### Response to Reviewer #2

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# Response to Reviewer # 2

We sincerely thank reviewer 2 for his helpful suggestions to improve the quality of the paper. The reviewer has suggested four areas for improvement:

- 1. Enhance the model description (reviewer 1 and 2)
- 2. Comment all the parameters related to physical processes
- 3. Discussion about input parameters, including erosion parameters
- 4. Effect of fluctuation energy and lubrication

To resolve these issues we have made considerable changes to original paper. We would like to emphasize that our goal was to write a short paper concerning the modelling of point release wet snow avalanches, without a long overview of the modelling equations which have already been published elsewhere. However, once we began the task of rewriting, we came to agree with the reviewers comments that more explanations were indeed necessary and that these explanations would greatly improve the overall quality of the paper. The paper is now longer, but certainly more self-consistent.

# Model Description:

To enhance the model description we extended the modelling section to include: Section 2.1 Model equations, Section 2.2 Entrainment of warm, moist snow, Section 2.3 Wet snow avalanche rheology, Section 2.4 Meltwater production and lubrication and Section 2.5 Initial and boundary conditions: point release areas. In each section we include a physical description of the model parameters we employ, including the parameter ranges. Of particular importance is the separation of constitutive relations (flow rheology, lubrication, entrainment parameterizations) from the specification of initial and boundary conditions. A primary goal of the paper is to underscore the fact that whatever constitutive relations are employed (and the reviewer is free to recommend another), detailed initial (release zone) and boundary conditions (snowcover along track) are needed. In fact we believe, that model performance is no longer 'constitutive', but rather related to the boundary conditions including temperature and snowcover moisture content

We include the entire modelling section at the end of this response letter.

# Parameters and physical process:

In the modelling section we have made a large effort to separate parameters related to the fluidization of the avalanche core  $(\alpha, \beta, \gamma)$  and parameters related to temperature and meltwater lubrication  $(h_m \text{ and } \mu_s)$  to obtain dense, wet, plug-like avalanche flows. To support our arguments we have provided a new figure in the results section where we model the avalanche flow density. We have also included figures identifying the onset of melting and show how different

meltwater parameters will influence the results. For example, we show and discuss how the height of the melting layer influences the model results.

For completeness we have included our extended Voellmy model with cohesion, which is based on actual measurements of snow flows at our experimental snow chute. We state very clearly what process each model parameter controls when using the extended Voellmy model. In the results section, we do not vary Voellmy model parameters, which are fixed for wet snow, but use the initial and boundary conditions, which are based on our field measurements and observations and SNOWPACK model results. Therefore, the simulation results are not based on fitting but actual experiments/observations with flowing snow. They are not free parameters. Of course, different constitutive relations can be employed, which implies different numerical values of the parameters.

Please see the updated modelling section at the end of the rebuttal letter.

## **Erosion parameters:**

As stated above, we have now included an entire section on the Entrainment of warm, moist snow. Furthermore we have included a new figure depicting the erosion zones, which are based on field measurements. These sections/visualizations should help clarify how entrainment is treated

In the discussion section we have provided a long discussion of the role of thermal energy entrainment which determines the flow regime of the avalanche.

Please see updated discussion section, which we have included at the end of the rebuttal letter.

## Effect of fluidization energy and lubrication:

This critique we addressed by (1) rewriting the modelling section and (2) rewriting the discussion section. In the first part of the modelling section we discuss the free mechanical energy R and how we can model both highly fluidized and dense-type flows in the frictional flow regime. We discuss how wet snow influences the fluidization process, both by increasing the decay of mechanical free energy and cohesive bonding. In the discussion section, we show that avalanche runout is not given by the fluidization, which is marginal for the wet flows, but rather by some meltwater lubrication. This result an hypothesis for now is one of the primary results of the paper. Lubrication, of course, is very much dependent on the snowcover temperature and moisture content. This is the primary reason for introducing temperature and moisture content as independent state variables in the model formulation.

# Rewritten modeling section:

To model wet snow avalanches in general three-dimensional terrain we extend the depth-averaged model equations of [Christen et al., 2010] to include streamwise density variations [Buser and Bartelt, 2015] and thermal effects [Vera et al., 2015]. In this model avalanche flow is described by nine state variables:

$$\mathbf{U}_{\Phi} = (M_{\Phi}, M_{\Phi}u_{\Phi}, M_{\Phi}v_{\Phi}, Rh_{\Phi}, Eh_{\Phi}, h_{\Phi}, M_{\Phi}w_{\Phi}, N_K, M_w)^T. \tag{1}$$

 $M_{\Phi}$  denotes the total mass (per unit area) of the avalanche core, including both snow and meltwater. The meltwater mass is tracked separately and denoted  $M_w$ . When  $M_w$ =0, the avalanche is termed 'dry'; 'wet' flows occur when  $M_w > 0$ . The mass of water is always bonded

to the moving snow which is moving in the slope parallel direction with velocity  $=(u_{\Phi}, v_{\Phi})^T$ . The flow height of the avalanche is designated  $h_{\Phi}$ . The model equations can be conveniently written as a vector equation:

 $\frac{\partial \mathbf{U}_{\Phi}}{\partial t} + \frac{\partial \mathbf{\Phi}_{x}}{\partial x} + \frac{\partial \mathbf{\Phi}_{y}}{\partial y} = \mathbf{G}_{\Phi}$  (2)

where the flux components  $(\Phi_x, \Phi_y)$  are:

$$\Phi_{x} = \begin{pmatrix}
M_{\Phi}u_{\Phi} \\
M_{\Phi}u_{\Phi}^{2} + \frac{1}{2}M_{\Phi}g'h_{\Phi} \\
M_{\Phi}u_{\Phi}v_{\Phi} \\
Rh_{\Phi}u_{\Phi} \\
Rh_{\Phi}u_{\Phi} \\
h_{\Phi}u_{\Phi} \\
M_{\Phi}w_{\Phi}u_{\Phi} \\
N_{K}u_{\Phi} \\
M_{w}u_{\Phi}
\end{pmatrix}, \quad
\Phi_{y} = \begin{pmatrix}
M_{\Phi}v_{\Phi} \\
M_{\Phi}u_{\Phi}v_{\Phi} \\
Rh_{\Phi}v_{\Phi} \\
Rh_{\Phi}v_{\Phi} \\
Rh_{\Phi}v_{\Phi} \\
Rh_{\Phi}v_{\Phi} \\
M_{\Phi}w_{\Phi}v_{\Phi} \\
M_{\Phi}w_{\Phi}v_{\Phi} \\
N_{K}v_{\Phi} \\
M_{w}v_{\Phi}
\end{pmatrix}$$
(3)

and

$$\mathbf{G}_{\Phi} = \begin{pmatrix} \dot{M}_{\Sigma \to \Phi} \\ G_x - S_{\Phi x} \\ G_y - S_{\Phi y} \\ \dot{P} \\ \dot{Q} + \dot{Q}_{\Sigma \to \Phi} + \dot{Q}_w \\ w_{\Phi} \\ N_K \\ 2\gamma \dot{P} - 2Nw_{\Phi}/h_{\Phi} \\ \dot{M}_{\Sigma \to w} + \dot{M}_w \end{pmatrix} . \tag{4}$$

The mathematical description of mountain terrain is defined using a horizontal X-Y coordinate system. The elevation Z(X,Y) is specified for each (X,Y) coordinate pair. We introduce a local surface (x,y,z) coordinate system with the directions x and y parallel to the metric geographic coordinates X and Y. The grid of geographic coordinates defines inclined planes with known orientation; the z-direction is defined perpendicular to the local x-y plane. The flowing avalanche is driven by the gravitational acceleration in the tangential directions  $\mathbf{G} = (G_x, G_y) = (M_{\Phi}g_x, M_{\Phi}g_y)$ . The acceleration in the slope perpendicular direction is denoted g' and is composed of gravity g, dispersive  $N_K$  [Buser and Bartelt, 2015] and centripetal accelerations f, [Fischer et al., 2012]. The total normal stress at the base of the avalanche is given by  $N = M_{\Phi}g'$ .

$$g' = g + N_K + f \tag{5}$$

The model extension includes the explicit calculation of the depth-averaged free mechanical energy R [Bartelt et al., 2006, Buser and Bartelt, 2009]. The mechanical free energy of the avalanche is partitioned into two parts, see [Buser and Bartelt, 2015]

$$R = R_K + R_V \tag{6}$$

where  $R_K$  is the kinetic energy associated with random particle movements and  $R_V$  is the potential or configurational energy associated with expansion of the avalanche core, defined by the height  $h_{\Phi}$ . Shearing in the avalanche core induces particle interactions that create a dispersive pressure  $N_K$  at the basal boundary which leads to fluidization of the avalanche core and streamwise variations of avalanche flow density,  $\rho_{\Phi}$ . The production of free mechanical

energy  $\dot{P}$  is given by an equation containing two model parameters: the production parameter  $\alpha$  and the decay parameter  $\beta$ , see [Buser and Bartelt, 2009]

$$\dot{P} = \alpha \left[ \mathbf{S}_{\Phi} \cdot \mathbf{u}_{\Phi} \right] - \beta R_K h_{\Phi}. \tag{7}$$

The production parameter  $\alpha$  defines the generation of free mechanical energy from the shear work rate  $[\mathbf{S}_{\Phi} \cdot \mathbf{u}_{\Phi}]$ ; the parameter  $\beta$  defines the decrease of the kinetic part  $R_K$  by inelastic interactions. In this model formulation the basal boundary plays a prominant role because particle motions in the slope-perpendicular motion are inhibited and reflected back into the flow. The basal boundary converts the production of random kinetic energy into an energy flux that changes the z-location of particles and therefore the potential energy and particle configuration within the avalanche core. The energy flux associated with the configurational changes is denoted  $\dot{P}_V$  and given by

$$\dot{P}_V = \gamma \dot{P}.\tag{8}$$

The parameter  $\gamma$  therefore determines the magnitude of the dilitation of the flow volume under a shearing action. When  $\gamma=0$  there is no volume expansion by shearing. Therefore, the model formulation we apply allows the simulation of both disperse and dense avalanche flow types. In this paper we are primarily concerned with dense, plug-like wet snow avalanche movements  $(R_V \approx 0)$ ; however, as we shall see in the case studies, even wet flows fluidize in steep, rough terain  $(R_V > 0, \gamma > 0)$ . The production  $\alpha$ , decay  $\beta$  parameters, which control the degree of the flow dilitation will be discussed in the next section when discussing wet snow avalanche flow rheology. The model extension also includes the explicit calculation of the depth-averaged avalanche temperature T [Vera et al., 2015]. The temperature T is related to the internal heat energy E by the specific heat capacity of snow c

$$E = \rho_{\Phi} c T. \tag{9}$$

The avalanche temperature is governed by (1) the initial temperature of the snow  $T_0$ , (2) dissipation of kinetic energy by shearing  $\dot{Q}$ , as well as (3) thermal energy input from entrained snow  $\dot{Q}_{\Sigma \to \Phi}$  and (4) latent heat effects from phase changes  $\dot{Q}_w$  (meltwater production), see [Vera et al., 2015]. Dissipation is the part of the shear work not going to free mechanical energy in addition to the inelastic interactions between particles (decay of random kinetic energy,  $R_K$ )

$$\dot{Q} = (1 - \alpha) \left[ \mathbf{S}_{\Phi} \cdot \mathbf{u}_{\Phi} \right] + \beta R_K h_{\Phi}. \tag{10}$$

The model equations are solved using the same numerical schemes outlined in [Christen et al., 2010].

#### 0.1 Entrainment of warm, moist snow

We treat the entrainment of warm, moist snow as a fully plastic collision between the avalanche core  $\Phi$  and snow cover  $\Sigma$ . Snow with temperature  $T_{\Sigma}$  is entrained at the rate  $\dot{M}_{\Sigma \to \Phi}$ . The entrained snow is initially at rest, but after the collision with the avalanche all the entrained mass is moving with the avalanche velocity  $\mathbf{u}_{\Phi}$  (definition of plastic collision). If the entrained snow is moist, water mass, in addition to the snow mass, is entrained at the rate  $\dot{M}_{\Sigma \to w}$ . The water mass is always at temperature  $T_w = 0^{\circ}$  C. The total entrainment rate is defined by the (1) density of the snowcover  $\rho_{\Sigma}$ , (2) the dimensionless erodibility coefficient  $\kappa$  and (3) the avalanche velocity, [Christen et al., 2010]:

$$\dot{M}_{\Sigma \to \Phi} = \rho_{\Sigma} \kappa \| \mathbf{u}_{\Phi} \| \tag{11}$$

The entrained water mass is found from the volumetric water content of the undisturbed snow-cover,  $\theta_w$ :

$$\dot{M}_{\Sigma \to w} = \theta_w \dot{M}_{\Sigma \to \Phi}.\tag{12}$$

The thermal energy entrained by the avalanche is therefore

$$\dot{Q}_{\Sigma \to \Phi} = \left[ (1 - \theta_w) c_{\Sigma} T_{\Sigma} + \theta_w c_w T_w \right] \dot{M}_{\Sigma \to \Phi} + \frac{1}{2} \dot{M}_{\Sigma \to \Phi} \|\mathbf{u}_{\Phi}\|^2. \tag{13}$$

where  $c_{\Sigma}$  is the mass heat capacity of dry snow and  $c_w$  is the mass heat capacity of water. The last term in this equation represents the heat produced during the plastic collision. In this entrainment model no random kinetic energy is generated during the entrainment process because of the entrainment process is considered completely inelastic.

### 0.2 Wet snow avalanche flow rheology

Wet snow avalanches are regarded as dense granular flows in the frictional flow regime [Bozhinskiy and Losev, 19 Measured velocity profiles exhibit pronounced visco-plastic like character and are often modelled with a Bingham-type flow rheology [Dent and Lang, 1983, Dent et al., 1998, Bartelt et al., 2005, Kern et al., 2009]. Granules in wet-avalanche flows are large, heavy and poorly sorted in comparison to granules in dry avalanches [Jomelli and Bertran, 2001, Bartelt and McArdell, 2009]. Sintered particle agglomerates and levee constructions with steep vertical shear planes are found in wet snow avalanche deposits, indicating that cohesive processes are an important element of wet snow avalanche rheology [Bartelt et al., 2012c, Bartelt et al., 2015]. To model wet snow avalanche flow we apply a Voellmy rheology with cohesion. Frictional resistance is given by a Voellmy-type shear stress  $\mathbf{S}_{\Phi} = (S_{\Phi x}, S_{\Phi y})$ , containing Coulomb stress  $S_{\mu}$  (coefficient  $\mu$ ) and velocity dependent stress  $S_{\xi}$  (coefficient  $\xi$ ):

$$\mathbf{S}_{\Phi} = \frac{\mathbf{u}_{\Phi}}{\|\mathbf{u}\|_{\Phi}} \left[ S_{\mu} + S_{\xi} \right]. \tag{14}$$

For wet snow avalanche flow, the Coulomb shear stress is modified to include the cohesion, parameterized by the coefficient  $N_0$ , see [Bartelt et al., 2015]:

$$S_{\mu} = \mu(R_V)N - \left[1 - \mu(R_V)\right]N_0 \exp\left[-\frac{N}{N_0}\right] + \left[1.0 - \mu(R_V)\right]N_0. \tag{15}$$

Note that when  $N_0=0$  and the standard Coulomb friction of the Voellmy model is retrieved:  $S_{\mu}=\mu(R_V)N$ . Experiments with flowing snow have identified cohesion values for wet snow to be as low as  $N_0=100$  Pa, but never exceeding  $N_0=2000$  Pa [Bartelt et al., 2015]. The speed dependent stress of the Voellmy friction model is

$$S_{\xi} = \rho_{\Phi} g \frac{\|\mathbf{u}\|_{\Phi}^2}{\xi(R_V)}.$$
 (16)

We define the functional dependency of the friction parameters  $(\mu, \xi)$  on the configurational energy  $R_V$  and cohesion  $N_0$  as

$$\mu(R_V) = \mu_d \exp\left[-\frac{R_V}{N_0 + R_0}\right] \tag{17}$$

and

$$\xi(R_V) = \xi_d \exp\left[\frac{R_V}{N_0 + R_0}\right]. \tag{18}$$

With this frictional model  $\mu_d$  and  $\xi_d$  are the static friction coefficients associated with dry, non-fluidized flowing snow,  $R_V$ =0. The parameter  $R_0$  defines the decrease in shear stress with increasing fluidization (lower flow density), see [Bartelt et al., 2012a]. Snow temperature and wetness have a strong influence on the mechanical properties of snow and therefore the amount of free mechanical energy in the avalanche. The primary difference between wet and dry flows is the production and dissipation of free mechanical energy, which controls the fluidization of the avalanche core. When the avalanche snow contains some free water, the hardness of the granules decreases [Voytokskiy, 1977], and they can be plastically deformed and sculptured into well-rounded forms [Bozhinskiy and Losev, 1998]. We model this effect by using production coefficients  $\alpha \geq 0.05$  and large free mechanical energy decay coefficients  $\beta = 1.0$  for wet snow. This ensures that only in very rough and steep terrain is fluidization of the wet avalanche core possible.

### 0.3 Meltwater production and lubrication

In wet snow avalanche flow, we must consider two additional physical processes: (1) the production of meltwater  $\dot{M}_w$  and (2) the decrease of Coulomb shear stress because of meltwater lubrication. Meltwater production is considered as a constraint on the flow temperature of the avalanche: the mean flow temperature T can never exceed the melting temperature of ice  $T_m = 273^{\circ}$  K. The energy for the phase change is given by the latent heat L

$$\dot{Q}_w = L\dot{M}_w \tag{19}$$

under the thermal constraint that within a time increment  $\Delta t$ 

$$\int_0^{\Delta t} \dot{Q}_w dt = M_{\Phi} c (T - T_m) \qquad \text{for} \qquad T > T_m. \tag{20}$$

Of course, when the flow temperature of the avalanche does not exceed the melting temperature, no latent heat is produced  $\dot{Q}_w = 0$ . The length of the time increment is defined by the numerical time integration scheme of the vector equations. Because we employ a depth-averaged model to calculate the bulk avalanche temperature T we have no information to define the depth in the avalanche flow core where melting occurs. The dissipation rate  $\dot{Q}_w$  depends on the internal shear distribution, which can be concentrated at the bottom surface of the avalanche, or distributed uniformly over the entire avalanche flow height. The spatial concentration of meltwater will therefore determine how the meltwater lubricates the flow. To account for the spatial distribution of meltwater in a depth-averaged model, we use the following two-parameter lubrication function to replace the dry Coulomb friction coefficient  $\mu_d$ ,

$$\mu(h_w, R_V) = \mu_s + (\mu_d - \mu_s) \exp\left[-\frac{h_w}{h_m}\right]. \tag{21}$$

The amount of meltwater in the avalanche core is characterized by the height  $h_w$  defined with respect to the density of water  $M_w = \rho_w h_w$ . This height (measured from the avalanche running surface) is compared to the height  $h_m$ , representing the height where the meltwater is concentrated. We approximate the height  $h_m$  using measured shear layers of wet avalanche flows which show 0.01 m  $\leq h_m \leq 0.1$  m, see [Dent and Lang, 1983, Dent et al., 1998, Bartelt et al., 2005, Kern et al., 2009]. The parameter  $\mu_s$  defines the Coulomb friction when the layer  $h_0$  is saturated,  $h_w \approx h_m$ . We take  $\mu_s = 0.12$ . This ensures that dense, non-fluidized wet snow avalanches will not stop on slopes greater than 9° when they contain fully saturated lubrication layers,  $\mu(h_w, R_V) \approx 0.15$  for  $h_w = h_m$ .

### 0.4 Initial and boundary conditions: point release areas

The initial release volume  $V_0$  is calculated by estimating a release area  $A_0$  and a mean fracture depth  $h_0$ . This is somewhat difficult for point release avalanches because the area is reduced to a single point with no surface area. In this work, point release avalanches are specified by defining a small triangular shaped release area where the upper apex of the triangle is located at the point release. The triangle together with the fracture height defines the initial release volume .

Fracture depth, erosion depths, surface eroded and snow properties along the avalanche paths were estimated from snow cover observations and meteorological data. The Codelco Andina mine has automatic weather stations which provide air temperature, snow surface temperature, pressure, wind, precipitation and radiation measurements. The meteorological data was used to run SNOWPACK simulations [Bartelt, 2002, Lehning et al., 2002] to define the release temperature, density and initial snowcover water content. Coupled with the field studies performed by the winter operation crew, provides accurate snow cover information. The distance between the chosen automatic weather station and the avalanches paths varies between 0.5 km and almost 4.0 km. The release areas in the case studies were between 3085 and 3600 m.a.s.l.; the used weather station is located at 3570 m.a.s.l. The small elevation difference between the release zones and the automatic weather station ensures accuracy in snow and meteorological data. However, snow surface temperature and surface energy fluxes might be influenced by the slope exposition. SNOWPACK allows the user to generate virtual slopes, specifying slope angle and exposition and coupling the measured meteorological and snow data to the virtual slopes [Bartelt, 2002, Lehning et al., 2002]. Meteorological data from the winter operation building at the valley bottom (Lagunitas building 2700 m.a.s.l.) is available. Thus, it was possible to estimate the precipitation and temperature gradients existing between the weather station location and the winter operation building and therefore to estimate the snow cover conditions along the selected avalanche paths. To estimate the fracture and erosion depths for each case study we considered field work measurements and the data provide from the automatic weather stations and the SNOWPACK simulations. The remaining snow input parameters are snow temperature, snow water content and snow density. These were specified directly using SNOWPACK simulations using the meteorological and snow data collected from the automatic weather station.

# **Rewritten Discussion**

Many existing avalanche dynamics models widely used in practice (e.g. [Christen et al., 2010, Sampl et al., 2004, Sheridan et al., 2005, Mergili et al., 2012]) do not include the role thermal temperature, fluidization or snow wetness in their mechanical description of avalanche motion. As such, wide ranging flow parameters are required to model avalanche runout and danger. These models therefore cannot be applied to forecast how avalanche activity will disrupt mining operations because they cannot take into account measured and observed snow conditions. Road closure is associated with severe financial costs and avalanche forecasters must deliver runout warnings based on daily, perhaps hourly, meteorological information.

To address this problem we developed a depth-averaged avalanche dynamics model that separates the properties of flowing snow from the specification of initial and boundary conditions, which can be supplied by avalanche forecasters using a combination of weather stations and snowcover modelling. The avalanche model requires input parameters for snow temperature, density and water content in the release area and along the avalanche path. The temperature

data provided by the automatic weather stations can be assumed to be reliable at the altitude and exposition where the weather stations are located. However, the difference in altitude and exposition of the four different cases studies requires a method to extrapolate temperature from the point locations of the automatic weather stations to the entire slope. For this purpose we applied the SNOWPACK model on virtual slopes matching the expositions with the studied slopes. When it was possible to enter the slopes we used hand measurements of validate the SNOWPACK model predictions for temperature and density.

As the SNOWPACK simulations predicted isothermal snowcover at  $T=0^{\circ}$  for the snow depth affected by the avalanches, the entrained snow temperature was set to zero degrees in all four cases studies. This approach could not be followed with the modelled snowcover water content which has no limiting value in an isothermal snowcover. Although SNOWPACK was used to predict snow water content [Wever et al., 2014] it was difficult to measure and validate the distribution of snow water content at lower altitudes and different expositions. For example, in the case CG-1 the snowfall was preceded by rain making it difficult to calculate the snowcover water content which depends on the variability of the rainfall.

The position of all release zones was obtained from the eyewitness reports and post-event surveys. Entrainment depths for the simulations were also obtained from field studies and event documentation. In the examples LGW-2 and BN-1 the erosion depths where measured along the path in several points. Because the avalanches disrupted road traffic, road clearance crews could estimate deposition depths allowing good estimates of avalanche mass balance. The temperature, snow density and water content of the eroded mass are the key input information to predict accurate avalanche deposition volumes and runout distances. The release mass does not play an important role apart from defining the location of release and the triggering the whole subsequent process.

The four examples contain mountain rock faces with well defined flow channels (CG-1, CCHN-3) as well as open slopes (BN-1, LGW-2). At release the avalanche mass spreads depending on the terrain features. In two of the four case studies, avalanche spreading is inhibited by the steep sidewalls of mountain gullies, a function of the topographic properties of the mountain. The remaining two examples are open slopes where the spreading angle is larger. Avalanche movement is therefore not only controlled by the hydrothermal state of the snow, but also by the slope geometry. High resolution digital elevation models that accurately represent mountain ravines and channels are thus necessary to apply avalanche dynamics models to simulate small avalanches, [Bühler et al., 2011].

The avalanche model simulates both fluidization and lubrication processes. This requires introducing depth-averaged equations for thermal energy [Vera et al., 2015], mechanical free energy [Buser and Bartelt, 2015] and meltwater [Vera et al., 2015]. The degree of fluidization characterizes the avalanche flow regime: dry snow avalanches being associated with more fluidized, less dense flows (mixed flowing/powder avalanches) and wet avalanches being associated with less fluidized, dense flows. The degree of fluidization is controlled by parameters governing the production and decay of free mechanical energy R ( $\alpha$ ,  $\beta$  and  $\gamma$  [Buser and Bartelt, 2015]). The production parameter  $\alpha$  is made dependent on terrain roughness and is independent of the avalanche temperature and moisture content. Highly plastic, wet particle interactions quickly dissipate any free mechanical energy leading to dense flows that can only fluidize in steep, rough slopes. We model this process by increasing the dissipation parameter  $\beta$  for warm, wet avalanches. This produces dense flows in the frictional flow regime. In the four case studies the flow density in the runout zone is close to the deposition density  $\rho_{\Phi} = 450 \text{ kg/m}^3$ , whereas in the steep track sections the flow density is somewhat lower  $\rho_{\Phi} = 300 \text{ kg/m}^3$ . Important is that the same model formulation is used for both dry and wet avalanches and fluidization is

controlled by a combination of terrain (production of free mechanical energy) and wet snow granule properties (dissipation of free mechanical energy). A single model parameter  $\beta$  controls the degree of fluidization. An important model assumption is that entrainment of moist wet snow is a completely dissipative process which does not introduce additional free mechanical energy into the avalanche core.

Therefore, our results indicate that fluidization cannot be responsible for long runout distances of wet avalanches. Snow chute experiments with wet snow, showing that cohesive interactions in the avalanche core further hinder fluidization [Bartelt et al., 2015], provide more evidence that wet snow avalanche mobility is strongly linked to the temperature and moisture dependent mechanical properties of wet snow [Voytokskiy, 1977]. To investigate this hypothesis, we postulate that temperature and lubrication effects lead to a significant reduction of the Coulomb part of the Voellmy friction. A two parameter empirical relation between water content and friction  $\mu$  was devised. A problem with depth-averaged models is that the distribution of meltwater in the avalanche height cannot be predicted from depth-averaged calculations of avalanche flow temperature, which depends on the slope perpendicular shear profile in the avalanche core. We assume that meltwater is concentrated in a shear layer of height  $h_m$ . When this layer becomes saturated with meltwater, Coulomb friction is reduced to a sliding value of  $\mu_s$ , which we take, for now, to be constant  $\mu_s=0.12$ . This value was selected based on our observations of wet snow avalanche runout in Switzerland. The layer height was set to  $h_m = 0.01$  m, indicating that shearing in wet avalanche flows is concentrated in a thin basal layer. This is in agreement with velocity profile measurements of wet avalanche flows [Dent et al., 1998, Kern et al., 2009].

The model calculates the depth-averaged flow temperature from initiation to runout. In the four case studies the avalanche reached the melting point of snow-ice immediately after release due to the warm initial conditions. The entrainment of warm, moist snow enhanced the lubrication process. The decrease of Coulomb friction due to lubrication effects was essential for the point release avalanches to develop into long-running wet snow avalanches. For practical applications it is important that lubrication processes due to the (1) initial snow water content, (2) snow melting by frictional dissipation and (3) heat energy of entrained snow must all be taken into account. The method used to simulate the avalanche point release requires defining a small triangular area. The ratio between the eroded snow volume and the initial snow volume is between 20 to 60 for the four case studies we studied in this paper. The initial area used to simulate the avalanche release does not affect the final run-out, velocity and avalanche deposit calculations. The model results emphasize that complete information of the snow cover is necessary to achieve accurate representations of the events. The model is sensible to variations in the initial snow cover conditions (temperature and water content). For example, when colder snow is specified at release, the simulated avalanches stop immediately after release and do not reach the valley bottom. Given accurate initial conditions the model was able to back calculate runout distances, flow perimeters and avalanche volumes. Therefore, with this model formulation, it is only possible to obtain realistic runout predictions with accurate snow cover data. The application of the avalanche dynamics model should be restricted to cases where accurate data is available.

## Specific comments

**Reviewer:** p. 2884, line 6: 'documented case studies'...I have the feeling that the quantitative available information remains very poor. This comment would need to be qualified.

**Response:** The quantitative information is much better than casual observations. We have

(1) meteo-information from nearby automatic weather stations (2) GPS measurements for mass balance/entrainment (3) snowpits in the release zone and deposition zones (4) measurements of deposits (width, runout, height, granulometry). The quantitative information that is missing are avalanche flow velocities. In the paper we included a new figure (fig 2) with the GPS points taken during the field campaigns and we clarified the methods used to perform these measurements.

**Reviewer:** p. 2884, line 10: the key role of snow entrainment should be primarily mentioned here.

Response: We do agree. Snow entrainment is added now among the main factors affecting avalanche runout and velocity. However, we want to point out that, as usual, we are probably not in agreement with why entrainment has a strong influence. We believe that there are two influences (1) due to the heat energy input (facilitating lubrication) and (2) due to elastic scattering processes that modify the free mechanical energy of the avalanche. The last point we do not discuss in this paper. We model snow entrainment as a completely plastic process, which we think is justifiable because of the deformable, plastic properties of wet snow.

**Reviewer:** p. 2885, lines 17-19: is it just an observation or is there any underlying physics supporting this statement?

Response: The mine operators are confronting wet snow avalanche cycles more often, but that it is just an observation made by the winter operation crew, since in their records they did not discriminate between wet snow avalanches and dry snow avalanches until the last three seasons. In Europe [Pielmeier and other, 2013] and in North and South America [McClung, 2013] have already written about this issue.

**Reviewer:** p. 2885, lines 25-30: these lines can be summarized by reminding that energy balances are proposed in depth-averaged forms, so we cannot expect more.

**Response:** We consider it necessary to write a comment about this model assumption in the introduction.

**Reviewer:** p. 2885, line30 and p. 2886, lines 2-5: what do you want to say? As snow entrainment is crucial in your study (very low ratios of released volume to final deposit volume), it seems obvious that the properties of snow cover along the track are much more important than properties of snow in the released area. I am not sure that result stems from your model.

Response: We do agree with the reviewer, but this is one of the main features of this work. It is the first time that a snow avalanche model has a dependency on the snowcover conditions (temperature, water content, density) at release and when entraining mass. The initial released mass is small in comparison with the final avalanche mass, another novelty of the simulation results. Unlike existing avalanche models used in presently in practice, we stress that our temperature-based mechanics depends on the entrained snow properties and not entirely on the released snow properties. We clearly want to emphasize the important role of entrained heat energy.

**Reviewer:** Section 2, Eq.(1):  $u_{\Phi}$  should be defined here (the reader should not wait for page 2889).

**Response:** Thank you. The model part has been re-written, in any case  $u_{\Phi}$  was defined in page 2887, line 17.

**Reviewer:** Eq. (4): many variables are not defined: all  $\dot{Q}$ ,  $\dot{E}$ , $\dot{W}$ ? The notation  $\dot{X}$  where  $\dot{X}$  (where X is the variable considered) should be properly defined. The paper must be self-contained.

**Response:** Thank you. It has been noted in the manuscript.

**Reviewer:** p. 2888, lines 6-8: what is the relation between g',  $g_z$ , and  $f_z$ ? The reader should not have to guess (Is g' the sum of  $g_z$  and  $f_z$ ?). Again the paper has to be self-contained!

**Response:** Thank you. The equation (5) has been added.

**Reviewer:** Eq.(5), p 2889, line 2: R refers to 'fluctuation energy' while R refers to the 'mechanical free energy' on page 2887, line 19. Please be more precise on the semantics used and the underlying physical processes.

**Response:** Thank you. We refer now to 'mechanical free energy' every time need it. Actually, we would much prefer using German: Random energy being defined as ungerichtete kinetische Energie oder elastische Streuungsenergie.

**Reviewer:** Eq.(6): how the values of  $\mu_{wet}$  and  $h_m$  are chosen? Do you have physical arguments for these values? Do you have physical arguments for these values? This equation established by Colbeck (1992) and arguments regarding its application to snow avalanches would merit much more discussion. A graph showing the variation of  $\mu$  against both R and  $h_w$  would be very useful for that purpose.

**Response:** We have added a section explaining the chosen parameters. We added a figure of  $\mu$  against R and  $h_w$  too.

**Reviewer:** p.2889, lines 17-18: very elusive...what about the snow cover distribution? Please discuss the assumption of a uniform depth distribution across the width of the avalanche path and along the avalanche path?

Response: We have included a section clarifying this issue. The erosion depths are inputs of the model. We got it from field measurements, Fig2. The snowcover temperature was 0 degrees at the upper layers (snowcover with water content only occurs with snow at 0 degrees). For the snow water content and density we have only values from SNOWPACK simulations at the AWS and the Lagunitas building, but the proximity of the avalanche paths allow us to assume those values as a good approximation.

**Reviewer:** p. 2889-2890 (up to line 14): more detailed information and discussion on how SNOWPACK calculations were made would be needed.

**Response:** The final SNOWPACK results (configuration file tested together with snow and meteo data from the last 5 seasons) was the one which best fitted the field measurements (snow pits). The snowpits were dug by the winter operation crew and the author. It is beyond the scope of this paper to discuss about a snowpack model successfully tested in different mountain ranges and climates, please see e.g. [Wever et al., 2014], [Monti, 2014], [Fierz...]

**Reviewer:** p. 2890, lines 20-22: very unclear...which variable are you comparing at the end between the field and the SNOWPACK simulations?

Response: These section has been rewritten. To test SNOWPACK we compared the snow pits (grain type, grain size, temperature, density, hardness, water content (qualitative)) done from the winter operation crew with the simulation results obtained from SNOWPACK.

**Reviewer:** p. 2890, lines 24-30 and Table 2: why  $\mu$ , and  $\beta$  are kept constant? Why  $\alpha$  is changed (0.07 instead of 0.08) for one avalanche? May I suspect a problem of convergence if alpha would be 0.08 for this avalanche... you must justify the choices made here for the values of  $\mu$ ,  $\beta$  and  $\alpha$ .

Response: We have added a section explaining the chosen parameters. The  $\alpha$  parameter accounts for the production of mechanical free energy. Steeper, rougher terrain fluidizes the avalanche motion more than gentle smoother slopes. In our case the avalanche path BN-1 ( $\alpha$ =0.07) is a bit more gentle and smoother than the others. The reader must note that the change of 0,07 to 0,08 is only one per cent change of the frictional work rate used to produce the free mechanical free energy. This is not much.

**Reviewer:** Table 2: given that the orientations of avalanche paths are different, I am surprised not to see any difference in the values of some parameters (snow properties, depth of eroded snow, etc.) between the LGW-2 and the other three avalanche tracks.

**Response:** The avalanches LGW-2 and BN-1 occurred the same day with 2 hours of difference. A snowfall occurred two days before, covering all the avalanche tracks, so the exposition

dependencies were not prominant. The other two avalanches released once the upper 30-40 cm became warm and wet. This can explain why the density values and erosion depths do not differ too much between expositions.

**Reviewer:** Table 2: some parameters are missing in table 2:  $R_0$ ? What is its value and how that value is chosen? You must define all parameters, give their value, and explain, justify your choice.

Response: Table 2 has been completed. Thank you.

**Reviewer:** Table 2: why mentioning the cohesion C here? I am not sure that cohesion is used in the model equations...

**Response:** The model section has been rewritten adding the cohesion part [Bartelt et al., 2015]. **Reviewer:** Table 2: snow densities are not so high meaning that great quantities of air are present (typically around 70

**Response:** The densities are not so high but at that time we did not have an old, isothermal, densified snowcover. It was the first wetting on the upper snowcover.

**Reviewer:** Section 3, p. 2891, lines 11-12: what is the information from mine staff in Fig.8?Do you refer to Fig. 2 instead? What were the techniques used by mine staff: eyewitness observation, expert knowledge of the site, survey after avalanche, instrumentation used, etc? The manuscript is generally very elusive regarding the field data available from mine staff.

Response: This section has been modified and the figure 2 added. Thank you.

**Reviewer:** p.2894, lines 6-7: this sentence looks very speculative. What is the relation between the dissipated heat energy and the maximum velocity fields shown in Fig. 10? I do not understand...

**Reviewer:** There was a mistake on the reference. This sentence should refer to figure 7 from the first manuscript version. It has been corrected. Thank you.

**Reviewer:** p.2894, lines 9-13: I am not sure that your conclusion directly stems from the results of your model. Your model includes many physical processes (erosion/deposition, fluctuation energy affecting  $\mu$  and  $\xi$ , production of melt water affecting  $\mu$ ) in addition to many input parameters. As a result it is very unclear to me to distinguish between the weights/contributions of each process and choices which you made in the final avalanche run-out.

Response: Firstly, we did our best to quantify the initial (release) and boundary (snowcover) conditions. These are not input parameters to be modified, rather given by observations and measurements to constrain our problem. Secondly, we do include many new observable processes. To ignore them would mean to go back to the block models of Voellmy (which we can do also by selecting the production coefficient  $\alpha=0$ ). Then we would play the game of tuning friction parameters. We chose not to do this rather to invoke a physical process causing the change of these parameters. One is fluidization, the change in friction parameters because of configurational changes (i.e. density,  $R_V$ ) changes in the avalanche core. The other is meltwater lubrication, which can determine because we track the avalanche flow temperature. This requires introduces additional mass and energy equations in order to maintain mass and energy conservation. Of course, in order to quantify the process we need an additional coefficient. We demonstrate showing the calculated avalanche densities that wet snow avalanches to not fluidize in comparison to their dry counterparts, flow densities are high. We now have the longsearched for parameters to characterize wet snow avalanche flow regime (beta, decay of free mechanical energy). We are left with the conclusion that lubrication, driven by the boundary conditions, plays a significant role in wet snow avalanche runout.

To answer these questions we have rewritten the discussions and conclusions. A figure clarifying the role played by the lubrication in the model has been added (Fig. 12).

**Reviewer:** Discussion, p. 2895, lines 18-22: I do not like this part of the text. Avalanche movement is firstly controlled by the balance between gravity force (proportional to the sinus of the slope) and friction force (proportional to the cosine of the slope). The main inclination angle and the slope geometry (lateral spreading), and the available volumes of snow along the tracks are of course the key factors: 'without mechanics, no avalanches!' Are the closure equations (such as Eq.(7) and Eq.(6)) well validated against well documented and controlled experiments for each process (fluctuation energy, production of melt water) to be able to be conclusive in the case of full-scale avalanche events which remain poorly documented?

Response: To summarize the reviewer: I dont believe in your model unless you have independent experimental measurements of the fluctuation energy, or meltwater production. Firstly, we invoke simple physical laws, validated by countless experiments: e.g. ice melts at zero Celcius. Of course, it would be really nice to have more experiments to show the temperature variations in the avalanche core. And (2) some partitioning between mechanical energy and heat must exist and the flow density of the avalanche is not constant. The experiments, when they arrive, can be used to determine the partitioning coefficient or even the functional dependency of the partitioning coefficient. In the sense of Popper, we have constructed a theory that can be falsified. For now, we must rely on our field observations, which is perhaps not so bad a research strategy as you would suggest.

**Reviewer:** p.2896, lines 24-25: I would add that using simple models (with a reduced number of both model and input parameters) but some good statistics (sensitivity of the results o parameters, confidence intervals, etc.) would have been a better strategy for very poorly documented avalanches.

**Response:** Simple models imply reduced physics, without process understanding. And furthermore, without the possibility of constructing well-thought out experiments. No. This is not our way.

Reviewer: p.2896 -2897, end of section 5: this is a very poor (obvious) conclusion while looking at the huge ratios of final deposit volumes to the released volumes for the four avalanches.. Response: Discussion and conclusions have been completely rewritten.

**Reviewer:** section 6 conclusion: yes, precondition 2 appears to be essential in your study but the erosion/deposition model which you are using is not described and an uniform eroded snow cover is assumed. Other assumptions regarding the distributions of the eroded snow cover would lead to a noticeable variability of the results in terms of avalanche run-out and velocities (before looking at the effect of melt water production). These points should be further discussed in the manuscript.

Response: Discussion and conclusions have been completely rewritten.

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