

## **Interactive comment on “Estimations of statistical dependence as joint return period modulator of compound events. Part I: storm surge and wave height” by Thomas I. Petroliaqkis**

**Thomas I. Petroliaqkis**

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### **General Comment**

*The paper addresses compound events defined by combined high surges and high wind waves along European coastlines, especially in estuaries/river mouths. Statistical methods are used to investigate joint probabilities of compound events and the statistical dependency, since flood risk is not a function of one parameter (storm surges with peak value and duration) but usually of more (e.g. wind waves, river runoff). Large scale weather systems can cause either high storm surges or high wind waves and further more high precipitation and river runoff/discharges. Two sets of almost 35-year hindcasts of storm surges and wave heights were used to analyse the correlation and statistical dependency. As expected the frequency of the occurrence of the top compound events in different coastal areas were found to be higher during the winter months. In the introduction the hydrological and meteorological conditions for high wind waves and extreme tidal surge events which can occur simultaneously with extreme precipitation events and high river flows (compound events) leading to increased flood risk is highlighted clearly. But the paper and the used methodology focused only on very few parameters. What is the background of the generalization? The subject of the paper is interesting yet a little confusing especially in the context of coastal engineering therefor the manuscript should be major improved. The paper and its structure is not easy to understand and the description of different data sets (and different time spans) of observed and modelled hindcast data is confusing (e.g. a lot of unusual abbreviations). The number of tables and especially the huge amount of data should be reduced as they are displayed in figures. The selected 32 stations at the end of the rivers or estuaries cover a wide variety of geographical areas and meteorological, oceanographical and hydrological (currents and tides) systems in coastal zones along European coasts. E.g. the tidal range varies from nearly zero to some meters and within the deterministic part of compound events in comparison to the stochastic part (surges, wind waves and river flow) of these compound events. Further discussion of the deterministic and the stochastic part of the compound events and the effects in the statistical analyses (dependency of different parameters) is recommended (page 41, line 26-30). In general I agree completely with reviewer # 1!*

I truly thank the reviewer for his/her comments on the manuscript.

Next, I will address all referee's comments specifically.

## Comments (Ref2)

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*Com\_Ref2\_01: The description whether storm surge and/or wind waves are capable of reproducing extreme values is incomplete (e.g. river runoff?). It has to be explained, why river runoff is not taken into account!*

This study is the first part (Part I: storm surge and wave height) of investigating how statistical dependence can act as modulator referring to the joint return period of compound events. It is clear that this is the case of surge and wave events, so, no river runoff was taken into account. For the preparation of Part II (storm surge and river discharge) and Part III (wave height and river discharge) the effect of runoff will be included and be given special emphasis. I truly believe that such a separate investigation (by parts) allows for a deeper and better understanding of the different components contributing to a compound coastal event. Further, a study including all three components would have become too lengthy and difficult for the reader to follow.

I will explain in more detail (in the Introduction) the reasoning behind this separate investigation of the different components contributing to coastal compound events. I will also refer to the preparation of Part II (storm surge and river discharge) and Part III (wave height and river discharge).

... ~~This study~~ The current work focuses on data preparation, parameter selection, methodology application and estimation of both correlation and statistical dependence between source variables. It also focuses on the prevailing (higher frequency) and dominant (higher intensity) low-level wind conditions over a set of preselected (top 80) extreme compound events. The critical time period during which such extremes take place is also analysed based on monthly frequency values of occurrence. The dependence analysis utilises 32 river ending points selected to cover a variety of geographical areas along European coasts. The variable-pairs presented in this report, which include enough information for calculations, are storm surge and wave height, relevant to most coastal flood defence studies. Two main time intervals were considered for the estimation of maximum values: the half-day interval (max12) and the one-day interval (max24) ...

... This study represents the first part (i.e., Part I) of the investigation while Part II (storm surge and river discharge) and Part III (wave height and river discharge) are to follow. The reasoning behind such a separate investigation (by parts) is to allow the reader for a deeper and better understanding of the interaction between different components contributing to a compound coastal event.

*Com\_Ref2\_02: In the context of the paper a very interesting problem is discussed where copula functions should be taken into account, so far only a simple approach for copula functions has been taken into consideration, the discussion of different copula functions within the scope of the addressed topic is to be considered, more references to copula functions could be helpful (e.g. Wahl, T., Jain, S., Bender, J., Meyers, S. D., & Luther, M. E.*

(2015). *Increasing risk of compound flooding from storm surge and rainfall for major US cities*. *Nature Climate Change*, 5(12), 1093-1097).

The study follows the methodology proposed by Coles et al. (2000) where the basic theory behind the utilisation of an optimal copula function refers to Nelsen (1998), Joe (1997) and Currie (1999). I agree that the inclusion of more references as the suggested one, i.e., Wahl et al. (2015) definitely helps the reader to get more insight in the use of copulas when joint probability methodologies are taken into account.

I will include the suggested reference (Wahl et al., 2015). In addition, I will include the extra references of Nelsen (1998), Joe (1997) and Currie (1999) in the main text.

### References

- Coles, S.G., Heffernan, J. and Tawn, J.A.: *Dependence measures for extreme value analyses*. *Extremes*, 2, 339-365, 2000.
- Currie, J.E., "Directory of coefficients of tail dependence," *Department of Mathematics and Statistics Technical Report, ST-99-06, Lancaster University, 1999*.
- Joe, H., *Multivariate Models and Dependence Concepts*, Chapman & Hall, London, 1997.
- Nelsen, R.B., *An Introduction to Copulas*, Springer-Verlag, New York, 1998.
- Wahl, T., Jain, S., Bender, J., Meyers, S. D., & Luther, M. E. (2015). *Increasing risk of compound flooding from storm surge and rainfall for major US cities*. *Nature Climate Change*, 5(12), 1093-1097).

Com\_Ref2\_03: A point of criticism is that the meteorological conditions and oceanographic system were not sufficiently described and the temporal developments of surges and wind waves are also not clearly described. E.g. in fig 10 and 12 it is shown that for the Weser (RIEN 29) the dependence for the prevailing (highest frequency) and dominant (highest intensity) wind during the top 80 extreme compound events are caused by wind direction from WSW. This is completely in contrast to my experience and has to be explained (same for e.g. RIEN 3, 12, : : : 23, 24, : : : and 32)! From my point of view, it would be advisable to consider a subarea, e.g. only the North Sea, and after a successful investigation of the statistical dependence then implicate other areas.

As a general comment: Directions falling in the WSW category do not count for the total percentage of the Top-80 events but besides this, there exists a logical explanation since the combined events had to be de-clustered. This means that a compound event lasting more than one day had to be counted as one (1) event even if this event could have lasted for a few days. An example of such a compound event lasting for three consecutive days can be seen in Table 2 and Table 3 of the new Technical Supplement (referring to the time interval between 2 to 4 January 2012). After de-clustering this event will count only once and it will refer to its first date (4 Jan 2012) since after the necessary de-clustering all cases of compound events are referring to the first day of the event (the first day that both storm

surge and wave height found to be above a predefined critical threshold). With such an approach, a compound event is considered only once and no other (another) event is taken into account for the next three days (even if the same event continues to exist longer than a day). Both prevailing and dominant directions are referring to the maximum daily intensity and if we consider the most common case of an approaching barometric low (storm) the wind in the beginning is more WSW whereas with the passage of the storm tends to veer to a more northwest (northern) direction. I have checked the validity of this during the second, third and even the fourth day of a compound event and such a distinct veering is true.

Another important point is that not only an incoming onshore perpendicular wind leads to a significant storm surge or even to compound event. As an example Mistral (of north direction) that is heading to the open sea – Marin (of south direction) that is heading toward the coast of Marseille are capable of producing extreme storm surge events of equal intensity (during distinct periods of rough seas) meaning that there exist other directions as well besides the ones blowing perpendicular to the coast relating to extremes as well.

I will stress this unavoidable disagreement due to the veering of the wind and provide necessary explanations for such discrepancy.

... Details of clima and Top-80 flow characteristics are contained in Table 7. A possible exploitation of such information referring to both prevailing and dominant low-level flow characteristics should be considered significant and kept in mind when such extreme events possibly driven by intense storm outbreaks are anticipated over the area of interest (in forecast mode) ...

... Not all prevailing and dominant directions contained in Table 7 fall in the perpendicular onshore category. Especially for the RIEN points of the south North Sea, wind directions appear to be more SWS instead of rather more northerly directions and this is because combined events had to be de-clustered. This means that a compound event lasting more than one day had to be counted as one (1) event even if this event could have lasted for a few days. After this necessary de-clustering all cases of compound events, are referring to the first day of the event (the first day that both storm surge and wave height found to be above a predefined critical threshold). With such an approach, a compound event is considered only once and no other (another) event is taken into account for the next three days (even if the same event continues to exist). Both prevailing and dominant directions are referring to the maximum daily intensity and if we consider the most common case of an approaching barometric low (storm) the wind in the beginning is more WSW whereas with the passage of the storm tends to veer to a more north-western (northern) direction ...

*Com\_Ref2\_04: (Length of observations/hindcasts) As I understood the water level data/storm surge/wind waves: The 32 RIEN (Table 1, page 10) were selected mainly because of their proximity to tidal gauges, although many of them cannot be evaluated due to lack of long-term measurements. For most RIENs, there are no data from nearby open wave buoys. Only for the Rhine (RIEN 28) are the tide and sea data (without*

data gaps) available from a nearby wave buoy for a period of 3 years. The validation of the combined hindcasts (tide and wind waves) was done on the basis of measured data at the Rhine (NL) was done on the tidal data at Hoek von Holland (HvH), wave buoy: Lichteiland (LiG) over a period of ~ 3 years on measurement data without gaps and comparison of daily and half-day maxima. The generation of the hindcast of storm surge data was done with Delft3D-Flow (according to Vousdoukas et al. (2016) and the generation of the hindcast of the wind waves data was done with ECWAM wave model (according to Bidlot et al. (2006), Bidlot (2012), ECMWF (2015), Philips (2017)), e.g. ~36 years, wind- and pressure fields from ERA-Interim (ERA-I) (time resolution: 1 h, spatial resolution: 28x28 km, fixed water level, signif. wave height, max. wave height, mean wave period, mean wave direction and validation based on available records from 101 wave buoys throughout Europe + North Atlantic (1996-2015) (Fig. 2)) The overlapping period of the two hindcasts (~ 35 years) was used in statistical analysis.

The methodology of the research (using the hindcast data sets and observed data) has to be explained more detailed and especially what that means for the interpretation of the results (for all 32 RIEN). A time series of observed water level and wave buoy of only 3 years and only for one station in the area at Hoek van Holland seems to me as being not sufficient and much too short for comparison/evaluation with the modelled (hind cast) data and the conclusions. There should much more field data (water level, surges, wind waves, river runoff) available around the 32 RIEN!

As a general comment:

when low resolution models are used (as in this case) for reproducing time series of significant weather parameters, extremes cannot be captured with their exact (high-impact) value but in most cases only their footprints can be resolved (as extremes of a lesser value). A previous example can be seen in Petroligis and Pinson (2012) where the footprints of extreme wind speed values over Bremen airport are captured by ERA-Interim (as footprint spikes) but they are considerably underestimated. In a similar approach, the scope of the study is to take (at least) into account such spikes (footprints) of extremes and study the statistical dependence of these spikes of storm surge and (significant) wave height.

Such footprints of extremes (resolved by hindcasts) can be found in Table 2 (Technical Supplement) where the 98.5% percentile extremes of storm surge observations are compared to their corresponding hindcast values (falling in the same 98.5% category). It becomes obvious that although hindcasts could not resolve the exact extremity of events at least their footprints were well captured. In a similar way in Table 3 (Technical Supplement) the footprints of significant wave height observation extremes are resolved by their corresponding hindcast (less intense) values.

It is important to point out that hindcasts above all were capable of identifying and resolving all seven (7) compound events that took place during the common time interval of 1,114 days.

On the same track, the set of storm surge hindcasts used in the current paper was already validated against 110 tidal gauge stations as described in Vousdoukas et al. (2016) reference paper. Vousdoukas et al. (2016) utilised both RMSE and relative (%) RMSE metrics.

Overall, the model showed to reproduce satisfactory the measurements as shown in examples given in Figure 3 (Vousdoukas et al., 2016) over four tide-gauge stations in various coastal points of European coasts (Saint-Nazaire in France, Millport in UK, Hirsthals in Denmark and Rorvik in Norway). Studying closely Figure 3 it becomes obvious that hindcasts were able to simulate quite well the available set of observations capturing also efficiently local extremes. Further, the period of validation (2008-2014) had been characterized by an increased marine storm activity including high impact events as mentioned in Bertin et al. 2014; Breilh et al. 2013; Met Office and Centre for Ecology and Hydrology 2014; Vousdoukas et al. 2012.

Referring to the suggestion of using percent maps a new reference in text will be made pointing to Figure 4 (Vousdoukas et al., 2016) scatter plot showing RMS error in m (a) and as a percentage of the SSL (Storm Surge Level) range (b) for all the available tidal gauge stations.

Concerning the validation of wave hindcasts, the set used in the study is considered as a validated set with further details to be provided in Philips et al. (2017). The data are based on a dedicated re-run of the European Centre for Medium-Range Weather Forecasts (ECMWF) ECWAM Wave Model (ECMWF, 2016) Cycle 41R1 at 28-km resolution. The model is forced by a six hourly ERA-interim (Dee et al., 2011) wind field with no wave data assimilation. The effect of water level change and surface current due to tides and surge is neglected. This global hindcast set has been produced in preparation of the ECMWF next reanalysis (ERA5).

I will add in the main text a reference to Figure 4 (Vousdoukas et al., 2016) scatter plot showing RMS error in m (a) and as a percentage of the SSL range (b) for all available tidal gauge stations. This reference will be in harmonisation with Figure 2 (current study) that is referring to the validation of wave hindcasts (RMSE values).

## References

- Bertin X., Li K., Roland A., Zhang Y.J., Breilh J.F., Chaumillon E.: *A modeling-based analysis of the flooding associated with Xynthia, central Bay of Biscay*. *Coastal Eng* 94:80–89, 2014.
- Breilh J.F., Chaumillon E., Bertin X., Gravelle M.: *Assessment of static flood modeling techniques: application to contrasting marshes flooded during Xynthia (western France)*. *Nat Hazards Earth Syst Sci* 13:1595–1612, 2013.
- Met Office, Centre for Ecology & Hydrology: *The recent storms and floods in the UK*. p 29, 2014.
- Petroligis, T. I. and Pinson, P.: *Early warnings of extreme winds using the ECMWF Extreme Forecast Index*, *Meteorol. Appl.*, 21, 171–185, 10 doi:10.1002/met.1339, 2014.
- Phillips, B.T., Brown, J.M. and Bidlot, J.-R. and Plater, A.J.: *Role of Beach Morphology in Wave Overtopping Hazard Assessment*. *J. Mar. Sci. Eng.* 5(1), 2017

- Vousdoukas M.I., Almeida L.P., Ferreira Ó.: Beach erosion and recovery during consecutive storms at a steep-sloping, mesotidal beach. *Earth Surf Process Landforms* 37:583–691, 2012.

- Vousdoukas, M.I., Voukouvalas, E., Annunziato, A., Giardino, A. and Feyen, L.: Projections of extreme storm surge levels along Europe. *Clim. Dyn.* 47(9): 3171-3190, doi:10.1007/s00382-016-3019-5, 2016.

*Approx. 2.3 "extreme events" (at least 3 days between peaks) per year (total 80 top events) were chosen. It has to be explained more detailed why 2.3 "extreme events" were chosen and what that means for the interpretation of the results*

Extreme value analysis can be carried out using two types of data series (Bezak et al., 2014), annual maximums (MA) or flows above a certain threshold (POT for Peak Over Threshold). The POT model used in this study can be composed of the Poisson, binomial and negative binomial distributions for modelling the annual number of events above threshold, and of exponential or generalized Pareto distributions for magnitudes of exceedances.

The selection is defined by the parameter alpha ( $\alpha$ ) representing the annual maximum non-exceedance probability taken equal to 0.1 following Defra TR3 Report suggestions. Such a value (0.1) of alpha corresponds to ~2.3 compound POT (Peaks-Over-Threshold) events per year exceeding the corresponding optimal selected percentile threshold (the one providing ~2.3 compound events).

Since both surge and wave time series are almost 35 years long this points to ~80 (~2.3 x 35) events over the total time period.

I will add a more detailed explanation (Section 6) in the new Statistical Supplement taking into consideration the basic guidelines documented in Defra TR3 Report (2005). I will also add the relevant reference (Bezak et al., 2014) the one referring to Defra TR3 Report (2005).

## **6 Selection of criterion thresholds resulting in the consideration of top-80 events**

Since values of dependence ( $\chi$ ) can be estimated for any lower or upper threshold, initial trials were performed studying the behaviour of  $\chi$  over a wide range of thresholds. Findings were similar to those contained in Defra TR3 Report (2005), justifying the selection of an optimal threshold for "alpha" ( $\alpha$ ) equal to 0.1 corresponding to an annual maximum being exceeded in 9 out of 10 years (see Sect. 2.2 of the main text for details).

Such a value (0.1) of alpha was considered for both mat\_chi and mat\_chibar routines when utilising POT (Peaks-Over-Threshold) methodology resulting in an annual maximum of ~2.3 compound events.

Such an annual threshold of ~2.3 events corresponds to the top 80 (Top-80) compound events taking place during any (POT separated) day of the total 12,753 days and it was dictated mainly by two factors: the threshold had to be low enough to allow a sufficient

number of data points to exceed it for estimating dependence reliably, while being high enough for the data points to be regarded as extremes.

### *References*

#### References

Bezak, N., Brilly, M., and Sraj, M., 2014. Comparison between the peaks-over-threshold method and the annual maximum method for flood frequency analysis. *Hydrological Sciences Journal*, 59 (5), 959-977.

Defra TR3 Report by Svensson, C. and Jones, D.A.: Joint Probability: Dependence between extreme sea surge, river flow and precipitation: a study in south and west Britain. Defra/Environment Agency R & D Technical Report FD2308/TR3, 62 pp. + appendices ([http://evidence.environment-agency.gov.uk/FCERM/Libraries/FCERM\\_Project\\_Documents/FD2308\\_3430\\_TRP\\_pdf.sflb.ashx](http://evidence.environment-agency.gov.uk/FCERM/Libraries/FCERM_Project_Documents/FD2308_3430_TRP_pdf.sflb.ashx)), 2005.

### **Improvements (Ref2)**

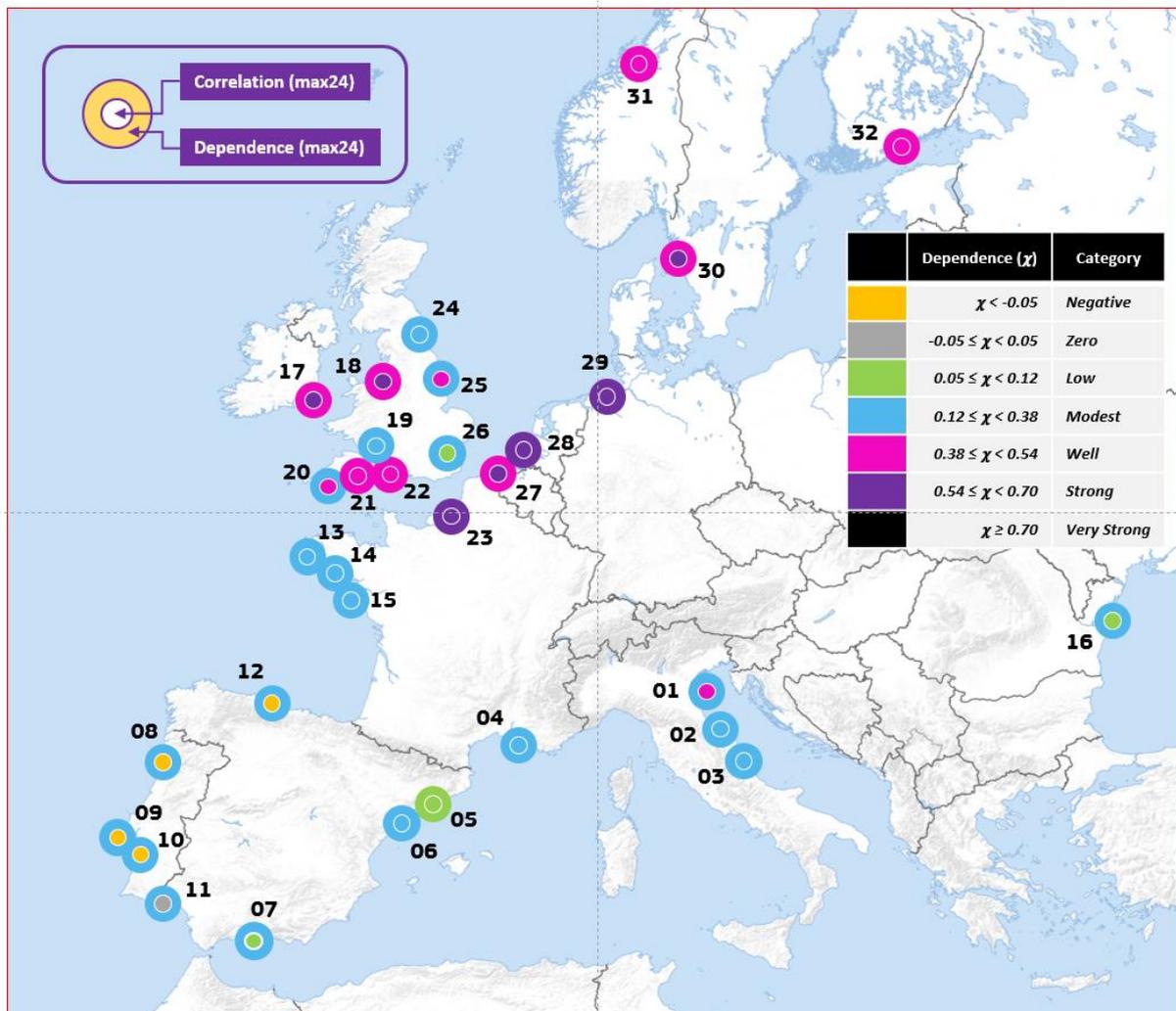
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*Impr\_Ref2\_01: The number of tables and graphs should be reduced and more summarized.*

I will reduce / convert tables and graphs to a more summarised form. List of changes in Figures and Tables are listed below:

- Table 1 will be moved in the new Technical Supplement to provide the reader with exact details of the selected RIENs (river ending points).
- New Table 1 (Details and Abbreviations of the main data sets) will be incorporated in the main text.
- Figure 9 (results based on statistical packages of R) will be moved in the new Statistical Supplement in Section 7 (Details and examples of the statistical packages used in the study)
- Table 3 will be moved in the new Statistical Supplement as new Table 4 (Section 3).
- Table 4 will be moved in the new Technical Supplement as new Table 6 (Section 3).
- Table 5 will be moved in the Technical Supplement as new Table 5 (Section 3).
- Table 6 will be moved in the Technical Supplement as new Table 7 (Section 3).
- Figure 12 will be skipped whereas old Table 7 will be kept as new Table 3 (in the main text) since it contains all relevant information of prevailing and dominant winds that was graphically presented in Figure 12 (upper and lower panels) over the selected 32 RIEN points.

- Old Figure 10 (now new Figure 9 in the main text) will include the spatial distribution of both correlation and statistical dependence (shown below) utilising seven (7) relevant / reference categories. Prevailing and dominant winds are to be left out of this new Figure 9 for more clarity. Their exact details contained in the old Table 7 can be found in the new Table 3.



*New Figure 9 (Old Figure 10)*

*Impr\_Ref2\_02: The paper is not easy to understand for a wide diversified audience, the length of the paper is too long and has partly too much redundancy (e.g. table 1 and fig. 1).*

I will reduce the length of the main paper by creating a separate Statistical Supplement and an additional Technical Supplement. These two new Supplements will help the reader to understand easier the main concept and findings of the current work. Redundant parts will be merged, shortened and improved.

*Impr\_Ref2\_03: The pure agreement between hindcast and observation of daily maximum of storm surges in Fig. 4 has to be explained.*

The pure agreement between hindcasts and observations is a clear indication of the model's (Delft3D-Flow) capability to simulate efficiently observations in hindcast mode having as input parameters (wind components and mean sea level pressure) from the ECMWF ERA-Interim reanalysis data set.

Indicative examples of such capabilities can be seen in Table 2 and Table 3 of Section 2 of the new Technical Supplement revealing that hindcasts above all were capable of identifying and resolving all seven (7) compound events (based on 98.5% percentile threshold) that took place during the common time interval of 1,114 days over HvH area of interest.

I will explain and stress this capability of Delft3D-Flow model of resolving daily maximum of storm surge observations in the main test referring also to Table 2 (Section 2) of the new Technical Supplement.

*Impr\_Ref2\_04: Why are small storm surges, e.g. below 0.5 m are taken in to account?*

In Figure 4, the capability of hindcasts to simulate correctly observations was done over the full range of observations, since it is important to show that model hindcasts are capable to perform well over any part of observations.

With the help of such models, it should be anticipated to have two validated sets of hindcasts resulting to the determination of the correct sign and strength of both correlation and statistical dependence.

I will point out that validation of both hindcast sets is done over the full spectrum of observations since the capability of the model to simulate correctly observations should refer to any part of the spectrum values.

*Impr\_Ref2\_05: What is the definition of a storm surge?*

Storm surge is the abnormal rise in seawater level during a storm, measured as the height of the water above the normal predicted astronomical tide <https://oceanservice.noaa.gov/facts/stormsurge-stormtide.html>.

Same wise the definition of significant wave height will be also included (see below).

In physical oceanography, the significant wave height (SWH or  $H_s$ ) is defined traditionally as the mean wave height (trough to crest) of the highest third of the waves ([https://en.wikipedia.org/wiki/Significant\\_wave\\_height](https://en.wikipedia.org/wiki/Significant_wave_height)).

I will include the definition of storm surge (and significant wave height) in the Introduction and provide the relevant (site) references.

*Impr\_Ref2\_06: What is the reason to use the storm between 25th December 2012 and 24th January 2013?*

It is just an example chosen for demonstrating how a compound event looks like and how it is related to the prevailing synoptic conditions (Storm Emil).

Further, it is an example of a compound event that lasts for three consecutive days (from 4 to 6 January 2012) as shown in Table 2 and Table 3 of the new Technical Supplement. During de-clustering this event will be counting only once and it will refer to its first date that this event took place (4 January 2012).

I will point out the concept of this multi-purpose demonstrating example and give emphasis in the de-clustering concept.

*Impr\_Ref2\_07: The pure agreement between hindcast and observation of daily maximum of the significant wave height in Fig. 6 has to be explained.*

As in the previous case (Imp\_02\_03), the pure agreement between hindcasts and observations is a clear indication of the model's (ECMWF / ECWAM) capability to simulate efficiently observations in hindcast mode having as input parameters (wind components) from the ECMWF ERA-Interim reanalysis data set.

Once again, indicative examples of such capabilities can be seen in Table 2 and Table 3 of Section 2 of the new Technical Supplement revealing that hindcasts above all were capable of identifying and resolving all seven (7) compound events (based on 98.5% percentile threshold) that took place during the common time interval of 1,114 days over HvH area of interest.

I will explain and stress this capability of ECMWF / ECWAM model of resolving daily maximum of significant wave height observations.

*Impr\_Ref2\_08: Fig. 8: The fairly pure agreement (chi) of the statistical dependence (chi) of storms surge and significant wave height between observation and hindcasts has to be explained.*

The fairly pure agreement between chi values estimated by observations (of surge and waves) and hindcasts (of surge and waves) is a clear indication that hindcasts were found capable of resolving and estimating both the correct type and strength of correlation and dependence between source variables.

I will point out the capability of the hindcasts to resolve and estimate the correct type and strength of correlation and dependence and stress the significance of such an agreement between dependence values estimated from observations and hindcasts.

*Impr\_Ref2\_09: Fig. 9: For the lower and higher quantiles the chi plots have to be explained and discussed.*

Values of dependence in the area of lower and higher quantiles seem (and somehow expected) to be quite unstable due the sparse of data.

I will explain and stress the behaviour of chi in lower and higher percentiles. Emphasis will be given on the stability of chi (graph) curves by identifying the area that dependence is clearly converging to a specific value (with no abrupt fluctuations).

*Impr\_Ref2\_10: Fig. 10: I do not find the category dependence “negative” and “zero”.*

The old Figure 10 (new Figure 9) contained only dependence values (no correlations). Zero and negative values refer to a certain number of correlations contained in the old Table 3 and Table 5 (now new Table 4 and Table 5 of the new Technical Supplement) valid for both max12 and max24 configurations.

In the new Figure 9 containing both correlations and dependence (max24) values, zero correlations are marked by a grey colour whereas negative correlations by a yellow one.

I will produce the new combined Figure 9 (in place of the old Figure 10) containing both correlation and dependence values. For more clarity, the prevailing and dominant components will be skipped since they are also presented analytically in the old relevant Table 7 (now new Table 3 in the main text).

*Impr\_Ref2\_11: Symbol and wind N to NNW is not necessary.*

I will keep the 16 main and secondary directions of the wind (N – NNE – NE – ENE – E – ESE – SE – SSE – S – SSW – SW – WSW – W – WNW – NW – NNW).

*Impr\_Ref2\_12: The description of tables and figures should be improved.*

I will improve the description of both tables and figures accordingly. This will be also applied for the new updated Tables and Figures. A full description of the updated Tables and Figures is contained in author’s reply to Impr\_Ref2\_01 comment (in improvements suggested by Ref 02 comments).

## Suggestions (Ref2)

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*Sugg\_Ref2\_01: I do not find a clear definition of highest intensity, page 34, row 2 and page 41, row 9, does it mean only the dominant wind? Direction and/or speed?*

Prevailing Wind is the most common wind direction over an area, i.e., the direction of wind with the highest frequency (AMS, 2017), whereas Dominant Wind is the direction of the strongest wind that might blow from a different direction than the prevailing wind, i.e., from a less common direction (Thomas, 2000). The periods most frequently used for the estimation of prevailing and dominant winds are the observational day, month, season, and year. Methods for determination vary from a simple count of periodic observations to the computation of a wind rose.

I will provide definitions of both prevailing and dominant wind and add the relevant references.

### References

- AMS (American Meteorological Society) Glossary: Prevailing Wind. Glossary of Meteorology (Available online at [http://glossary.ametsoc.org/wiki/Prevailing\\_wind\\_direction](http://glossary.ametsoc.org/wiki/Prevailing_wind_direction)), 2017.
- Thomas, DG. 2000. Dictionary of physical geography. Blackwell.

*Sugg\_Ref2\_02: I do not find a clear definition of negative bias: Systematically underestimated parameter?*

Bias is the difference between the mean of the forecasts and the mean of the observations. It could be expressed as a percentage of the mean observation. Also known as overall bias, systematic bias, or unconditional bias (<http://www.cawcr.gov.au/projects/verification/>).

I will provide the definition and include the relevant (site) reference.

## Minor Improvements (Ref2)

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*Mimp\_Ref2\_01: page 2 row 18 "This is"*

I will correct it.

*Mimp\_Ref2\_02: page 5/6 row 19/1 "Matlab"*

I will correct it.

*Mimp\_Ref2\_03: page 7 row 22 "also uses"*

I will correct it.

*Mimp\_Ref2\_04: page 8 row3 "... Good (1994)"*

I will correct it.

*Mimp\_Ref2\_05: page 14 row 14 "to the"*

I will correct it.

*Mimp\_Ref2\_06: page 16 rows 10-14 "Storm Emil" as well as page 18 rows 1 and 30*

I will correct it.

*Mimp\_Ref2\_07: p.42, row 10: providing "us"?*

I will delete the word "us".

*Statistical Supplement of*

**Estimations of statistical dependence as joint return period modulator of compound events. Part I: storm surge and wave height.**

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*8 References*

## 1 Statistical dependence ( $\chi$ )

The main concept of the so-called dependence measure  $\chi$  (chi) is related to two or more simultaneously observed variables of interest – such as in our case storm surge and wave height – known as observational pairs. If one variable exceeds a certain extreme (high-impact) threshold, then the value of  $\chi$  represents the risk that the other variable will also exceed a high-impact threshold as explained in Hawkes (2004), Svensson and Jones (2004a & 2004b), Petroligkis et al. (2016).

Following Coles et al. (2000), if all of the extreme observations of two variables exceed a given threshold at the same time, this indicates total dependence ( $\chi = 1$ ). If the extreme observations of one variable exceed a given threshold but the second variable does not, this indicates total independence ( $\chi = 0$ ). Similarly, if the extreme observations of one variable exceed a given threshold but the other variable produces lower observations than would normally be expected, this indicates negative dependence ( $\chi = -1$ ). In practice, hydro-meteorological analyses based on real data often lead to an assessment of complete independence that could result to an under-estimation of the joint probability of concurrent extreme events, whereas, an assumption of complete dependence could result to an over-estimation of joint probabilities (Beersma and Buishand, 2004). In reality, as variables reach their extreme values, a special methodology of estimating statistical dependence could be utilised. This methodology has been documented by Buishand (1984) and Coles et al. (2000). A brief description of the method based on Coles et al. (2000) is given below.

## 2 Estimation of dependence ( $\chi$ )

For bivariate random variables ( $X, Y$ ) with identical marginal distributions, the dependence measure ( $\chi$ ) can estimate the probability of one variable being extreme provided that the other one is extreme:

$$\chi = \lim_{z \rightarrow z^*} \Pr(Y > z | X > z) \quad (1)$$

where  $z^*$  is the upper limit of the observations of the common marginal distribution.

For obtaining identical marginal distributions, each set of observations is ranked separately and each rank is then divided by the total number of observations resulting in a data transformation with Uniform  $[0, 1]$  margins. At this point, it is convenient to consider the bivariate cumulative function  $F(x, y) = \text{Prob}(X \leq x, Y \leq y)$  that describes the dependence between  $X$  and  $Y$  completely. The effect of different marginal distributions can be diminished by assuming the copula function  $C$  in the domain  $[0, 1] \times [0, 1]$  such as:

$$F(x, y) = C \{ F_x(x), F_y(y) \} \quad (2)$$

where  $F_x$  and  $F_y$  can be any marginal distributions. **Such utilisation of the copula function has the same effect as if observations were ranked separately and divided by the total number of observations. In addition, The the copula  $C$  contains the complete information about the joint distribution of  $X$  and  $Y$  and it is invariant to marginal transformation. This means that  $C$  is invariant to marginal transformation and it can be described as the joint**

distribution function of X and Y. Further, X and Y are transformed to new variables U and V with Uniform [0, 1] margins. It follows that the dependence measure  $\chi(u)$  for a given threshold u can be given by:

$$\chi(u) = 2 - \frac{\ln \Pr(U \leq u, V \leq u)}{\ln P(U \leq u)} \quad \text{for } 0 \leq u \leq 1 \quad (3)$$

Taken into account the upper limit of the observations (previously defined as  $z^*$  in Eq. 1), the dependence measure  $\chi(u)$  will be given by:

$$\chi = \lim_{u \rightarrow 1} \chi(u) \quad (4)$$

Details of deriving Eq. 3 can be found in Coles et al. (2000). Based on Eq. 3, a set of  $\chi$  values can be evaluated at different quantile levels u (~~for details see Coles et al., 2000~~). The selection of a particular level u corresponds to threshold levels ( $x^*$ ,  $y^*$ ) for the two different data series. For applying Eq. 3, the number of appropriate observation-pairs (X, Y) is counted for estimating the numerator and denominator terms (~~Eq. 4 & Eq. 5~~ Eq. 5 & Eq. 6):

$$P(U \leq u, V \leq u) = \frac{\text{Number of } (X, Y) \text{ such that } X \leq x^* \text{ and } Y \leq y^*}{\text{Total number of } (X, Y)} \quad (\rightarrow) (5)$$

and

$$\ln P(U \leq u) = \frac{1}{2} \ln \left[ \frac{\text{Number of } X \leq x^*}{\text{Total number of } X} \cdot \frac{\text{Number of } Y \leq y^*}{\text{Total number of } Y} \right] \quad (\rightarrow) (6)$$

In this study, a set of routines (mat\_chi) based on [matlab Matlab](#) software were coded following Eq. 3 to ~~5~~ 6 for estimating  $\chi$ . Additional modules and routines based on the integrated statistical package R were also used for estimating dependence terms and inter-comparing various parameters. Emphasis was given on the routine “taildep” of the module “extRemes” (<https://cran.r-project.org/web/packages/extRemes/extRemes.pdf>) that is capable of estimating  $\chi$  values when a critical percentile (extreme) threshold is considered. Another “powerful” routine capable of providing a variety of dependence graphs and plots (besides single estimated values of  $\chi$ ) has been the routine “chiplot” of the module “evd” (Extreme Value Distributions) of R (<https://cran.r-project.org/web/packages/evd/evd.pdf>). The routine chiplot is also capable of providing confidence intervals at any preselected level.

Besides estimating values of  $\chi$ , similar routines (mat\_chibar) were coded in [matlab Matlab](#) following Coles et al. (2000) for calculating the “sister” attribute of  $\chi$ , namely chibar ( $\bar{\chi}$ ). Chibar (**chi\_bar** parameter) refers to the statistical dependence of asymptotically independent variables whereas chi ( $\chi$ ) refers to the statistical dependence of asymptotically dependent ones. **Details on the estimation of chibar are documented in Coles et al. (2000) whereas examples and how to utilise ( $\bar{\chi}$ ) can be found in Coles (2001).** The ~~latter~~ class of asymptotic dependence appears to be the case in Literature, having reached a consensus that there is strong, although not overwhelming, evidence for asymptotic dependence between wave height and surge (Wadsworth et al., 2017).

The concept of asymptotic dependence ( $\chi$ ) is stated with adequate details in Coles et al. (2000). In brief,  $\chi$  is on the scale [0, 1] with the set (0, 1] corresponding to asymptotic dependence whereas the measure  $\bar{\chi}$  falls within the range [-1, 1] with the set [-1, 1) corresponding to asymptotic independence. That is why the complete pair of  $\chi$  and  $\bar{\chi}$  is required as a summary of extremal dependence:

- $\chi > 0$  &  $\bar{\chi} = 1$  reveals asymptotic dependence, in which case the value of  $\chi$  determines a measure of strength of dependence within the class

- $\chi = 0$  &  $\bar{\chi} < 1$  reveals asymptotic independence, in which case the value of  $\bar{\chi}$  determines the strength of dependence within the class.

For estimating both  $\chi$  and  $\bar{\chi}$  parameters, the general POT (Peaks-Over-Threshold) methodology was followed. Such an approach (POT) is considered as giving a more accurate estimate of the probability distribution than using the annual maximum series (see details in Stedinger et al., 1993). Applying POT as described in detail in Defra TR1 Report (2005), the selection of an optimal threshold for the data pairs (~2.3 events per year) was adopted as suggested in Defra TR3 Report (2005). Care was taken to force two POT extreme compound events not occurring on consecutive days, but separated by at least three days from each other. Emphasis was also given on the stability of  $\chi$  (graph) curves ~~as strongly recommended by Prof Pieter Van Gelder of Delft University, Netherlands (personal communication, 2016)~~ identifying the area that dependence was clearly converging to a specific value (no abrupt fluctuations).

Relatively small differences among various estimates made by `chiplot` of `evd` (R), `taildep` of `extRemes` (R) and `mat_chi` (`matlab` Matlab) were found. This most probably is due to the unavoidable dissimilarities between the criteria being imposed on data pairs when applying POT methodology (selection of different critical thresholds).

### 3 Selection of critical thresholds

For selecting a threshold  $u$  (referring to a critical percentile) as required in Eq. 5.3, it seems appropriate to transform the Uniform distribution to an annual maximum non-exceedance probability scale (Defra TR3 Report, 2005). Then the annual maximum non-exceedance probability ( $\alpha$ ) is defined as:

$$\alpha = \text{Prob}(\text{Annual maximum} \leq x) \quad (6) (7)$$

where  $x$  is the magnitude of the source variable. Such non-exceedance probability relates to the return period,  $T_\alpha$ , as:

$$T_\alpha = 1 / (1 - \alpha) \quad (7) (8)$$

For a transformation from annual maximum to POT series (see details and scope in the previous Sect. 2.1.2), we define the “new” non-exceedance probability, the so-called  $p$ , referring to a rate of  $\lambda$  events per year, relating to the annual maximum of Eq. 6.7, as:

$$\alpha = \exp(-\lambda(1-p)) \quad (8) (9)$$

where  $1-p$  is the “new” exceedance probability of the POT series. The term  $(1-p)$  can be estimated graphically leading to Equation 9 10:

$$\lambda(1-p) = (N_e / N) * (i - 0.5) / N_e = (i - 0.5) / N \quad (9) (10)$$

where  $i$ , represents the rank of the independent POT events,  $N_e$  is the number of POT events while  $N$  represents the number of years (see details in Defra TR3 Report, 2005). The independence criterion of two POT events to be separated by at least three days (six half-day intervals in the max12 case) was applied for all river ending points. Combining ~~Eq. 8 and Eq. 9~~ Eq. 9 and Eq. 10, an estimation of  $\alpha$  is possible as given by Eq. 10 11:

$$\alpha = \exp(-(i - 0.5) / N) \quad (10) (11)$$

Therefore, going after the magnitude of  $x$  in Eq. 6 7 ~~is equivalent to~~ as trying to define the magnitude of the POT element with rank  $i$  in Eq. 10 11 for the same maximum non-exceedance annual probability, alpha ( $\alpha$ ). After the selection of an optimal threshold ( $u$ ) based on alpha ( $\alpha$ ), the estimation of  $\chi$  is straightforward (Eq. 3). The main idea here is to use  $\chi$  in a relatively simple formula that ~~uses also~~ uses as input the individual return periods  $T_X$  and  $T_Y$  for estimating the joint return period ( $T_{X,Y}$ ), like the formula described by Eq. 11 following White (2007), Australian Rainfall & Runoff Project 18 (2009).

$$T_{XY} = \sqrt{T_X * T_Y / \chi^2} \quad (11) (12)$$

Studying Eq. 11 12 closely it becomes obvious that dependence is capable of substantially modulating the joint return period. For details and potential limitations of Eq. 11 12, see discussions in White (2007), Hawkes (2004), Meadowcroft et al. (2004), Australian Rainfall & Runoff Project 18 (2009). Furthermore, in cases of totally dependent variables, Eq. 11 12 yields the common individual return period of source variables as an estimation of the joint return period. An example of how to utilise the formula of Eq. 11 12 is given in Sect. 4.2 of the main text for the river ending point of Rhine (NL). Some limitations of Eq. 11 12 could be overcome if a more complete formula is used such as Eq. 2.15 for instance taken from White’s thesis (2007) but this is above the scope of the current study.

#### 4 Significance

The values of dependence ( $\chi$ ) corresponding to the 5% significance level were estimated using a permutation method as described by ~~Good, 1994~~ Good (1994). As in Defra TR3 Report (2005), 199 permutations of the data were made for each surge-wave pair and a new value of  $\chi$  was calculated each time. All 199 values of  $\chi$  were subsequently ranked in descending order and the 5% significance level was defined by selecting the 10th largest value representing the 95% point of the null distribution (the hypothetical distribution occurring if data-pairs were indeed independent). Care was taken to preserve the seasonality since permutation of data was performed by randomly reshuffling intact blocks of one year time period.

## 5 Confidence intervals

For the estimation of confidence intervals, a well-tested bootstrapping method was applied similar to the permutation method already used for estimating significance (for details see Defra TR3 Report, 2005). This bootstrapping resulted in the generation of many new data-sets (resamples). The original sample of observation-pairs was used as the main (reference) distribution from which the resamples were chosen randomly. A large number of data sets were generated for calculating  $\chi$  for each of these new data sets. This provided a sample of what would occur for a range of situations. Seasonality was kept intact by sampling in blocks of one year, rather than using individual observation-pairs. The balanced resampling as documented by Fisher (1993) was applied ensuring that each year occurs equally often overall among the total number of bootstrap samples. In total, 199 bootstrap samples of the data were made for each station-pair and a new  $\chi$  value was calculated each time. The 199 values were subsequently ranked in descending order and the 10 and 190 largest values were accepted as determining the 90% confidence interval.

## 6 Selection of critical thresholds resulting in the consideration of top-80 events

Since values of dependence ( $\chi$ ) can be estimated for any lower or upper threshold, initial trials were performed studying the behaviour of  $\chi$  over a wide range of thresholds. Findings were similar to those contained in Defra TR3 Report (2005), justifying the selection of an optimal threshold for “alpha” ( $\alpha$ ) equal to 0.1 corresponding to an annual maximum being exceeded in 9 out of 10 years (see Sect. 3 of the accompanying Statistical Supplement). This value (0.1) of alpha was considered for both `mat_chi` ( $\chi$ ) and `mat_chibar` ( $\bar{\chi}$ ). routines when utilising POT (Peaks-Over-Threshold) methodology resulting in an annual maximum of ~2.3 compound events.

Such an annual 25 threshold of ~2.3 events corresponds to the top 80 (Top-80) compound events taking place during any (POT separated) day of the total 12,753 days and it was dictated mainly by two factors: the threshold had to be low enough to allow a sufficient number of data points to exceed it for estimating dependence reliably, while being high enough for the data points to be regarded as extremes.

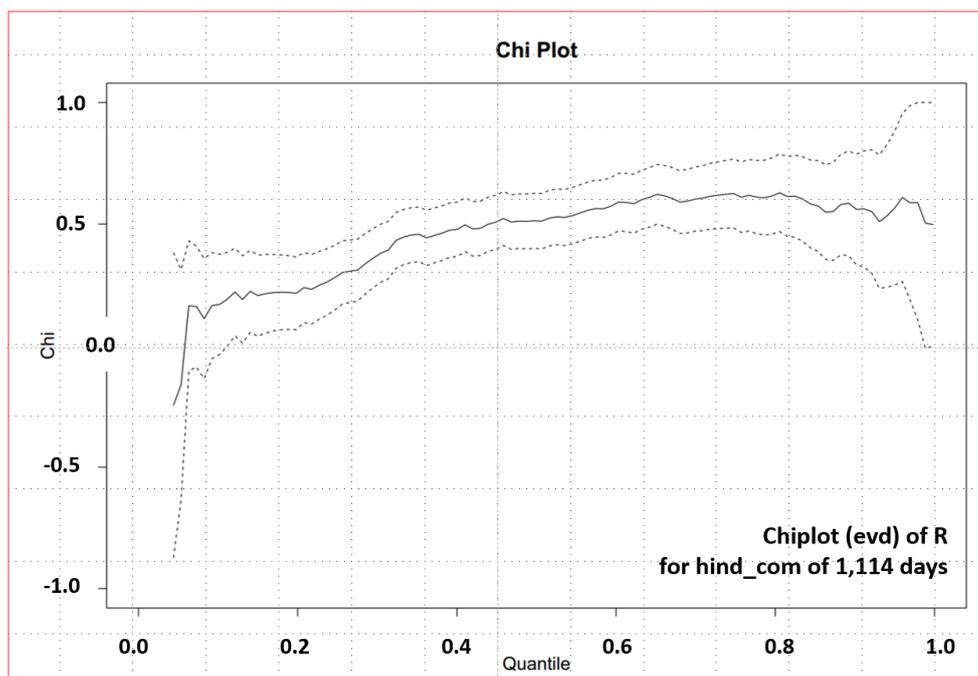
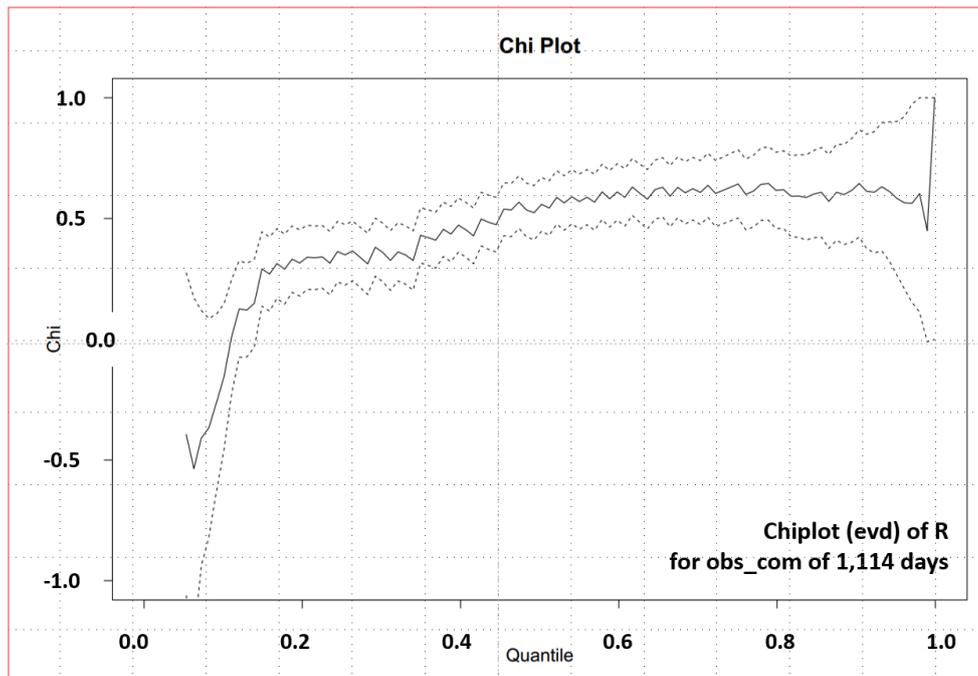
## 7 Details and examples of the statistical packages used in the study

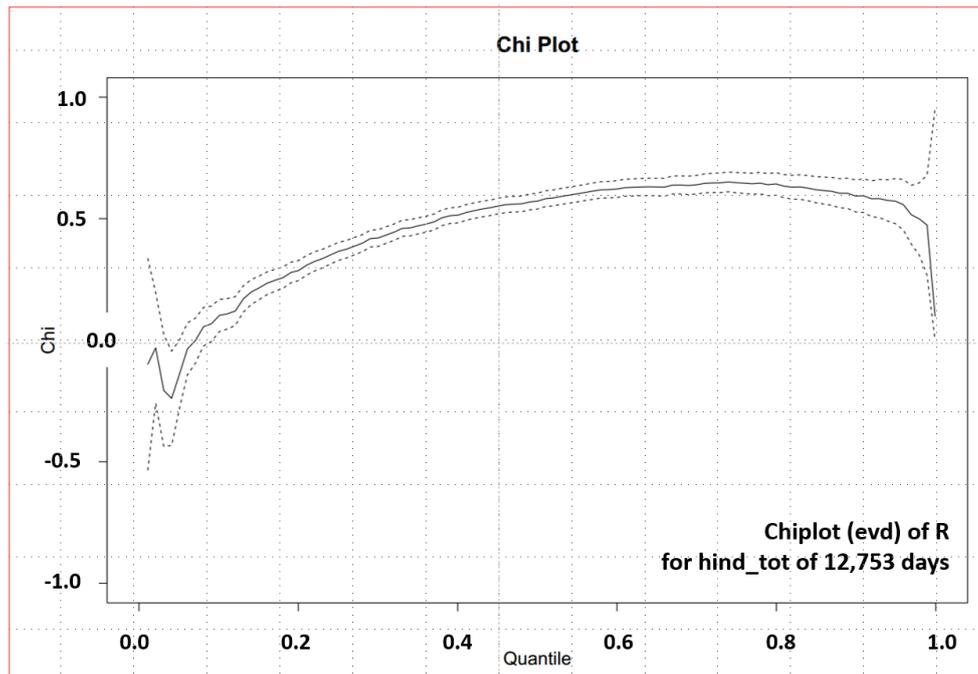
In this study, a set of routines (`mat_chi`) based on [matlab Matlab](#) software were coded following Eq. 3 to 5 for estimating  $\chi$ . Additional modules and routines based on the integrated statistical package R were also used for estimating dependence terms and inter-comparing various parameters. Emphasis was given on the routine “`taildep`” of the module “`extRemes`” (<https://cran.r-project.org/web/packages/extRemes/extRemes.pdf>) that is capable of estimating  $\chi$  values when a critical percentile (extreme) threshold is considered.

Another “powerful” routine capable of providing a variety of dependence graphs and plots (besides single estimated values of  $\chi$ ) has been the routine “`chiplot`” of the module “`evd`” (Extreme Value Distributions) of R (<https://cran.r-project.org/web/packages/evd/evd.pdf>). The routine `chiplot` is also capable of providing confidence intervals at any preselected level. As mentioned above (Section 2) relatively small differences among various estimates made by `chiplot` of `evd` (R), `taildep` of `extRemes` (R) and `mat_chi` ([matlab Matlab](#)) were found and this

most probably is due to the unavoidable dissimilarities between the criteria being imposed on data pairs when applying POT methodology.

Examples of estimated statistical dependence ( $\chi$ ) values between surge (HvH) and wave (LiG) max24 values in obs\_com (upper panel), hind\_com (middle panel) and in hind\_tot (lower panel) mode by chiplot routine of evd module (R) are given in Fig. 1.





**Fig. 1.** Estimated  $\chi$  values between surge (HvH) and wave (LiG) max24 values in c (upper panel) & hind\_com (middle panel) and in hind\_tot (lower panel) mode by chiplot routine of evd module (R).

Studying closely Fig. 1 it becomes obvious that considerable high values of dependence are estimated over all three (obs\_com, hind\_com & hind\_tot) modes. The importance and implications of such high values of dependence can be demonstrated with an example by considering the total hindcast (hind\_tot) series for surge (HvH) and wave (LiG). Utilising the [matlab Matlab](#) function “gevfit” an estimation of the return levels having a 100-year return period for surge and wave height variables was made (1.78 and 6.05 metres respectively). Inserting the common return period value (100-year) together with the estimated  $\chi$  value (0.56) in Eq. 11, the Joint Return Period (JRP) of such a compound event (surge  $\geq 1.78$  metres and significant wave height  $\geq 6.05$  metres) was estimated at ~179 years.

Such a value (~179 years) is significantly different from the value of 10,000 years representing the estimated JRP assuming that surge and wave variables were totally independent. In a case like this (of independent events), the dependence would have been equal to zero and the JRP would be given by the product of their individual probabilities (Blank, 1982).

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*Technical Supplement of*

**Estimations of statistical dependence as joint return period modulator of compound events. Part I: storm surge and wave height.**

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*2 Capability of hindcasts to identify and resolve compound events of surge and wave*

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## 1 Details of RIEN (RIver ENding) point positions

The current statistical (dependence) analysis is focused over 32 river ending points that have been selected to cover a variety of riverine and estuary areas along European coasts. The sea areas used in the study refer to the Mediterranean Sea (central and north Adriatic Sea, Balearic Sea, Alboran Sea and Gulf of Lion), West Iberian, North Iberian, Bay of Biscay, Irish Sea, Bristol Channel, English Channel, North Sea, Norwegian Sea, Baltic Sea and Black Sea. A map showing the position of RIEN (RIver ENding) points used in the study is shown in Fig. 1 (of the main text). Additional details can be found in Table 1 (current Technical Supplement) containing the exact location (lat, lon) of **all** RIEN points

**Table 1.** Positions (lat, lon) of 32 RIEN points used in the study. Names refer to river ending areas.

	RIEN	lat	lon		RIEN	lat	lon
1	Po Della Pila	44.96	12.49	17	Muir Eireann	52.65	-6.22
2	Madonna Del Ponte	43.83	13.05	18	Wallasey	53.44	-3.04
3	Martinsicuro	42.84	13.93	19	Severn Bridge	51.61	-2.65
4	Aries	43.34	4.84	20	Fort Picklecombe	50.34	-4.17
5	El Foix	41.20	1.67	21	Exmouth	50.62	-3.42
6	Illa de Buda	40.71	0.89	22	Christchurch District	50.72	-1.74
7	Rio De Velez	36.72	-4.11	23	Dieppe	49.91	1.09
8	Matosinhos	41.18	-8.71	24	South Tynesid	55.01	-1.43
9	Carcavelos	38.69	-9.26	25	Spurm Point	53.57	0.11
10	Setubal	38.53	-8.89	26	Sheerness	51.45	0.74
11	San Bruno	37.18	-7.39	27	Western Scheldt	51.43	3.55
12	Punta Del Arenal	43.47	-5.07	28	Rockanje	51.87	4.01
13	Concarneau	47.86	-3.92	29	Wurster Arm	53.65	8.14
14	Riviere De Belon	47.81	-3.72	30	Kattegat	57.77	11.76
15	Larmor-Plage	47.71	-3.38	31	Trondheimsfjord	63.32	9.82
16	Musura Bay	45.22	29.73	32	Vanhankaupunginselka	60.24	24.99

## 2 Capability of hindcasts to identify and resolve compound events of surge and wave.

As already mentioned long-period water level data coinciding with wave observations directly or very close to the exact sites of interest (RIEN points) were not available with the exception of the Rhine River (RIEN). For this RIEN, concurrent (close-by) observations with no gaps of sea level, astronomical tide, storm surge, and wave height from a close-by wave buoy were available for a period of about 3 years (1,114 days).

In Table 2, extreme storm surge (above 98.5% percentile) values for both observations and hindcasts for HvH tide gauge station over the common time interval of 1,114 days are shown. Same way extreme significant wave height (above 98.5% percentile) values for both observations and hindcasts for LiG wave buoy station over the common time interval are contained in Table 3.

**Table 2.** Extreme storm surge (above 98.5% percentile) values for both observations and hindcasts for HvH tide gauge station over the common time interval of 1,114 days. Compound events of surge and wave are marked by yellow shade.

#	Date	Observations	hindcasts
1	12 Nov 2010	1.38	1.10
2	4 Feb 2011	1.20	1.00
3	27 Nov 2011	1.25	1.04
4	28 Nov 2011	0.98	0.93
5	3 Dec 2011	1.08	1.03
6	7 Dec 2011	1.10	0.95
7	9 Dec 2011	1.45	1.23
8	29 Dec 2011	1.23	1.03
9	3 Jan 2012	1.07	0.47
10	4 Jan 2012	1.46	1.16
11	5 Jan 2012	1.66	1.59
12	6 Jan 2012	1.37	1.57
13	21 Jan 2012	1.09	1.02
14	22 Jan 2012	1.00	1.07
15	30 Jan 2013	1.07	0.73
16	10 Sep 2013	0.96	0.59

Compound events of surge and wave are marked by orange shade (in both Table 2 & 3) based on joint observations of storm surge and significant wave height. It becomes obvious that hindcasts were able to resolve all seven (7) compound events that took place during the common time period of 1,114 days.

**Table 3.** As in Table 2, but for significant wave height for LiG wave buoy station.

#	Date	Observations	hindcasts
1	12 Nov 2010	4.79	3.99
2	14 Jul 2011	4.61	3.34
3	7 Oct 2011	4.34	3.34
4	7 Dec 2011	5.06	4.83
5	8 Dec 2011	4.49	3.87
6	9 Dec 2011	4.17	3.53
7	24 Dec 2011	4.37	3.27
8	29 Dec 2011	4.18	3.46
9	30 Dec 2011	4.66	3.84
10	4 Jan 2012	4.31	4.02
11	5 Jan 2012	5.14	4.79
12	6 Jan 2012	4.55	4.90
13	20 Jan 2012	4.15	2.81
14	31 Aug 2012	4.11	3.24
15	24 Sep 2012	4.61	3.43
16	25 Nov 2012	4.36	4.09

An extra investigation based on extreme values of observations (during the common time interval of 1,114 days) exceeding a variety of percentile values (for the RIEN of Rhine River) showed that both storm surge and their corresponding wave height hindcasts were able to capture almost all of the 24-hour extremes on the same (correct) day but with a weaker intensity (i.e., with a correct footprint of lesser intensity).

### 3 Analytical values of correlation and statistical dependence based on Matlab routines.

A necessary split of results had to be made for a better and easier visualisation due to the relatively large amount of RIEN points to fit in one single Table. This split also revealed the distinct differences between southern and northern coastal European areas. Details of both correlations and dependencies found over southern RIEN points are presented analytically in Table 4 and Table 5 based on matlab routines. In Table 4 and Table 5, correlation (corr) and dependence (chi) values for both max12 and max24 intervals are presented together with critical threshold (thrs), significance (sig) and 95% confidence level (lower & upper) max24 values. Referring to correlation values, a large amount of variability is evident in both max12 and max24 modes

**Table 4.** Correlation and statistical dependence values for storm surge and significant wave heights over Mediterranean (ADR: Adriatic Sea – GOL: Gulf of Lion – BAL: Balearic Sea – ALB: Alboran Sea), West and North Iberian coasts (WIB & NIB), Bay of Biscay (BOB) and Black Sea (BLK) based on ~~matlab~~ Matlab routines.

	RIEN	sea	max12			max24						
			corr	thrs	chi	corr	thrs	chi	chibar	sig	lower	upper
1	Po	ADR	0.26	97.4	0.28	0.39	97.1	0.29	0.43	0.02	0.21	0.37
2	Metauro	ADR	0.23	96.8	0.26	0.35	95.7	0.22	0.30	0.05	0.03	0.35
3	Vibrata	ADR	0.23	96.6	0.35	0.37	96.5	0.32	0.36	0.04	0.23	0.37
4	Rhone	GOL	0.08	94.6	0.20	0.13	93.8	0.21	0.17	0.04	0.13	0.30
5	Foix	BAL	0.09	92.2	0.03	0.10	91.2	0.03	0.05	0.03	0.00	0.08
6	Ebro	BAL	0.04	94.7	0.19	0.12	94.5	0.22	0.22	0.03	0.10	0.30
7	Velez	ALB	0.02	93.9	0.19	0.06	93.1	0.11	0.13	0.04	0.05	0.17
8	Douro	WIB	-0.18	97.0	0.30	-0.06	95.7	0.30	0.30	0.05	0.11	0.38
9	Tagus	WIB	-0.30	94.3	0.05	-0.22	93.7	0.14	0.16	0.03	0.09	0.22
10	Sado	WIB	-0.26	94.9	0.10	-0.19	93.9	0.13	0.17	0.03	0.06	0.21
11	Guadiana	WIB	-0.04	95.9	0.22	0.03	95.7	0.28	0.29	0.02	0.15	0.36
12	Sella	NIB	-0.25	93.2	0.10	-0.17	86.2	0.14	0.07	0.05	0.07	0.19
13	Moros	BOB	0.07	96.2	0.32	0.22	96.2	0.30	0.34	0.03	0.17	0.39
14	Aven	BOB	0.13	97.0	0.34	0.25	96.7	0.35	0.39	0.01	0.23	0.42
15	Blavet	BOB	0.11	96.5	0.33	0.25	96.7	0.34	0.39	0.02	0.22	0.40
16	Danube	BLK	-0.01	96.7	0.21	0.09	96.3	0.24	0.35	0.05	0.07	0.38

**Table 5.** As in Table 4 but for Irish Sea (IRS), Bristol Channel (BRC), English Channel (ENC), North Sea (NRS), Norwegian Sea (NOS) and Baltic Sea (BAS). Owena stands for Owenavarragh RIEN (IE) while Goeta is Goeta Aelv RIEN (ES).

			<b>max12</b>			<b>max24</b>						
	<b>RIEN</b>	<b>sea</b>	<b>corr</b>	<b>thrs</b>	<b>chi</b>	<b>corr</b>	<b>thrs</b>	<b>chi</b>	<b>chibar</b>	<b>sig</b>	<b>lower</b>	<b>upper</b>
17	Owena	IRS	0.50	98.4	0.46	0.59	97.9	0.45	0.53	0.05	0.30	0.55
18	Mersey	IRS	0.45	98.2	0.43	0.56	97.4	0.43	0.48	0.03	0.29	0.52
19	Severn	BRC	0.19	96.1	0.29	0.30	94.9	0.30	0.24	0.04	0.22	0.35
20	Tamar	ENC	0.28	97.8	0.35	0.39	96.9	0.35	0.41	0.02	0.24	0.49
21	Exe	ENC	0.31	97.9	0.38	0.41	97.1	0.40	0.43	0.03	0.29	0.54
22	Avon	ENC	0.37	98.1	0.44	0.50	97.9	0.48	0.55	0.04	0.35	0.58
23	Bethune	ENC	0.59	99.1	0.62	0.68	98.8	0.64	0.77	0.02	0.55	0.73
24	Tyne	NRS	0.14	91.7	0.31	0.28	94.5	0.26	0.21	0.05	0.10	0.39
25	Humber	NRS	0.18	97.3	0.35	0.38	96.6	0.35	0.37	0.04	0.20	0.49
26	Thames	NRS	-0.10	92.6	0.22	0.06	92.7	0.22	0.11	0.05	0.11	0.31
27	Schelde	NRS	0.31	97.6	0.54	0.54	97.5	0.53	0.50	0.01	0.45	0.61
28	Rhine	NRS	0.52	98.5	0.57	0.67	98.0	0.56	0.57	0.03	0.41	0.64
29	Weser	NRS	0.56	99.0	0.58	0.65	98.5	0.56	0.69	0.02	0.42	0.63
30	Goeta	NRS	0.43	97.2	0.53	0.55	96.8	0.51	0.39	0.05	0.44	0.61
31	Orkla	NOS	0.35	97.6	0.46	0.46	97.0	0.41	0.43	0.03	0.33	0.50
32	Vantaa	BAS	0.30	97.0	0.43	0.44	96.9	0.44	0.42	0.03	0.36	0.50

#### 4 Analytical values of correlation and statistical dependence based mainly on R routines.

Details of both correlations and dependencies found over southern RIEN points are presented analytically in Table 6 and Table 7 based mainly on R routines.

**Table 6.** As in Table 4, but based mainly on R (chiplot & taildep) routines. Ensemble mean (comb) values of dependence are also shown (last column).

			R				MAT	ENS
	RIEN	sea	lower	upper	chiplot	taildep	mat_chi	comb
1	Po	ADR	0.13	0.34	0.23	0.27	0.29	0.26
2	Metauro	ADR	0.08	0.26	0.17	0.22	0.22	0.20
3	Vibrata	ADR	0.13	0.32	0.23	0.36	0.32	0.30
4	Rhone	GOL	0.06	0.21	0.14	0.22	0.21	0.19
5	Foix	BAL	0.01	0.13	0.07	0.16	0.03	0.09
6	Ebro	BAL	0.14	0.30	0.22	0.28	0.22	0.24
7	Velez	ALB	0.03	0.18	0.10	0.16	0.11	0.12
8	Douro	WIB	0.17	0.33	0.26	0.31	0.30	0.29
9	Tagus	WIB	0.07	0.21	0.14	0.22	0.14	0.17
10	Sado	WIB	0.08	0.21	0.14	0.21	0.13	0.17
11	Guadiana	WIB	0.19	0.34	0.27	0.32	0.28	0.29
12	Sella	NIB	0.05	0.19	0.12	0.18	0.14	0.15
13	Moros	BOB	0.14	0.32	0.23	0.28	0.30	0.27
14	Aven	BOB	0.18	0.37	0.27	0.31	0.35	0.31
15	Blavet	BOB	0.17	0.36	0.27	0.30	0.34	0.30
16	Danube	BLK	0.13	0.32	0.23	0.26	0.24	0.24

**Table 7.** As in Table 5, but based mainly on R (chiplot & taildep) routines. Ensemble mean (comb) values of dependence are also shown (last column).

			<b>R</b>				<b>MAT</b>	<b>ENS</b>
	<b>RIEN</b>	<b>sea</b>	<b>lower</b>	<b>upper</b>	<b>chiplot</b>	<b>taildep</b>	<b>mat_chi</b>	<b>comb</b>
17	Owena	IRS	0.26	0.52	0.39	0.40	0.45	0.41
18	Mersey	IRS	0.26	0.48	0.38	0.38	0.43	0.40
19	Severn	BRC	0.16	0.32	0.24	0.30	0.30	0.28
20	Tamar	ENC	0.21	0.41	0.31	0.34	0.35	0.33
21	Exe	ENC	0.25	0.46	0.36	0.38	0.40	0.38
22	Avon	ENC	0.33	0.57	0.45	0.46	0.48	0.46
23	Bethune	ENC	0.49	0.80	0.64	0.66	0.64	0.65
24	Tyne	NRS	0.11	0.27	0.19	0.26	0.26	0.24
25	Humber	NRS	0.20	0.40	0.30	0.33	0.35	0.33
26	Thames	NRS	0.08	0.22	0.15	0.25	0.22	0.21
27	Schelde	NRS	0.36	0.58	0.47	0.48	0.53	0.49
28	Rhine	NRS	0.41	0.64	0.52	0.54	0.56	0.54
29	Weser	NRS	0.40	0.67	0.55	0.54	0.56	0.55
30	Goeta	NRS	0.35	0.53	0.44	0.46	0.51	0.47
31	Orkla	NOS	0.25	0.45	0.35	0.38	0.41	0.38
32	Vantaa	BAS	0.27	0.48	0.37	0.40	0.44	0.40

For the analysis of results, the ensemble mean value of  $\chi$  (by averaging mat\_chi, chiplot and taildep values) is taken as a reference value (contained in the last column of Table 6 and Table 7). The different categories of correlation and dependence used later in the text (and in Figure 9) refers to the categorisation adapted by Defra TR1 Report (2005).

## References

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