



# A hazard model of subfreezing temperatures in the United Kingdom using vine copulas

Symeon Koumoutsaris

Guy Carpenter, Tower Place, London, EC3R 5BU, UK

*Correspondence to:* Symeon Koumoutsaris ([symeon.koumoutsaris@guycarp.com](mailto:symeon.koumoutsaris@guycarp.com))

**Abstract.** Extreme cold weather events, such as the winters of 1962/63, the third coldest winter ever recorded, or more recently the winter of 2010/11, have significant consequences for the society and economy. This paper assesses the probability of such extreme cold weather across the United Kingdom. For that, a statistical model is developed in order to model the extremes of the Air Freezing Index (AFI), which is a common measure of magnitude and duration of freezing temperatures. A novel approach in the modelling of the spatial dependence of the hazard has been followed which takes advantage of the vine copula methodology. The method allows to model complex dependencies especially between the tails of the AFI distributions which is important to assess reliably the extreme behaviour of such events. The model suggests that the extreme winter 1962/63 has a return period of approximately once every 89 years, but the relative short record length together with the unclear effects of anthropogenic forcing on the local climate add considerable uncertainty to this estimate. This model is used as part of a probabilistic catastrophe model for insured losses caused by the bursting of pipes.

## 1 Introduction

Extended periods of extreme cold weather can cause severe disruptions in human societies; on human health, by exacerbating previous medical conditions or due to reduction of food supply which can lead to famine and disease; agriculture, by devastating crops particularly if the freeze occurs early or late in the growing season; on infrastructure, e.g. severe disruptions in the transport system, burst of residential or system water pipes (Bowman et al., 2012). All these consequences lead to important economic losses.

Of particular interest for the insurance industry are the economical losses that originate as a result of bursting of pipes due to freeze events. Water pipes burst because the water inside them expands as it gets close to freezing which causes an increase in pressure inside the pipe. Whether a pipe will break or not, depends on the water temperature (and consequently on the air temperature), the freezing duration, the pipe diameter and composition, the wind chill effect (due to wind and air leakage on water pipes), and the presence of insulation (Gordon, 1996; McDonald et al., 2014).

Insurance losses from burst pipes have a significant impact on the UK insurance industry. They amount to more than £900 million in the last 10 years, representing around 10% of the total insured losses, mainly due to flood and windstorm, in the United Kingdom (UK) during the same period (ABI, 2017). Particular years can be very damaging, such as, for example, the winter of 2010/2011 where losses from burst pipes have exceeded £300 million in UK making it the peril with the largest



losses that year (ABI, 2017). Moreover, much more extreme cold winters have actually occurred in the UK in the last 100 years, such as the winters of 1946/47 and 1962/63. It is crucial for the insurance business to be able to anticipate the likelihood of occurrence of similar and even more extreme events so that they can adequately prepare for their financial impact (AIR, 2012). In fact, the capital requirements in (re)insurance is estimated in a 1 in 200 year return period (RP) loss basis, which is usually much larger than the available historical records.

Probabilistic catastrophe modelling is generally agreed to be the most appropriate method to analyze such problems. The main goal of catastrophe models is to estimate the full spectrum of probability of loss for a specific insurance portfolio (i.e. comprised by several residential, auto, commercial or industrial risks). This requires the ability to extrapolate the possible losses at each risk to high return periods which is usually achieved by simulating synthetic events that are likely to happen in the near future (typically a year). More importantly, it requires to consider also how all risks relate to each other and their potential synergy to create catastrophic losses. Such spatial dependence between risks can result from various sources, for example due to the spatial structure of the hazard (e.g. the footprint in a windstorm or the catchment area in a flood event) or due to similar building vulnerabilities between risks in the same geographical area (e.g. due to common building practices) (Bonazzi et al., 2012).

Modelling the spatial dependence of the hazard is usually achieved by taking advantage of certain characteristic properties of the hazard footprint, like for example the track path and the radius of maximum wind for and windstorms or the elevation in the case of floods. In the case of temperature, however, such a property cannot be easily defined; an alternative solution is to use multivariate copula models.

Based on Sklar's theorem (Sklar, 1959), the joint distribution of all risk sources can be fully specified by the separate marginal distributions of the variables and by their copula, which defines the dependence structure between the variables. However, the choice of adequate multivariate copulas is limited for more than two dimensions. For example, standard multivariate copula models such as the elliptical and Archimedean copulas do not allow for different dependency models between pairs of variables. Vine copulas provide a flexible solution to this problem based on a pairwise decomposition of a multivariate model into bivariate copulas. This approach is very flexible, as the bivariate copulas can be selected independently for each pair, from a wide range of parametric families, which enables modelling of a wide range of complex dependencies (Czado, 2010; Dißmann et al., 2013)

In this paper, I use the vine copula methodology in a novel application to develop a catastrophe model on insurance losses due to pipe bursts resulting from freeze events in the United Kingdom. The focus here is on the hazard component (section 2) which is modeled using the Air Freezing Index (AFI), an index which takes account both the magnitude and duration of air temperature below freezing, calculated from temperature data from the last 51 years. Extreme value analysis is performed on the historical AFI values in order to extrapolate to longer return periods. Stochastic winter-seasons are simulated taking into account the correlation of the hazard between all pair-cells with the help of regular vine copulas (section 3) . The resulting exceedance probabilities of extreme cold winters in UK are discussed in section 3.1. Concluding remarks are found in section 4.



## 2 Hazard

### 2.1 Temperature data

The hazard component of the catastrophe model is based on the gridded dataset of observed daily average temperature developed from the UK Met Office (Perry et al., 2009). The dataset covers the entire UK for the period from 1960 to 2011 at 5km x 5km resolution and georeferenced in the British National Grid projection. It is based on rigorously quality-checked station data interpolated to a regular grid using inverse-distance weighting, as described in Perry et al. (2009). For computational reasons, we regrid the data to a lower resolution of 50 km x 50km, which leads to a total of 170 cells over land.

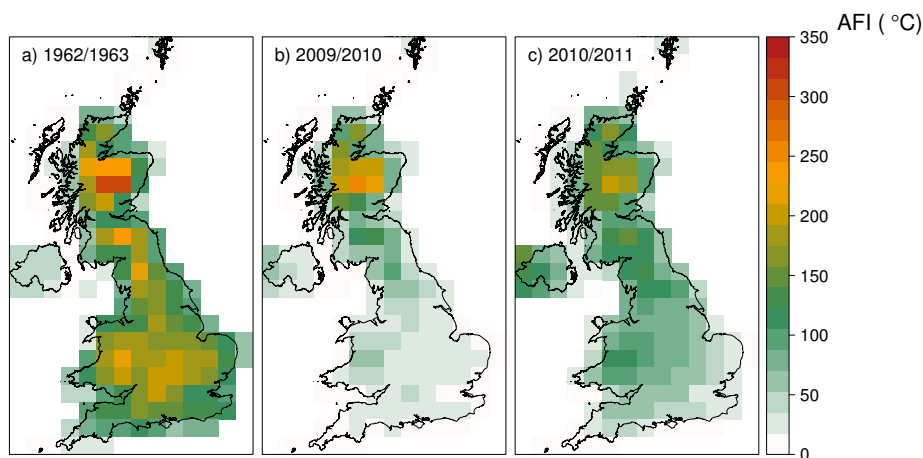
The use of a coarser horizontal resolution is expected to have relatively small influence in most cases given that winter climate anomalies are often coherent across large parts of the UK as they are primarily associated with large-scale atmospheric circulation patterns (Scaife and Knight, 2008). Nevertheless, local temperature may be subtly different in certain micro-climates, such as upland and urban regions. In particular over urban regions, which are most important from an insurance perspective, lower resolution may lead to temperatures that are biased towards lower values, leading though to a conservative view on the severity of extreme freeze events. In upland regions, on the other hand, extreme cold temperatures are most probably underestimated, although it is reasonable to expect that their damaging effects are somewhat mitigated from increased protection levels. For example, water pipes in properties located in mountainous regions are usually better protected against cold spells.

#### 2.1.1 Air-Freezing Index and historical events

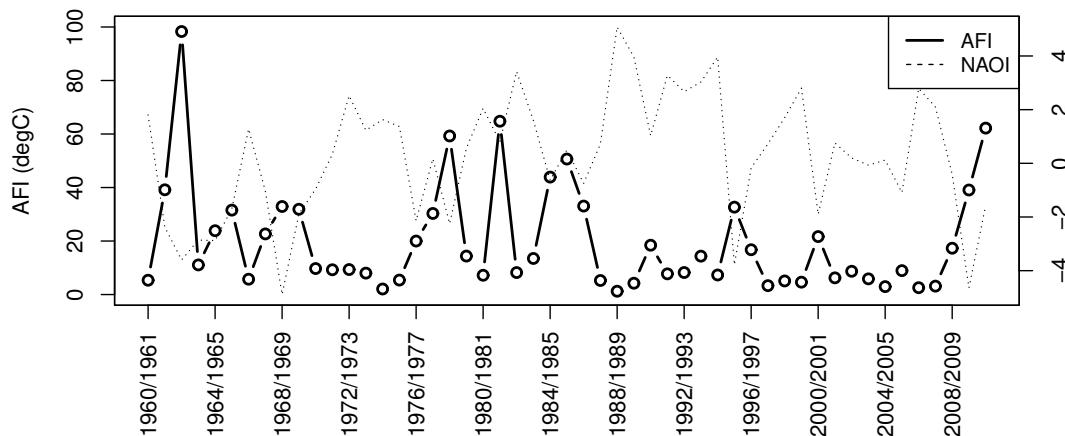
The daily temperature data are used to compute the AFI as the sum of the average daily temperatures of all days with below 0°C temperatures during the freezing period. The freezing period in this study is defined from first of June to end of May of the following year, in order to include the entire winter season. Because AFI accounts both for the magnitude and duration of the freezing period, it is commonly used for determining the freezing severity of the winter season (Frauenfeld et al., 2007; Bilotta et al., 2015).

Figure 1 shows a map of the AFI values for the season 1962/1963 (i.e. season starting from 1st June 1962 to 31st of May 1963), which was one of the coldest on the record in the United Kingdom (Walsh et al., 2001). The "Big Freeze of 1962/63", as it is also known, began on the 26 of December 1962 with heavy snowfall and went on for nearly three months until March 1963. The cause of the cold conditions has been the development of a large "blocking" anticyclone over Scandinavia and north-western Russia. Easterly winds on the southern edge of this system transported cold continental air westwards, displacing the more usual mild westerly influence from the Atlantic Ocean on the British Isles. Over the Christmas period, the Scandinavian High collapsed, but a new one formed near Iceland, bringing Northerly winds. Based on the AFI, the mean value in the entire UK (i.e. average of AFI values across all gridcells) mounted up to 98.3°C, which represents four standard deviations larger than the average of the entire 51-year period (19.6°C). The event affected the entire country with peak AFI values exceeding 200°C both in the South and in the North of the country (Figure 1).

After 1962/63, a long run of mild winters followed until late 1978 and early 1979 (Figure 2). However, temperatures in 1978/79 were not as low and the cold weather was interrupted frequently by brief periods of thaw (Cawthorne and Marchant,



**Figure 1.** Map of AFI values (in °C) for the the winter-seasons of a) 1962/63, b) 2009/10, and c) 2010/11.



**Figure 2.** Interannual variation of AFI over the study period.

1980). The mean AFI value of that winter reached 59.2°C. The 1980s stands out as a decade with several cold spells in UK, with mean annual AFI values above 40°C for the winters 1981/82, 1984/85, and 1985/86 (64.8, 43.9, and 50.6 °C, respectively). Finally, the winters of 2009/2010 and 2010/2011 brought frigid temperatures to parts of Europe and the UK (Guirguis et al., 2011; Osborn, 2011; Seager et al., 2010), with average AFI values across UK of 39.1 and 62.2°C. As mentioned previously, the latter one had a significant financial impact on the UK insurance industry. The relation between AFI and the North Atlantic Oscillation (NAO), a large-scale mode of natural climate variability, is discussed in detail in section 3.1.



## 2.2 Extreme value analysis

Since the historical data only extends for 51 years and our interest lies in very rare events (such as 1 in 200 years), it is necessary to extrapolate by fitting an extreme value distribution. The Generalized Extreme Value (GEV) family of distributions has been chosen, which includes the Gumbel, the Frechet, and Weibull distributions. An additional term was included, the probability of no hazard ( $P_0$ ), in order to account for the cells mainly on the south England coast that have years with no negative temperatures at all. The probability therefore that the AFI value ( $X$ ) inside a cell  $j$  will exceed a certain value ( $x$ ) is defined by the GEV cumulative distribution function  $F(x)$  which has the form:

$$F(x) = P(X \leq x) = P_0 + (1 - P_0) \exp \left\{ - \left( 1 + \xi \frac{x - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right\} \quad (1)$$

where  $\mu$ ,  $\sigma$ , and  $\xi$  represent the location, scale, and shape of the distribution, respectively and  $F(x)$  is defined when  $1 + \xi \frac{x - \mu}{\sigma} > 0$ ,  $\mu \in \mathfrak{R}$ ,  $\sigma > 0$ , and  $\xi \in \mathfrak{R}$ .

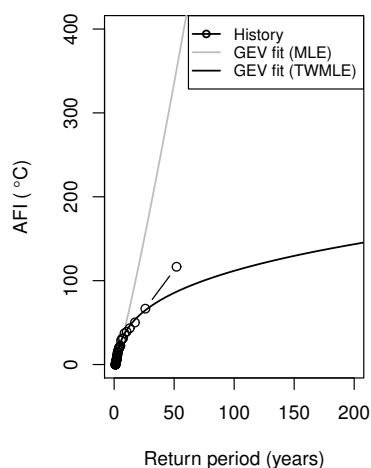
There are various methods of parameter estimation for fitting the GEV distribution, such as least squares estimation, maximum likelihood estimation (MLE), probability weighted moments, and others. Traditional parameter estimation techniques give equal weight to every observation in the dataset. However, the focus in catastrophe modeling is mainly on the extreme outcomes and, thus, it is preferable to give more weight to the long return periods. I therefore use the Tail-Weighted Maximum Likelihood Estimation (TWMLE) method developed by (Kemp, 2016) to estimate the GEV parameters which introduces ranking dependended weights ( $w_{(i)}$ ) in the maximum likelihood. The weights are defined for each cell based on the historical winter-season AFI values, i.e. the lowest historical AFI value in the cell (rank  $i=1$  out of  $n$  observations) has the lowest weight, while the largest historical AFI value (rank  $i=n$ ) has the largest weight, as follows:

$$w_{(i)} = AFI_{(i)} / \sum_{i=1}^n AFI_{(i)} \quad (2)$$

As an example, the GEV fit for a single cell over London is shown in Figure 3. The grey line represents the GEV fit without any weighting applied, while the black curve is estimated using the TWMLE method with an improved fit towards the tail of the distribution (i.e. the more extreme events). The largest AFI historical point in Figure 3 represents the 1962/63 exceptional winter which is estimated to be a more rare event than what the historical data suggests (i.e. larger than 1 in 52 years), as further discussed in the following sections.

## 2.3 Return period maps

The obtained GEV fits for each cell from the previous section are used to create return period maps, as shown in Figure 4. The top panels represent the AFI values that occur once every 10, 25, and 50 years. The largest AFI values follow, as expected, the orography of UK peaking in the northern region of the highlands where the elevation reaches 1000 meters. In this region, values of AFI greater than  $200^\circ\text{C}$  are reached often, approximately once every 5 years. The southern part of UK shows much

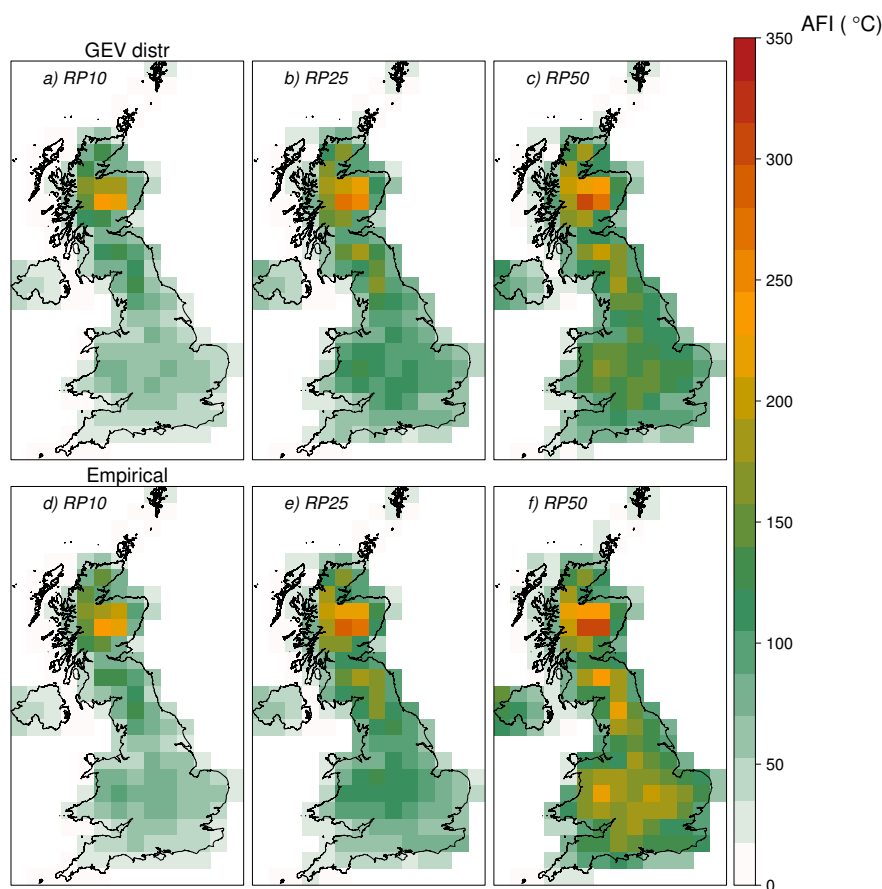


**Figure 3.** Exceedance probability curve for a single cell over London, based on the historical AFI GEV fit (black circles), the GEV fitted with MLE (grey line), and the GEV fitted with TWMLE (black line).

lower values, not exceeding 200°C even at the 50 year RP. Slightly milder temperatures are also evident around the London area denoting the urban micro-climate effect. Other urban regions (e.g. Manchester or Midlands area) do not stand out as much as a result of the low grid resolution.

The empirical return periods are also plotted for comparison (bottom panels, Figure 4). These are calculated for each cell as  $1/(1-P)$ , where P represents the cumulative probabilities of the ranked values and is calculated based on the Weibull formula  $P=i/(n+1)$  (Makkonen, 2006). The AFI values from the GEV fits correspond well with the empirical estimates, apart from the southern part of UK where the empirical values are approximately 20-30% larger at 50 years RP. This difference is driven by the 1962/63 event which empirically is estimated at 1 in 52 years while it is estimated to be less frequent according to the GEV fits. The probability of such an event happening today and its influence in the inhabited areas is discussed in detail in section 3.1.

At higher return periods (100, 200, and 500 years, top panels of Figure 5), AFI values exceeding 300°C are predicted to be able to occur not only in the north but in the southern part of UK, as well. The extreme AFI values in the south are again driven by the 1962/63 winter; excluding this winter from the analysis results in almost two times lower AFI values in most of the region (bottom panels in Figure 5).

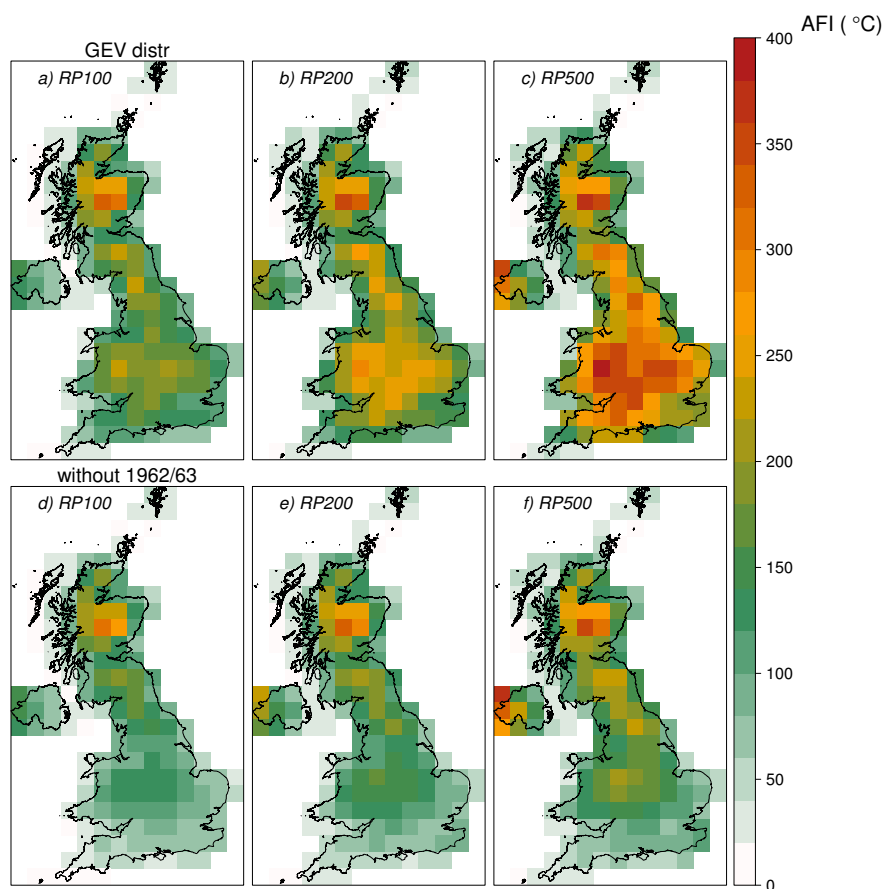


**Figure 4.** Top panels: Maps of AFI values (in °C) for return periods of a) 1 in 10, b) 1 in 25, and c) 1 in 50 years calculated assuming a GEV (Generalized Extreme Value) distribution. Bottom panels: Maps of AFI values (in °C) for the same return periods but computed empirically (see text).

### 3 Vine copulas

The stochastic behaviour of the hazard (i.e. AFI) at each cell is fully described by its corresponding GEV probability distribution, as described in section 2.2. However, insurance portfolio loss analysis requires the calculation of the combined stochastic behaviour of the hazard across all the model domain (i.e. all cells). This is described by the joint probability distribution of the hazard, which, according to Sklar's theorem, can be fully specified by the separate marginal GEV distributions and by their copula, which models the dependence between the hazard between cells.

Nevertheless, identifying the appropriate copula family is not an easy task since, in higher dimensions, the choice of adequate families is rather limited (Brechmann and Schepsmeier, 2013). Standard multivariate copulas, either do not allow for tail dependence (i.e. multivariate Gaussian) or have only a single parameter to control tail dependence of all pairs of variables



**Figure 5.** Top panels: Maps of AFI values (in °C) for return periods of a) 1 in 100, b) 1 in 200, and c) 1 in 500 years calculated assuming a GEV (Generalized Extreme Value) distribution. Bottom panels: Same as top panels but without taking into account the extreme winter of 1962/63.

(Student-t and archimedean multivariate copulas). This is particularly problematic for catastrophe modeling applications, where a flexible modeling of tails is vital to assess reliably the extreme behaviour of natural events.

Vine copulas provide a flexible solution to this problem based on a pairwise decomposition of a multivariate model into bivariate copulas, where each pair-copula can be chosen independently from the others. In particular, asymmetries and tail dependence can be taken into account as well as (conditional) independence to build more parsimonious models. Vines thus combine the advantages of multivariate copula modeling, that is separation of marginal and dependence modeling, and the flexibility of bivariate copulas (Brechmann and Schepsmeier, 2013).

In this study, the joint multivariate hazard distribution of AFI across all the model domain (170 cells) is decomposed as a product of marginal and pair-copula densities. The decomposition is not unique and Bedford and Cooke (2002) introduced a graphical structure called regular vine (R-Vine) structure to represent this decomposition with a set of nested





trees. The pair-copulas are fitted using the R (<https://www.r-project.org/>) package VineCopula (Schepsmeier et al., 2017; Brechmann and Schepsmeier, 2013). The method follows an automatic strategy of jointly searching for an appropriate R-Vine tree structure, its pair-copula families, and estimating their parameters developed by Dißmann et al. (2013). The fits are estimated sequentially and, for computational reasons, the two-parameter Archimedean copulas are excluded from this analysis (which however has only a negligible impact in the results, see Figure 1 of the Appendix). The large majority of the pairs (82%) are estimated to be independent, but these pairs occur mainly at the higher tree levels, since the most important dependencies are captured in the first trees (Brechmann and Schepsmeier, 2013; Dißmann et al., 2013). At the first level, 49% of the selected bivariate copulas are found to be Gumbel which implies greater dependence at larger AFI values. Larger dependencies, with Kendall's tau coefficients greater than 0.90, are found as expected between neighbouring cells.

Goodness-of-fit is performed for the final selected R-Vine Model (RVM) based on the RVineGofTest algorithm of the same R package (Schepsmeier, 2013). The Cramer von Mises test, which compares the empirical copula with the RVM, has a value of 0.019 and a p.value = 1, which indicates that the fitted RVM cannot be rejected at a 5% significance level.

The RVM is used to simulate 10,000 years of winter-seasons in the UK. This amount of realisations should be long enough in order to estimate with enough confidence the 200 year RP hazard, which is commonly associated with capital and regulatory requirements. For each year, the simulated AFI values at each grid cell depend on the other cells based on the fitted RVM. As an example, the AFI maps for the first six simulated winter-seasons are shown in Figure 6.

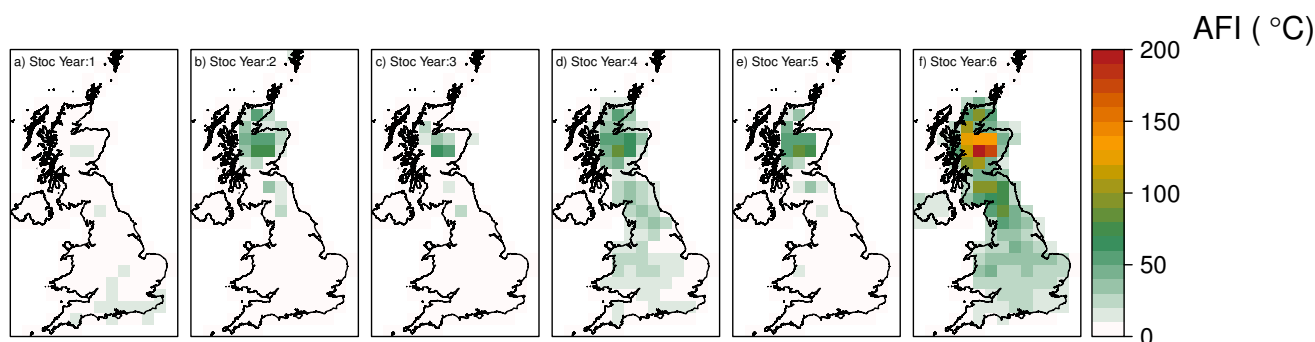
Since our focus is mainly on inhabited areas, for each simulation year, I compute the "weighted AFI" (wAFI), where the AFI value at each cell  $j$  is weighted by the corresponding number of residential properties ( $n_j$ ), as shown in equation 3. The weighted AFI thus places more weight on the hazard over large populated urban areas than agricultural or mountainous areas. The number of residential properties in the UK is taken from the PERILS Industry Exposure Database (<https://www.perils.org/>), which contains up-to-date high quality insurance market data at Cresta level ("Catastrophe Risk Evaluation and Standardizing Target Accumulations", <https://www.cresta.org/>) based on data directly collected from insurance companies writing property business in the UK. Exceedance probability plots based on the UK average AFI (without weighting) can be found in the Appendix (Figure 2).

$$wAFI_{year} = \frac{\sum AFI_{j,year} \cdot n_j}{\sum n_j} \quad (3)$$

### 3.1 Results and discussion

The exceedance probability (EP) curve of wAFI is shown in Figure 7, both for the historical and the stochastic data. The uncertainty intervals in the historical data are computed as the 5<sup>th</sup> and 95<sup>th</sup> quantile of the probability density function (Folland and Anderson, 2002).

Sensitivity tests are performed in order to evaluate the influence of selected RVM families and parameters to the resulting exceedance probabilities. EP curves based on RVMs that are fitted using only a single copula family are shown in Figure 1 of the Appendix. Apart of the case of Gaussian and Frank copulas, the choice of only a single copula family has a small impact

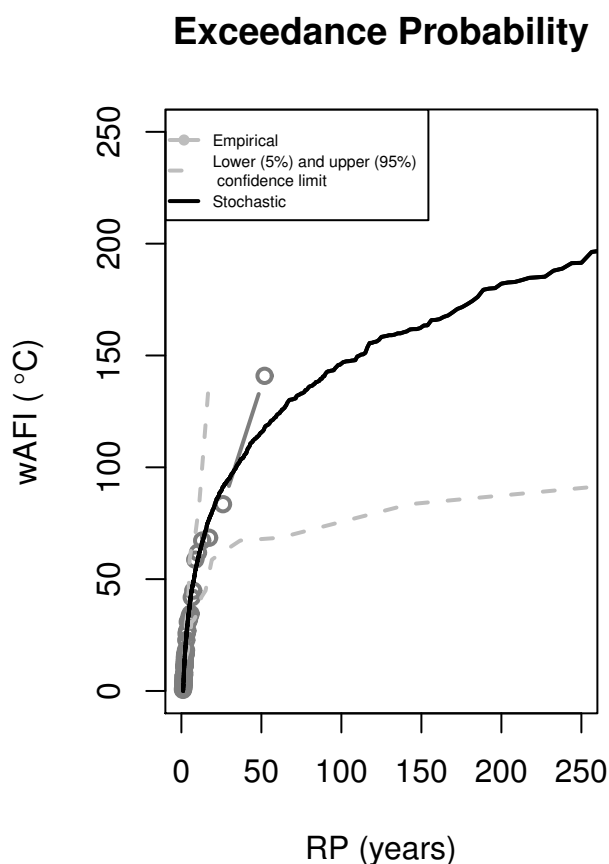


**Figure 6.** AFI maps (in °C) for the first six (out of 10K) years of the stochastic set.

on the resulting probabilities. The differences in the case of the Gaussian and, especially, Frank copulas are related to their low tail dependence. On the other hand, the low impact of the other copula families is due to the fact that the extreme hazard values are mainly driven by the large dependencies between nearby cells, especially at the first tree levels. This low sensitivity of the stochastic EP curve results in the selected RVM families and parameters, provides confidence on the robustness of the model.

5 The stochastic EP curve (black line, Figure 7) follows closely the empirical one (grey line and circles) apart from the last historical point (with a wAFI of 141°C), which corresponds to the 1962/63 severe cold winter. This event is estimated empirically as 1 in 52 years event, but with a high uncertainty around this estimate due to the small size of the historical record, as shown by the uncertainty lines in Figure 7. In the stochastic set, such an extreme winter represents a larger return period of 89 years. Especially for Southern England (Figure 8), this winter has been particularly rare; the model suggests a return period of 10 of 1 in 96 years, which corresponds well with other independent point measurements. For example, according to the Central England Temperature (CET) record, the oldest continuously running temperature dataset in the world (Manley, 1974), only two other winters (1683/84 and 1739/40) have been colder than 1962/63 in the last 350 years, suggesting a return period in the range of 110-120 years.

15 However, recent studies suggest that cold weather in the UK is likely to be less severe, to occur less frequently, and to last for a shorter period of time than was historically the case due to anthropogenic induced climate change (on Climate Change, 2017). Massey et al. (2012) used climate model simulations to demonstrate that cold December temperatures in the UK are now half as likely as they were in the 1960s. Christidis and Stott (2012) also indicate that human influence has reduced the

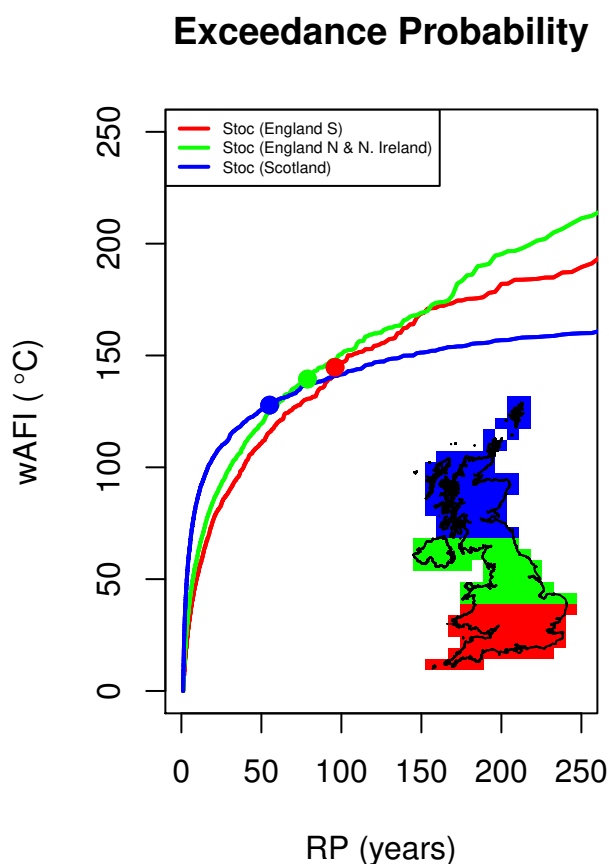


**Figure 7.** Exceedance probability curve of weighted AFI (in °C) based on the stochastic model (black line). The empirical estimates are shown in grey, including their 5%/95% confidence intervals.

probability of such a severe winter in UK by at least 20% and possibly by as much as 4 times, with a best estimate that the probability has been halved.

On the other hand, recent studies have argued that warming in the Arctic could favor the occurrence of cold winter extremes, and might have been also responsible for the unusually cold winters in the UK of 2009/10 and 2010/11 (Francis and Vavrus, 2012; Tang et al., 2013). This hypothesis though is still largely under debate, see for example Barnes and Screen (2015) and Wallace et al. (2014).

As shown in Figure 8, South England is in general warmer than the North England and Northern Ireland region, partially driven by the urban micro-climate effect of the London area. The 1962/63 winter was less extreme in this region (wAFI of 139 °C) with an estimated return period of 1 in 79 years. On the other hand, Scotland is usually significantly colder than the rest of UK, reaching for example AFI values of 100 °C almost 2 times more often. However, the curve flattens out shortly after



**Figure 8.** Exceedance probability curve of weighted AFI (in °C) for South England (red), North England & Northern Ireland (green), and Scotland (blue). The points denote the historical AFI values and estimated return periods for the winter 1962/63 for the respective regions.

that and more extreme winters are estimated to be less probable than in the South. The winter of 1962/63 was less extreme in comparison to the south and therefore shows a lower return period of approximately once every 55 years.

The winters 1962/63, 1985/86, 2009/10, 2010/11 were associated with a negative NAO phase (Murray, 1966; Osborn, 2011). The NAO has a profound effect on winter climate variability around the Atlantic basin, accounting more than half of the year-to-year variability in winter surface temperature over UK (Scaife et al., 2005; Scaife and Knight, 2008). Not surprising, the average AFI over the entire UK is found to be significantly anti-correlated ( $\rho = -0.59$ ,  $pval=4.810^{-6}$ ) with the winter (December through March) station-based NAO index (NAOI) (Hurrell, 2017), as shown in Figure 2. In order to investigate this further, I incorporated a generalized linear model (GLM) into the statistical distribution parameter estimates of the GEV fits. More precisely, I defined the location parameter of the fits for each cell as a function of the NAOI:  $\mu = \beta_0 + \beta_1 NAOI$  (see equation 1 in section 2.2). However, the non-stationary fits were statistically similar to the stationary ones, with  $\beta_1$  parameters not



significantly different from zero. This is probably related to the quite noisy character of the phenomenon and the relatively short historical record used in this study, which makes it difficult to discern the statistical differences in the extreme temperatures between positive and negative NAO winters. Because of its intrinsic chaotic behaviour, NAO is difficult (if even possible) to be predicted (Kushnir et al., 2006). Nevertheless, numerical seasonal forecast systems are currently rapidly improving and have even shown some success in the past (Graham et al., 2006; Folland et al., 2006). Incorporating such information in models could be very useful from the catastrophe risk management perspective.

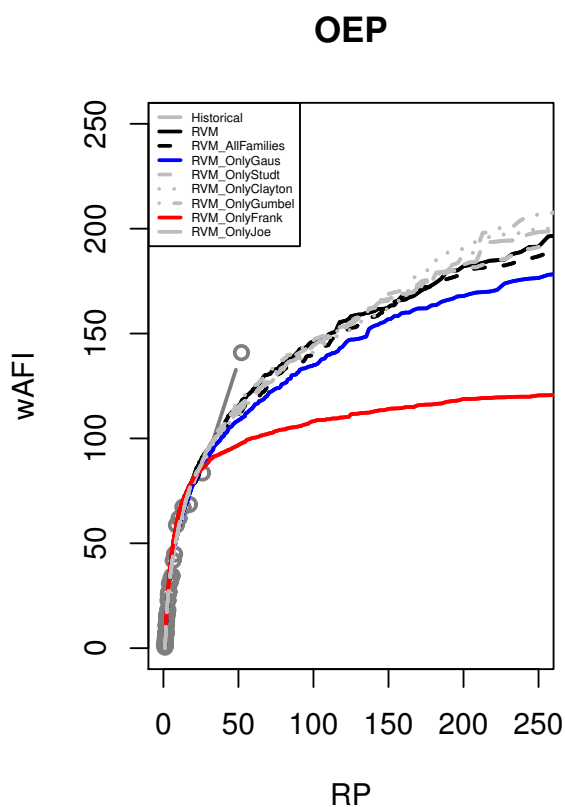
#### 4 Conclusions

This paper describes a hazard model of subfreezing temperatures in the United Kingdom. Extreme value theory has been applied in order to estimate the probability of extreme cold winters spatially across the UK. More importantly, the spatial dependence between regions in the UK has been assessed through a novel approach which takes advantage of the vine copula methodology. This approach allows the modeling of concurrent low temperatures across the country which is necessary in order to assess reliably the extreme behaviour of such events. A stochastic set of 10K years is generated which is used to estimate the exceedance probability of extreme cold winters in UK, such as the "Big Freeze of 1962/63". According to the model, such a cold winter is estimated to occur once every 89 years in UK. Especially for South England, this winter has been particularly rare with a return period close to 100 years. It is important to note, though, that considerable uncertainty exists in this estimate. First and foremost, the 52-year historical record used in this study is short in order to estimate with enough confidence the frequencies of such extreme events. A longer record of temperature data would be necessary in order to reduce the uncertainty. Meteorological reanalysis datasets could provide a comprehensive and consistent gridded temperature dataset over a very long period (e.g. Poli et al. (2016); Compo et al. (2011)), but higher spatial and temporal resolution is required in order to accurately calculate the air freezing index. Additional uncertainty in the model stems from the impacts of our changing climate due to anthropogenic forcing, but further research is necessary in order to discern how exactly winter temperature and circulation is affected in the UK. Significant improvements are expected to come with increasing availability of data, increasing understanding of the science, and with advancements in computing capability and technology. This model is part of the first probabilistic catastrophe model for insurance losses due to burst pipes resulting from freezing temperatures.

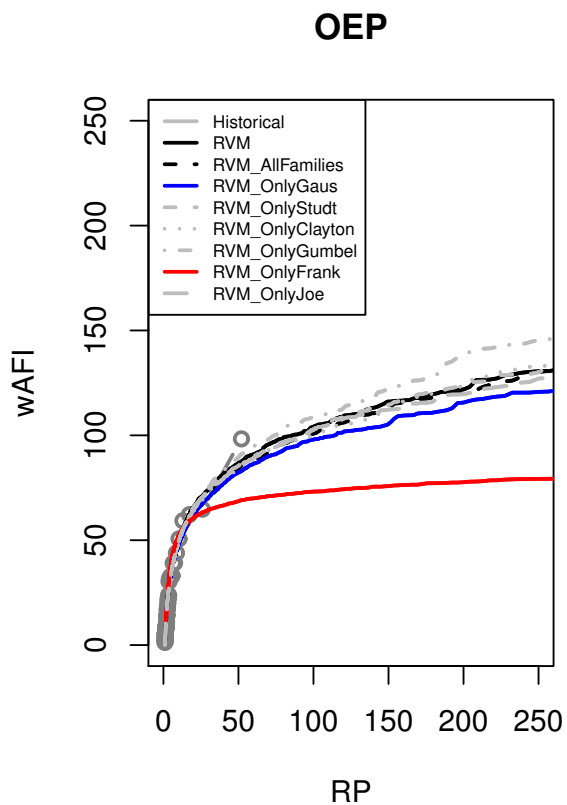
#### 25 Appendix A

##### A1

*Disclaimer.* TEXT



**Figure 1.** Sensitivity tests for the exceedance probability curve of weighted AFI based on RVM fitted using: all but the two-parameter Archimedean copulas (black line), all available copula families (black dotted line), only one Copula family each time, i.e. Gaussian (blue line line), Student's  $t$  (dashed grey), Clayton (dotted grey line), Gumbel (dotdash grey line), Frank (red line), and Joe (grey line). The Gaussian and Frank based RVMs which differ from the base RVM used in the text (black line) are shown in blue and red lines respectively, while the rest are shown in grey lines.



**Figure 2.** Same as Figure 1, but based on the UK average AFI (without weighting).

*Acknowledgements.* TEXT



## References

- ABI: Industry Data Downloads, Tech. rep., Association of British Insurers, <https://www.abi.org.uk/data-and-resources/industry-data/free-industry-data-downloads/>, 2017.
- AIR: About Catastrophe Models, Tech. rep., AIR Worldwide, <https://www.air-worldwide.com/Publications/Brochures/documents/About-Catastrophe-Models/>, 2012.
- Barnes, E. A. and Screen, J. A.: The impact of Arctic warming on the midlatitude jet-stream: Can it? Has it? Will it?, *Wiley Interdisciplinary Reviews: Climate Change*, 6, 277–286, doi:10.1002/wcc.337, <http://dx.doi.org/10.1002/wcc.337>, 2015.
- Bedford, T. and Cooke, R. M.: Vines—a new graphical model for dependent random variables, *Ann. Statist.*, 30, 1031–1068, doi:10.1214/aos/1031689016, <https://doi.org/10.1214/aos/1031689016>, 2002.
- 10 Bilotta, R., Bell, J. E., Shepherd, E., and Arguez, A.: Calculation and Evaluation of an Air-Freezing Index for the 1981–2010 Climate Normals Period in the Conterminous United States, *Journal of Applied Meteorology and Climatology*, 54, 69–76, doi:10.1175/JAMC-D-14-0119.1, <http://dx.doi.org/10.1175/JAMC-D-14-0119.1>, 2015.
- Bonazzi, A., Cusack, S., Mitas, C., and Jewson, S.: The spatial structure of European wind storms as characterized by bivariate extreme-value Copulas, *Natural Hazards and Earth System Sciences*, 12, 1769–1782, doi:10.5194/nhess-12-1769-2012, <https://www.nat-hazards-earth-syst-sci.net/12/1769/2012/>, 2012.
- 15 Bowman, G., Coburn, A., and Ruffle, S.: Freeze - Profile of a Macro-Catastrophe Threat Type, Cambridge centre for risk studies working paper series, Cambridge Risk Framework, <https://hazdoc.colorado.edu/handle/10590/6787?show=full>, 2012.
- Brechmann, E. C. and Schepsmeier, U.: Modeling Dependence with C- and D-Vine Copulas: The R Package CDVine, *Journal of Statistical Software*, doi:10.18637/jss.v052.i03, <https://www.jstatsoft.org/article/view/v052i03>, 2013.
- 20 Cawthorne, R. A. and Marchant, J. H.: The effects of the 1978/79 winter on British bird populations, *Bird Study*, 27, 163–172, doi:10.1080/00063658009476675, <http://dx.doi.org/10.1080/00063658009476675>, 1980.
- Christidis, N. and Stott, P. A.: Lengthened odds of the cold UK winter of 2010/2011 attributable to human influence [in "Explaining Extreme Events of 2011 from a Climate Perspective"], *Bulletin of the American Meteorological Society*, 93, 1060–1062, doi:10.1175/BAMS-D-12-00021.1, <https://doi.org/10.1175/BAMS-D-12-00021.1>, 2012.
- 25 Compo, G. P., Whitaker, J. S., Sardeshmukh, P. D., Matsui, N., Allan, R. J., Yin, X., Gleason, B. E., Vose, R. S., Rutledge, G., Bessemoulin, P., Brönnimann, S., Brunet, M., Crouthamel, R. I., Grant, A. N., Groisman, P. Y., Jones, P. D., Kruk, M. C., Kruger, A. C., Marshall, G. J., Mauerer, M., Mok, H. Y., Nordli, O., Ross, T. F., Trigo, R. M., Wang, X. L., Woodruff, S. D., and Worley, S. J.: The Twentieth Century Reanalysis Project, *Quarterly Journal of the Royal Meteorological Society*, 137, 1–28, doi:10.1002/qj.776, <http://dx.doi.org/10.1002/qj.776>, 2011.
- 30 Czado, C.: Pair-Copula Constructions of Multivariate Copulas. In: Jaworski P., Durante F., Härdle W., Rychlik T. (eds) *Copula Theory and Its Applications*. Lecture Notes in Statistics, Springer, doi:[https://doi.org/10.1007/978-3-642-12465-5\\_4](https://doi.org/10.1007/978-3-642-12465-5_4), 2010.
- Dißmann, J., Brechmann, E., Czado, C., and Kurowicka, D.: Selecting and estimating regular vine copulae and application to financial returns, *Computational Statistics & Data Analysis*, 59, 52 – 69, doi:<https://doi.org/10.1016/j.csda.2012.08.010>, <http://www.sciencedirect.com/science/article/pii/S0167947312003131>, 2013.
- 35 Folland, C. and Anderson, C.: Estimating Changing Extremes Using Empirical Ranking Methods, *Journal of Climate*, 15, 2954–2960, doi:10.1175/1520-0442(2002)015<2954:ECEUER>2.0.CO;2, 2002.





- Folland, C. K., Parker, D. E., Scaife, A. A., Kennedy, J. J., Colman, A. W., Brookshaw, A., Cusack, S., and Huddleston, M. R.: The 2005/06 winter in Europe and the United Kingdom: Part 2 –Prediction techniques and their assessment against observations, *Weather*, 61, 337–346, doi:10.1256/wea.182.06, <http://dx.doi.org/10.1256/wea.182.06>, 2006.
- Francis, J. A. and Vavrus, S. J.: Evidence linking Arctic amplification to extreme weather in mid-latitudes, *Geophysical Research Letters*, 5 39, n/a–n/a, doi:10.1029/2012GL051000, <http://dx.doi.org/10.1029/2012GL051000>, 106801, 2012.
- Frauenfeld, O. W., Zhang, T., and McCreight, J. L.: Northern Hemisphere freezing/thawing index variations over the twentieth century, *International Journal of Climatology*, 27, 47–63, doi:10.1002/joc.1372, <http://dx.doi.org/10.1002/joc.1372>, 2007.
- Gordon, J. R.: An Investigation into Freezing and Bursting Water Pipes in Residential Construction, Research report 96-1, Building Research Council. School of Architecture. College of Fine and Applied Arts. University of Illinois at Urbana-Champaign, <http://hdl.handle.net/2142/54757>, 1996.
- 10
- Graham, R. J., Gordon, C., Huddleston, M. R., Davey, M., Norton, W., Colman, A., Scaife, A. A., Brookshaw, A., Ingleby, B., McLean, P., Cusack, S., McCallum, E., Elliott, W., Groves, K., Cotgrove, D., and Robinson, D.: The 2005/06 winter in Europe and the United Kingdom:Part 1 –How the Met Office forecast was produced and communicated, *Weather*, 61, 327–336, doi:10.1256/wea.181.06, <http://dx.doi.org/10.1256/wea.181.06>, 2006.
- 15
- Guirguis, K., Gershunov, A., Schwartz, R., and Bennett, S.: Recent warm and cold daily winter temperature extremes in the Northern Hemisphere, *Geophysical Research Letters*, 38, n/a–n/a, doi:10.1029/2011GL048762, <http://dx.doi.org/10.1029/2011GL048762>, 117701, 2011.
- Hurrell, J.: The Climate Data Guide: Hurrell North Atlantic Oscillation (NAO) Index (station-based)., Tech. rep., National Center for Atmospheric Research, <https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-station-based>, 2017.
- 20
- Kemp, M.: Tail Weighted Probability Distribution Parameter Estimation, Tech. rep., Nematrian Limited, <http://www.nematrian.com/Docs/TailWeightedParameterEstimation.pdf>, 2016.
- Kushnir, Y., Robinson, W. A., Chang, P., and Robertson, A. W.: The Physical Basis for Predicting Atlantic Sector Seasonal-to-Interannual Climate Variability, *Journal of Climate*, 19, 5949–5970, doi:10.1175/JCLI3943.1, <http://dx.doi.org/10.1175/JCLI3943.1>, 2006.
- Makkonen, L.: Plotting Positions in Extreme Value Analysis, *Journal of Applied Meteorology and Climatology*, 45, 334–340, doi:10.1175/JAM2349.1, <https://doi.org/10.1175/JAM2349.1>, 2006.
- 25
- Manley, G.: Central England temperatures: Monthly means 1659 to 1973, *Quarterly Journal of the Royal Meteorological Society*, 100, 389–405, doi:10.1002/qj.49710042511, <http://dx.doi.org/10.1002/qj.49710042511>, 1974.
- Massey, N., Aina, T., Rye, C., Otto, F., Wilson, S., Jones, R., and Allen, M.: Have the odds of warm November temperatures and of cold December temperatures in Central England changed. *Bulletin of the American Meteorological Society* 93, 1057–1059, *Bulletin of the American Meteorological Society*, 93, 1057–1059, 2012.
- 30
- McDonald, A., Bscheiden, B., Sullivan, E., and Marsden, R.: Mathematical simulation of the freezing time of water in small diameter pipes, *Applied Thermal Engineering*, 73, 142 – 153, doi:<https://doi.org/10.1016/j.applthermaleng.2014.07.046>, <http://www.sciencedirect.com/science/article/pii/S135943111400622X>, 2014.
- Murray, R.: A note on the large scale features of the 1962/63 winter, *Meteorol. Mag.*, 95, 339–348, 1966.
- 35
- on Climate Change, C.: UK Climate Change Risk Assessment 2017. Synthesis report: priorities for the next five years, Tech. rep., Committee on Climate Change, <https://www.theccc.org.uk/wp-content/uploads/2016/07/UK-CCRA-2017-Launch-slidepack.pdf>, 2017.
- Osborn, T. J.: Winter 2009/2010 temperatures and a record-breaking North Atlantic Oscillation index, *Weather*, 66, 19–21, doi:10.1002/wea.660, <http://dx.doi.org/10.1002/wea.660>, 2011.



- Perry, M., Hollis, D., and Elms, M.: The generation of daily gridded datasets of temperature and rainfall for the UK, Tech. rep., National Climate Information Centre, 2009.
- Poli, P., Hersbach, H., Dee, D. P., Berrisford, P., Simmons, A. J., Vitart, F., Laloyaux, P., Tan, D. G. H., Peubey, C., Thépaut, J.-N., Trémolet, Y., Hólm, E. V., Bonavita, M., Isaksen, L., and Fisher, M.: ERA-20C: An Atmospheric Reanalysis of the Twentieth Century, *Journal of Climate*, 29, 4083–4097, doi:10.1175/JCLI-D-15-0556.1, <https://doi.org/10.1175/JCLI-D-15-0556.1>, 2016.
- 5 Scaife, A. A. and Knight, J. R.: Ensemble simulations of the cold European winter of 2005-2006, *Quarterly Journal of the Royal Meteorological Society*, 134, 1647–1659, doi:10.1002/qj.312, <http://dx.doi.org/10.1002/qj.312>, 2008.
- Scaife, A. A., Knight, J. R., Vallis, G. K., and Folland, C. K.: A stratospheric influence on the winter NAO and North Atlantic surface climate, *Geophysical Research Letters*, 32, n/a–n/a, doi:10.1029/2005GL023226, <http://dx.doi.org/10.1029/2005GL023226>, 118715, 2005.
- 10 Schepsmeier, U.: Estimating standard errors and efficient goodness-of-fit tests for regular vine copula models, Ph.D. thesis, Fakultät für Mathematik Technische Universität München, <http://mediatum.ub.tum.de/doc/1175739/document.pdf>, 2013.
- Schepsmeier, U., Stoeber, J., Brechmann, E. C., Graeler, B., Nagler, T., and Erhardt, T.: VineCopula-package: Statistical Inference of Vine Copulas, Tech. rep., R package, <https://cran.r-project.org/web/packages/VineCopula/VineCopula.pdf>, 2017.
- Seager, R., Kushnir, Y., Nakamura, J., Ting, M., and Naik, N.: Northern Hemisphere winter snow anomalies: ENSO, NAO and the winter of 2009/10, *Geophysical Research Letters*, 37, n/a–n/a, doi:10.1029/2010GL043830, <http://dx.doi.org/10.1029/2010GL043830>, 114703, 2010.
- 15 Sklar, A.: Fonctions de répartition à n dimensions et leurs marges, *Publications de l'Institut de Statistique de L'Université de Paris*, 8, 229–231, 1959.
- Tang, Q., Zhang, X., Yang, X., and Francis, J. A.: Cold winter extremes in northern continents linked to Arctic sea ice loss, *Environmental Research Letters*, 8, 014036, <http://stacks.iop.org/1748-9326/8/i=1/a=014036>, 2013.
- 20 Wallace, J. M., Held, I. M., Thompson, D. W. J., Trenberth, K. E., and Walsh, J. E.: Global Warming and Winter Weather, *Science*, 343, 729–730, doi:10.1126/science.343.6172.729, <http://science.sciencemag.org/content/343/6172/729>, 2014.
- Walsh, J. E., Phillips, A. S., Portis, D. H., and Chapman, W. L.: Extreme Cold Outbreaks in the United States and Europe, 1948–99, *Journal of Climate*, 14, 2642–2658, doi:10.1175/1520-0442(2001)014<2642:ECOITU>2.0.CO;2, 2001.