n increases but R_t decreases. The percentage increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples.

To special comments

Abstract: - l14: the impact angle 'with respect to the slope' page2, L2: define the coefficient of restitution

Reply: Thank you very much! In the revised abstract, the related sentence has been rephrase as your suggestion. Section 1 and 2 in the original manuscript are merged together and restructured as the introduction section in the new manuscript. The definition of the coefficient of restitution are given first, and then the previous study are illustrated.

Introduction: - l 26: 'the similitude requirements: : : cannot be easily matched': I do not understand this sentence. Please rewrite.

Reply: When conducting a model test, the similarity ratio is usually important. While, a matured similarity theory is absent for those laboratory tests on the coefficients of restitution. The main reason is that the various factors are involved, such as the material properties, the shape of the rocks, the roughness, and the kinematic parameter. Thus, it is questionable whether the existing conclusions that the impact angle affects the coefficients of restitution based on small scale tests are valid when the test scale changes. In the new manuscript, the related sentence is rewritten. "Therefore, the existing results are restrained by the small scale of the laboratory tests. Influence factors are much more than the material properties and sizes, which induces the absence of the matured similarity theory for the model test on the coefficient of restitution (Heidenreich, 2004)."

page 2 L32: define the energy coefficient of restitution. 'The kinetic coefficient of restitution' is more appropriate.

Reply: Thank you a lot! We have inspected the related literatures. Sometimes R_E is called as the kinetic energy coefficient of restitution, and in some papers it is directly called as the energy coefficient of restitution. Of course, the first is more appropriate and it has been revised in the new manuscript.

page 2 l.34: Please do not give the same results as that given in the abstract. Please raise the general questions that require you to conduct additional experiments and that you answer in this paper, and answer these specific questions in the conclusion section. Sections 1.2 and section 2 should be merged with 1.Introduction and this whole section should lead to the problematic of the paper: what new contribution are you bringing to this research subject? To what questions are you answering?

Reply: Thank you for your suggestion! Section 1 and 2 in the original manuscript are merged together and restructured as the introduction section in the new manuscript. And the purpose of this study include: (1) to verify whether the test scale influence the laws that the impact angle affects the coefficients of restitution, (2) to determine the role of rotation in the effect of the impact angle on the coefficients of restitution. In the revised manuscript, the purpose has been stated in the ending of the introduction section.

- Page 3, L.15: n_{cor} and t_{cor} are never used in the following of the paper thus they should not be introduced.

Reply: Thank you for your reminding! We have noticed the issue, and in the revised manuscript they are removed.

- Page 3, L.20: it could be also interesting to present the results for R_v as a function of the impact angle and the kinetic energy lost because lots of people are using this definition. Is it varying differently than R_n with the impact angle?

Reply: In the original manuscript R_v wasn't presented because it is the square root of R_E when the rotational energy isn't involved in R_E. In the revised manuscript, the effect of the impact angle on R_v is also investigated, and the trend of R_v versus the impact angle is plotted as Fig. 5c.

- Page 4, l.2: ratio of kinetic energies

Reply: Thank you for your reminding! In the revised manuscript it is revised.

- Page 4, l.26: 'the impact angle can influence the rebound angle': be more precise. Does the rebound angle increase or decrease when impact angle increases?

Reply: Thank you a lot! In their paper, Cagnoli and Manga (2003) stated "The rebound angles are relatively larger at small and large impact angles with smaller values in between." In the original manuscript, we try to give a concise restatement while the meaning maybe unclear. In the new manuscript it has been revised as your suggestion.

Page 4, l.29: 'the kinematic coefficient of restitution R_v was more appropriate than the normal COR for use in correlations with the impact angles'. The relation between R_v and the impact angle should be also represented in this paper to check whether this statement is also true with the present experiments.

Reply: Thank you for your suggestion! In the revised manuscript, the effect of the impact angle on R_v is also investigated, and the trend of R_v versus the impact angle is plotted as Fig. 5c. In this study this statement is not valid. Various functions have considered to match data points, but no function can provide a correlation coefficient R^2 more than 0.40 in terms of R_v for all options considered. Power function provides the best R^2 in matching data points of R_n, which reaches 0.80.

- Page 5, l.3-4: These are poor sentences to sum up the previous results and motivate your work. Please clearly state at the end of the introduction what is missing from the previous work and requires you to do additional experiments.

Reply: You are right! This is caused by the poor structure of the original manuscript. In the new manuscript, Section 1 and 2 in the original manuscript are merged together and restructured as the introduction section. And our motivation are illustrated.

- Page 5, l.14: what is the 'rebound hardness value'? Does not it have units? I think it could be more useful to give the Poisson's ratios and Young's moduli of the materials composing the impactors and the slabs. For example, people may want to use your data to compute impact forces (for example using Hertz's impact model) and compare the impact forces to the coefficients of restitution and impact angles and such computations require the Poisson's ratios and Young's moduli.

Reply: Thank you for your suggestion! In the first place, the 'rebound hardness value' represent the hardness value measured by Schmidt hammer method, and it has no units. Some scholars considered the hardness as the key factor in the determination of the coefficient of restitution. In the revised manuscript, we provided the Poisson's ratios and Young's moduli for the material, and replaced "rebound hardness value" by "Schmidt Hardness R", which is a more formal name.

- Fig 6: The coefficient of restitution does not seem to depend on diameter, except 2 data points of higher value for D=10cm at low impact angle. In fact, the theory says that the coefficient of restitution should not depend on impactor size for impacts on a thick block (when the thickness of the impacted slab is large compared to the size of the impactor) and that the COR decreases as the impactor size increases when the impact is on a substrate whose thickness is small compared to the impactor size (cf Farin et al. (2015) Characterization of rockfalls from seismic signal: Insights from laboratory experiments, JGR:Earth Surface). The slab you are using could be considered as thin compared to the impactor size but because the slabs seem to be a bit buried in ground, they may be considered as thick substrates, thus the coefficient of restitution does not depend on the impactor size. A comment on this could be interesting to explain the fact that the measured COR is independent of the impactor size in your experiments.

Reply: Thank you for your suggestion! The law that the impact angle influence the coefficients of restitution appears independent of the sample sizes in this study. Your excellent study supports our results and we have list it as a reference. It is very interesting that in this study the sample size can affect the percentage of the total kinetic energy converted to rotational energy, but cannot affect the effect of the impact angle on the coefficients of restitution. We have checked the related literature till now, and we can't find similar works. Because more detailed information, such as the erosion caused by each impact and the impact orientation during collision, is not recorded when performing the test, the further research is absent. It is a pity and we would like to investigate this problem in the future.

- All Figures in general: Please use a larger and sans-serif font to improve figures readability.

Reply: Thank you for your suggestion! In the new manuscript the figures are redrawn as your suggestion.

- Figures 6, 7 and 8: I would use the same kind of scaling law (power law) for the 3 coefficients of restitution to compare them. A 2nd order polynomial law for figure 8 makes no sense because (1) you could fit everything why it and (2) you change your mind after that and use a linear law in figure 5c because it compares better with the previous results.

Reply: You are right! Considering comments by all reviewers, the best-fit curve is replaced by the mean value line for data points in the related figures in the new manuscript. Considering the discreteness in data points, a general trend is more appropriate than a fitting cure to illustrate the effect of the impact angle on the coefficients of restitution.

Although Wu (1985) suggested the linear correlation between the impact angle and R_n, R_t, a few literatures adopted the linear function to fit data points. In most literatures, data are not matched. In this study, test performed by by Chau et al. (2002), Cagnoli and Manga (2003), Asteriou et al. (2012) are selected to make a comparison. Cagnoli and Manga adopted a second-order polynomial to fit R_n, and adopted the linear function to fit R_t and R_E. Asteriou adopted the power function to fit R_n and R_v. As your comment, we should pay more attention on the sence of the function adopted in fitting rather than their imitative effect. In the new manuscript, our efforts in matching data points are briefly described, and the related formula is removed. A conclusion which type of function should be recommended is not given, because the previous study and this study haven’t provide sufficient evidence.

Please merge some of the figures together (e.g Fig. 2,3,4; Fig. 6,7,8; Fig. 12,13: : :)
Reply: In the new manuscript the figures are merged as your suggestion and other comments.

Page 9, L.14: the sentence ‘The values of Rt : : :’ is unnecessary, one can read the values on the figure.
Reply: In the new manuscript the sentence has been removed.

Page 10, L.5: the sentence ‘The values of Re : : :’ is unnecessary, one can read the values on the figure.
Reply: In the new manuscript the sentence has been removed.

Page 10, L.16: Have you measured the depth of erosion created by the impacts? Maybe the largest impactor have caused more erosion of the slabs and thus lose more energy in deformation of the slab than the smallest impactors. A figure showing the energy lost as a function of the depth of erosion due to the impact could be interesting if you can do it.
Reply: I am very sorry that more detailed information, such as the erosion caused by each impact and the impact orientation during collision, is not recorded when performing the test. We would like to investigate this problem in the subsequent studies.

Page 12, L.3: ‘The data points are stably located above the 45_ line until the impact angle reaches 36_’ may be a clearer sentence.
Reply: Thank you for your suggestion! In the new manuscript the sentence has been revised as your suggestion.

The ‘45_ line’ is misleading because the compared variables are angles. The ‘equality line’ or ‘y = x line’ are other possibilities.
Reply: Thank you for your reminding! In the new manuscript it is replaces by the “α=β line”.

Page 12, l.5: ‘the kinetic energy loss constituted 50-75% of the total kinetic energy’ This is false: total energy also includes the rotation energy.
Reply: You are right! Now the rotational energy is involved in the calculation of the kinetic energy coefficient of restitution. Therefore, the percentage is reduced. In the new manuscript, this mistake has been revised.

Page 12, l.6: ‘the energy loss level cannot be assessed by comparing the rebound and impact angle’: not clear
Reply: Thank you for your suggestion! Results of our test shows that for a given impact angle, larger rebound angle doesn’t means more kinetic energy dissipation than smaller rebound angle. The original sentence is not very clear. In the new manuscript, it has been revised as “Therefore, the

ratio between the rebound angle and the impact angle cannot be directly used as a reference in estimating whether the energy loss level is high or low."

Page 12, l.18: Maybe you should directly compare your results with that of previous studies before drawing conclusions because your conclusions seem to change a bit after the comparison with the other studies (for example you say later than R_t does not depend on the impact angle and you change the scaling law for Re), thus sections 4.1, 4.2 and 5 are redundant and confusing.
Reply: Thank you very much for your suggestion! Considering the purpose of this study, the paper is restructured. In the new manuscript, the results comparison between this study and the existing small scale tests follows the test results of this study, and they compose Section 3. The conclusion is given after the comparison. "Various experimental conditions induce different results for R_n, R_t, R_v and R_E, although there are certain trends that occur regardless of the test conditions. The normal coefficient of restitution R_n, kinematic coefficient of restitution R_v and kinetic energy coefficient of restitution R_E all decrease with increasing in the impact angle, while the tangential coefficient of restitution R_t increases as the impact angle increases in most cases."

Page 12, L.23 to Page 13 L.2: This should be in the introduction.
Reply: Thank you for your reminding! In the introduction, tests conducted by Chau et al. (2002), Cagnoli and Manga (2003), Asteriou et al. (2012) had been briefly introduced. Here, the detailed test conditions of those studies are provided in Table 2.

Page 13, L.9: what is the 'ideal state'? If you observe rebound angles larger than 1.2 times the impact angle, there is a chance that we can also observe this in nature. You should not exclude data points just because they do not compare well to the previous work. On contrary, you should keep these points and interpret why you observe such situation in your experiments and why it is not observed in the previous work.
Reply: Thank you very much for your reminding! In the revised manuscript, all data points are reserved.

Table 2 and Fig. 11: please replace 'Wang 2018' by 'this study' to avoid confusion.
Reply: The words in the figure have been revised as your suggestion.

Page 15, L.3: 'The minimum Rn occurred: : : erosion and particle breakage'. This explanation that stronger kinetic energy dissipation due to erosion may explain the lower Rn value for Cagnoli's experiment does not work because (1) you also observe erosion by the impacts and the Rn in your experiments are larger and (2) you state later that the normal coefficient of restitution does not correlate with kinetic energy loss: : :
Reply: You are right! This study verifies that the test scales don't alter the general law regarding the effect of the impact angle on the coefficients of restitution. The reason that causes the magnitude difference is still questionable. Cagnoli suggested that "The small R_n values in Cagnoli and Manga (2003) are due to the weak strength of pumice whose damage upon impact dissipates energy" in the comment. The existing studies and this study cannot provide sufficient evidence to determine the reason, because the test conditions are different in multiple aspects. Asteriou indicated that R_n reduces when increasing the impact velocity, and increases as the material become harder in the

latest paper (Asteriou, P. and Tsiambaos, G.: Effect of impact velocity, block mass and hardness on the coefficients of restitution for rockfall analysis, Int J Rock Mech Min Sci, 106, 41-50, 2018). This problem is proposed in the ending of Section 3.

Page 15, L.9-12: the exact scaling law that describe best the data is not very important given the large scattering in the data. What matters more is if you can explain the general trend. Also, if you give a scaling law for you data, you should also try to fit the data of the previous work with the same kind of scaling law. If the scaling law works for your data and not with the other work, its usefulness is very limited: :

Reply: Thank you very much for your reminding! Considering comments by all reviewers, the best-fit curve is replaced by the mean value line for data points of this study in the related figures in the new manuscript. In section 3.2, the trend line for the existing small scale tests are drawn as the original literature. The lines with data markers are the mean value lines, while those lines without data markers are fitting lines.

Page 15, L.15: The variation of the kinetic energy COR with impact angle may be better understood if you also show the rotation energy (more energy dissipated in rotation means less energy restituted in kinetic energy for the rebound). You should not remove data points just because they do not compare well with previous work. Explain the difference otherwise the same conclusions could have been drawn by just comparing the previous work together and this present work contribution is limited.

Reply: Thank you very much for your suggestion! It is unreasonable to exclude those “non-ideal data points” for a better fitting curve. When the rotational energy is involved in this study, some interesting phenomenon is observed. When the impact angle is small, two sample sizes appear a clear distinction in the percentage of the total kinetic energy converted to rotational energy. Small samples always induce bigger percentage than large samples. Considering that a higher percentage will results in a larger R_n and a lower R_t, the magnitude difference in the coefficients of restitution within the first impact angle interval is reasonable between two sample sizes.

Page 10, l.16: ‘The impact velocity is an important: : : resulting coefficients of restitution’: please show a figure of the CORs as a function of the impact speed (even including the previous work data) to support your conclusion.

Reply: The magnitude difference in the coefficients of restitution between the tests compared was attributed to the difference in their impact velocity in the original manuscript. But, this deduction is arbitrary, considering the various test conditions. The mean value of R_n versus the impact velocity are drawn with different slope angles in the new manuscript as Fig. 7, and no determined trend is observable. The previous work data is not involved. In our opinion, to determine the effect of one specific factor on the magnitude of the coefficient of restitution using data from the tests under various test conditions together may be unreasonable. We cannot make a definitive conclusion which factor is the main reason for the magnitude difference in the coefficients of restitution between the tests compared.

Discussion section. Different things are discussed here, please add subsections to make the discussion clearer.

Reply: Thank you very much for your suggestion! In the new manuscript, Discussion section is composed by three subsections.

Page 16, L. 24: I do not understand what you mean by ‘with a parallel motion’
Reply: I am sorry for the poor sentence. In the original paper, “with a parallel motion” means that only translational motion is involved. But we considers that this expression is also confusing. So, in the new manuscript, the related sentences are rewritten. “When the impact angle is sufficiently large to generate a rebound angle as the solid arrow, the border imposes no constraints on the rebound motion, and the sample can leave with the default rebound angle. But, when the impact angle is small and generate a default rebound angle as the dashed arrow, rotation motion must be involved to overcome the constraint.”

Page 16, L. 27: ‘Therefore,Â˘a: : :’ I do not understand the logical link with the previous sentence. If rotation speed has an important effect on rebound angle and coefficient of restitution, you should show it on Figures.
Reply: Thank you very much for your suggestion! In the new manuscript, the direction transitions of translational velocities and the rotation are regarded as two consequence of the impact in Section 4. And the effect of the rotation on the coefficient of restitution is investigated. In the original paper, the logical link is not clear.

Page 17: I understand that basal roughness can lead to higher angles of rebound, but in this case, the impactors on intact slabs should have in average lower angles of rebound than impactors on eroded slabs. Can you draw a figure or give the average rebound angles on intact vs eroded slabs to support your discussion? If you measured the depth of erosion on the slabs, maybe the rebound angle could be correlated to with erosion depth (?).
Reply: I am very sorry for the information isn’t recorded. When one slab are too eroded, it is replaced by another one. For one specific slope angle, the data points from intact slabs and eroded slabs are mixed together, and we can’t distinguish them now. And the depth of erosion is not measured for each impact. It is a pity. We would like to verify this phenomenon in the subsequent studies.

Page 20, l. 5: This conclusion does not bring anything new to the research. I believe you could draw much more results from you experimental data.
Reply: Thank you very much for your encouragement!
In the new manuscript, the contribution of this study is concluded as two points: (1) verified that several general laws occur when accounting for the effect of the impact angle, regardless of the test scales and conditions, (2) indicated that the rotation plays an important role in the effect of the impact angle on the coefficient of restitution. A higher percentage of kinetic energy converted to rotational energy always induces a higher normal coefficient of restitution R_n and a lower tangential coefficient of restitution R_t.

Replies to the comments by B. Cagnoli

I, on behalf of other co-authors, would like to express our gratitude for the reviewer's attitude towards the reviewing.

General comments: This is a good set of experiments. I encourage the authors to take some time to improve their manuscript. Here some comments that can be useful.

Reply: Thank you very much for your encouragement!

To date, restrained by the measure devices, the existing laboratory test are mainly small scale tests. The initial purpose of this study is to investigate whether the existing conclusions regarding the effect of the impact angle on the coefficients of restitution are valid when the test scale changes.Considering comments by all reviewers, the rotation is involved in this study. And, in the new manuscript, the role of the rotation in the effect of the impact angle is also investigated. Because the magnitudes of the total kinetic energy before impact varies, the percentage of the total kinetic energy converted to rotational energy is used as a reference. Results show that the percentage increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples. A higher percentage always induces a higher R_n and a lower R_t. While, no clear correlations occurs between the percentage and the other two coefficients, R_v and R_E. In the revised manuscript, this has been listed as another contribution of this study.

The main changes in manuscript:

Considering all comments, the structure of this paper is rearranged. All figures are modified and rearranged.

The purpose of this study is described as: (1) to verify whether the test scale influence the laws regarding the effect of the impact angle on the coefficients of restitution, (2) to determine the role of rotation in the effect of the impact angle on the coefficients of restitution.

Rotation is involved in this study. As a consequence, the kinetic energy coefficient of restitution R_E is recalculated, and results of the kinematic coefficient of restitution R_v is added in this study.

The fitting curve are replaced by mean value lines of data points, and the fitting formula is removed.

In the original manuscript, we considered the impact velocity difference as the main reason for the magnitude difference in the coefficients of restitution between the tests compared. In the revision, we withdraw this deduction.

The role of rotation in the effect of the impact angle on the coefficients of restitution is investigated. As the percentage of the total kinetic energy converted to rotational energy increases, R_n increases but R_t decreases. The percentage increases as the impact angle decreases, and large samples are more likely to have a stead and small percentage than small samples.

To special comments

LINE 4 PAGE 4. Please note that this R_E value omits the rotational kinetic energy and as such, it simplifies the description of the collisions. I do expect that your spherical polyhedrons rotated both before and after their impacts. This should be mentioned in the discussion since it affects the plot in Fig 8. For example, in our experiments (Cagnoli and Manga, 2003), our cylindrical particles did have a rotational kinetic energy but only after the collision with the target as the high-speed video camera confirmed.

Reply: Thank you very much for your suggestions! Considering comments by all reviewers, the rotation is involved in the calculation of the energy coefficient of restitution R_E in the new manuscript, and the role of rotation in the effect of the impact angle on the coefficients of restitution is investigated. In this study, the samples has little rotation before impact, while has the observable rotation when leaving the slope.

LINE 18 PAGE 5. I think that a drawing of the apparatus with vertical and horizontal length scales would improve the readability of the paper.

Reply: Thank you very much for your suggestions! A general view of the apparatus has been added in the revised manuscript. All figures are rearranged according to their logical link, and some figures are merged.

FIG 8 PAG 11. Here, it seems to me that you felt the obligation to have to find one single best-fit curve even if your data points illustrate a much more complex situation. Rather than concave-down best-fit curves (which are truly not convincing), this plot shows two features: 1) the maximum values decrease as the impact angle increases and 2) the spread of the data points decreases as the impact angle increases. This is true for both your grain sizes. We obtained these same features as shown by Fig 4A in Cagnoli and Manga (2003). I strongly suggest to remove these concave-down curves because they are truly misleading.

Reply: Thank you very much for your reminding! Considering comments by all reviewers, the best-fit curve is replaced by the mean value line for data points in the related figures in the new manuscript. Considering the discreteness in data points, a general trend is more appropriate than a fitting cure to illustrate the effect of the impact angle on the coefficients of restitution.

FIG 9 PAG 11. It would be useful to identify in this figure each experiment with its own characteristics.

Reply: Thank you very much for your reminding! In the new manuscript, two markers are adopted to represent data points for two sample sizes, respectively. And the original “45° line” is renamed as “α=β line” considering other comments.

TABLE 2 PAG 13. Please note that our cylinders are 0.89 cm long and with a basal diameter equal to 0.55 cm (Cagnoli and Manga, 2003). However, rebound angles of larger cylinders are also shown in Fig. 2A.

Reply: Thank you a lot! In the new manuscript, the size of the cylinders are noted in Table 2.We noticed that the rebound angles of larger samples are presented in your excellent paper. It is not involve in the results comparison because the results comparison focuses on the effect of the impact angle on the coefficients of restitution. In Section 4.1 “Direction transitions of translational velocities”, we noted that your test also observed this phenomenon.

LINE 9 PAG 13. The rebound angles can be larger than the impact angles for two reasons. First, the surface of your concrete slabs cannot be perfectly flat in particular after the target has been damaged by previous impacts. Second, the surface of your particles has a curvature that varies from place to place (i.e., they have edges and corners). In other words, the true impact angle is not known. In our Fig 2A, some rebound angles are also larger than the impact angles. Even if this seems to be a flaw of the experiments, it has to be accepted as the inevitable complexity of rock fragment collisions and it is still useful to understand this complexity. For this reason, it is not correct to exclude what you call “non-ideal data points” when computing best-fit curves. The truth is that a single best-fit curve of the entire set of data points in Fig 8 does not exist. You can plot only a trend line for the maximum values if you really want to.

Reply: Thank you very much! In the revised manuscript, all data points are reserved. It is unreasonable to exclude those “non-ideal data points” for a better fitting curve. Considering comments by all reviewers, the mean value line for data points is adopted in the new manuscript.

FIG 11 PAG 14. Please, remove curves 5 from Figs 11a, 11b and 11c, because, in nature, beta can be larger than alpha. Do Figs 11a and 11b display mean values? If yes, state this clearly. In Figure 11c, draw only curves showing the maximum RE values.

Reply: Thank you for your reminding! All data points are reserved in the revised manuscript, so curve 5 is removed naturally. Considering comments by all reviewers, the mean value line is adopted to represent the trend for our study. The meaning of every trend line for the tests compared and our study in figures are stated in the new paper.

LINE 3 PAG 15. What you say here is true. However, I would rephrase the sentences. The small R_n values in Cagnoli and Manga (2003) are due to the weak strength of pumice whose damage upon impact dissipates energy.

Reply: Thank you a lot! In the new manuscript, the sentence has been rewritten as your suggestion.

LINE 25 PAG 15. What do you mean with “nadir”? Please find a more appropriate word.

Reply: I am very sorry for the inexact word used. Actually, in the original manuscript, we noticed that when the impact angle is less than 40°, your test provided the lowest R_t. And it increases as the impact angle increases. We considered the impact velocity as the main reason for the magnitude difference in the coefficients of restitution between different tests. However, in the new manuscript we didn’t make a determined conclusion about this. Multiple factors can affect the magnitude, thus, it is unreasonable to appraise the effect of one specific factor on the magnitude of the coefficient of restitution using data from the tests under various conditions together.

LINE 30 PAG 15. As explained above, curve 1 in Fig 11c is not useful and should be removed from the plot.

Reply: Thank you for your reminding! Considering comments by all reviewers, the mean value line is adopted to represent the trend for our study.

LINE 8 PAGE 16. This is not correct. Both your Fig 7 and our Fig 3B confirm that R_t increases as the impact angle increases. The problem is that the data spread is large. But this is due also to irregularity on the surfaces of target and particles, for example.

Reply: You are right! One purpose of this study is to verify whether some general laws occur when accounting for the effect of the impact angle, regardless of the test scales and conditions. The results comparison shows that the tangential coefficient of restitution R_t increases as the impact angle increases in most cases.

LINE 21 PAGE 16. This is the same explanation we have provided in our paper (see our Fig 1), but no credit is given.
Reply: I am very sorry that more detailed information, such as the erosion depth caused by each impact and the impact orientation during each collision, is not recorded when performing the test. In the new manuscript, one figure of the damaged surface is provided as a credit in Discussion.

LINE 18 PAGE 18. The use of the coefficient of restitution does not provide a good description of rock fragment collisions. But credit should be given to who has already said it (e.g., Stronge, 1991). Both your and our data sets show that: 1) there is no such as thing as a single value of the coefficient of restitution, and 2) also the more informative ratio of the kinetic energy is not a constant.
Reply: Thank you very much! In the revised manuscript, Stronge’s conclusion has been cited as a credit, and the paper has been listed as a reference.

Effects of the impact angle on the coefficient of restitution based on a medium-scale laboratory test

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Abstract. The reliability of a computer program simulating rockfall trajectory depends on the ascertainment of reasonable values for the coefficients of restitution, which typically vary with the kinematic parameters and terrain conditions. The effects of the impact angle with respect to the slope on the coefficients of restitution have been identified and studied using small scale laboratory tests. To investigate whether the existing conclusion based on small scale laboratory tests is valid when the test scale changes and the role of rotation in the effect of the impact angle on the coefficients of restitution, this study performed a medium-scale laboratory test using spherical limestone polyhedrons impacting concrete slabs. Free fall test are conducted and the velocities before and after the impact are obtained by a 3D motion capture system. The results comparison between our test and the existing small scale tests verified that several general laws occur when accounting for the effect of the impact angle, regardless of the test scales and conditions. Increasing the impact angle will induce reductions in the normal coefficient of restitution R_n, the kinematic coefficient of restitution R_v and the kinetic energy coefficient of restitution R_E, whereas it will lead to increases in the tangential coefficient of restitution R_t. The rotation plays an important role in the effect of the impact angle. A higher percentage of kinetic energy converted to rotational energy always induces a higher normal coefficient of restitution R_n and a lower tangential coefficient of restitution R_t. As the impact angle decreases, the ratio between the rebound angle β and the impact angle α increases, and the percentage of kinetic energy dissipated in rotation during the collision becomes higher. Considering that the effect of block shape and the detailed impact orientations are not involved in the present study, the test results is valid for trajectory simulation codes based on a lumped-mass model, and can be referenced in the trajectory predication of spherical rocks impacting hard surface using a rigid body model.

1 Introduction

In mountain areas, rockfall is a frequent natural disaster that endangers human lives and infrastructure. Numerous examples of fatalities or infrastructure damage due to rockfall have been reported (Guzzetti, 2003; Pappalardo, 2014). Various protective measures, such as barrier fences, cable nets and rockfall shelters, have been widely used to reduce rockfall hazards. To ensure the efficiency of mitigation techniques, the motion trajectory of the rockfall must be estimated. The trajectory can provide important information, such as the travel distances of possible rockfall events, the bouncing height and kinetic energy level of the rockfall at various positions along the slope.

Numerous algorithms have been developed to solve this problem, and the progress up to the end of the last century has been summarized by Dorren (2003) and Heidenreich (2004). Due to these efforts, computer simulation codes, such as RockFall (Stevens, 1998), CRSP (Jones et.al, 2000) and Stone (Guzzetti et.al, 2002), RAMMS::Rockfall (Christen et al, 2007), Rockyfor3D (Dorren, 2010) and Pierre (Valentin et al., 2015; Andrew and Oldrich, 2017), are developed to acquire motion information for rockfall. A main feature that allows one to distinguish between different rockfall trajectory codes is the representation of the objective rock. The first approach, a lumped-mass model, treats the rock as a single and dimensionless point, and assigns all of the properties of the rock to that point. The second one is a rigid body model, which considers the rock as a body with its own shape and volume, and accounts for all types of block movement, including rotation. Finally, a hybrid model adopts a lumped mass model to calculate the free fall of the rock, and simulates other types of block motion using a rigid body model. In most codes the trajectory of falling rocks was described as combinations of four types of motion: free fall, rolling, sliding and rebound. The rebound motion, a succession of rockfalls impacting the slope surface, is the least understand and the most difficult to predict of the four types of motion (Volkwein A et.al, 2011), which is controlled by the coefficients of restitution in computer simulation. Thus, the reliability of the estimation of the coefficient of restitution must be ensured.

1.1 Definition for the coefficient of restitution

The coefficient of restitution is a dimensionless value representing the ratio of velocities or energies of a boulder before and after it impacts the slope. Various definitions for the coefficient of restitution have been proposed in previous studies, but no consensus was reached on which definition is more appropriate for rockfall prediction. As shown in Fig. 1, when one boulder impacts the slope surface, the impact velocity v_i can be resolved into a normal component v_{ni} and a tangential component v_{ti} according to the slope angle θ. Then, the boulder leaves the surface with a rebound velocity v_r, which similarly has a v_{nr} and a v_{tr}. The angular velocities of the boulder before and after impact are denoted as ω_i and ω_r, respectively. The impact angle α and rebound angle β are drawn in Fig. 1.

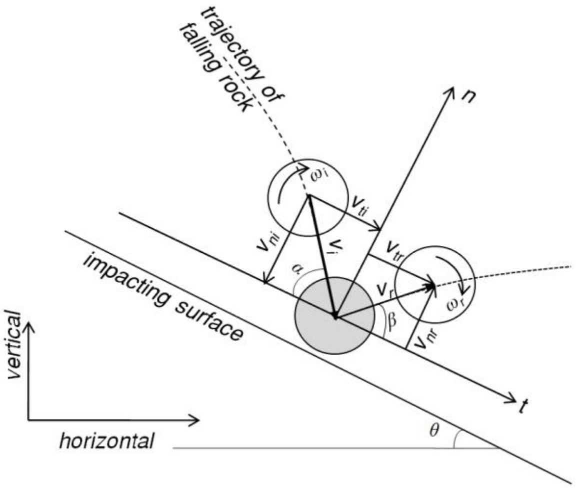


Fig. 1. Related quantities adopted in definitions for the coefficient of restitution

The normal and tangential coefficients of restitution are the most used definitions, and the two coefficients of restitution are typically denoted as R_n and R_t, respectively. The mathematical expressions of R_n and R_t are

$$R_n = v_{nr} / v_{ni}, R_t = v_{tr} / v_{ti} \tag{1}$$

Another common definition is the kinematic coefficient of restitution, R_v, representing the ratio between the magnitudes of the rebound and impact velocities:

$$R_v = v_r / v_i \tag{2}$$

This definition originated from Newton's theory of particle collision, and had been used by Habib (1976), Paronuzzi (1989) and other scholars. When R_v is used in the trajectory predication, an assumption regarding the rebound direction is necessary to fully determine the velocity vector after impact.

In addition, the ratio of kinetic energies before and after impact is used to define the kinetic energy coefficient of restitution R_E, which is written as

$$R_E = E_r / E_i = (E_{rr} + E_{rt}) / (E_{ir} + E_{it}) \tag{3}$$

in which E_i and E_r are the kinetic energy before and after the impact, respectively. E_{ir} and E_{rr} are the rotational energy before and after the impact; E_{it} and E_{rt} denote the translational energy before and after the impact. E_{ir}, E_{it}, E_{rr} and E_{rt} are computed as

$$E_{ir} = 0.5I\omega_i^2, E_{it} = 0.5mv_i^2, E_{rr} = 0.5I\omega_r^2, E_{rt} = 0.5mv_r^2 \tag{4}$$

Here, m is the mass, I is the moment of inertia. R_E can reflect the kinetic energy loss caused by the impact, and had been used by Bozzolo and Pamini (1986), Azzoni et al. (1995) and Chau et al. (2002).

In these definitions, R_n and R_t get more popularity in engineering practice for the simplicity in computer simulation software. R_n and R_t are used conjointly and characterize the variation in the tangential and normal components of the boulder velocity, respectively. Given an impact velocity, the rebound velocity and direction can be completely determined using this definition without any further assumption. So, R_n and R_t attracted most attentions in the previous studies, and some typical values of R_n and R_t had been summarized (Agliardi and Crosta, 2003; Heidenreich, 2004; Scioldo, 2006).

1.2 Previous studies on the effects of the impact angle on the coefficient of restitution

Various techniques, such as laboratory tests (Buzzi et.al, 2012; Asteriou et.al, 2012), field tests (Dorren et.al, 2006; Spadari et al. 2012), back analysis of field evidence (Paronuzzi, 2009) and theoretical estimation (He et.al, 2008), have been used to determine the coefficient of restitution. Variations in the impact conditions, e.g., the material properties of both the rocks and slopes (Wu, 1985; Fornaro et.al, 1990; Robotham et.al, 1995; Richards et.al, 2001; Chau et.al, 2002; Asteriou et.al, 2012), the shape of the rocks (Chau et.al, 1999; Buzzi et.al, 2012), the roughness of the slope surface (Giani et.al, 2004) and the impact angle, influence the coefficient of restitution considerably. While, in those existing summaries for typical values, the coefficients of restitution were determined mainly accounting for the terrain conditions.

The impact angle, the angle between the directions of the impact velocity and the slope segment, is a kinematic parameter of the falling rock, indicating only that the terrain conditions involved in estimating the value of the coefficient of restitution may be unreliable. Since Broili (1973) first identified this problem, numerous experiments have been performed to acquire a comprehensive picture of the effects of the impact angle. In situ tests are expensive and not suitable for statistical and parameter analysis; thus, existing studies have largely been performed in the laboratory. In some literatures, the impact angle was referred to as the slope angle θ (or the impact surface angle) in free-fall tests. While, the impact surface angle is only another expression because the slope angle θ and impact angle α sum up to 90 °under these conditions.

Wu (1985) conducted laboratory tests using rock blocks on a wooden platform and rock slope, and suggested that there is a linear correlation between the impact surface angle and the mean value of the restitution coefficient. He proposed that increasing the angle of the impact surface causes the normal coefficient R_n to increase regardless of the block mass and causes the tangential coefficient R_t to decrease slightly.

Richards et al. (2001) executed free-falling tests considering different types of rock and slope conditions and established a correlation between the coefficient of restitution and the Schmidt hammer rebound hardness. The impact surface angle was added to the correlation to reflect its linear improvement effect on the normal coefficient R_n.

Chau et al. (2002) conducted experiments using spherical boulders and a rock slope platform, both made of dental plaster. The free-falling tests indicated that the normal coefficient increases with increases in the impact surface angle, whereas there was no clear correlation with the tangential coefficient.

Cagnoli and Manga (2003) studied oblique collisions of lapilli-size pumice cylinders on flat pumice targets and determined that the impact angle can influence the rebound angle, the kinetic energy loss and the ratios of the velocity components. The normal coefficient decreases as the impact angle approaches 90 °.

Asteriou et al. (2012) performed laboratory tests using five types of rocks from Greece. The result of the parabolic drop tests indicated that the kinematic coefficient of restitution R_v was more appropriate than the normal coefficient of restitution for use in correlations with the impact angles. Then, the normal coefficient of restitution could be estimated accounting for the rebound–impact angle ratio.

Buzzi et al. (2012) conducted experiments using flat concrete blocks in four different forms and determined that a combination of low impact angle, rotational energy and block angularity may result in a normal coefficient of restitution in excess of unity.

James (2015) evaluated restitution coefficients using milled aluminium blocks and a planar wooden slope. Three different shapes of blocks were custom made and the slope surface was carpeted. Both the first impact under free fall conditions and the series impacts during runout were recorded. It was observed that R_n shows a positive correlation with increasing slope angle while R_t shows a negative correlation.

These efforts have highlighted the importance of the impact angle with regard to the coefficient of restitution. Most of the existing tests indicated that increasing the impact angle induces a reduction in the normal coefficient of restitution R_n, but an improvement in the tangential coefficient of restitution R_t. However, there are two issues still unsolved. In the first place, whether the laws regarding the effect of the impact angle on the coefficient of restitution are influenced by the test scale is uncertain. Up to now, the existing laboratory tests commonly captured the trajectory of small samples using a high-speed video camera, which means that the existing results are based on small scale laboratory tests. As Heidenreich (2004) noticed, the matured similarity theory regarding the model test on the coefficient of restitution is still absent, for the influence factors are much more than the material properties and sizes. Therefore, laboratory tests with larger scales should be performed to confirm the validity of the existing conclusions, which is benefit for further interpreting the results of small-scale laboratory tests. Secondly, in free fall tests the rotation after impact plays an important role in kinetic energy dissipation of the falling block (Chau et al., 2002), and it was supposed to affect the variation of R_n and R_t (Broili, 1973; Cagnoli and Manga, 2003). But, few work has been done to reveal the effect of the rotational motion on the coefficient of restitution through quantitative analysis. Whether a correlation occurs between the rotation and the effect of the impact angle deserves our attention, which may offer some insights into the effect of the impact angle on the coefficients of restitution.

Hence, this study employs a 3D motion capture system and a special releasing device to perform a medium-scale laboratory experiment. Spherical polyhedrons made of limestone were selected as samples, with a maximum diameter of 20 cm. The landing plate consisted of C25 concrete slabs. To address the effect of the impact angle, different inclined plate angles and releasing heights were used in free fall tests. The resulting coefficients of restitution, R_n, R_t, R_v and R_E, were calculated, and their trends in terms of the impact angle were explored to provide a complete picture. Then, the results are compared with three existing small scale experiments to determine whether the test scale affects the law that the impact angle influences the coefficients of restitution. The percentage of the total kinetic energy before impact converted to rotational energy was investigated, and the role of rotation in the effect of the impact angle on the coefficient of restitution was analysed. For only spherical polyhedrons are taken as the samples in this study, the test results may have some limitations.

2 Laboratory investigation

2.1 Rock specimens and concrete slabs

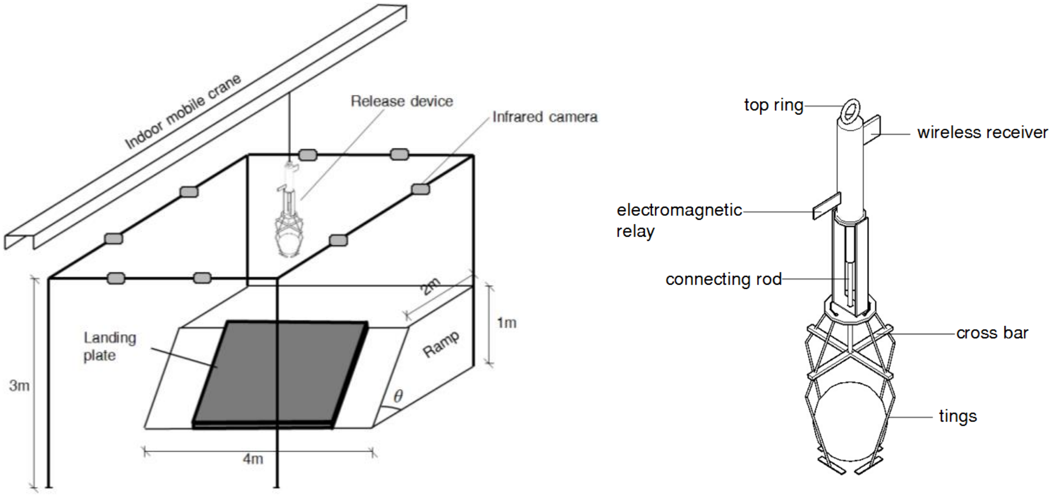
All falling rock specimens in this study were natural limestone from the China Three Gouges area and were customized in accordance with the required sizes. As shown in Fig. 2a, irregular artificial cutting facets constituted the surface of the specimens, and the edges were not smoothed; thus, the shape is called a spherical polyhedron in this study to distinguish it from the standard sphere used in other research studies. To appraise the effect of rock size on the rebound characteristics, two different diameters were considered (D=10 cm and D=20 cm), with corresponding average masses of 1.2 kg and 10 kg. The C25 concrete slabs came from a prefabricated concrete factory. As shown in Fig. 2b, each concrete slab had dimensions of 1,200 mm×500 mm×150 mm. The mechanical properties of the materials adopted in test are determined beforehand. The limestone has the Young's modulus E=41 GPa, the Poisson's ratio v=0.21 and Schmidt Hardness R=36.0. And the concrete has E=28 GPa, v=0.20 and R=32.5.

(a) Limestone rocks **(b) Concrete slabs**

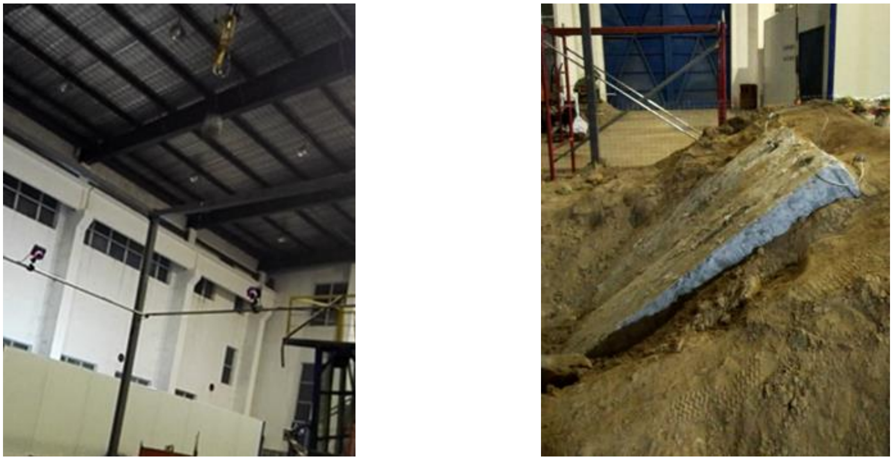
Fig. 2. Materials used in this study

2.2 Testing apparatus

The apparatus used in this study consisted of a ramp, landing plate and releasing device (Fig. 3a). The ramp was built by compacting gravelly soil and had an inclined surface with planned angles produced by artificial excavation. Then, two concrete slabs were placed upon the inclined surface to form the landing plate. One device was designed and manufactured specially to catch and release specimens of various sizes. As shown in Fig. 3b, the device had four adjustable tongs at the bottom, which could grasp spherical blocks with diameters from 8 cm to 25 cm. A wireless receiver and electromagnetic relay were installed in the upper portion of the device, offering a wireless method of altering the tong status, grasping or loosing. The device could be connected to an indoor mobile crane using the top ring, which means that the device could go up and down by managing the crane.



(a) General view **(b) Release device**



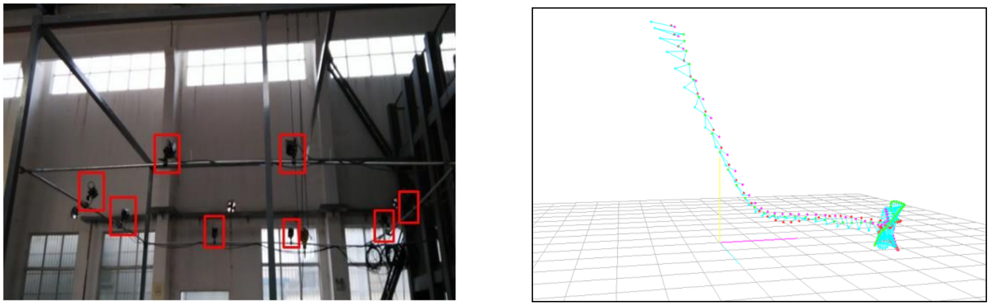
(c) A falling block **(d) Damaged slabs to be replaced**

Fig. 3. The testing apparatus

A free-fall test was performed in the experiment, and the complete process of one test is as follows. First, when one spherical polyhedron is prepared to be tested, the tongs are adjusted to accommodate the polyhedron by moving the cross bar up and down. After the sample is in the tongs, the grasping state is selected. Then, the device hanging on the indoor crane is moved to the position above the landing plate and lifted to the planned height. Next, by operating the wireless switch, the tongs are loosened, and the sample begins to fall (Fig. 3c). Finally, the sample impacts the landing plate, and its motion is recorded. The surface of the concrete slabs becomes worn with successive impacts. Once the surface occurs excessive damage as shown in Fig. 3d, the used slabs are replaced with new slabs.

2.3 Data acquisition

The spatial motion information of falling samples was obtained by the Doreal DIMS-9100(8c) Motion Capture System. This system has eight near-infrared cameras (see Fig. 4a) with an operating speed of 60 fps and can capture the spatial trails of markers attached to the surface of the sample, as shown in Fig. 4b. Then, the motion analysis program provides the spatial motion information of the sample, e.g., its position and velocity. Finally, the coefficient of restitution can be calculated according to Eq. (1-4) for subsequent analysis.



(a) Distribution of 8 infrared cameras **(b) Trails of marked points in one test**

Fig. 4. Motion capture instruments

2.4 Experimental program

Four different inclined angles θ of the landing plate (30°, 45°, 60° and 75°) were considered in this study to determine the effect of the impact angle on the coefficients of restitution. The impact angles are approximately related to the incline angle θ of the landing plate under free-fall test conditions. Limestone specimens were released at three different heights of 2.5 m, 3.5 m and 4.5 m upon the inclined concrete slabs. While, two tests do not necessarily have identical release conditions even if they have the same release height and use the same specimen because the positions on which the tongs catch the specimen may differ slightly in any two tests.

In Table 1, the initial conditions of our experiment are presented, in addition to the resulting impact velocities and angles, R_n, R_t, R_v, R_E and the rebound angles. The inertia moment of the sample was approximated to a full sphere in the calculation of rotational energy. Before impact, the angular velocities didn't exceed 3 rad/s, and the rotational energy of the rock only took up 0.01%-0.03% of the total kinetic energy in this study.

Table 1. Initial conditions and results of our experiments

Size (cm)	Inclined Angle θ (°)	Release Height (m)	Impact Velocities (m/s)		Impact Angles α (°)		R_n		R_t		R_v		R_E		Rebound Angles β (°)		Test Numbers N
			Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	

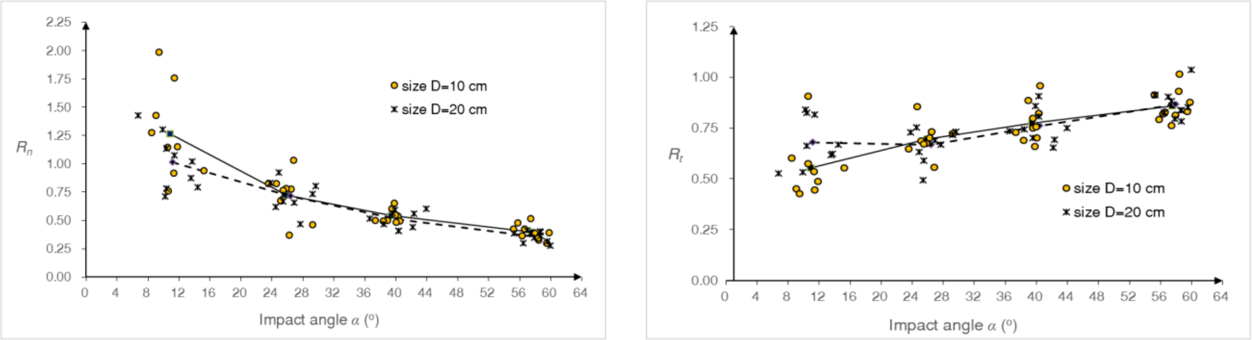
10	*30*	*2.5*	*6.50*	*0.11*	*57.25*	*1.41*	*0.42*	*0.09*	*0.80*	*0.03*	*0.56*	*0.04*	*0.36*	*0.05*	*38.31*	*6.21*	*4*
		3.5	*7.75*	*0.06*	*57.21*	*1.44*	*0.38*	*0.04*	*0.92*	*0.08*	*0.59*	*0.04*	*0.39*	*0.04*	*32.88*	*3.94*	*3*
		4.5	*9.06*	*0.16*	*58.27*	*1.33*	*0.39*	*0.03*	*0.88*	*0.04*	*0.57*	*0.01*	*0.37*	*0.02*	*35.49*	*3.23*	*3*
	45	*2.5*	*6.41*	*0.17*	*39.42*	*1.24*	*0.53*	*0.04*	*0.82*	*0.09*	*0.72*	*0.05*	*0.57*	*0.07*	*28.47*	*3.52*	*4*
		3.5	*7.72*	*0.09*	*39.07*	*0.66*	*0.52*	*0.02*	*0.78*	*0.08*	*0.69*	*0.06*	*0.52*	*0.08*	*28.48*	*2.85*	*3*
		4.5	*8.76*	*0.10*	*39.77*	*0.20*	*0.58*	*0.07*	*0.82*	*0.06*	*0.67*	*0.04*	*0.50*	*0.05*	*30.47*	*3.76*	*3*
	60	*2.5*	*6.51*	*0.21*	*25.06*	*1.21*	*0.76*	*0.06*	*0.69*	*0.03*	*0.70*	*0.03*	*0.59*	*0.04*	*27.17*	*1.77*	*3*
		3.5	*7.94*	*0.16*	*25.76*	*0.91*	*0.88*	*0.11*	*0.70*	*0.12*	*0.74*	*0.08*	*0.64*	*0.10*	*32.13*	*8.13*	*3*
		4.5	*8.80*	*0.17*	*26.99*	*1.62*	*0.53*	*0.17*	*0.70*	*0.02*	*0.67*	*0.02*	*0.51*	*0.03*	*20.92*	*5.70*	*3*
	75	*2.5*	*6.47*	*0.16*	*12.79*	*1.71*	*1.00*	*0.11*	*0.53*	*0.03*	*0.56*	*0.02*	*0.44*	*0.02*	*23.35*	*3.16*	*3*
		3.5	*7.91*	*0.08*	*10.06*	*0.73*	*1.11*	*0.27*	*0.65*	*0.19*	*0.67*	*0.17*	*0.58*	*0.21*	*18.65*	*7.38*	*3*
		4.5	*9.12*	*0.20*	*9.76*	*1.21*	*1.68*	*0.30*	*0.49*	*0.08*	*0.57*	*0.04*	*0.45*	*0.05*	*31.24*	*9.73*	*3*
20	*30*	*2.5*	*6.47*	*0.20*	*57.69*	*0.75*	*0.40*	*0.01*	*0.87*	*0.03*	*0.58*	*0.02*	*0.38*	*0.02*	*35.77*	*1.49*	*3*
		3.5	*7.80*	*0.11*	*58.09*	*1.44*	*0.31*	*0.03*	*0.89*	*0.11*	*0.54*	*0.03*	*0.34*	*0.02*	*29.41*	*4.18*	*3*
		4.5	*8.89*	*0.19*	*58.02*	*1.63*	*0.37*	*0.03*	*0.85*	*0.05*	*0.55*	*0.04*	*0.34*	*0.05*	*34.90*	*3.15*	*4*
	45	*2.5*	*6.47*	*0.21*	*40.97*	*3.14*	*0.56*	*0.03*	*0.73*	*0.02*	*0.66*	*0.02*	*0.53*	*0.04*	*33.89*	*4.48*	*3*
		3.5	*7.96*	*0.05*	*40.05*	*1.55*	*0.49*	*0.05*	*0.70*	*0.04*	*0.62*	*0.04*	*0.46*	*0.06*	*30.54*	*3.03*	*3*
		4.5	*8.74*	*0.13*	*40.16*	*0.22*	*0.47*	*0.09*	*0.86*	*0.04*	*0.72*	*0.04*	*0.56*	*0.05*	*24.68*	*3.77*	*3*
	60	*2.5*	*6.33*	*0.05*	*26.76*	*2.40*	*0.76*	*0.08*	*0.72*	*0.02*	*0.73*	*0.03*	*0.65*	*0.06*	*28.15*	*2.71*	*3*
		3.5	*7.57*	*0.09*	*24.96*	*0.38*	*0.74*	*0.13*	*0.66*	*0.07*	*0.68*	*0.05*	*0.56*	*0.07*	*27.64*	*5.60*	*3*
		4.5	*8.76*	*0.19*	*27.44*	*1.57*	*0.64*	*0.12*	*0.63*	*0.10*	*0.63*	*0.07*	*0.49*	*0.11*	*28.29*	*6.11*	*3*
	75	*2.5*	*6.08*	*0.16*	*11.39*	*1.52*	*0.79*	*0.07*	*0.67*	*0.13*	*0.71*	*0.09*	*0.65*	*0.09*	*14.32*	*5.67*	*3*
		3.5	*7.30*	*0.29*	*11.56*	*2.05*	*1.08*	*0.21*	*0.64*	*0.16*	*0.69*	*0.11*	*0.59*	*0.15*	*19.58*	*5.83*	*3*
		4.5	*7.97*	*0.26*	*10.63*	*2.90*	*1.17*	*0.18*	*0.62*	*0.16*	*0.68*	*0.12*	*0.58*	*0.15*	*19.41*	*3.23*	*3*

3 Analysis of the results and comparison with existing studies

3.1 Effect of the impact angle on the coefficients of restitution based on our tests

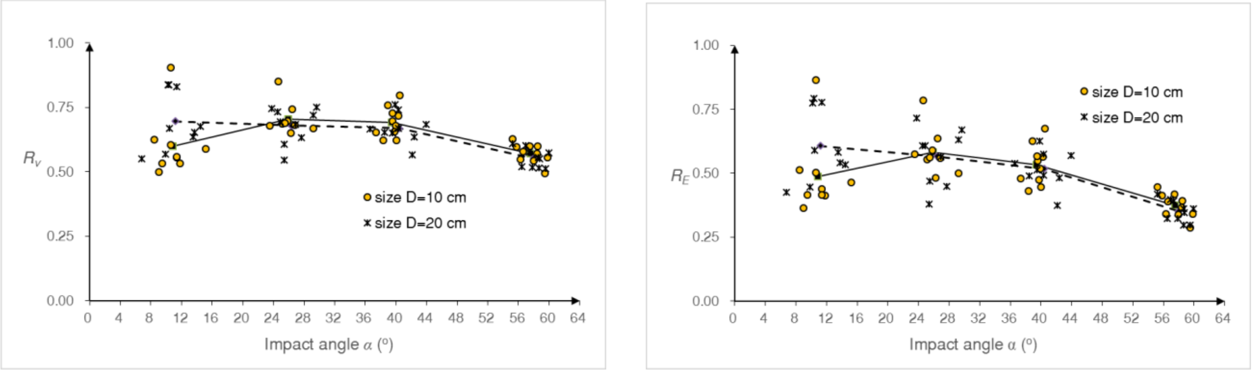
Although the mean values and standard deviations have been calculated in terms of various release conditions, data points are considered in this section to provide a broad perspective for an evaluation of the effect of the impact angle. Four different inclined angles of the landing plate (θ=30°, 45°, 60° and 75°) induce four intervals of impact angles, $55° \leq \alpha < 60°$, $36° \leq \alpha < 44°$, $23° \leq \alpha < 30°$ and $6° \leq \alpha < 15°$. The mean value of the coefficients of restitution are computed for the four intervals. In Fig. 5, solid lines are adopted to represent the mean values for samples with size D=10 cm, and dashed lines represent the mean values for size D=20 cm.

The effect of impact angle on the normal coefficient of restitution R_n is shown in Fig. 5a. When the impact angle is smaller than 15°, the values of R_n range from 0.709 to 1.989, and more than 60% of the values of R_n are larger than 1.0. A larger impact angle tends to produce a smaller value of R_n and reduce the discreteness. Initially the solid line is above the dashed line, although the gap narrows with increasing in the impact angle. When the impact angle is larger than 30°, these two lines do not exhibit a clear difference. Therefore, small specimens are more likely to have a higher R_n than large specimens with small impact angles, and the effect of the rock size on R_n can be neglected when the impact angle is more than 30°.



(a) R_n versus the impact angle *a*

(b) R_t versus the impact angle *a*



(c) R_v versus the impact angle *a*

(d) R_E versus the impact angle *a*

Fig. 5. Effect of the impact angle on the coefficients of restitution

As Fig. 5b shows, the impact angle has a different effect on the tangential coefficient of restitution R_t compared to R_n. In the first place, the discreteness of data points hasn't been reduced as the impact angle increases. Then, R_t increases slightly with increasing in the impact angle. In the first impact angle interval, the solid line is below the dashed line, which implies that small specimens gain a lower R_t than large specimens with small impact angles. Until the impact angle reaches 23°, the two lines have no distinct difference to be distinguished. Overall, the mean value lines in Fig. 5b seem to accord with the linear correlation between R_t and the impact angle α (Wu, 1985).

Furthermore, the kinematic coefficient of restitution R_v versus the impact angle is plotted in Fig. 5c. As the impact angle increases, the peak values of R_v of the four impact angle intervals fall down gradually, and R_v become concentrated. However, the mean values present a more complicated trend in Fig. 5c. Except that the solid line rises from the first impact angle interval to the second, the mean values have a decline in general. The decline is tiny from the second impact angle interval to the third, while it is apparent from the third to the fourth. Taking the small gap between the mean lines into account, bigger specimens are easier to gain a small R_v than small specimens when the impact angle is more than 23 °.

Finally, the effect of the impact angle on the coefficient of kinetic energy restitution R_E is illustrated in Fig. 5d. Similar to Fig. 5c, the peak values of R_E of the four impact angle intervals decrease with increasing in the impact angle. But the discreteness of data points does not disappear clearly until the fourth impact angle interval. The trends for the mean value of R_E are similar to R_v. However, the decline in the mean values of R_E is more intuitive than R_v from the second impact angle interval to the third. And the gap between the mean lines of R_E is narrower than R_v with larger impact angles. Although some scholars (Chau 2002; Asteriou 2012) suggested that smaller impact angles induce less kinetic energy loss and higher R_E, the deduction may be not suitable for small impact angles in this study.

Besides the effect of the impact angle on the four coefficients of restitution, other interesting phenomenon can be observed in Fig. 5. Two sizes are adopted in this experiment to evaluate the effect of rock size on the rebound characteristics. Except for smaller impact angle, the gaps between the two mean lines in Fig. 5 are much tiny compared to the magnitudes of restitution coefficients. Therefore, the four coefficients of restitution seem to be independent of the sample sizes in our test when the impact angle exceeds 23 °, which could be attributed to the test conditions. As Farin et al. (2015) noted, the thickness of the impacted objective is an important factor in determining whether the coefficients of restitution change with the boulder size. When the impacted objective has a large thickness compared to the boulder size, the coefficient of restitution is independent of the boulder size. In this study, the impacted objective is concrete slabs fixed in the ground, which has enough thickness to eliminate the effect of rock size on the coefficients of restitution in case of large impact angles.

In addition, data points for all coefficients except R_t become concentrated as the impact angle increases. In the first impact angle interval, the diversity of R_n is clearly larger than the other three coefficients. However, when the impact angle exceeds 23 °, the lowest diversity occurs for R_n, and the second is for R_v. Various functions were considered to match data points, but no function can provide a correlation coefficient R^2 more than 0.60 in terms of R_t, R_v and R_E for all options considered. Power function provides the best R^2 in matching data points of R_n, which reaches 0.80. So, the scaling law to describe data points is abandoned in this study. Although Asteriou et al. (2012) suggested that R_v was more suitable than R_n for use in correlations with the impact angles, it is invalid in this study, which is caused by the variations of test conditions.

3.2 Comparison with existing small scale experiments

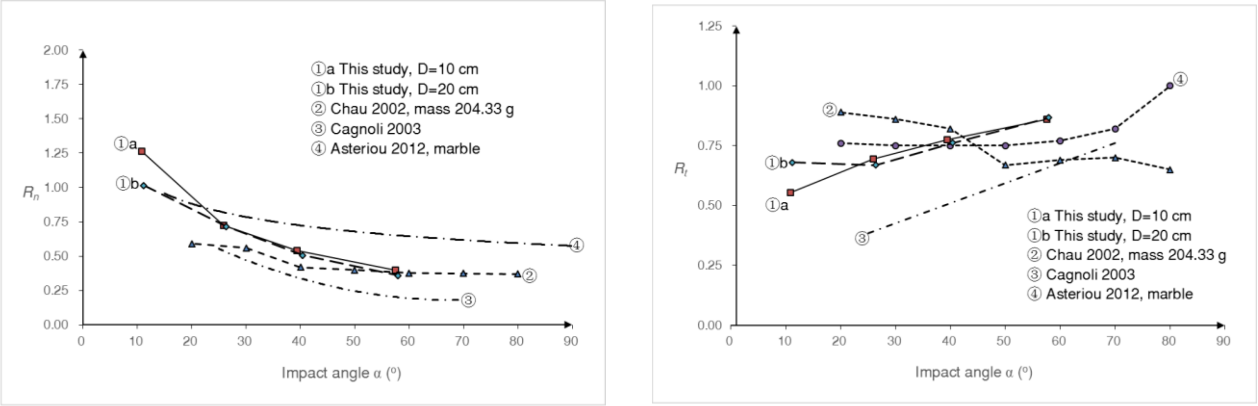
In this section, the effect of the impact angle on the coefficient of restitution obtained in this study is compared with some existing small scale experiments, to determine the effect of the test scale. Tests conducted by Chau et al. (2002), Cagnoli and Manga (2003), Asteriou et al. (2012) are selected here for the data availability. The test conditions of those studies are provided in Table 2 in comparison with this study. This study mainly differs from the other studies in terms of the size and mass of the samples.

Table 2. Test conditions of the previous studies versus our experiment

	This study	Chau et al, 2002	Cagnoli and Manga, 2003	Asteriou et al., 2012
Sample shape	Spherical Polyhedron	Sphere	Cylinder	Cubic
Sample material	limestone	dental plaster	pumice	marble/sandstone/marl
Landing plate material	concrete	dental plaster	pumice	marble
Sizes (mm)	Diameter: 100/200	Diameter: 18.35/60/60	Basal diameter: 5.5 Length: 8.9	Edge:20
Mass (g)	1200/10000	6.05/153.64/204.33	0.11	19.9/20.3/16.5
Impact velocities (m/s)	6.7-9.3	4-5.65	24.88	3
Impact angles	6°-60°	20°-80°	18º-74°	16°-77º
Roughness of the target surface	uneven	flat	flat	smooth
Sharpness of the specimens	rear edges	no edges	rear edges	smooth edges

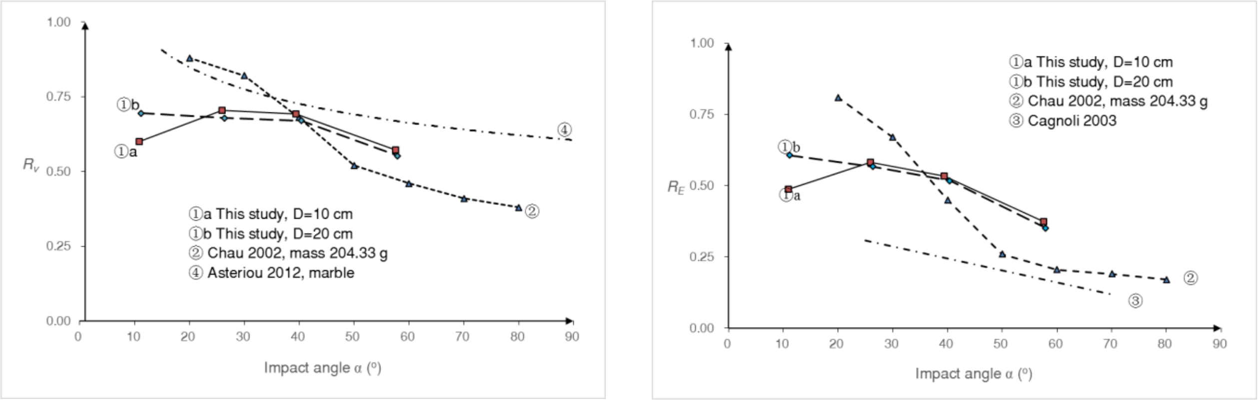
Although the previous studies imposed various test conditions, they provided references for us to evaluate the effect of the test scale. The effects of the impact angle on R_n, R_t, R_v and R_E provided by Chau et al. (2002), Cagnoli and Manga (2003), Asteriou et al. (2012) are plotted in Fig. 6 with this study. In Chau's results, the specimen with a mass of 204.33 g was selected because that mass is closest to those of our samples. In Asteriou's results, we chose a marble specimen as the reference because marble and limestone have nearly the same hardness. In the absence of detailed data, only the trend line results in the related literature are extracted and redrawn in Fig. 6 to make a comparison. Different line styles are adopted for trend lines in Fig. 6. The lines with data markers are the mean value lines, while those lines without data markers are fitting lines. Two lines, R_v versus the impact angle a for Cagnoli and Manga's test, and R_E versus the impact angle a for Asteriou's test, are absent in Fig 6 because the literatures didn't provide them. In addition, ①a and ①b are used to represent results for D=10 cm and D=20 cm in this study, respectively.

Although results of the previous studies and our tests are quite different when they are plotted in one figure, some general trends could be observed. First, all of the trends in R_n versus the impact angle are consistent (see Fig. 6a): R_n decreases with increasing in the impact angle. Asteriou's tests offered the maximum R_n in most cases, which can be attributed to the lighter mass and lower impact velocities adopted in the tests. The small R_n values in Cagnoli and Manga's tests were due to the weak strength of pumice whose damage upon impact dissipates kinetic energy, and the impact velocity in this test is much higher than the others. Compared to the other tests, our results produce the steepest descent in the beginning of the trend line. Linear function had been suggested to be used to describe the correlation of R_n and the impact angle α in several reports (Wu, 1985; Richards et al., 2001), although we cannot make a definitive conclusion that linear functions are the best choice. In Fig. 6a, the fitting curve ③ is a second-order polynomial, and the fitting curve ④ is a power function. As mentioned above, it is also found that the best correlation coefficient R^2 is provided by power function when matching R_n in this study.



(a) R_n versus the impact angle *a*

(b) R_t versus the impact angle *a*



(c) R_v versus the impact angle *a*

(d) R_E versus the impact angle *a*

Fig. 6. Comparison with existing small scale experiments

Next, Fig. 6b indicates that the trends of R_t versus the impact angle are scattered. Except for Cagnoli's result, the R_t obtained in the tests are all in the range from 0.5 to 1.0. Wu (1985) suggested that R_t may decrease linearly with increasing in the slope angle θ. In other words, R_t may experience an improvement with increasing in the impact angle α, which is in line with results of all experiments except Chau's. And Cagnoli and Manga matched R_t using a linear function, which resulted in the fitting curve ④ in Fig. 6b. Therefore, the improvement effect of the impact angle α on R_t is valid in most cases. Besides, if the impact angle is less than 45°, there is an apparent gap between Cagnoli and Manga's result and the other tests. Although the results of the other three experiments are different, the variation ranges of R_t occurs regardless of the test conditions.

Finally, the trends of R_v and R_E versus the impact angle are shown in Fig. 6c and 6d, respectively. Cagnoli and Manga's result is not involved in Fig. 6c for its absence, and for the same reason Asteriou's result is not involved in Fig. 6d. Four unique trend lines are plotted in Fig. 6c, although R_v exhibits a descending trend overall, which means that R_v is reduced in most cases as the impact angle α increases. Similar to R_v, all experiments produce downward trend lines for R_E, except the initial ascent stage in line ①a, which implies that increasing the impact angle induce more kinetic energy dissipation. But, the trend lines in Fig. 6d are scattering. Clearly, the trend lines for R_v and R_E are more likely to be influenced by the test conditions than R_n and R_t. In Fig. 6c the fitting curve ④ is a power function, and in Fig. 6d the fitting curve ③ is a linear function. The difference in the trend lines is apparent for the listed experiments, and we cannot have a conclusion which type of functions should be recommended to match R_v and R_E.

In conclusion, various experimental conditions induce different results for R_n, R_t, R_v and R_E, although there are certain trends that occur regardless of the test conditions. The normal coefficient of restitution R_n, kinematic coefficient of restitution R_v and kinetic energy coefficient of restitution R_E all decrease with increasing in the impact angle, while the tangential coefficient of restitution R_t increases as the impact angle increases in most cases. Power function appears suitable to be used in fitting data points of R_n, while its validity is worth to be further verified by other studies.

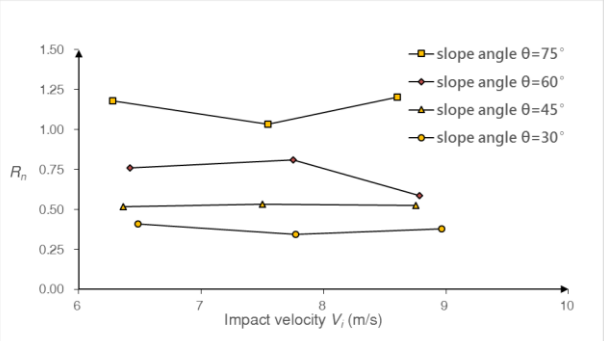


Fig. 7. R_n versus the impact velocity v_i in this study

In addition, Asteriou's test provided the highest trend lines of R_n and R_v in Fig. 6, while Cagnoli and Manga's test provided the lowest trend lines of R_n, R_t and R_E. Asteriou's experiment was conducted using the lowest impact velocity in Table 2, while the highest impact velocity are adopted by Cagnoli and Manga. And Cagnoli and Manga employed pumice, which has a much weaker strength compared to sample materials in other tests. Asteriou and Tsiambaos (2018) has just noticed that R_n reduces when increasing the impact velocity, and increases as the material become harder, which partly accounts for the difference between Asteriou's and Cagnoli and Manga's test. However, we cannot make a definitive conclusion which factor in Table 2 is the main reason for the magnitude difference in the coefficients of restitution between the tests compared. The tests compared differ from each other in multiple test conditions as Table 2 lists, therefore the estimation of the effect of one specific factor on the magnitude of the coefficient of restitution is unreasonable using their data together. To evaluate the effect of the impact velocity in this study, Fig. 7 plots the mean value of R_n versus the impact velocity with different slope angles. No determined trend of R_n appears, for the limited variation range of the impact velocity.

4 Direction transitions of translational velocities and rotation

4.1 Direction transitions of translational velocities

Taking the ratio between the rebound angle and the impact angle β/α as a reference, the direction transition of the translational velocity versus the impact angle are illustrated in Fig. 8. Assuming that the falling rock is spherical and no energy dissipation occurs during the collision, the rebound angle should theoretically be equal to the impact angle, which would result in the data points lying on the red line $\beta/\alpha=1$ in Fig. 8.

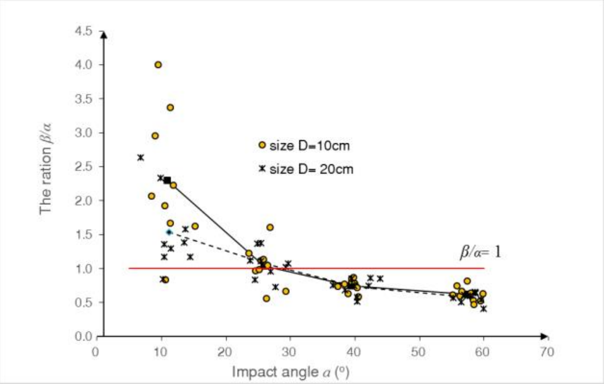
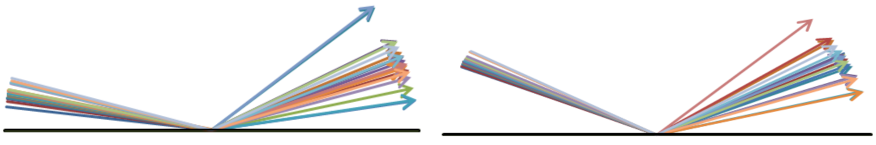


Fig. 8. The ratio between the rebound angle and the impact angle β/α versus impact angle

However, the test results are almost entirely located above the line in the first impact angle interval, and nearly 50% of the test results are above the line in the second interval. The data points are stably located below the line until the impact angle reaches to 36 °. As the impact angle increases, the ratio between the rebound angle and the impact angle β/α appears a clear reduction, and the discreteness of data points decreases. The mean values of β/α are still represented by a solid line for samples with size D=10 cm, and a dashed line for size D=20 cm. The two mean values line have little difference from the second interval to the fourth. But, under small impact angle conditions, a smaller sample is more likely to have a larger β/α than a bigger sample.

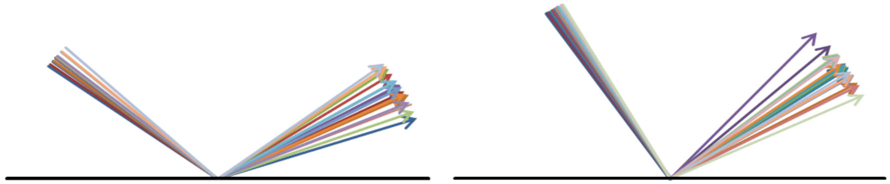
A rebound angle greater than the impact angle was also observed by Cagnoli and Manga (2003), which does not violate the energy dissipation rule. The experimental results presented in Section 3.1 demonstrated that in this study the kinetic energy loss constituted 40-65% of the total kinetic energy for many data points in the first impact angle interval, and constituted 35-55% in the third interval. Therefore, the ratio between the rebound angle and the impact angle cannot be directly used as a reference in estimating whether the energy loss level is high or low.

This phenomenon implies that the rebound motion may have an unexpected direction of translational velocity. Fig. 9 plots the direction transition of translational velocity caused by the impact, in which diagrams are individually drawn for four impact angle intervals. For a uniform expression, the landing plate is denoted as the bottom black line. Although the impact velocity directions are concentrated for each impact angle interval, the rebound velocity directions vary considerably. The variation for the interval $36°<\alpha< 44°$ is the smallest of all intervals.



(a) Impact angle $6°<\alpha<15°$

(b) Impact angle $23°<\alpha<30°$

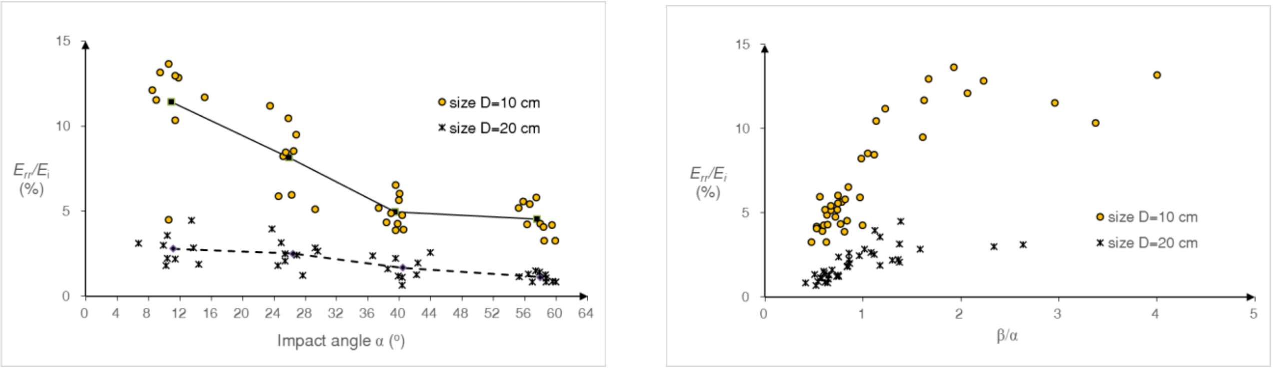


(c) Impact angle $36^\circ \leq \alpha < 44^\circ$ **(d) Impact angle $55^\circ \leq \alpha < 60^\circ$**

Fig. 9. Direction transitions of translational velocities induced by impacts

4.2 The rotation caused by the impact

Except the direction transition of translational velocity, the rotation is another significant consequence of the impact. Despite little rotation before impact, the samples occurred an observable rotation after impact in this study, and the angular velocities were recorded and involved in the calculation of the kinetic energy coefficient of restitution R_E in Table 1. Considering that the magnitudes of kinetic energy before impact varied in this study, the percentage between E_{rr} and E_i is used to denote how much kinetic energy is dissipated in rotation after the impact. As mentioned in Section 1.1, E_i is the total kinetic energy before impact, and E_{rr} is the rotation energy after impact.



(a) E_{rr}/E_i versus the impact angle **(b) E_{rr}/E_i versus β/α**

Fig. 10. Rotational energy caused by the impact

Fig. 10a shows the effect of impact angle on E_{rr}/E_i. A solid line represents the mean values for specimens with size D=10 cm, while a dashed line is for size D=20 cm. At firstly, the difference is most remarkable between two sample sizes in Fig. 10a. E_{rr}/E_i ranges from 3.3% to 13.7% for size D=10 cm, and ranges from 0.7% to 4.5% for size D=20 cm, which means that small samples are more likely to have a high E_{rr}/E_i than larger samples. Next, E_{rr}/E_i reduces as the impact angle increases. For size D=10 cm, E_{rr}/E_i experiences a steep decline from the first impact angle interval to the third, then decreases gently to the fourth. For size D=20 cm E_{rr}/E_i has a gradual reduction all the time, which may be attributed to its small variation range. At last, the improvement of the impact angle results in more concentrated data points, although data points for size D=10 cm are always more scattering than size D=20 cm. Both the impact angle and the sample size have an important impact on E_{rr}/E_i

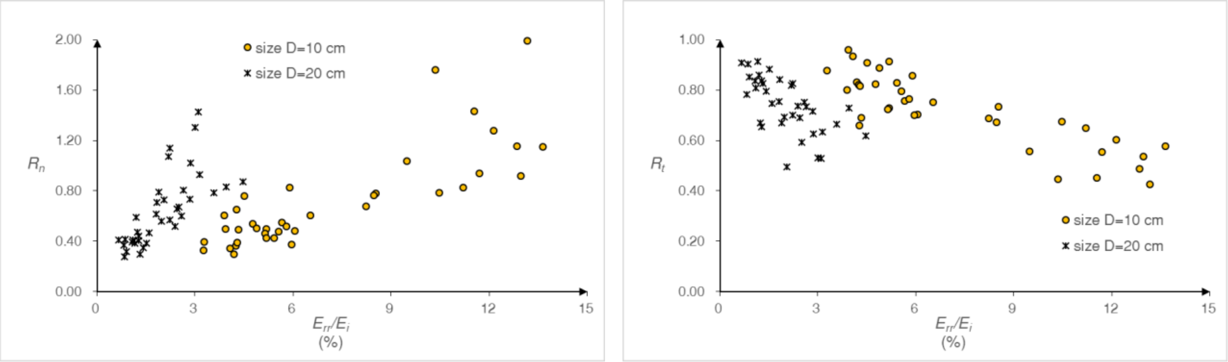
in this study. In conclusion, larger samples are more likely to have a stead and small E_{rr}/E_i than small samples, and a large impact angle leads to a small E_{rr}/E_i.

As mentioned in Section 4.1, the unexpected direction transition of translational velocity always happens when the impact angle is small. The correlation between the direction transition of translational velocity and E_{rr}/E_i is investigated by using β/α (the ratio between the rebound angle and the impact angle) as a reference. Fig. 10b illustrated the trends for E_{rr}/E_i versus β/α. E_{rr}/E_i increases as the ratio β/α increases. If β/α is smaller than 1.0, E_{rr}/E_i are much concentrated, which ranges from 3.3% to 6.5% for size D=10 cm and ranges from 0.7% to 2.6% for size D=20 cm. With increasing in β/α, the data points become scattering. Moreover, the improvement of E_{rr}/E_i appears being terminated when β/α reaches a specific value. So, we can have a conclusion that a strong correlation occurs between E_{rr}/E_i and β/α. For a given impact angle, larger rebound angles means that more kinetic energy be converted to rotational energy during the collision.

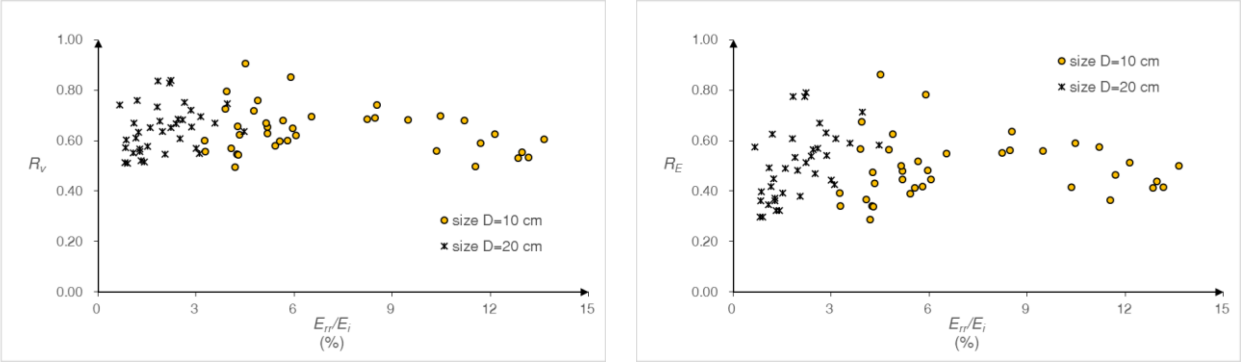
4.3 The correlation between the coefficients of restitution and the rotation

The rotation plays an important role in energy dissipation during impact, especially for small samples. The percentage between the resulting rotational energy and the original total kinetic energy decreases as the impact angle increases. The correlation between E_{rr}/E_i and the coefficients of restitution is investigated in this section, to evaluate the effect of rotation.

Fig. 11 plots the coefficients of restitution versus E_{rr}/E_i for this study. The four coefficients fall into two categories according to their responses to E_{rr}/E_i. The first category includes R_n and R_t, two most commonly used coefficients of restitution, which appears a strong correlation with E_{rr}/E_i. As E_{rr}/E_i increases, R_n increases but R_t decreases, which verified Broili's deduction (1973). The rotation generated from impact results in an increased normal velocity and reduced tangential velocity. Furthermore, more kinetic energy being converted to rotational energy during the collision, higher R_n and lower R_t we have. In Fig. 11a and 11b, data points for two sizes are not mixed, which can be attributed to the effect of sample sizes on the magnitude of E_{rr}/E_i. Besides that, data points become more scattering with increasing in E_{rr}/E_i. R_v and R_E belong to the second category. There is no remarkable correlation between them and E_{rr}/E_i as shown in Fig. 11c and 11d, so R_v and R_E are independent of the rotation motion in this study. In conclusion, the improvement in the percentage of kinetic energy converted to rotational energy leads to a larger R_n and a smaller R_t, while it has no distinct influence on R_v and R_E.



(a) R_n versus E_{rr}/E_i **(b) R_t versus E_{rr}/E_i**



(c) R_v versus E_{rr}/E_i **(d) R_E versus E_{rr}/E_i**

Fig. 11. The influence of E_{rr}/E_i on the coefficients of restitution

As illustrated in Fig. 10a, more kinetic energy is converted to rotational energy during the collision with a smaller impact angle. Considering the effect of E_{rr}/E_i on R_n and R_t, a smaller impact angle is more likely to have a high R_n and a low R_t than a larger impact angle. Therefore, R_n typically decrease with increasing in the impact angle, and R_t increases as the impact angle increases. When the impact angle is small, two sample sizes appear a clear distinction in E_{rr}/E_i as shown in Fig. 10a, which results in the difference in the mean values of R_n and R_t between two sizes in the first impact angle interval in Fig. 5a and 5b.

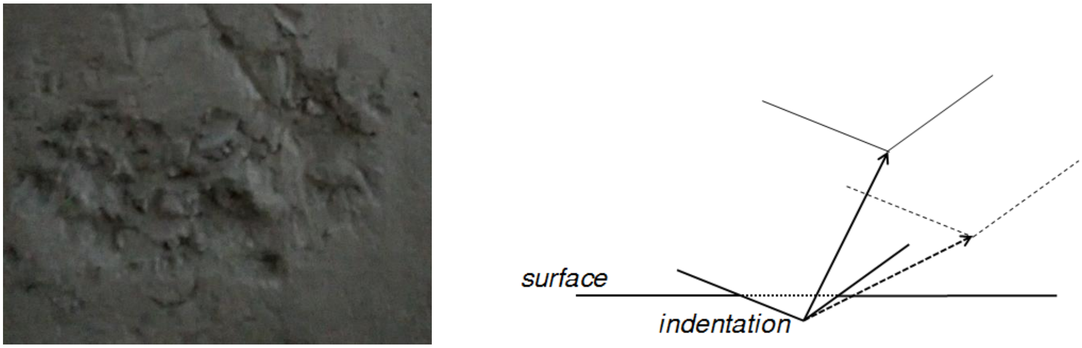
5 Discussion

The test results demonstrated the correlation between the rotation and the effect of the impact angle on the coefficients of restitution. Under free fall conditions, a higher percentage of kinetic energy converted to rotational energy always induces a higher R_n and a lower R_t. The percentage can be associated with the ratio between the rebound angle and the impact angle β/α. As the impact angle decrease, the ratio β/α increases, then more kinetic energy is converted to rotational energy. In this section, the reason why a small impact angle is easier to have a high β/α and its consequences are discussed.

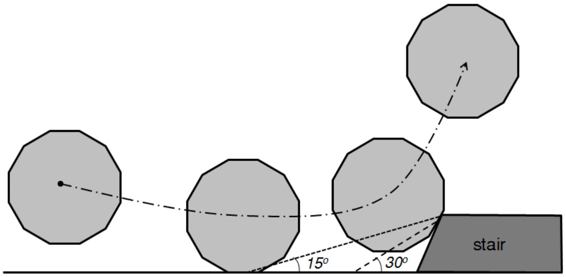
5.1 The main reason for the high β/α in case small impact angles

When the impact angle is small, the rebound angle is easier to exceed the impact angle and causes a high β/α, which can be associated with the impact orientation and the damages caused by the impact. The sample has irregular cutting facets and rear edges in this study, while the landing plate was made of concrete slabs of a smaller hardness compared to the falling samples. Suppose that the spherical polyhedron impacts the landing plate with a corner or an edge, the damages will happen. Fig. 12a shows the indentations on the surface caused by the impacts. The configuration of indentation is simplified as Fig. 12b to evaluate the effect of the indentation on the rebound direction.

Once the impact compression ends, the indentation is formed completely and the rebound motion starts. The rebound angle is constrained by the border of the resulting indentation. The restriction is less susceptible to a small rebound angle, because a translational motion along the dashed arrow indicates additional penetration. Theoretically, the rebound angle will be less than the impact angle accounting for energy loss. When the impact angle is sufficiently large to generate a rebound angle as the solid arrow, the border imposes no constraints on the rebound motion, and the sample can leave with the default rebound angle. But, when the impact angle is small and generate a default rebound angle as the dashed arrow, rotation motion must be involved to overcome the constraint, then an unexpected larger rebound angle happens. Thus, the penetration caused by the impact may contribute to the high β/α in case of small impact angles.



(a) Indentations caused by the impacts **(b) Simplified configuration of indentation and its restriction on motion**



(c) Effect of macro roughness on the rebound angles

Fig. 12. Effect of indentation on the rebound angles

Another important factor to generate a high β/α is the macro roughness of the landing plate, which comes from repeated damages on the slab surface. Assuming that the macro roughness of the landing plate is represented as a small stair in Fig. 12c, the interaction between the falling sample and surface may have two stages in certain situations. The sample impacts the surface before the stair and starts leaving in the first stage. Then, the sample contacts the stair and the velocity changes again in the second stage. The time interval between the two stages is so short that the two stages appear to finish simultaneously. The probability that two stages interaction happens is related to the magnitude of the impact angle. As Fig. 12c illustrates, if the default rebound angle is 15°, the stair can affect the rebound motion if the sample contacts the surface within 3.73 times the stair height before the stair. As the default rebound angle increases, the surface region that the stair can affect the rebound motion decreases. Considering that a smaller impact angle will, in theory, induce a smaller rebound angle, the reduction of impact angle must improve the risk of the sample contacting the stair. When the impact angle is small, the sample has more possibility to have a two stages interaction and leave the plate with a rebound angle larger than the impact angle, and a high β/α will happen.

In conclusion, the restriction from the configuration of the indentation, as well as the macro roughness caused by repeated damages, are more likely to affect the rebound motion when the impact angle is small. As a consequence, the rebound angle is easy to exceed the impact angle in case of small impact angle, which results in a high β/α ultimately.

5.2 Interpretation of normal coefficient of restitution R_n larger than 1.0

Of the various consequences of the rebound angle being greater than the impact angle, high values of the normal coefficient of restitution R_n may be remarkable. Engineers usually take 1.0 as the upper bound of R_n in computer codes, whereas several scholars had reported R_n values larger than 1.0 (Azzoni et al., 1992; Paronuzzi 2009; Spadari et al., 2012). In this section the relationship between R_n and the direction transition of translational velocity is investigated.

Considering that the rotation before impacting is little in this study, the normal coefficient of restitution R_n can be expressed as Eq. (5) based on the basic definition in Section 1.1.

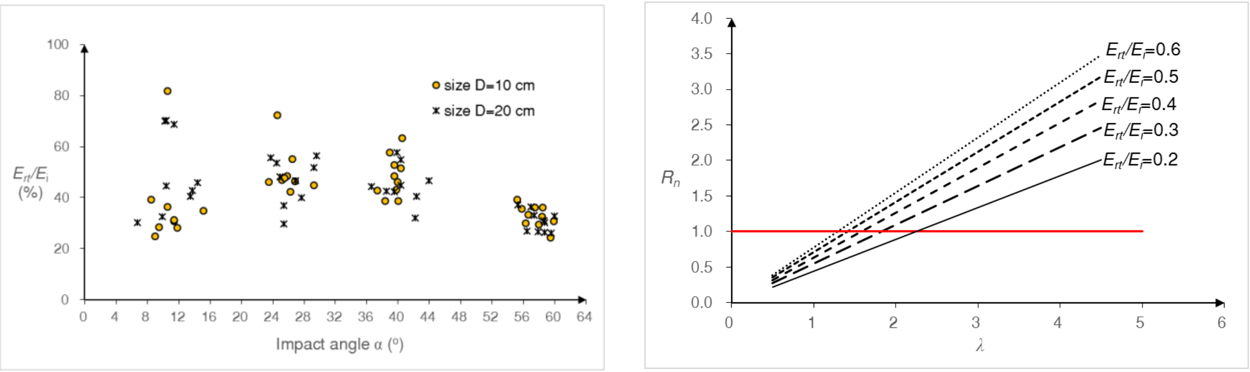
$$R_n = v_{nr} / v_{ni} = \sqrt{E_{rt} / E_{it}} \times \left(\sin\beta / \sin\alpha \right) = \sqrt{E_{rt} / E_i} \times \left(\sin\beta / \sin\alpha \right) \quad (5)$$

By introducing an angle coefficient

$$\lambda = \sin\beta / \sin\alpha \quad (6)$$

Eq. (6) can be simplified as

$$R_n = \lambda \sqrt{E_{rt} / E_i} \quad (7)$$



(a) E_{rt}/E_i versus the impact angle **(b) R_n versus the angle coefficient λ**

Fig. 13. The conditions of R_n larger than 1.0

E_{rt}/E_i, the ratio between the translational energy after impact and the total kinetic energy before impact, is plotted in Fig 13a with respect to the impact angle. As the impact angle increases, the mean value of E_{rt}/E_i increases when the impact angles are smaller than 36°, then it decreases. While, the peak values of E_{rt}/E_i of the four impact angle intervals fall down gradually with increasing the impact angles. In this study, the values of E_{rt}/E_i are located in the range (0.20, 0.60) in most cases, which provides us with a reference to explore the conditions of R_n larger than 1.0.

Fig. 13b plots the relationship between R_n and the angle coefficient λ under different E_{rt}/E_i. The value of R_n increases when increasing the angle coefficient λ. Even if E_{rt}/E_i is only 0.2, R_n is greater than 1.0 when $\lambda>2.24$. An extremely large rebound angle is not needed to generate such a λ when the impact angle is small. For example, when the impact angle is 12° and 15°, a rebound angle of 27.8° and 35.5° is sufficient to obtain $\lambda>2.24$. Assuming that E_{rt}/E_i is unchanged, a case that the rebound angle is larger than the impact angle must lead to a higher R_n. Although the value of λ corresponding to R_n=1.0 varies with E_{rt}/E_i, the condition $\lambda>1.0$ is required to obtain R_n greater than 1.0. As shown in Fig. 13b, R_n cannot exceed 1.0 if the rebound angle is lower than the impact angle. As discussed in previous sections, small impact angles are easy to result in unexpected large rebound angles. If the angle coefficient λ formed by the rebound and the impact angle is sufficiently large, R_n will exceed 1.0 even though E_{rt}/E_i is small. Furthermore, assuming a constant E_{rt}/E_i, the reduction in the impact angle decreases the threshold value of the rebound angle that should be satisfied to achieve an R_n in excess of unity, which means that smaller impact angles are more likely to yield R_n larger than 1.0.

5.3 Relation between the normal coefficient of restitution and the kinetic energy loss

A smaller impact angle is easier to have a high β/α and a high percentage of kinetic energy converted to rotational energy, then induces a higher R_n. However, the kinetic energy coefficient of restitution R_v appears independent of the percentage of kinetic energy converted to rotational energy. Therefore, simply treating a higher R_n as a symbol of lower kinetic energy loss maybe unreasonable. Stronge (1991) had indicated that in the valuation of kinetic energy dissipation, the normal coefficient of restitution is only reliable for nonfrictional collisions. Under frictional collisions conditions, the total kinetic energy may have a paradoxical increase, if the normal coefficient of restitution is adopted as the unique reference. As shown in Fig. 14, the correlation between R_n and R_E is more complicated in this study, which verifies Stronge's argument.

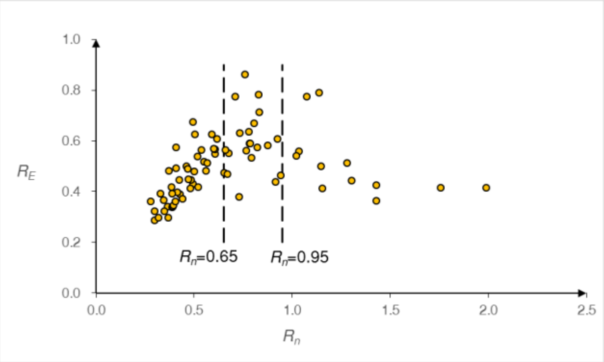


Fig. 14. R_E versus R_n

Increasing R_n will increase R_E initially but decrease it overall. For simplicity, two boundaries (R_n=0.65 and R_n=0.95) are added in Fig. 14. The kinetic energy coefficient of restitution R_E increases with increasing R_n when $R_n<0.65$, in agreement with the relationship between R_n and the energy loss level based on the elastic-plastic response analysis. If R_n is greater than 0.95, a larger R_n indicates a smaller R_E. High values of R_n are associated with unexpected large rebound angles in this study, which means that the unexpected large rebound angles can be related to higher level of kinetic energy loss. R_E is disordered if R_n lies in (0.65, 0.95), which is caused by the two different trends meeting. Therefore, the normal coefficient of restitution R_n cannot be directly used in the evaluation of the kinetic energy dissipation level.

5.4 The difficulty in introducing the effect of impact angle into trajectory simulation

This study, as well as the previous experiments, has demonstrated that the variation of the coefficients of restitution in terms of the impact angle are significant. For this reason, the impact angle should be involved in determining the coefficients of restitution in rockfall trajectory simulation. However, some problems cause a barrier to develop a reasonable way to account for the effect of the impact angle in computer simulation.

First of all, although the test scales and conditions have little influence on the general laws that the impact angle affects the coefficients of restitution, it is difficult to construct a uniform formula to reflect the effect under various test conditions. Take R_n as an example, the effect of the impact angle on R_n had been formulated by several different functions, such as the linear function (Wu, 1985; Richards et al., 2001), power function (Asteriou et al. 2012) and second-order polynomial (Cagnoli and Manga, 2003). In this study, power function provides the best correlation coefficient in fitting data points of R_n. Furthermore, the mathematic expression regarding the effect of the impact angle on the coefficients of restitution is abandoned in more experiments (Chau et al., 2002; James, 2015). Therefore, we can’t have a conclusion which type of functions is the best choice to describe the effect, and whether a uniform expression occurs is questionable.

Another problem comes from the discreteness of data points. Given the impact angle, the discreteness of data points determines the reliability of the rebound velocity estimated by adopting a typical value of the coefficients of restitution. For all coefficients of restitution, the discreteness of data points experiences a reduction if the impact angle increases in this study. When the impact angle is large, it may be acceptable to predict the rebound using a typical value of the coefficients of restitution, e.g. the mean value. However, the data points are extremely scattering under small impact angle condition, which means that using a typical value in the simulation maybe unreliable.

Therefore, further researches should be carried on to establish a reasonable and comprehensive method to reflect the effect of the impact angle on the coefficients of restitution in rockfall trajectory simulation. The stochastic model has more potential in achieving this target, because it accounts for the variation of the coefficients of restitution in terms of various factors based on data collection (Jaboyedoff et al., 2005; Frattini et al., 2008; Bourrier et al., 2009; Andrew and Oldrich, 2017).

6 Conclusions

The coefficients of restitution are critical parameters in the predication of rockfall trajectory by computer codes. Both the terrain characteristics and kinematic parameters can significantly affect the coefficients of restitution. The effect of the impact angles on the coefficients of restitution have been observed and some laws have been concluded in a series of tests. Until now, the existing laboratory tests have largely been limited to small scale tests, and whether the previous conclusion is valid for different scale tests is uncertain. The role of rotation is still unresolved in the effect of the impact angle on the coefficient of restitution.

In the present study, laboratory tests were performed using a 3D motion capture system. Spherical limestone polyhedra with diameters of 10 cm and 20 cm were taken as samples, and C25 concrete slabs were adopted to form the landing plate. By altering the release height and the inclined angle of the landing plate, the effects of the impact angle on the coefficients of restitution were estimated under freefall test conditions. The result comparison between our test and the existing small scale tests indicated that several general laws occur when accounting for the effect of the impact angle, regardless of the test scales and conditions. The normal coefficient of restitution R_n, the kinematic coefficient of restitution R_v and the kinetic energy coefficient of restitution R_E all decrease when increasing the impact angle, while tangential coefficient of restitution R_t increases as the impact angle increases in most cases. However, the reason for the magnitude difference in the coefficients of restitution between the tests compared is unidentified, for the tests differ from each other in multiple test conditions.

In free fall test, the rotation after impact dissipates part of the kinetic energy of the sample, and play an important role in the effect of the impact angle on the coefficient of restitution. Test results shows that the percentage of kinetic energy converted to rotational energy can be associated with the ratio between the rebound angle β and the impact angle α. When the impact angle is small, the rebound angle is more likely to exceed the impact angle and yields a high β/α, for the indentations and macro roughness caused by the impacts. As the impact angle decreases, the ratio β/α increases, and the percentage of kinetic energy converted to rotational energy become higher. Given a β/α, large samples are more likely to have a stead and small percentage than small samples. A higher percentage of kinetic energy converted to rotational energy always induces a higher R_n and a lower R_t. But no correlations are observed in this study between the rotation energy and the other two coefficients of restitution, R_v and R_E. In additional, R_n being larger than 1.0 can be related to the rebound angle greater than the impact angle under small impact angle condition.

Although it is verified in this study that several general laws regarding the effect of the impact angle on the coefficients of restitution are independent of the test scales and conditions, we are still lacking a reliable method to introduce the effect of the impact angle into rockfall trajectory simulation, which is caused by the discreteness of the measured data under small impact angle condition and the absence of a uniform and reasonable function describing the effect of the impact angle.

Last but not least, only the spherical limestone polyhedrons are taken as the samples, and the detailed impact orientations during impact are not involved in this study. Whether the conclusions are valid for the boulder with other shapes should be further investigated through more elaborated experiments. In views of this, the test results is valid for trajectory simulation codes based on a lumped-mass model, and can be referenced in the trajectory predication of spherical rocks impacting hard surface using a rigid body model.

Acknowledgements

Sponsored by Research Fund for Excellent Dissertation of China Three Gorges University, the Open Research Programme of the Hubei Key Laboratory of Disaster Prevention and Mitigation (No. 2017KJZ03), the National Natural Science Funds (No. 51409150), and the CSC Scholarship (No. 201707620009).

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