<u>Referee #2</u>

1. **P.1. 1.Introduction: more literature for assessment methods (numerical models)**

   P.1. Line 35-50.: Literature review for assessment methods (numerical models) is added

   Risk management associated with safety is a fundamental focus in railway operations. It has been integrated into global safety management system of railways (Berrado et al., 2010) and developed to allow a rapid risk assessment using a common risk score matrix (Braband, 2011). As roadbed settlements exceed the allowable limits, it may result in track irregularity and derailments of trains causing heavy loss of life. Therefore, risk management tools are developed to deal with track safety by controlling and reducing the risk of derailments (Zarembski et al., 2006). In this study, methods to secure the stability of roadbeds have been examined using numerical analysis.

   Numerical analyses have been widely used for risk assessment. Numerical analyses using three-dimensional geotechnical codes were carried out to predict the subsidence area and its interaction with buildings (Castellanza et al., 2015) and a three-dimensional groundwater flow model for risk evaluation was developed to be an effective management strategy (Ashfaque et al., 2017). The coupling of numerical models and monitoring data contribute to undertake efficient risk reduction policies (Bozzano et al., 2013). Especially using FLAC, which is a finite-difference numerical code especially specialized in the area of geotechnical engineering, numerical computations to simulate the influence of rainfall (Pisani, 2010), both acoustic emission (AE) activities at AE sensor locations of the Kannagawa cavern (Cai et al., 2007), and a comprehensive pump test at Sellafield (Hakami, 2001) showed good agreement with field monitoring results. In this study, FLAC^{3D}, which is a three-dimensional finite-difference numerical code especially specialized in the area of geotechnical engineering, is adopted for numerical analysis.

2. **P.2. 2. Case studies of ground subsidence, what kind of the cases are the simulated target in this paper?**

   P.2. Line 62-70.: The cases of ground subsidence occurred at nearby urban railways in South Korea are quite similar. Therefore, no specific case is selected for numerical analysis but the simulated cases cover historical events.

3. **P.3. 3. Numerical analysis, please add a section to briefly introduce this three-dimensional model such as theory base, essential parameters, input/output, boundary conditions, initial conditions, etc.**

   P.3. Line 97-189: FLAC3D is briefly introduced.

   2 Numerical analysis

   In the following sections, the FLAC^{3D} given in this work are briefly described in the following sections by paraphrasing from those of Itasca Consulting Group (2002).

   2.1 Theoretical background of FLAC^{3D}

   FLAC^{3D} (Fast Lagrangian Analysis of Continua in three Dimensions) is numerical modeling software for advanced geotechnical analysis of soil, rock, groundwater, and ground support in three dimensions. FLAC is used for analysis, testing, and design by geotechnical, civil, and mining engineers (Itasca Consulting Group Inc., 2002). It is designed to accommodate any kind of geotechnical engineering project that requires continuum analysis.

   The mechanics of the medium are derived from general principles (definition of strain, laws of motion), and the use of constitutive equations defining the idealized material. The resulting mathematical *expression is a set of partial differential equations, relating mechanical (stress) and kinematic (strain rate, velocity) variables, which are to be solved for particular geometries and properties, given specific boundary and initial conditions.*

An important aspect of the model is the inclusion of the equations of motion, although FLAC3D is primarily concerned with the state of stress and deformation of the medium near the state of equilibrium. It will be shown, in the numerical implementation section, that the inertial terms are used as means to reach, in a numerically stable manner, the equilibrium state.

2.1.1 Conventions

In the Lagrangian formulation adopted in FLAC^{3D}, a point in the medium is characterized by the vector components x_i, u_i, v_i and d_{vi}/d_t, i=1,3 of position, displacement, velocity and acceleration, respectively. As a notation convention, a bold letter designates a vector or tensor, depending on the context. The symbol a_i denotes component i of the vector $[a]$ in a Cartesian system of reference axes; A_{ij} is component (i, j) of tensor $[A]$. Also, a, i is the partial derivative of a with respect to x_i. (α can be a scalar variable, a vector or tensor component.) By definition, tension and extension are positive. The Einstein summation convention applies, but only on indices i, j and k, which take the values 1, 2, 3.

2.1.2 Stress

The state of stress at a given point of the medium is characterized by the symmetric stress tensor σ_{ij}. The traction vector $[t]$ on a face with unit normal $[n]$ is given by Cauchy's formulae (tension positive):

$$t_i = \sigma_{ij} \tag{1}$$

2.1.3 Rate of Strain and Rate of Rotation

Let the particles of the medium move with velocity $[v]$. In an infinitesimal time dt, the medium experiences an infinitesimal strain determined by the translations $v_i dt$, and the corresponding components of the strain-rate tensor may be written as

$$\xi_{ij} = 1/2(v_{i,j} + v_{j,i}) \tag{2}$$

where partial derivatives are taken with respect to components of the current position vector $[x]$. For later reference, the first invariant of the strain-rate tensor gives a measure of the rate of dilation of an elementary volume. Aside from the rate of deformation characterized by the tensor ξ_{ij}, a volume element experiences an instantaneous rigid-body displacement, determined by the translation velocity $[v]$, and a rotation with angular velocity,

$$\Omega_i = -1/2 e_{ijk}\omega_{jk} \tag{3}$$

where e_{ij} is the permutation symbol, and [ω] is the rate of rotation tensor whose components are defined as

$$\omega_{ij} = 1/2(v_{i,j} + v_{j,i}) \tag{4}$$

2.1.4 Equations of Motion and Equilibrium

Application of the continuum form of the momentum principle yields Cauchy's equations of motion:

$$\sigma_{ij,j} + \rho b_i = \rho(d_{vi}/d_t) \tag{5}$$

where ρ is the mass per unit volume of the medium, [b] is the body force per unit mass, and $d[v]/dt$ is the material derivative of the velocity. These laws govern, in the mathematical model, the motion of an elementary volume of the medium from the forces applied to it. Note that in the case of static equilibrium of the medium, the acceleration $d[v]/dt$ is zero, and Eq. (5) reduces to the partial differential equations of equilibrium:

$$\sigma_{ij,j} + \rho b_i = 0 \tag{6}$$

2.1.5 Boundary and Initial Conditions

The boundary conditions consist of imposed boundary tractions (see Eq. (1)) and/or velocities (to induce given displacements). In addition, body forces may be present. Also, the initial stress state of the body needs to be specified.

2.1.6 Constitutive Equations

The equations of motion Eq. (5), together with the definitions Eq. (2) of the rates of strain, constitute nine equations for fifteen unknowns — the unknowns being the 6 + 6 components of the stress- and strain-rate tensors and the three components of the velocity vector. Six additional relations are provided by the constitutive equations that define the nature of the particular material under consideration. They are usually given in the form

$$[\dot{\sigma}]_{ij} = H_{ij}(\sigma_{ij}, \xi_{ij}, k) \tag{7}$$

in which $[\dot{\sigma}_{ij}]$ is the co-rotational stress-rate tensor, [H] is a given function, and k is a parameter that takes into account the history of loading. The co-rotational stress rate $[\dot{\sigma}]$ is equal to the material derivative of the stress as it would appear to an observer in a frame of reference attached to the material point and rotating with it at an angular velocity equal to the instantaneous value of the angular velocity [Ω] of the material. Its components are defined as

$$[\dot{\sigma}]_{ij} = (d\sigma_{ij}/d_t) - \omega_{ik}\sigma_{kj} + \sigma_{ik}\omega_{kj} \tag{8}$$

in which d[σ]/dt is the material time derivative of [σ], and [ω] is the rate of rotation tensor.

4. **P.3. 3. Numerical analysis, please add a section of model's verification by historical events to properly demonstrate the reliability of the model's performance**

P.1. Line 41-50: Model's verification by historical events are added to demonstrate the reliability of the model's performance.

Numerical analyses have been widely used for risk assessment. Numerical analyses using three-dimensional geotechnical codes were carried out to predict the subsidence area and its interaction with buildings (Castellanza et al., 2015) and a three-dimensional groundwater flow model for risk evaluation was developed to be an effective management strategy (Ashfaque et al., 2017). The coupling of numerical models and monitoring data contribute to undertake efficient risk reduction policies (Bozzano et al., 2013). Especially using FLAC, which is a finite-difference numerical code especially specialized in the area of geotechnical engineering, numerical computations to simulate the influence of rainfall (Pisani, 2010), both acoustic emission (AE) activities at AE sensor locations of the Kannagawa cavern (Cai et al., 2007), and a comprehensive pump test at Sellafield (Hakami, 2001) showed good agreement with field monitoring results. In this study, $FLAC^{3D}$, which is a three-dimensional finite-difference numerical code especially specialized in the area of geotechnical engineering, is adopted for numerical analysis.

5. **P.3, 3.1 Conditions for numerical analysis, Ln. 103-104, how to decide the scenario such as diameter 4-10 m, distance 15-25 m and various groundwater levels? Based on any field cases?**

P.5. Line 194-197: Diameters of the cavity are determined by historical events described in Introduction. Distance and groundwater level are arbitrarily determined with respect to diameter of the cavity.

A circular cavity below the ground surface has been modeled with respect to diameters (D) of 4-10 m, which is selected by historical events as described in previous section. Distances of 15-25 m from the cavity to the center of the roadbed and various groundwater levels are arbitrarily selected for roadbed settlement influenced by given size of cavity.

6. **P.3, In additions, please add a table to list total computational runs.**

   P.11.: Total computation time is added in Table 2.

7. **P.3, Figure 4, what is the meaning of the roller attached on the left side and two sides of bottom?**

   P.5. Line 198-200: The meaning of the roller is added.

   As shown in the figure, roller supports prevent normal translations, but capable of tangential translations and/or rotations. There is a single linear reaction force in either vertical or horizontal directions.

8. **P.3, Figure 5, the legend texts are too small and unclear. Is it possible to merge this figure with Figure 4 as a single figure?**

   P.7.: Figure 5 (original manuscript) is erased because a similar description is given in Figure 4 (P.7. revised manuscript).

9. **P.4, 3.1.2 Physical properties of rail, rail pad, and prestressed, too many tables in this section, I suggest to reorganize these tables to reduce table numbers**

   P.6.: Tables 1-4 are reorganized and merged to Table 1.

10. **P.5, Figure 7 - The legend texts are too small and unclear. – Please use the same color interval of vertical displacement value of (a) and (b) in order to clearly to show "ground settlement increases as the diameter of the cavity increases". – Please keep the same geometric scale and view angle of the model display.**

    P.7.: In Figure 4, the legend texts are changed to be clearly visible and the same color interval of vertical displacement and the same geometric scale and view angle of the model are used.

11. **P.6, 4.1.1 Regression analysis of roadbed settlement, too short descriptions. What's the meaning of the regression analysis? Why the groundwater level is absent in the regression?**

    P.7. Line 280-287: Meaning of regression analysis is described in detail. In dry condition, regression analysis of roadbed settlement associated with distance and diameter is carried out. If the groundwater level is included for the analysis, there are too many parameters to define its relationships.

    Roadbed settlement increases as the diameter (D) of the cavity increases and the distance (d) between the roadbed and the cavity decreases. Therefore, in this study, the roadbed settlement is examined with respect to D normalized by d (Fig. 7). The regression analyses results show medium to high correlations of r^2*=0.72. As D/d is greater than 0.2 and less than 0.3, the roadbed settlement is approximately 5 mm. It* *requires that a database of measurement sensors should be established for real-time monitoring of the roadbed, structures and groundwater to prevent disasters in advance. As D/d exceeds 0.35, the roadbed settlement substantially increases and is greater than 10 mm. Since it may result in highly probable traffic accident, train operation should be stopped and the roadbed should be reinforced or repaired.*

12. **The better description for R-squared=0.72 probably is “medium to high correlation” instead of “high correlation”.**

    P.7. Line 282: It is changed to “medium to high correlation” instead of “high correlation”.

13. **P.7, Figure 9, a linear equation in legend, editing error?**

    P.9.: In Figure 6, It is corrected to exponential equation.

14. **P.7, Figure 10 – The legend texts are too small and unclear. – Why the vertical displacement is symmetry along the centerline of roadbed since only cavity on one side**

    P.9.: Figures 4 & 7 are corrected and the right and the left side of roadbed settle down and heave, respectively, because the cavity is on the right side.

15. **P.9, It’s difficult to understand the risk level through Table 5-Table 7 since the risk level is based on the combination of cavity diameter, distance and groundwater level. I suggest to reorganize these tables to perform more systematical outcome.**

    P.11.: Tables 5-7 (original manuscript) are merged to Table 2 (revised manuscript) and reorganized in systematic pattern.

16. **P.10, 5 Conclusions, conclusions should include vital or quantitative findings of this paper.**

    P.11. Line 358-376: Quantitatively findings are described in conclusions.

The number of occurrences of ground subsidence induced by a leakage of aged pipelines for water and sewage in urban areas resulting in various sizes of cavity near the urban railway in Seoul City has been found to increase and it may cause the roadbed settlement to exceed the allowable value. A large-scale cavity is rarely found, but if it is close to the roadbed, the roadbed is highly influenced by the cavity and may cause train derailment.

In this study, numerical analyses are carried out to estimate roadbed stability and its risk level associated with various groundwater levels, sizes of cavities. The analyses results show that roadbed settlement increases as the diameter (D) of the cavity increases and the distance (d) between the roadbed and the cavity decreases. The regression analyses results show that, as D/d is greater than 0.2 and less than 0.3, a database of measurement sensors should be established for real-time monitoring of the roadbed, structures and groundwater to prevent disasters in advance. As D/d exceeds 0.35, the roadbed settlement, which substantially increases and is in the status of danger, may result in highly probable traffic accident. Therefore, train operation should be stopped and the roadbed should be reinforced or repaired. The effects of groundwater level on the roadbed settlement are examined at the distance of 20 m for both 4 and 6 m diameter cavities and at 25 m for both 8 and 10 m diameter cavities. Ground settlement for 4 and 6 m diameter cavities located at a distance of 20 m from the roadbed satisfies the allowable value for GWL = (-) 4 and (-) 12m, respectively. The ground settlement for 8 and 10 m diameter cavities located at a distance of 25 m from the center of the roadbed has substantially decreased as GWL is 8 and 15 m below the *ground surface, respectively, and satisfies the allowable value as its level is 18 and 22 m below the ground surface, respectively. It indicates that a roadbed settlement is highly influenced by groundwater levels to an extent greater than even the influence of the size of the cavity.*