Response to the Comments by Reviewer #2

Review of the paper "A Study of Earthquake Recurrence based on a One-body Spring-slider Model in the Presence of Thermal-pressurized Slip-weakening Friction and Viscosity" by Jeen-Hwa Wang

This paper studied earthquake recurrence by numerical simulations of a one-degree-of-freedom spring-slider model with thermal-pressurized slip-weakening friction. The paper investigated the effects of the viscosity and the wear process on the recurrence time, slip amount for each event, slip velocity, and so on.

The many parts of the main results stated in the manuscript would not be obtained or read from the simulation results shown in Figs. 4-12. The main reasons of this were the assumption of the constant U_c in the simulations for the examination of the wear effect (Figs. 8-12) and the way of drawing Figs. 4-12.

Regarding the following specific comments [1]-[6] at least, the numerical simulations should be conducted appropriately and the manuscript and figures should be modified before the publication.

[Answer] I would like to express my thanks to you for carefully reading my manuscript and giving me many valuable comments and suggestions to improve the manuscript. The revisions are marked with red.

Major comments

[1] L.23-28 (Abstract), L.385, etc.: The Author stated that the effect of the wear process increases with C. However, the dependency of C on T_R or D is not obtained from the simulation results shown in Figs. 8-12. This is because U_c was assumed to be constant and the same in (a)-(d) for each figure, as stated in L.266-269 and captions of Figs. 8-12, which means that the other parameters (at least one among ρ_f, C_v, μ_f, Λ, and D_0) varied with C in (a)-(d) for each figure. In order to investigate the effect of C solely, the other parameters (ρ_f, C_v, μ_f, Λ, and D_0) should be constant and the same in (a)-(d), and thus U_c should change in (a)-(d) and vary with h(t) (i.e., the cumulated slip). It is better to calculate U_c using h=CS(t) for every time step in the simulations.

[Answer] Actually, U_c is not constant and varies with h in panels (a)–(d) of Figs. 8–12. The value of U_c written in each figure caption is the initial value, U_{co}, in the relationship: $U_c=U_{co}+C\Sigma U$ assumed by me. This point has been explained in the revised manuscript. The re-written statements are "Simulation results for four values of C are shown in Figs. 8–12: (a) for C=0.0001; (b) for C=0.001; (c) for C=0.01; and (d) for C=0.05 when η=0 in Figs. 8–10 and when η=1 in Figs. 11–12. The initial values of U_c are 0.1 for Fig. 8, 0.5 for Fig. 9, 0.9 for Fig. 10, 0.1 for Fig. 11, and 0.5 for Fig. 12." Meanwhile, the value of U_c shown in each figure caption has been replaced by "U_{co}."

[2] ・L.285-286: "The left-handed-side panels in Figs. 5–7 show that V_m and D decrease when … U_c … increases"

・L.290-293

・L.309-310: "The phase portraits shown in the right-handed-side panels of Figs. 5–7 exhibit that … the size associated with D decrease with increasing η."

The values of V_m, D, and the slope at the two fixed points cannot be compared among Figs. 5-7 because V and U would be normalized by different values of V_{max} and U_{max} among the figures. I guessed that V_{max} and U_{max} correspond to the maximum values of V and U in (a) for each figure and that the maximum values decreases with increasing U_c when $\eta \neq 0$, similar to the cases with $\eta=0$ (Fig. 4). I suggest that V and U should be normalized by V_p and $V_p \tau_{max}$, respectively, where τ_{max} is the maximum value of the horizontal axis (1300) in Figs. 4-12.

[Answer] It is very sorry that the notations were not well explained in the original manuscript. The values of V_m and D, respectively, represent the peak value of velocity and final slip of an event in each figure and have been displayed in Fig. 1 which has been re-drawn and different from the original one. The quantities V_{max} and U_{max} are the maximum values of V and U, respectively, in the first panel marked by "a" of a figure with four panels. The normalization scales in the left-handed-side panels of Figs. 4–12 are V_{max} for the velocities and the final value of $\Sigma U/U_{max}$ for the cumulative displacements in the computational time. Hence, the upper bound scales are "1" for both the velocity and the displacement. Hence, only the patterns of temporal variations of velocity and cumulative slip are concerned in these figures. The above-mentioned explanations have been added to the revised manuscript

[3] ・L.17-18: "T_R increases when U_c decreases or η increases"

・L.286-287: "T_R increases when either η increases or U_c decreases"

T_R increases when η increases for U_c=0.8 (Fig. 7), while T_R decreases when η increases for smaller U_c (Figs. 5 and 6). The behaviors of stick-slips should be investigated more carefully.

[Answer] The related description has been improved.

[4] ・L.276-277: "The value of τ_D increases with U_c"

・L.286: "while τ_D increases with η and U_C"

The τ_D values are unclear in the left panels of Figs. 4-6. Please add the enlarged figures for only one event.

[Answer] Figure 1 has been re-drawn to include the temporal variation in particle velocity to meet your request.

[5] L. 278-282, L.288-292, L.405-407: I cannot understand what "the slope values at the two fixed points" means

$(V/V_{max})/(U/U_{max})$? Or $\frac{U_{max}}{V_{max}}\frac{dV}{dU}$?

[Answer] The slope means $d(V/V_{max})/d(U/U_{max})=(U_{max}/V_{max})(dV/dU)$. This statement has been added to the revised manuscript. By the way, the term "the slope value" has been replaced by "the absolute value of slope" in the revised manuscript.

[6] Some characters in the numerical formulas are very confusing.

- About slip and cumulative slip
  - u and U in the friction law (equation 2, the second term of the right side of equations 3 and 4, Figure 3, etc.) would represent the time-varying slip amount for one event.
  - u and U in $u-u_0$ and $U-V_p\tau$ (the first term of the right side of equations 1, 3, and 4, Figure 1, etc.) show the time-varying cumulative slip.
  - Also ΣU in Figs.4-12 correspond to the time-varying cumulative slip.

- The (maybe time-varying) cumulative slip used in the wear effect is S(t). Is S(t) the same as ΣU in Figs.4-12?
- Is D(t) in S(t)=ΣD(t) different from D (final slip of each event, defined at L.16)?

[Answer] The u and U only represent the time-varying slip and time-varying normalized slip, respectively, for one event and they do not denote the time-varying cumulative slip. The parameter ΣU represents the time-varying cumulative slip. D(t), which represents the final slip of an event, could be constant in the time history as displayed in Figs. 4–7 when all model parameters do not change with time; while it could vary with time as shown in Figs. 8–12 when one of the model parameters does change with time. Hence, D(t) in S(t)=ΣD(t) is merely D. This point has been explained in the revised manuscript.

- About friction, is *f* in L.155 the same as μ_f (L.156 etc.)?

[Answer] The "*f*" in L.155 has been replaced by "μ_f".

<u>Minor comments</u>

[7] The topic on the wear process starts abruptly at L.20 in Abstract and the last paragraph of Section 4 (Simulation Results, p.11). To clarify the subjects of this paper, it would be better to add the statement that this paper investigated the wear process to the first sentence in Abstract and to the Introduction. In addition, the statements on the wear process in p.11 should be moved to somewhere before Section 4.
[Answer] The statement "the wear process" has been moved to the places you suggested in the revised manuscript

[8] L.53: 'the Nankaido trough' → 'the Nankaido segment of the Nankai Trough'?
[Answer] The statement has been re-written in the revised manuscript.

[9] "$T_R=\Delta\sigma^{2/3}M_o^{1/3}/1.81\mu v_l$ ": The assumption of constant $\Delta\sigma$ and v_l is not needed to derive this relation. If $\Delta\sigma$ or v_l varies with time, also T_R varies with time.
[Answer] The related statement has been deleted in the revised manuscript.

[10]L.71-72: I cannot understand the meaning of 'the distribution of T_R'. The probability density distribution of T_R?
[Answer] Yes, you are right. It is the probability density distribution of T_R. The related statement has been added in the revised manuscript

[11]L.87: 'the Nankaido trough' → 'the Nankai trough'? Or 'the Nankaido segment of the Nankai Trough'?
[Answer] The statement "the Nankaido trough" has been replaced by "the Nankai trough" in the revised manuscript.

[12]L.152-153: "The latter is not appropriate in this study because of the request of constant velocity. " The equations of SOP model for variable velocity are shown in Rice (2006), which can be solved numerically. It should be noted that I agree to adopt AUD model in this study in order to examine the wear effect.
[Answer] The statement has been re-written in the revised manuscript.

[13] L.163 (equation 2): How did the Author treat equation 2 for the stable sliding (e.g. cases shown in Figs. 11d and 12d)? u=0?
[Answer] The value of F(u) at u=0 if F_o, i.e., the static friction force. The statement has been re-written in the revised manuscript.

[14] ・L.167-168: "The force drop is lower for larger u_c than for smaller u_c."

・L.399: "larger U_c yields a lower ΔF than smaller U_c"

The final friction drop is 1, regardless of u_c and U_c (Fig. 3). Did the Author mean "the force drop for a certain displacement"?
[Answer] You are right. The statement "for the same final slip" has been added to the revised manuscript.

[15] p.8: υ decreases with increasing T and η is proportional to υ. However, η was assumed to be constant in this study. I wonder if the simulations with η depending on T are possible. The Author does not have to conduct such simulations in this study, but the comments on this may be interesting.
[Answer] The statements "Since *υ* decreases with increasing T, *η* decreases with increasing *T*. Hence, *η* can vary with time during faulting. This point has been studied by Wang (2017b) for the generation of nuclear phase before an earthquake ruptures. In this study, constant *η* is considered for each case" have been added to the revised manuscript.

[16] L.222: "V_p must be much smaller than 1": The value of V_p depends on $D_o\omega_o$. How large is $D_o\omega_o$?
[Answer] In this study, is considered to be about 1 m/s.

[17] L.223: "V_p is taken to be 10^{-2}" Do the results change if V_p is another value?
[Answer] The statements "Since the value of V_p can only influence the recurrence time, T_R, between two events and cannot influence the pattern of time variations in velocities and displacements of events. In order to study earthquake recurrence, there must be numerous modelled events with clear and visualized time functions of displacements and velocities for an event in the computational time period. If $V_p=10^{-10}$ is considered, T_R is very long and thus τ_D is much shorter than T_R. This makes the time function of an event displayed in the variation in slip looks like a step function for the displacements and an impulse for the velocities. Hence, $V_p=10^{-2}$ is taken in this study."

[18] Section 4 (Simulation Results): The results of the numerical simulations stated in pp.12-13 and L.381-414 should be moved to Section 4.
[Answer] The related statements shown in the section of "Discussion" have been moved to the section of "Simulation Results".

[19] L.252-253: The references are needed.
[Answer] The related references have been added to the revised manuscript.

[20] L.264-265, L.377-378 etc.: "U_c is proportional to C": This phrase seems to be strange for me because the variable is S(t) and C is the proportion coefficient.
[Answer] The statements have been re-written in the revised manuscript.

[21]L.265: “This”: What does the word “this” show? The sentence just before this word? The fact “the more mature the fault is, the thicker its slip zone is” comes only from h(t)=CS(t).
[Answer] The sentence has been re-written as “Based on *h(t)=CS(t)*, the more mature the fault is, the thicker its slip zone is.” in the revised manuscript.

[22]L.274-276: “the force drop, ΔF, decreases with increasing U_c, thus indicating that larger ΔF yields higher V_m and larger D” I cannot understand the logic of this sentence. The Author’s intention may be “ΔF decreases with increasing U_c for a certain finite displacement” because the friction drop reaches 1 when displacement is ∞ regardless of U_c (Fig.3). If so, however, this phrase have no relation to “larger ΔF yields larger D”.
[Answer] The statement “” has been behind the sentence in the revised manuscript.

[23]L.292-293: The U_c values are different from those in L.248 and figure captions.
[Answer] The values of Uc shown in L.292-293 are wrong and must the same as those in L.248. The original sentence has been re-written to be “Clearly, like Fig. 4 the final slip decreases with increasing U_c.”

[24]L.301-305: In a one-degree-of-freedom spring-slider model with constant friction parameters, the system reaches limiting cycles even in the previous studies listed in L.304-305, although I have not checked the results by Kostić et al. (2013a) and Franović et al. (2016). The Author may consider the initial transient phase, but the phase depends on the assumed initial state before the spring starts to be pull with the driving velocity of V_p. The behaviors of the limiting cycle reflect the parameters of the friction and of the system properly. It should be noted that the very small transient phase was also observed in Rice and Tse (1986) (the reference in L.298).
[Answer] Your viewpoint is correct. In this study, I mainly focus on the effect on recurrence. The phase portrait is just used to express the possible change of fixed points due to either a use of different values of or a use of time-varying values of model parameters. Nonlinear behavior, including very small transient phase which was not observed in this study, of the system will be my next study.

[25]L.309-310: I cannot understand that the right panels show T_R.
[Answer] The related statements have been deleted in the revised manuscript.

[26]L.314: I cannot understand why larger η generates chaos.
[Answer] The word “chaos” has been re-written as “an attractor” in the revised manuscript.

[27]L.318: The slope values at V=0 and U=0 decrease with increasing η more drastically for the larger U_c. As pointed out in my comment [2], the slope values should not be compared among the figures because U_{max} and V_{max} values are different among the figures.
[Answer] As mentioned in my answer of your comment [2], for a certain figure we can the absolute values of slope in the four right-handed-sides panels because their values of V_{max} and U_{max} are the same. Of course, it is not good to compare the values in different figures due to different values of V_{max} and U_{max} in use.

[28]L.319: The references are needed.

[Answer] "The previous study" means the simulation results of this study. Hence, the words "The previous study" have been re-written as "The simulation results as mentioned previously".

[29]L.321: "the effects" The effects of temporal variations of η and U_c?
[Answer] The following statement "the effects of time-dependent η and U_c" has been added to the revised manuscript.

[30]L.329-330: "$\Lambda=(\lambda_f-\lambda_n)/(\beta_f+\beta_n)$"
It would be better to move this to p.7, adding the definition of λ_f, λ_n, β_f, and β_n.
[Answer] The statements have been re-written and added in the revised manuscript.

[31]L.338: "μ_f" → "μ_f"
[Answer] "μ_f" is replaced by "μ_f" in the revised manuscript.

[32]L.347: "ρ_f" and "n" → "ρ_f" and "n"
[Answer] "ρ_f" and "n" are replaced, respectively, by "ρ_f" and "n" in the revised manuscript.

[33]L.362-364: The Author used the words "time-varying". However, "the increase in permeability can result in the increase in pore pressure due to slip" would be better because "This" in the sentence "This can reduce the frictional resistance" obviously means an increase in the pore pressure.
[Answer] The statement "The time-varying permeability can result in the time-varying pore pressure, p_f" has been re-written as "An increase in permeability can result in an increase in pore pressure, p_f".

[34]L. 410: "$\underline{C}$" → "C"?
[Answer] "$\underline{C}$" is replaced by "C" in the revised manuscript.

[35]L.411: "approaches unity" The slope values seem to become smaller than unity in Figs. 9-12. Plotting the slope values (with time or slip) may clarify this point.
Why does the unity important? The slope values depend on the V_{max} and U_{max} values.
[Answer] Actually, the slope values become smaller than unity in Figs. 9-12. Hence, the statement "approach to unity" has been deleted in the revised manuscript.

[36]Bizzarri (2010) showed the effects of the wear process on the stick-slip behaviors, assuming the friction law with thermal pressurization, and thus the results on the wear processes in this study are not new. I suggest that the statements on the results of the simulations including both the wear processes and the viscous effects (Figs. 11 and 12) are added.
[Answer] I agree with you. Related information has been added to the revised manuscript.

[37]Are the η and C values used in the simulations consistent with those estimated by observations or laboratory experiments in previous studies (e.g., Boneh et al., 2014, pageoph)?
[Answer] This study is merely my first step to theoretically explore the earthquake recurrences caused by time-varying model parameters through numerical simulations. In this study, I just want to theoretical explore the possible effects on

earthquake recurrences caused by time-varying of fault width. Hence, only the assumed values have been taken into account. I have not yet compared my values with those obtained by others. I will approach the problem for real faults in near future, and thus it is necessary to take the values of model parameters obtained from field observations and laboratory experiments into account.

[38] Vertical aces in Figs. 4-12: Please add the scales of the $\Sigma U/U_{max}$ aces. The maximum of $\Sigma U/U_{max}$ must be larger than 1 because U/U_{max} reaches 1 or larger in the right panels of (a).
[Answer] In Figs. 8–12, the velocity waveforms and displacements are normalized by the maximum values of each figure. Hence, the upper bound value of the vertical axis is 1. The statements have been added to the revised manuscript.

[39] Fig.8-12: Why do the behaviors of the stick-slips (e.g., T_R, D, and V_m) vary with time in spite of the constant U_c?
[Answer] The values of U_c are not constant and vary with time in Figs. 8–12. The values of U_c shown in the text and figure captions of have been re-written to be the initial values of U_c.

[40] Figs.11(a) and 12(a): Why $V_m/V_{max}\neq 1$? I guessed that V_{max} was defined as V_m in (a) for each figure in Figs. 4-10. Why is the maximum of U/U_{max} larger than 1? I guessed that U_{max} was defined as the maximum of U in each figure in Figs. 4-10.
[Answer] It is very sorry that the numerical computation cannot work when C is larger than a certain value which depends on U_c. The normalization of original Fig. 11 and Fig 12 was made based on the maximum values of panel (d), which are wrong. The re-computed results are displayed in the revised manuscript.

[41] Figs. 11(d) and 12(d): Why are there two thin solid lines? Why is $\Sigma U/U_{max}$ constant (thick solid line)?
[Answer] It is very sorry that the numerical computation cannot work when C is larger than a certain value. Originally, I kept the bad simulation results for C=0.05 in Figs. 11–12 with η=1, because I wanted to retain the same four values of C for Figs. 8–12. This idea sounds not good. Hence, I have re-done the numerical computations to find out the upper bound value of C for a certain U_c with η=1. The re-computed results are displayed in Figs. 11–12 of the revised manuscript.