# Acceptance of Lightning Detectors and Localization Systems under Different Damping Conditions

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ABSTRACT

The acceptance of individual lightning detectors, idealized detection networks using both loop antenna and time of arrival techniques, and the Swedish lightning localization network have been investigated. The calculations were based on Weibull-distributed lightning signal strengths and the model of an exponentially damped spherical wave for the lightning pulse propagation.

For a real network, account has been taken for the different damping of pulse paths due to the land/water distribution.

The influence of different damping of the lightning pulses on the performance of lightning detection and localization was found to be very strong. The time of arrival method was more sensitive to the damping effects.

The technique presented in this paper can be used for maximizing the acceptance quality of a planned network and for weighting lightning density maps obtained by a lightning localization system.

## 1. Introduction

The acceptance of lightning detectors, defined as the percentage of detected cloud-to-ground lightning flashes, can be calculated knowing the lightning signal-strength distribution, the receiver thresholds and the damping conditions for the propagation of lightning pulses, as shown in a previous investigation (Schütte et al., 1987b). The main assumption was that the signal-strength distribution could be satisfactorily represented by a Weibull distribution.

The formulae from this work will now be used for calculating the acceptance under different conditions of exponential damping, for single stations or networks of stations, idealized or real. For a lightning detection and localizing system in Sweden, the dependence of the damping on the land/water distribution along the path of the lightning signal must be taken into account. Because of the high conductivity of water, there is negligible damping over the sea and lakes, but high damping occurs over the rocky Scandinavian soil.

There are two fundamentally different ways of locating lightnings using a detector network: 1) the magnetic direction-finding (MDF) method (Krider et al., 1980), where at least *two* detectors have to record the direction to a lightning; and 2) the time-of-arrival (TOA) method (Bent and Lyons, 1984), where at least *three* detectors have to record the onset time of a lightning pulse. One of the aims of this study is to compare the performance of both methods as regards the acceptance.



## 2. Acceptance as a function of damping

### *a. Acceptance for one station*

The acceptance, $A$, as a function of the distance, $r$, assuming an arbitrary damping function expressing the signal-strength attenuation with distance, $s = D(s_0, r)$, is (Schütte et al., 1987b):

$$A(r) = F[D^{-1}(s)]_{s_{\min}}^{s_{\max}} = F[D^{-1}(s_{\max})] - F[D^{-1}(s_{\min})] \quad (1)$$

where $s_{\min}$ and $s_{\max}$ are the lower and upper threshold values, respectively; $s_0$ is the signal strength normalized to a standard distance $r_0$, i.e. $s_0 = s \cdot r/r_0$; and

$$F(x) = \begin{cases} 0 & \text{for} \quad x \leqslant c \\ 1 - \exp\left[-\left(\dfrac{x-a}{a}\right)^b\right] & \text{for} \quad x > c \end{cases} \quad (2)$$

is the cumulative three-parameter Weibull distribution of the variable $x$ (Weibull, 1939) where $a$, $b$ and $c$ are the scale, shape, and location parameters, respectively. The signal-strength distribution for negative lightnings over sea (no damping) from Schütte et al. (1987b), based on data from Pisler (1984), will be used throughout this paper.

Assuming an exponentially damped spherical wave of the form

$$s = D(s) = \frac{s_0 r_0}{r} \exp\left[-\left(\frac{\lambda r}{r_0}\right)\right] \quad (3)$$

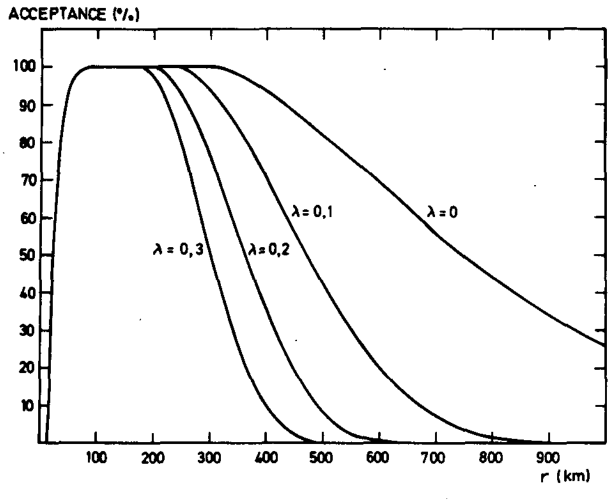



FIG. 1. Acceptance of a lightning detector as function of distance for different damping parameters λ.

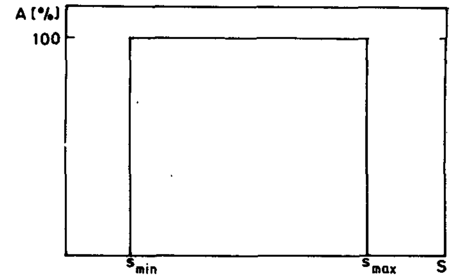



FIG. 3. As in Fig. 2 except for an ideal lightning detector.

where $\lambda$ is an empirical damping coefficient, it is possible to calculate the relationship between $A$, $r$ and $\lambda$ (Fig. 1). The value of $r_0$ will be 100 km. In all the calculations, $s_{\min}$ and $s_{\max}$ will be 20 and 600 arbitrary units (a.u.) respectively. These units are used by the MDF system installed in Sweden (Krider et al., 1980); 150 a.u. are equivalent to 5 V $\mathrm{m}^{-1}$ of the peak radiation field and approximately equivalent to 45 kA peak current for a lightning stroke at 100 km distance. The arbitrary units are proportional to the peak value of the lightning pulse, which is approximately proportional to the maximum current of the lightning. We use these values since higher or lower values are rare in our records, because the threshold values are angle-dependent, and we expect that the MDF system behaves more as in Fig. 2 than as in Fig. 3. More statistics, as well as more realistic information about the dynamics of the system, are desirable.

The effective radius $\rho$ of a lightning detector was defined in Schütte et al. (1987b) as

$$\pi\rho^2 = 2\pi \int_0^\infty A(r) r dr. \tag{4}$$

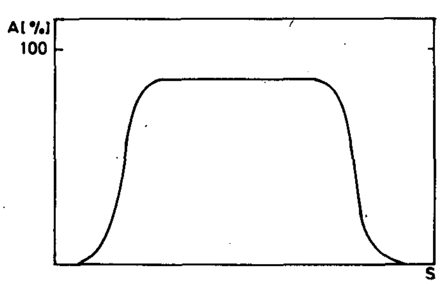



FIG. 2. Acceptance as function of signal strength for a real lightning detector (from Schütte et al., 1987b).

The solution can be made analytic for undamped signal propagation (Fig. 4). In other cases, the effective radius can be calculated, as a function of the damping parameter, by means of numerical integration (Fig. 5).

### b. *Acceptance for a network*

In order to locate a lightning, the signal has to be accepted by at least two stations for the MDF system and three stations for the TOA system, respectively. If a lightning pulse within the thresholds is accepted, the station governing the acceptance in a MDF system would be the station with the second highest acceptance:

$$A = a_2 \quad a_1 \text{ ranked } a_1 \geq a_2 \geq a_3 \geq \cdots \tag{5}$$

where $a_i$ denotes the acceptance of one detector and $A$ the acceptance of the network.

In reality, a fraction $(1 - \beta)$ of the lightnings within the limits will be rejected for different reasons. Equation (5) then becomes

$$A = \beta^2 a_2 + \beta^2(1-\beta)a_3 + \beta^2(1-\beta)^2 a_4 + \cdots. \tag{6}$$

From our observations, $\beta \approx 0.95$, so we truncate this expression after the linear term in $(1 - \beta)$.

The analogous expression for a TOA system is

$$A = \beta^3 a_3 + \beta^3(1-\beta)a_4 + \cdots. \tag{7}$$

The acceptance of a network is a function of the location of the stations and the damping. In our cal-

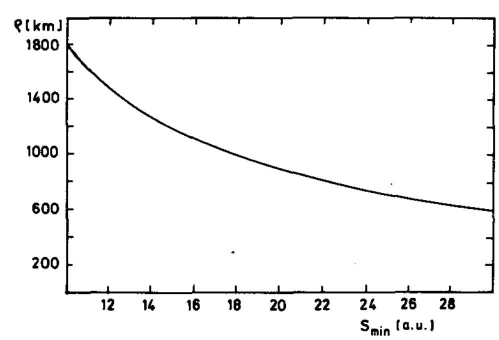



FIG. 4. Effective radius of an ideal lightning detector as a function of the lower threshold (from Schütte et al., 1987b).

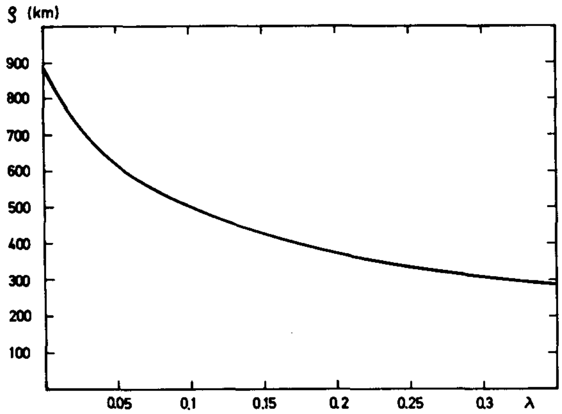



FIG. 5. As in Fig. 4 except for a function of the damping parameter λ.

culations, the coordinates were simplified as Cartesian, thus neglecting curvature of the earth. In the model cases, the damping was considered as isotropic and representable by the damping parameter $\lambda$. The $A(x, y)$ was calculated for different values of $\lambda$. The damping along the lightning pulse paths to the detector will be treated below in the section concerning the Swedish MDF system.

## 3. Examples of acceptance maps for different networks and systems

Three networks were modeled:

- two MDF stations with 400 km separation,
- three MDF stations on an equilateral triangle with 400 km side, and
- four TOA stations in a square with 400 km side.

The map region in all the cases is $1000 \times 1000$ km$^2$.

For each network, three different degrees of damping have been assumed, viz. $\lambda = 0$, 0.15 and 0.3. These are values of reasonable order of magnitude, as shown by theoretical calculations on damping of lightning pulses (Kawasaki and Cooray, personal communication) and real observations (Pišler, 1984). Additional observations and statistics would be desirable, however.

### a. *Two MDF stations (Figs. 6–11)*

This is the simplest arrangement; it shows three important features of a MDF system:

1) The region of maximum acceptance is situated between the stations.

2) There is an overflow minimum around each station due to signal overflow at the nearest station. This is magnituded five times in Figs. 9–11.

3) The introduction of damping dramatically reduces regions having an acceptance above a certain acceptance level.

### b. *Three MDF stations (Figs. 12–14)*

Using three stations, we still have the highest acceptance between the stations, but there are no overflow minima at the stations. The influence of damping is strong in this case, too.

### c. *Four TOA stations (Figs. 15–17)*

TOA networks always consist of at least four stations even if, in principle, three would be enough for the localization of a lightning. But even with one redundant station, the requirement for accepting a lightning by at least three stations reduces the acceptance to levels comparable with those of a two-station MDF system or even less using the same thresholds. Note that there are "overflow minima" around the stations. The acceptance in these minima is 62% and 8% for $\lambda = 0$ and 0.15, respectively. But apart from this behavior very close to the stations, the acceptance is higher near the stations than between them. The influence of the damping is very strong.

A comparison between the three networks is made in Table 1, showing the fraction of map region with acceptance over a certain level.

Any configuration of an arbitrary number of stations can easily be tested with the method outlined in this investigation.

## 4. How to include geography

In reality, the ground conductivity, and thus the damping, will not be constant for the entire map region. The minimum requirement is to take account of the distribution of land and water by assuming a constant damping coefficient, $\lambda$, over land and no damping over water. We define the distance to the lightning as $r = a + b$, where $a$ and $b$ are the path lengths over water and land, respectively.

A signal, initially propagating over water and later over land will be attenuated according to

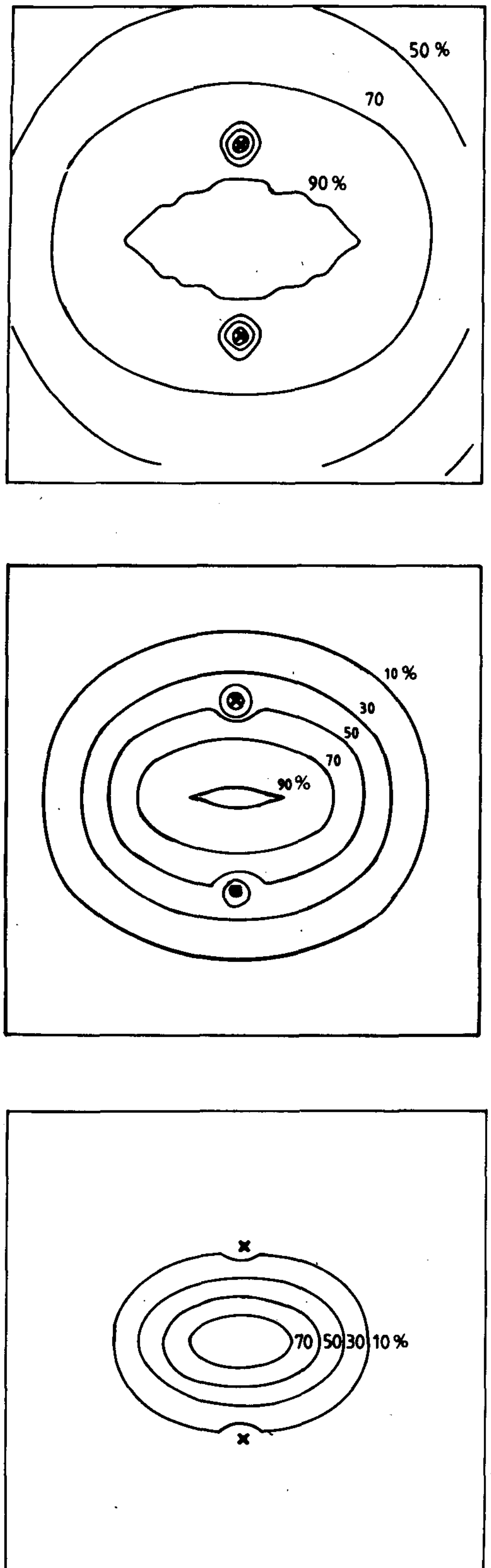
$$s(r) = \frac{s_0 r_0}{a} \cdot \left[\frac{a}{a+b} \exp\left(-\frac{\lambda b}{r_0}\right)\right]$$
$$= \frac{s_0 r_0}{a+b} \exp\left(-\frac{\lambda b}{r_0}\right) = \frac{s_0 r_0}{r} \exp\left(-\frac{\lambda b}{r_0}\right). \quad (8)$$

Similarly, for a signal initially traveling over land and later over water:

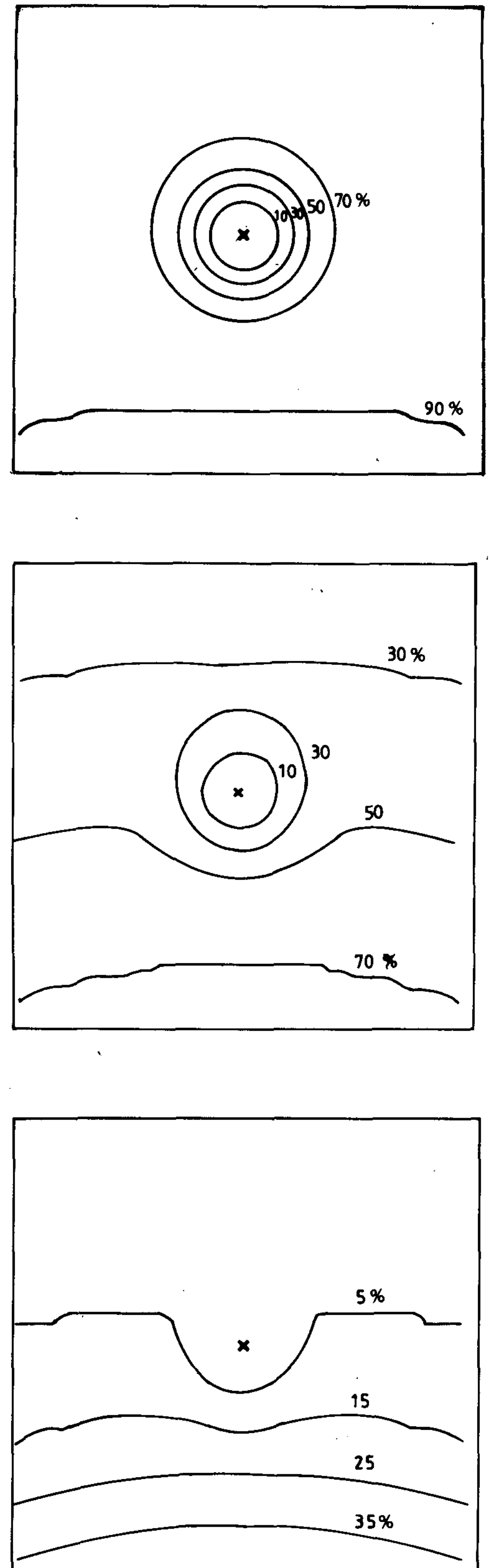
$$s(r) = \left[\frac{s_0 r_0}{b} \exp\left(-\frac{\lambda b}{r_0}\right)\right] \cdot \frac{b}{a+b} = \frac{s_0 r_0}{a+b} \exp\left(-\frac{\lambda b}{r_0}\right)$$
$$= \frac{s_0 r_0}{r} \exp\left(-\frac{\lambda b}{r_0}\right), \quad (9)$$

which is the same as Eq. (8).

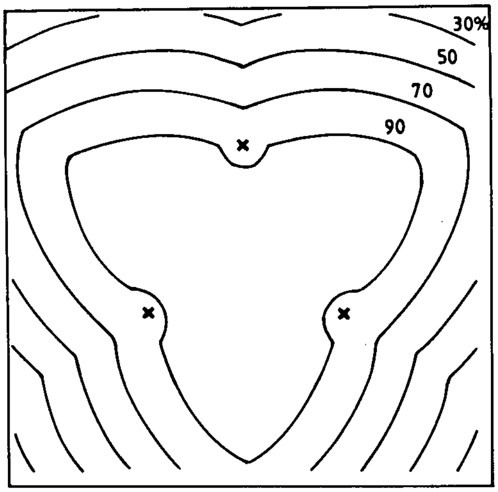
This linearity allows the introduction of a factor $\alpha = b/r$, the ratio between the propagation distance over land and the length of the entire path:

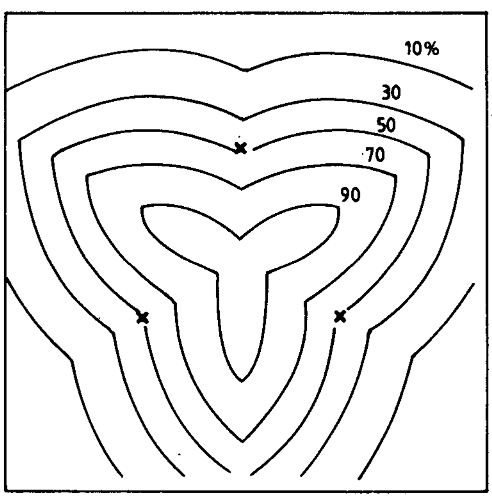


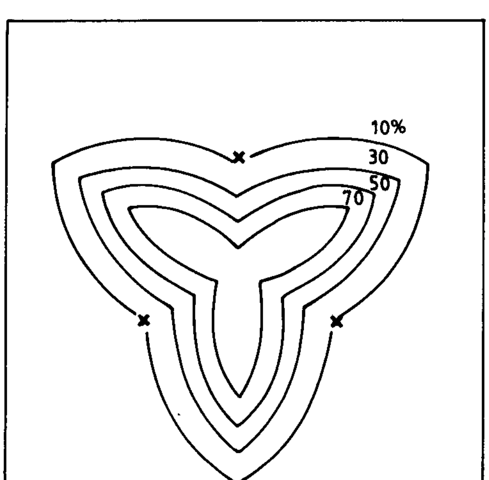

FIGS. 6–8 (from top to bottom). Acceptance maps for two MDF stations at 400 km distance. Fig. 6, λ = 0; Fig. 7, λ = 0.15; Fig. 8, λ = 0.3.



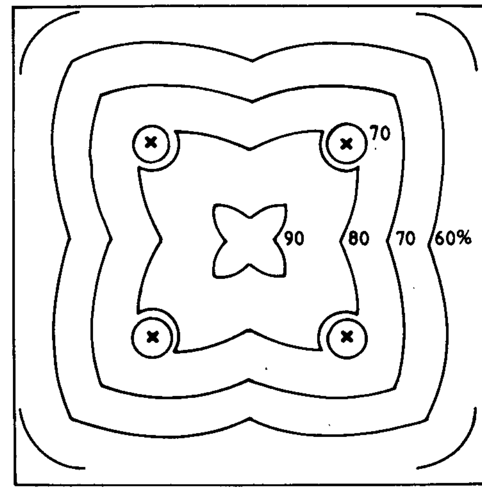

FIGS. 9–11 (from top to bottom). As in Figs. 6–8 except with the surrounding of one station amplified five times.

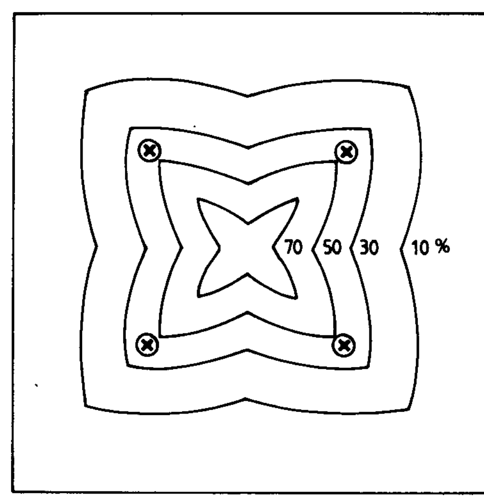


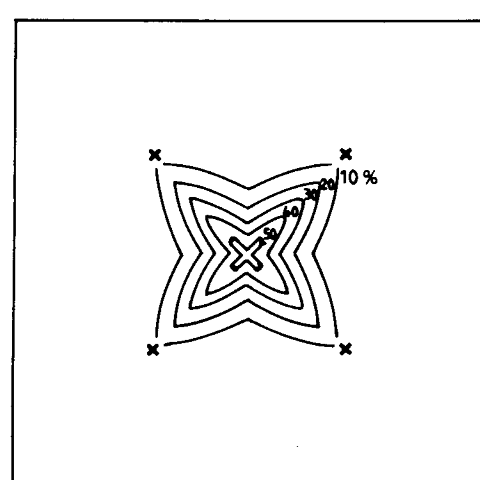




FIGS. 12–14 (from top to bottom). Acceptance maps for three MDF stations at 400 km distance. Fig. 12, $\lambda = 0$; Fig. 13, $\lambda = 0.15$; Fig. 14, $\lambda = 0.3$.




FIGS. 15–17 (from top to bottom). Acceptance maps for four TOA stations at 400 km distance. Fig. 15, $\lambda = 0$; Fig. 16, $\lambda = 0.15$; Fig. 17, $\lambda = 0.3$.

TABLE 1. Comparisons among three different detector networks for three values of λ.

| | Acceptance level (%) | | | | |
|---|---|---|---|---|---|
| | 10 | 30 | 50 | 70 | 90 |
| | | | $\lambda = 0$ | | |
| MDF 2 stations | 100 | 100 | >95 | 45 | 10 |
| MDF 3 stations | 100 | 100 | 100 | 78 | 33 |
| TOA 4 stations | 100 | 100 | 97 | 42 | 3 |
| | | | $\lambda = 0.15$ | | |
| MDF 2 stations | 50 | 28 | 17 | 9 | 1 |
| MDF 3 stations | >70 | 55 | 37 | 22 | 7 |
| TOA 4 stations | 50 | 25 | 12 | 3 | 0 |
| | | | $\lambda = 0.3$ | | |
| MDF 2 stations | 17 | 10 | 5 | 2 | 0 |
| MDF 3 stations | 37 | 23 | 14 | 7 | 0 |
| TOA 4 stations | 13 | 4 | 0.3 | 0 | 0 |

$$s(r) = \frac{s_0 r_0}{r} \exp\left(-\frac{\alpha \lambda r}{r_0}\right). \qquad (10)$$

It is even possible to account for the influence of different conductivity of different parts of land because of the linearity (Fig. 18).

## 5. Acceptance for the present MDF system in Sweden

In order to model the damping conditions for the MDF network in southern Scandinavia, a grid of 11 × 11 points was used, approximately forming a square with 1000 km side (Fig. 19). The deviation from a square is due to curvature of the earth. For each of these grid points the factor $\alpha$ was determined according to Fig. 18 for each of the four stations. The parameter $\eta$, defined in Fig. 18, was chosen to be 1 for Sweden, Norway and Finland and 0.5 for the other countries, because of their higher ground conductivity. The $\alpha$ values for the four stations are shown in Figs. 20–23. The effects of coastlines, the large lakes Vänern and Vättern, and the island of Gotland are evident. In this way $\alpha$ actually becomes a tensor

$$\alpha_{ijk}, i = 1 \cdots 11, \quad j = 1 \cdots 11, \quad k = 1 \cdots 4. \quad (11)$$

Linear interpolation was used for points between the grid points.

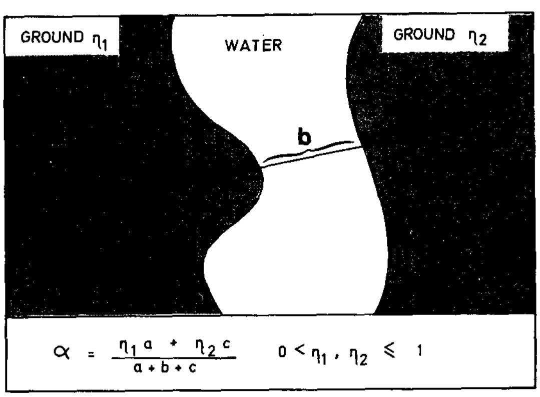



FIG. 18. The method used for determining the weight factor $\alpha$.

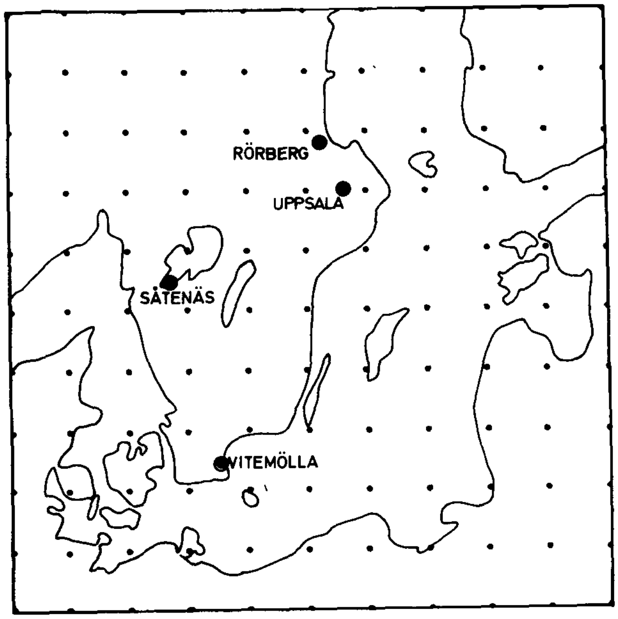



FIG. 19. Map of the region for Figs. 20–23 and 25–30 with grid points used for determining $\alpha$.

Before calculating acceptance maps for the Swedish MDF network, we have to take account of the angle dependence of the over- and underflow thresholds of crossed loop antennas. The LLP direction finders are working in a way so that the over- or underflow of the antenna loop with the higher induced voltage is determining if the signal will be accepted or not. Taking the dipole characteristics for each of the loops, the low and higher thresholds, $s_{min}$ and $s_{max}$, and the related radius for a given lightning strength are depending on angle according to

$$s(\varphi) = s_0/w \quad r(\varphi) = r_0 w \quad w = \max(|\cos\varphi|, |\sin\varphi|) \quad (12)$$

where $s_{min_0} = 17.2$ and $s_{max_0} = 517$, which is equivalent to an effective $\bar{s}_{min}$ and $\bar{s}_{max}$ of approximately 20 and 600 (Fig. 24). According to Krider (personal communication), the new LLP stations use Pythagoras' rule before testing for over- or underflow. This will not give any angle dependence. Recent information from the manufacturer states that the angle dependency of the *new* direction finder model follows the shape of Fig. 24, having the highest sensitivity at 0°, 90°, 180° and 270°. The older stations check overflow using the sum of the absolute voltage values of both loops. This will give a rotation of the figure by 45°, yielding the highest sensitivity at angles 45°, 135°, 225° and 315°. Figures

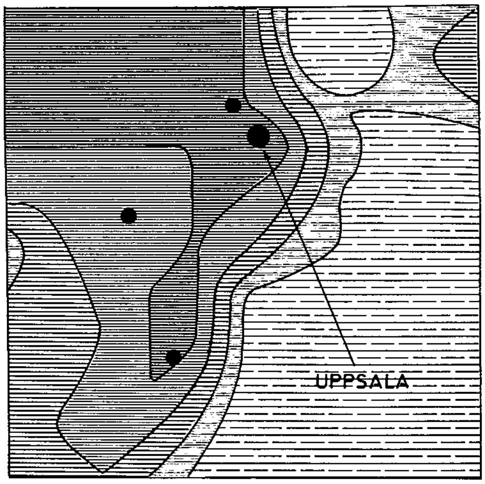



FIG. 20. Map of $\alpha$ for Uppsala.

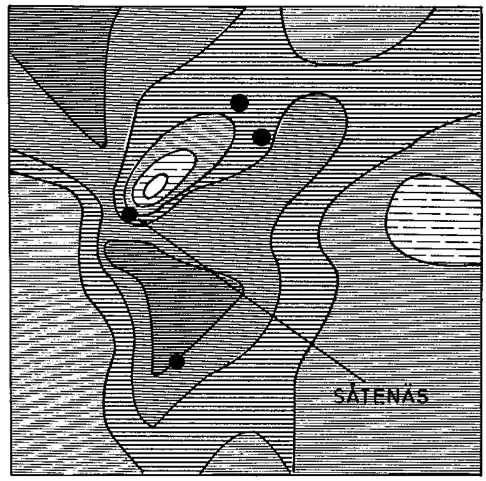



FIG. 22. Map of $\alpha$ for Såtenäs.

25 and 26 are thus representative for the *new type* of direction finders.

Acceptance maps with different maximal damping $\lambda$ as parameter were calculated, both for the three-station network (excluding Rörberg north of Uppsala) (Figs. 25a–c) and the four-station network presently used (Figs. 26a–c).

The most essential features are

- The large difference in acceptance decreases with distance over land (NW corner, Norway) and water (SE corner, Baltic Sea). This agrees well with the observations of the MDF system performance.
- A great improvement of the acceptance in the NE corner was made by introducing the fourth station (neglecting the bad localization quality due to the short baseline Uppsala–Rörberg).
- Overflow minima were observed at Vitemölla and Uppsala, and for the pair Uppsala/Rörberg, but not at Såtenäs. This can be explained by the location of Såtenäs between the other stations. Lightnings which cause overflow at Såtenäs will still be accepted by at least two other stations. For lightnings close to one of the other stations, the remaining stations are often too far away in order to receive a strong enough signal, or too near each other (Uppsala/Rörberg), with overflow occurring at both stations. This behavior has been observed very often in practice.

## 6. Discussion and conclusions

Before discussing the results, we have to be aware of the following three weak points of this investigation:

1) *The uncertainty of the thresholds, i.e. in the dynamic range of the detectors.* Figures 4 and 27 show the large influence of the lower threshold $s_{\min}$. The effective radius is almost proportional to $1/s_{\min}$ (Schütte et al., 1987b). Changes in the assumptions will alter the results. They will not alter the principal behavior,

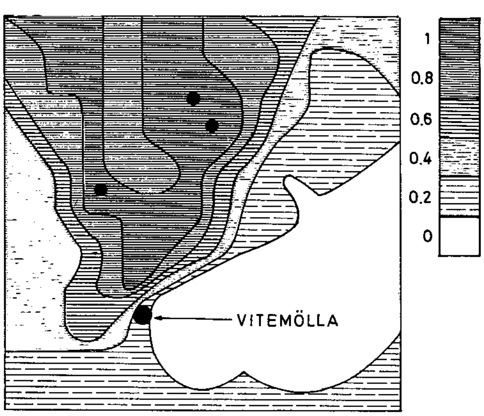



FIG. 21. Map of $\alpha$ for Vitemölla.

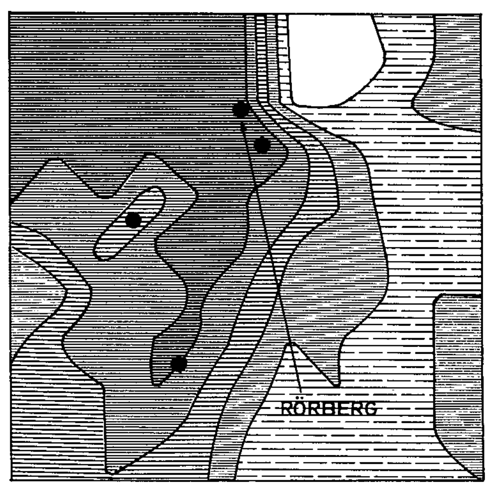



FIG. 23. Map of $\alpha$ for Rörberg.

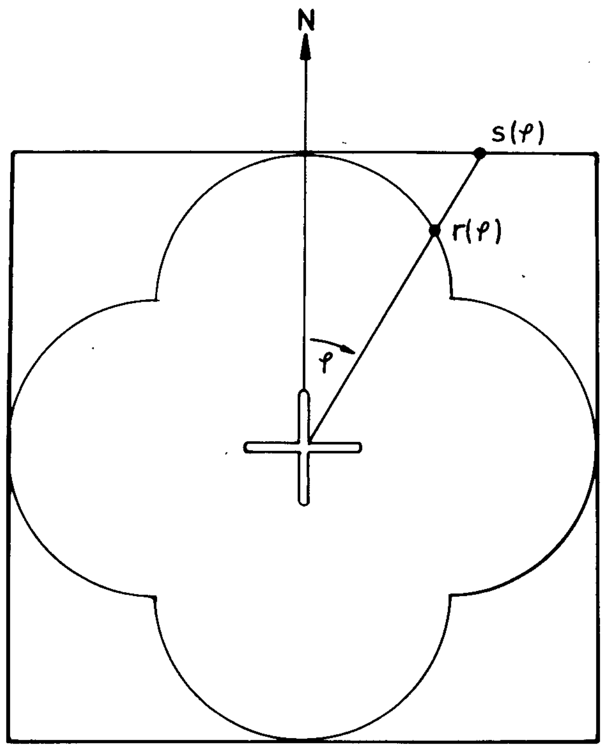



FIG. 24. Threshold signal strength $s$ and corresponding radius $r$ as function of the angle $\varphi$ for crossed loop antennas.

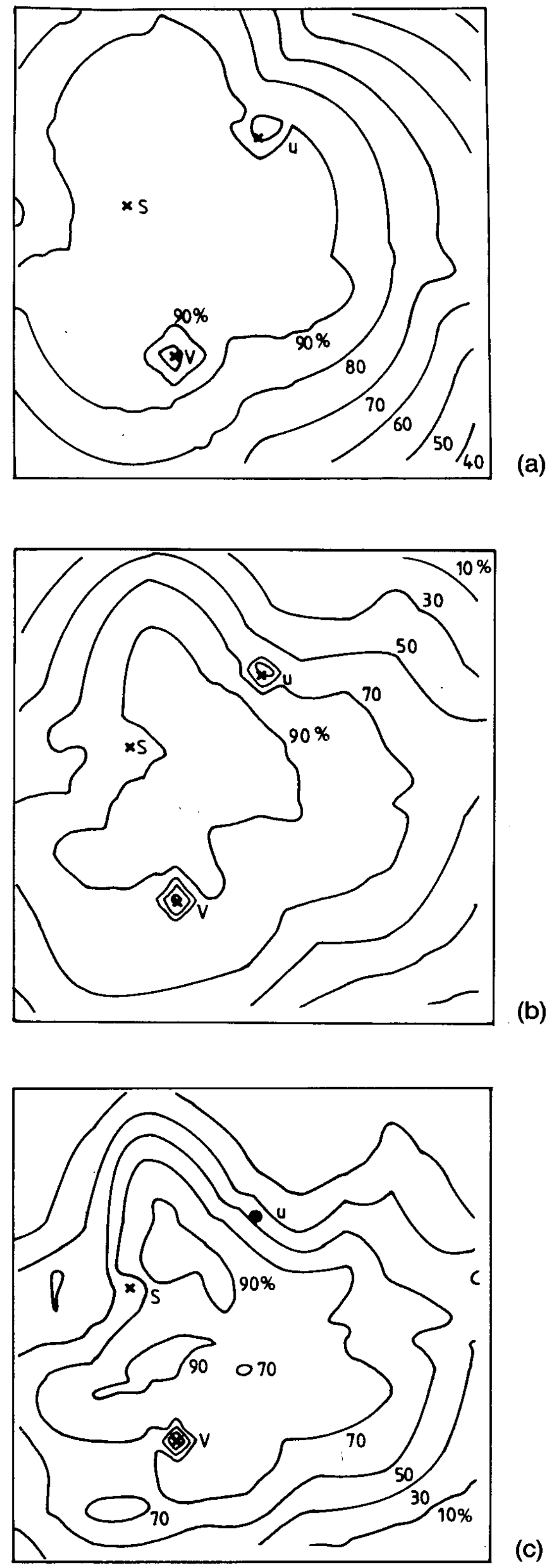



FIG. 25. Acceptance maps for the Swedish MDF system, three stations. (a) $\lambda = 0$, acceptance at Uppsala and Vitemölla 77%. (b) $\lambda = 0.15$, acceptance at Uppsala 17%, at Vitemölla 26%. (c) $\lambda = 0.3$, acceptance at Uppsala 0% and at Vitemölla 1%.

though the effective radius can be visualized as a kind of length scale, influenced by $s_{min}$. Changes in $s_{max}$ will alter the size of the "overflow holes," but not their shape.

2) *The signal-strength distribution* turns out to be dependent both on polarity and the meteorological situation. This also implicitly comprises the location of the station in different climates. We still do not have enough statistics on the signal-strength distribution of positive lightnings in Sweden. Work has been made elsewhere (Orville et al., 1985), and a rough assumption is that the mean strength of positive lightnings is about twice that of negative lightning. This will hence double the scale length. The bias toward positive lightnings received from longer distances, as a consequence of this, agrees well with our observations.

Some preliminary analyses on a few cases of negative lightning showed that the distribution becomes more long tailed under colder weather conditions, so that the probability of very strong lightning increases. The change in shape of the distribution can influence the acceptance conditions, but it will be less than for the positive lightnings.

3) *The parameterization of the damping.* The assumption of an exponentially damped spherical wave is based on a frequency-independent damping. In reality, the damping increases with frequency. The lightning pulse will hence become smoother with distance, because the higher frequencies will be more damped.

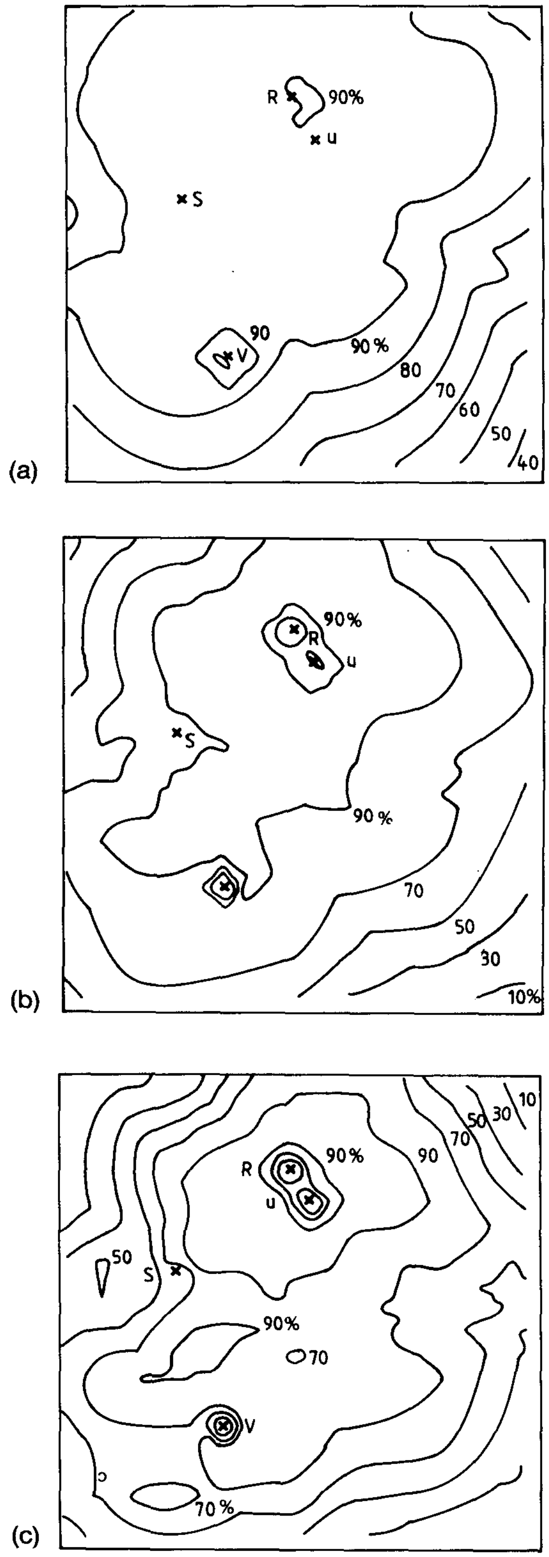



FIG. 26. Acceptance maps for the Swedish MDF system, four stations. (a) $\lambda = 0$, acceptance at Rörberg 85%, at Vitemölla 79%. (b) $\lambda = 0.15$, acceptance at Uppsala 67%, at Vitemölla 27%, at Rörberg 52%. (c) $\lambda = 0.3$, acceptance at Uppsala 30%, at Vitemölla 1%, at Rörberg 38%.

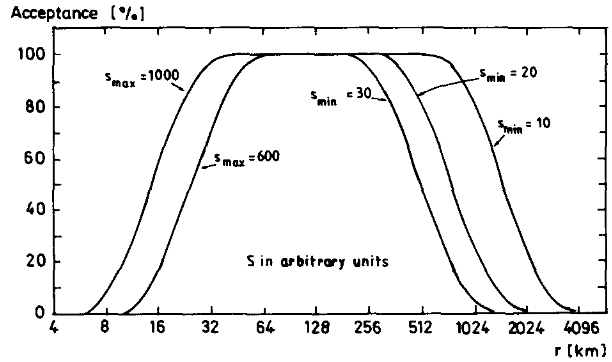



FIG. 27. Acceptance of an ideal lightning detector as function of distance for different low and high threshold values (from Schütte et al., 1986).

The damping of the whole pulse thus decreases with distance (Kawasaki and Cooray, personal communications).

Problem 1 can be solved in principal by making statistics of a very large number, $N$, of recorded lightnings. A plot of $(dN/ds)/p(s)$ against the unnormalized signal strength, $s$, will then converge against the acceptance/signal-strength function given in Fig. 2, where $p(s)$ describes the theoretical distribution and $dN/ds$ the measured distribution of the unnormalized signals. If this function turns out to have long tails, the while function has to be included into the integrations of the acceptance calculations. In other cases, one can determine effective $s_{\min}$ and $s_{\max}$ instead. A complication when using the data from the Swedish MDF system is the direction dependence of the threshold discussed previously. In this case, the plot $(dN/ds)/p(s)$ against $s$ will also contain this effect in an integrated way, making it smoother.

Testing the stations with simulated lightning pulses of different strengths (a large number is necessary for good significance), along with more detailed information from the manufacturer, would also be helpful.

The only way to solve problem 2 is to compute statistics on lightning data for different meteorological situations and polarities. Perhaps it will be enough to calculate the acceptance for positive and negative lightnings separately. Otherwise one has to divide lightning data in, for example, four cases with warm or cold weather conditions, viz. negative/warm, negative/cold, positive/warm and positive/cold.

Problem 3 is more difficult to solve. Discarding the exponential damping model will remove the linearity, which is fundamental for this approach. To obtain a more exact solution one has to follow a standard lightning pulse from the striking point to the detector over a conductivity map, while calculating the differential damping with adequate resolution.

The necessity of such large effort is doubtful. The differential damping of lightning pulses produces an-other acceptance decrease. Becoming smoother with distance, the rise time of the pulses may increase so that they no longer fulfill the rise time criterion and are rejected. For a sufficient parameterization of this effect, very extensive studies of the statistics of lightning pulse shapes and their dependence on the signal strength are necessary.

A close examination of the acceptance maps, obtained using the assumptions in this investigation, gives a great deal of information about the performance of lightning detection networks:

- The optimum distance between detecting stations is a function of the damping conditions. This distance can easily be deduced for different damping from Fig. 1, but considering the distance where the acceptance passes through, for example, the value 90%. Using this criterion, the optimum distance varies from approximately 450 km without damping to about 230 km with a damping coefficient, λ, of 0.3.
- Each station has to be within the optimum distance to at least two stations in order to avoid overflow minima.
- If one has to construct a network using less stations than necessary to fulfill the optimum distance criterion, one can optimize the system performance by testing different locations of the available stations before installing them. Together with the site-error detection method described in previous works (Pišler and Schütte, 1985, and Schütte et al., 1987a) which makes it possible to test a site as soon as the antenna has been installed, this will be a powerful tool for the construction of new lightning localization networks.

After improving the input parameters' dynamic range and signal-strength distribution, and after finding the most probable damping parameter λ (both from statistical and theoretical calculations), the method in this study is considered to be sufficient for the correction of lightning density maps obtained from the Swedish MDF system.

The method also opens a way to the use of relative acceptance data from regions where two lightning location networks overlap (for example the Swedish network with the Norwegian or Finnish). If, in one sufficiently small region, the ratio between the number of lightnings observed by network 2 to the number observed by network 1 is

$$q = N_2/N_1, \tag{13}$$

then one has a relation between this relative acceptance $q$ and the acceptances of both networks:

$$q = A_2/A_1. \tag{14}$$

Using this as an equation for the damping coefficient $\lambda$ [$A_1$ *and* $A_1(\lambda)$, $A_2 = A_2(\lambda)$], one obtains an equation which can be transformed into the form

$$g(\lambda) = 0, \tag{15}$$

which has to be solved numerically with respect to λ. This method may be a way to extract damping information from lightning data.

## APPENDIX

### Notes on The Map Plotting and Calculation Procedure

In all cases, the acceptance was calculated for $41 \times 41 = 1681$ points. This gives a step width of 25 km or 5 km for Figs. 9–11. The $11 \times 11 \times 4$ tensor $\alpha_{ijk}$ was first doubled to $21 \times 21 \times 4$ points using linear interpolation in the rows, the columns and the diagonals. The algorithm was then repeated, yielding $41 \times 41 \times 4$ values. The values $\alpha_{ijk}$ were determined manually on a map from 0 to 1 in steps of 0.1. More sophisticated interpolation methods (spline interpolation or development in Legendre polynomials) were not considered necessary.

The calculations were performed on the IBM computer at the University of Uppsala computer centre (UDAC). Due to format reasons, the output was a $39 \times 39$ matrix, i.e., the boundary values have been skipped. These data were read into a personal computer (McIntosh). By scanning the rows and columns, points of constant acceptance were found using linear interpolation. They form lines $A(x, y) =$ constant, i.e. "isoacceptance lines."

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