

This discussion paper is/has been under review for the journal Natural Hazards and Earth System Sciences (NHESD). Please refer to the corresponding final paper in NHESD if available.

Hazard function theory for nonstationary natural hazards

L. K. Read and R. M. Vogel

Department of Civil and Environmental Engineering, Tufts University, 200 College Avenue, Medford, MA, 02155, USA

Received: 5 October 2015 – Accepted: 12 October 2015 – Published: 13 November 2015

Correspondence to: L. K. Read (laurareads@gmail.com)

Published by Copernicus Publications on behalf of the European Geosciences Union.

NHESD

3, 6883–6915, 2015

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Abstract

Impact from natural hazards is a shared global problem that causes tremendous loss of life and property, economic cost, and damage to the environment. Increasingly, many natural processes show evidence of nonstationary behavior including wind speeds, landslides, wildfires, precipitation, streamflow, sea levels, and earthquakes. Traditional probabilistic analysis of natural hazards based on peaks over threshold (POT) generally assumes stationarity in the magnitudes and arrivals of events, i.e. that the probability of exceedance of some critical event is constant through time. Given increasing evidence of trends in natural hazards, new methods are needed to characterize their probabilistic behavior. The well-developed field of hazard function analysis (HFA) is ideally suited to this problem because its primary goal is to describe changes in the exceedance probability of an event over time. HFA is widely used in medicine, manufacturing, actuarial statistics, reliability engineering, economics, and elsewhere. HFA provides a rich theory to relate the natural hazard event series (X) with its failure time series (T), enabling computation of corresponding average return periods, risk and reliabilities associated with nonstationary event series. This work investigates the suitability of HFA to characterize nonstationary natural hazards whose POT magnitudes are assumed to follow the widely applied Generalized Pareto (GP) model. We derive the hazard function for this case and demonstrate how metrics such as reliability and average return period are impacted by nonstationarity and discuss the implications for planning and design. Our theoretical analysis linking hazard event series X , with corresponding failure time series T , should have application to a wide class of natural hazards with rich opportunities for future extensions.

NHESSD

3, 6883–6915, 2015

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



1 Introduction

Studies from the natural hazards literature indicate that certain hazards show evidence of nonstationary behavior through trends in magnitudes over time. Such trends in the magnitudes of natural hazards have been attributed to changes in climate patterns, e.g. for wind speeds (de Winter et al., 2013), wildfires (Liu et al., 2010), typhoons (Kim et al., 2015), and extreme precipitation (Roth et al., 2014), and also linked directly to human activities, e.g. increase in earthquakes from wastewater injection associated with hydraulic fracturing (Ellsworth, 2013). Other natural hazards such as floods resulting from streamflow (Di Baldassarre et al., 2010; Vogel et al., 2011) and from sea level rise (Obeysekera and Park, 2012) may be a result of a myriad of anthropogenic influences including climate change, land use change and even natural processes such as land subsidence. In addition to evidence of magnitude changes of many natural hazards, recent reports document a corresponding surge in human exposure to natural hazards (Blaikie et al., 2014), along with a 14-fold increase in economic damages due to natural disasters since 1950 (Guha-Sapir et al., 2004). Given evidence of trends and the consequent expected growth in devastating impacts from natural hazards across the world, new methods are needed to characterize their probabilistic behavior and communicate event likelihood and the risk of failure associated with infrastructure designed to protect society against such events. The existing rich and evolving field of hazard function analysis (HFA) is ideally suited to problems in which the probability of an event is changing over time, yet to our knowledge has not been applied to natural hazards. Our primary goal is to apply HFA to characterize the likelihood of nonstationary natural hazards and to better understand the expected time until the next natural hazard event occurrence for design and planning purposes.

Probabilistic analysis of natural hazards normally takes one of two approaches in fitting a probability distribution to hazard event data series. As we summarize in Table 1, a very common approach to probabilistic analyses of natural hazards employs the peaks over threshold (POT) dataset, also commonly referred to as the partial duration

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



series (PDS), to characterize exceedances above some defined magnitude (threshold) that occur over an interval of time. The second approach is to fit a probability distribution to the annual maximum series (AMS), common practice in hydrology (Gumbel, 2012; Stedinger et al., 1993) and also appropriate for earthquakes (Thompson et al., 2007) and many other processes (Beirlant et al., 2006; Coles et al., 2001). Hydrologists have extensively studied the theoretical relations between POT and AMS methods (Stedinger et al., 1993; Todorovic, 1978) and compared the two for characterizing the probabilities of flood events (Madsen et al., 1997). In general the POT approach appears to provide a larger dataset to draw from, however as the threshold of exceedance used to define the POT series is lowered, the series of maxima begin to exhibit temporal dependence which complicates the probabilistic analysis considerably. Further complexities arise in POT analyses due to subjectivity of the threshold and difficulty in confirmation of independence between events (Stedinger et al., 1993).

Still the POT method is the most widely used approach for many natural phenomena either because there is an intuitive choice for the threshold of exceedance, as in the case of earthquakes where the magnitude of completeness is often selected, or because the analyst wishes to maximize the use of data, and does not always understand the tradeoff between the length of the POT series and the inherent increase in its temporal dependence structure and the associated consequences. Due to the wide-spread application of the POT method, a substantial number of textbooks and articles have studied methods for minimizing the difficulty of implementation, with emphasis on the subjectivity of threshold selection and evaluation of independence of events (Davison and Smith, 1990, 2003). This study assumes a POT approach is taken as is so often the case for the probabilistic analysis of magnitudes of natural hazards including extreme winds (Palutikof et al., 1999), earthquakes (Pisarenko and Sornette, 2003), wildfires (Schoenberg et al., 2003), and wave heights (Lopatoukhin et al., 2000), among others. Future work will extend our analyses to AMS series as has been attempted recently by Read and Vogel (2015ab) for floods.

1.1 Application of general POT model in natural hazards

Here we focus on the POT approach to characterize exceedance events and their frequencies for natural hazards. The GP distribution, a generalization of the exponential distribution, was first introduced as a limiting distribution for modelling high level exceedances by Pickands (1975), and later developed by Hosking and Wallis (1987), who discuss its theory and an application for extreme floods. Davison and Smith (1990) provide techniques for dealing with serially dependent and seasonal data for modeling exceedances above a threshold with the GP distribution. Hosking and Wallis (1987) discuss the fundamental properties of the GP distribution. See Pickands (1975) and Davison and Smith (1990) for further theoretical background on the application of the GP distribution for modeling POT series. In general, the GP distribution arises for variables whose distributions are heavy-tailed, in cases where the lighter-tailed exponential distribution does not provide sufficient robustness (Hosking and Wallis, 1987).

The GP distribution has been widely applied to natural hazards (see Table 1) and in many other fields including financial risk, insurance, and other environmental problems (Smith, 2003) to characterize the magnitude of exceedances above a threshold. Hosking and Wallis (1987), Stedinger et al. (1993) and others show that if the time between the peaks corresponding to a POT follow a Poisson distribution and the POT magnitudes follow an exponential distribution, then the AMS follow a Gumbel distribution. Similarly, Hosking and Wallis (1987) and others show that if time between the peaks of the POT series are Poisson and the POT magnitudes follow a 2-parameter GP (GP2) distribution then the AMS follows a Generalized Extreme Value (GEV) distribution. Table 1 lists many natural hazards problems that apply the Poisson-GP model for their probabilistic analysis. Here we consider a POT that follows the GP model (the exponential distribution is a special case of the GP model when the limit of the shape parameter approaches zero; see Davison and Smith (1990) for details). For example the Gutenberg–Richter model developed for earthquake magnitudes is a 2-parameter exponential model (Gutenberg and Richter, 1954).

NHESSD

3, 6883–6915, 2015

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



For the natural hazards listed in Table 1, the common approach is to assume that the probability of exceedance (p) for a given magnitude event is constant from year to year, i.e. stationary through time. Under the assumption of stationarity in the time series and resulting exceedance probabilities associated with a particular design event, the theoretical relationships between POT and AMS enable straightforward computation of summary and design metrics such as the quantile or percentile of the distribution associated with a particular average return period and/or reliability (Stedinger et al., 1993). When evidence of nonstationarity, or a trend in either or both the frequency or the magnitude of the exceedance events occurring through time is present (Table 1), then p can no longer be assumed as a constant and the traditional Poisson-GP (or other) model must be modified to account for dependence on time and/or some other explanatory co-variate. Not adjusting the probabilistic analysis for a positive trend when it is present can lead to gross over-estimation of the expected return period and reliability of a system, as shown for floods by several recent studies (Salas and Obeysekera, 2014; Read and Vogel, 2015a). Importantly, Vogel et al., (2013) document that without a rigorous probabilistic analysis of trends in natural hazards, we may overlook and fail to prepare for a wide range of societal outcomes which they document may have occurred repeatedly in the past

We conclude from a brief review of the literature that the GP POT model is widely used to model natural hazards, and that it can provide a foundation for a nonstationary analysis. What distinguishes this work from the expanding literature on nonstationary natural hazards, is that we attempt to draw a formal linkage between the magnitude of the natural hazard event and the waiting time or failure time until we experience another hazard in excess of some design event.

1.2 Introduction to hazard function theory and implication for natural hazards

Despite the similarity in name, the application of the theory of hazard function analysis (HFA), also commonly referred to as survival analysis, is practically absent from general literature in the field of natural hazards. HFA is a well-established set of tools useful

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



for conducting a “time-to-event” analysis, or for understanding the distribution of survival (failure) times for a given process (e.g. survival rate of a chronic disease, time until electrical burnout of a device, age-specific mortality rate). This is precisely the concern of those modeling natural hazards that are changing over time. Generally, HFA is comprised of three primary functions: Eq. (1) the hazard function, $h(\tau)$, which is defined as the failure rate, or as the likelihood of experiencing a failure at a particular point in time, τ ; Eq. (2) the survival function, $S_T(t)$, defined as the exceedance probability for the random variable time to failure, T , or in reliability engineering as the cumulative function (cdf) with realization t , of T , $F_T(t)$, where $S_T(t) = 1 - F_T(t)$; and Eq. (3) the cumulative hazard function, $H(\tau)$, interpreted as the total number of failure events over a period of time. Note that in natural hazards work, we normally begin with the POT random variable X , which denotes the magnitude of the natural hazard of interest above some threshold. Thus a connection is needed between the variable of interest X , and the time to failure, T , associated with some design event chosen from the probability distribution of X . This is the focus of our work and distinguishes it from most previous work in HFA as well as most previous studies in natural hazards, because normally HFA only focuses on the random variable T , without any formal connection to the variable of interest X .

Most applications of HFA are interested in computing design metrics based on knowledge of $h(\tau)$, $S_T(t)$ and $H(\tau)$, e.g. the mean time to failure (MTTF), or the reliability of *surviving* a certain amount of time without at least one failure event. In nearly all of the literature on HFA, the process for defining these three functions begins in one of two ways: either by first identifying an appropriate hazard function $h(\tau)$, a possible path if sufficient knowledge (or empirical evidence) of the failure process is known, (e.g. does the probability of failure increase, decrease, or is it constant over time); or, by estimating the survival function $S_T(t)$ by fitting a set of survival time data to a distribution (Klein and Moeschberger, 1997). Neither of these two approaches are suited to natural hazards, because until this paper, there has not been any guidance on how to choose a suitable hazard function $h(\tau)$ for a natural hazard event, and in natural hazards work,

we do not have adequate empirical data on the time to failure to enable fitting a survival function to data.

The theory of hazard function analysis is derived elsewhere and summarized in numerous textbooks (Finkelstein, 2008; Kleinbaum and Klein, 1996; Klein and Moeschberger, 1997), hence we only summarize the fundamental and useful results here including the relationships among the hazard rate function $h(\tau)$, the probability distribution function of the time to failure $f_T(t)$, the cumulative distribution function of the time of failure $F_T(t)$, and its corresponding survival function $S_T(t)$, as well as the cumulative hazard function $H(\tau)$:

$$h(\tau = t) = \frac{f_T(t)}{1 - F_T(t)} \quad (1)$$

$$S_T(t) = 1 - F_T(t) = \exp\left(-\int_0^t h(s)ds\right) \quad (2)$$

$$H(\tau) = -\ln(S_T(t)) = \int_0^t h(s)ds \quad (3)$$

Note that since the variable of interest is the time to failure, we elect to use τ solely for deterministic time and t as the realization of the time to failure corresponding to random variable T . HFA has been applied in the fields of bio-statistics and medicine (Cox, 1972; Pike, 1966) as well as many other disciplines including economics (Kiefer, 1988) and engineering (Finkelstein, 2008; Hillier and Lieberman, 1990). Similar to the field of natural hazards, these fields need summary metrics associated with the time to failure, such as the concepts of reliability and the average return period (known in HFA as $S_T(t)$ and MTTF, respectively). Very little attention has been given to the use of HFA to natural hazards (Katz and Brown, 1992; Lee et al., 1986), with the exception of the recent paper by Read and Vogel (2015b), who apply HFA to nonstationary floods.

Hazard function theory

L. K. Read and R. M. Vogel

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Concepts from hazard function theory were applied to develop dynamic reliability models for characterizing evolving risk of hydrologic and hydraulic failures in conveyance systems in the 1980s (Landsey, 1989; Tung, 1985; Tung and Mays, 1981).

The primary goal of this work is to use the theory of HFA to link the probabilistic properties of T with properties of the probability distribution for a nonstationary natural hazard event X . We begin by explicitly relating the properties of $h(t)$ to the event magnitudes for a general natural hazard, X , assuming the natural hazard POT follows a GP distribution. We then derive $S_T(t)$, $H_T(t)$ and $MTTF = E[T]$ for the case of 2-parameter Generalized Pareto (GP2) model. Since an exponential model is a special case of the GP model, our results also apply to POT series which follow an exponential model. Recall that the exponential distribution is of interest when the AMS is Gumbel, which is applicable for many natural hazards. See Read and Vogel (2015b) for an application of HFA to POT hazards which follow an exponential distribution, in which case very elegant analytical expressions for $S_T(t)$ and $E[T]$ result. Since both the exponential and GP2 models are widely used for representing natural event magnitudes in a POT, the analysis presented here is relevant for a wide range of nonstationary natural hazards. We hope to demonstrate that HFA can be a useful methodology for characterizing nonstationary natural hazards, for communicating natural hazard event likelihood under nonstationarity, and for computing corresponding design metrics that reflect the changing behavior of the both the magnitude and frequency of a natural hazard through time.

2 Hazard function analysis for nonstationary natural hazard magnitudes

To relate the properties of $h(t)$ with the magnitudes of a particular natural hazard (X), we first consider the stationary situation in which the exceedance probability p_o for a particular natural hazard event to exceed some threshold magnitude value is constant through time. In the stationary case, the hazard failure rate, $h(t)$, is constant so that $h(t) = p_o$. For the stationary case, the time to failure, T , always follows a 1-parameter exponential distribution (or the geometric distribution for a discrete random

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



variable); and, computation of the average return period (or MTTF) is easily obtained from probability theory as the expected value of the exponential series $E[T] = 1/\rho$.

If the magnitudes of a natural hazard exhibit an increasing trend through time, this indicates that the exceedance probability associated with a particular design event is changing with time, which we denote as ρ_τ . In such a situation, the expectation $E[T]$ or MTTF, is no longer a sufficient statistic for the distribution of T , and a more complex analysis is needed.

For a nonstationary natural hazard, $h(t)$ is no longer constant and can be computed directly from a probabilistic analysis of the natural hazard of interest. For example, suppose our interest is in the probability distribution of the time to failure for a natural hazard which has been designed to protect against an event with exceedance probability ρ_o at time $t = 0$. Such a design event can be expressed using the quantile function x_p for the natural hazard of interest. We term this design event $x_o(\rho_o)$. Suppose we also have defined a nonstationary cumulative probability distribution for the natural hazard X , which we term $F_x(x, \tau)$. Then it is possible to compute the changing exceedance probability associated with this design event, ρ_τ , using the fact that $\rho_\tau = 1 - F_x(x_o, \tau)$. This forms the fundamental linkage between the probabilistic properties of X and T , because the changing exceedance probabilities associated with the design event are identical to the hazard rate function so that $h(t) = \rho_\tau$. Our use of probabilistic theory to link the properties of X and T , rather than empirical evidence (fitting) to explicitly relate $h(t)$ to the event magnitudes (X) of a natural hazard is the primary difference in our analysis compared with other HFA applications in the literature. We believe this is a fundamental difference which should enable future researchers to formulate even more general conclusions concerning the probabilistic behavior of nonstationary natural hazards.

Hazard function theory

L. K. Read and R. M. Vogel

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



3 Two-parameter generalized pareto (GP2) model for magnitudes of natural hazards

As discussed earlier, the GP distribution is widely used in modeling the magnitudes above a pre-defined threshold for a variety of natural hazards. In this section we present the GP2 stationary and nonstationary models, reviewing literature to support the selection of our nonstationary natural hazard model formulation. We then use HFA theory to derive $h(t)$, $S_T(t)$ and $H(t)$ for the GP2 nonstationary model and discuss the findings and interpretations for each function. Our goal is to show that HFA is ideally suited for modeling the probabilistic behavior of a wide range of natural hazards whose behavior is changing through time.

3.1 Stationary generalized pareto two-parameter model

We assume that the POT natural hazard series follows the GP2 distribution. The definitions of the stationary probability density function (pdf) and cumulative distribution function (cdf) for a GP2 distribution for random variable X , introduced by Hosking and Wallis (1987) are:

$$f_x(x) = \frac{1}{\alpha} \left[1 - \kappa \left(\frac{x}{\alpha} \right) \right]^{\frac{1}{\kappa} - 1} \quad \text{for } \kappa \neq 0 \quad (4)$$

$$F_x(x) = 1 - \left[1 - \kappa \left(\frac{x}{\alpha} \right) \right]^{\frac{1}{\kappa}} \quad \text{for } \kappa \neq 0 \quad (5)$$

where α is the scale parameter and κ is the shape parameter. Note that when $\kappa = 0$ $C_x = 1$, Eqs. (4) and (5) reduce to the exponential distribution with a mean of α ; this form corresponds to a Gumbel distribution for the AMS. The reliability function is simply $\text{Rel}(x) = 1 - F_x(x)$, or the probability of nonexceedance associated with X . The first and



second moments of X are:

$$\mu_x = \frac{\alpha}{1 + \kappa} \quad (6)$$

$$\sigma_x^2 = \frac{\alpha^2}{(1 + \kappa)^2(1 + 2\kappa)} \quad (7)$$

We use the coefficient of variation $C_x = \sigma_x / \mu_x$ to represent the variability of the system.

5 Combining Eqs. (6) and (7) for the GP2 model yields

$$C_x = \frac{1}{\sqrt{2\kappa + 1}} \quad (8)$$

The quantile function for the GP2 distribution for a design event, x_p , associated with exceedance probability, p , is written as

$$x_p = \frac{\alpha}{\kappa} [1 - p^\kappa] \quad (9)$$

10 These equations serve as the foundation for developing a nonstationary GP2 model, discussed in the next section.

3.2 Nonstationary GP2 model

Although we could not locate any previous research combining HFA and nonstationary natural hazards, there are numerous papers that employ a nonstationary GP model for the POT magnitudes of specific natural hazards (shown in Table 1). We briefly review those models to provide context for the trend model adopted here. Literature on nonstationary GP models for specific natural hazards has employed a variety of parameterizations. For example, Roth et al. (2012, 2014) considered models of changes in the POT threshold over time and Strupczewski et al. (2001) modeled the arrival time distribution of the POT with time-varying Poisson parameters. Strupczewski et al. (2001)

20

also modeled the changes in the magnitudes of the POT events over time by modeling changes in the GP model parameters over time as we do here.

Nearly all previous studies that employed nonstationary POT models in the context of natural hazards adopt some form of the Poisson-GP model, and many whose concerns regard increasing magnitudes have been specific to extreme rainfall. With respect to extreme daily rainfall, most have built nonstationary GP2 models assuming a trend in the scale parameter (α), either modeled linearly (Beguería et al., 2011; Sugahara et al., 2009), or log-linearly (Tramblay et al., 2013). Roth et al. (2014) notes that modeling a trend in the threshold level itself indicates a comparable trend in the scale parameter. Tramblay et al. (2013) used time-varying co-variates in the Poisson arrivals (occurrence of seasonal oscillation patterns) and in the magnitudes (monthly air temperature) to model heavy rainfall in Southern France and found improvement from the stationary model. As pointed out by Khaliq et al. (2006) and Tramblay et al. (2013) and others, it is less common to vary the shape parameter (κ) through time due to difficulty with precision and a lack of evidence on model improvement with a time-varying shape parameter.

Studies from other natural hazards are consistent with those in extreme rainfall for nonstationary Poisson-GP model formulations, though with more examples of time-variation in the shape parameter. For example, Strupczewski et al. (2001) used linear and parabolic trends in both α and κ to model flood magnitudes; others have explored linear models in κ for extreme winds (Young et al., 2011), and in sediment yield (Katz et al., 2005). For wave height, several assumed a trend in the location parameter either as linear (Ruggiero et al., 2010) or log-linear (Méndez et al., 2006) formulations. Renard et al. (2006) used a Bayesian approach to explore step-change and linear trend models in α for general purpose with an application to floods. The Bayesian framework was also used by Fawcett and Walshaw (2015) to present a new hybridized method for estimating more precise return levels for nonstationary storm surge and wind speeds.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



3.3 Derivation of nonstationary GP2 hazard model

Our approach is to derive the primary HFA functions $f_T(t)$, $F_T(t)$, $S_T(t)$, $h(t)$ and $H(\tau)$ from the probability distributions of the random variable X , $f_x(x)$ and $F_x(x)$ of the GP2 distribution. To create a nonstationary GP model, we employ an exponential trend model in the scale parameter $\alpha_x(\tau)$, so that:

$$\alpha_x(\tau) = \alpha_o \exp(\beta\tau) \quad (10)$$

The model in Eq. (10) is equivalent to a model of the conditional mean of the natural hazard X and has been found to provide an excellent representation of changes in the mean annual flood for flood series at thousands of rivers in the United States (Vogel et al., 2011) and in the UK (Prosdocimi et al., 2014). This model is described by Khaliq et al. (2006) and was also used in Trambly et al., (2013) for extreme rainfall.

We assume that the shape parameter κ is constant through time as consistent with previous studies discussed earlier. This assumption implies that C_x is fixed (Eq. 10), or that the variability of the system is assumed constant over the time period, defined at $\tau = 0$, and thus the standard deviation changes in step with the mean (parameterized by α). Again, there is reasonable evidence that this is the case for floods (see Vogel et al., 2011; and Prosdocimi et al., 2014).

Following Vogel et al. (2011), Prosdocimi et al. (2014), and Read and Vogel (2015), we replace the trend coefficient β in Eq. (10) with the more physically meaningful magnification factor M to represent the ratio of the magnitude of the natural hazard quantile at time period $(\tau + \Delta\tau)$ to the natural hazard quantile at time T . For the model developed here, the magnification factor, M , can be derived by combining the GP2 quantile function in Eq. (9) and the trend model in Eq. (10), inserting into the expression below:

$$M = \frac{x_p(\tau + \Delta\tau)}{x_p(\tau)} = \frac{\frac{1}{\lambda} \exp[\beta(\tau + \Delta\tau) \ln(p_t)]}{\frac{1}{\lambda} \exp[\beta t \ln(p_t)]} = \exp[\beta\Delta\tau] \quad (11)$$

Thus M reflects the change in the magnitude of the natural hazards over time. So for example, a magnification factor of two corresponding to particular time interval $\Delta\tau$, indicates that the natural hazard has increased twofold over that time period, for all values of p .

We consider a magnification factor M corresponding to ten time periods, ($\Delta\tau = 10$), since that is what others have done and because it provides a physically meaningful interpretation of the degree of change in the design events over time. For example, if the time periods were equal to a year as in an AMS series, this would correspond to a decadal magnification factor. In this section we derive $h(t)$, $S_T(t)$, $H(\tau)$, and $f_T(t)$ for the nonstationary GP2 model. First recall that the hazard function is equal to the exceedance probability through time for a natural hazard event series, $h(t) = p_\tau(\tau)$. Using the relationships above we can now derive an expression for $h(t)$ dependent only on those fundamental parameters M , p_o , and C_x , describing the behavior of the natural hazard X and our design exceedance probability to protect against future hazards.

Consider that the design event in Eq. (9) is fixed and set at time $\tau = 0$, and denoted as x_o , and associated with p_o and α_o ; we can use the fact that $p_\tau = 1 - F_x(x)$ in Eq. (5), combined with $x_p = x_o$ from Eq. (9) and the definition of M in Eq. (10) which yields,

$$H(\tau) = p_\tau = \left[\frac{1 - (1 - p^{\frac{1-C_x^2}{2C_x^2}})}{\exp(M \cdot \tau)} \right]^{\frac{2C_x^2}{1-C_x^2}} \quad (12)$$

where κ is replaced with C_x after rearranging Eq. (8). Combining the theoretical relationships in Eqs. (1–3) with Eq. (12), leads to expressions for $S_T(t)$, $H(\tau)$, and $f_T(t)$

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



which are solved easily by numeric integration.

$$S_T(t) = \text{Reliability}_T(t) = \exp \left(- \int_0^t \left[\frac{1 - (1 - \rho \frac{1 - c_x^2}{2c_x^2})}{\exp(M \cdot s)} \right]^{\frac{2c_x^2}{1 - c_x^2}} ds \right) \quad (13)$$

$$H(\tau) = \int_0^\tau \left[\frac{1 - (1 - \rho \frac{1 - c_x^2}{2c_x^2})}{\exp(M \cdot s)} \right]^{\frac{2c_x^2}{1 - c_x^2}} ds \quad (14)$$

Finally, the pdf of the time to failure distribution for the GP2 nonstationary model is

$$f_T(t) = b \cdot \exp \left(- \int_0^t b ds \right) \quad (15)$$

where $b = \left[\frac{1 - (1 - \rho \frac{1 - c_x^2}{2c_x^2})}{\exp(M \cdot s)} \right]^{\frac{2c_x^2}{1 - c_x^2}}$

For the case of the 1-parameter exponential, these functions simplify to $h(t) = \rho_0^{M-t/\Delta\tau}$,

$S_T(t) = \exp \left[- \int_0^t \rho_0^{M-s/\Delta\tau} ds \right]$, $H(\tau) = \int_0^\tau \rho_0^{M-s/\Delta\tau} ds$, and $F_T(t) = \frac{d}{dt} \left[\exp \left(\int_0^t \rho_0^{M-s/\Delta\tau} ds \right) \right]$ (see Read and Vogel, 2015b for a complete derivation).

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



3.4 Investigation of impacts of nonstationarity on probabilistic analysis of natural hazards using HFA

In this section we explore how HFA can characterize the behavior of nonstationary natural hazards whose PDS magnitudes follow a GP2 model. Our results are exact (within the limitations of numerical integration) because they result from the derived analytical equations in Eqs. (12–15) for the HFA functions $f_T(t)$, $F_T(t)$, and $S_T(t)$, $h(t)$ and $H(\tau)$ corresponding to a natural hazard X which follows a GP2 model. With no loss in generality, we assume the mean of the GP2 natural hazard of unity. We investigate the impact of small and large trends (corresponding to magnification factors, M , ranging from 1 to 1.25 with $\Delta\tau = 10$) for a range of physical systems characterized by a range in variability corresponding to a range in the coefficient of variation of X , C_x , from 0.5 to 1.5 (corresponding to a range in the GP2 shape κ between -0.28 to 1.5) for three event sizes ($p_o = 0.01, 0.002, 0.001$).

Figure 1 presents the hazard function $h(t)$, examining how it is influenced by the variability of the natural hazard and the magnitude of the trend for a particular design event $p_o = 0.002$ (500 year event): (a) increasing variability, $C_x = 0.75, 1.25, 1.5$ for a set $M = 1.1$, and (b) increasing trend values, $M = 1.1, 1.25, 1.5$ for a set $C_x = 0.75$. Note that in the stationary case, $p_t = p_o = 0.002$ results in a constant horizontal line, and as the magnitude of the trends increases (as M increases), the hazard rate $h(t)$ tends toward unity earlier in time. Even from this initial relatively simplistic investigation we find that the hazard functions exhibit complex shapes, with some exhibiting inflection points, and other without an inflection. This is extremely important, because most applications of HFA assumes a particular hazard function without deriving their generalized shapes in advance as we do here in Fig. 1. Another important point is that the variability of the hazard magnitudes (characterized by the shape of the pdf of X) impacts the rate at which $h(t)$ increases, so that less variable hazards tend to have higher hazard rates than more variable hazards. This point is perhaps initially counter-intuitive, our interpretation is that if a hazard is more consistent (with less variability),

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



a larger trend ensures exceedance more so than a less consistent system that has a wider range of small and large events. This finding is relevant for planning purposes as it indicates which systems may be greater impacted by nonstationarity.

Typically in HFA work, the survival function $S_T(t)$ is presented as a primary figure in understanding risk of failure and likelihood of experiencing an exceedance event within a given period of time. Since $S_T(t)$ also represents the relationship between system reliability and time and because many fields employ the concept of reliability to protect against natural hazards, the $S_T(t)$ function is also very relevant for planning purposes in this context (see Read and Vogel, 2015a, b for further discussions relating to flood management and design). Figure 2 illustrates $S_T(t)$ for a $p_o = 0.002$ event with a fixed $C_x = 0.75$ representing a slightly lower variable system, and a range of increasing trends ($M = 1.02, 1.1, 1.25$) compared with stationary conditions ($M = 1$). Clearly even a small trend significantly reduces the system reliability compared with our expectations under stationary conditions. For example, the reliability of a structure designed to protect against a 500 year event under stationary conditions after 50 time periods is quite high (Rel = 0.90), however, as M increases, the reliability decreases significantly, approaching zero for $M = 1.1$ and 1.25 at $\tau = 50$. This suggests that if one was designing infrastructure to withstand a particularly large magnitude event over a planning period, under nonstationary conditions, the design would need to be significantly larger, and it may not even be possible to design a structure to achieve the same reliability as expected under stationary conditions.

A unique tool offered by HFA that can provide advancements in planning for non-stationary natural hazards is the cumulative hazard function $H(\tau)$ which represents the total hazard over a given amount of time (Wienke, 2010). For example if $p_o = 0.002$, as expected under stationary conditions $H(\tau) = 1$ for $\tau = 500$, or we will experience, on average, one exceedance event every 500 years. However, if a trend with a magnification factor $M = 1.1$ ($\Delta\tau = 10$) is introduced in the same system, the time it takes for $H(\tau) = 1$ is about 36 time periods, or another view, $H(\tau) = 333$ events for $\tau = 500$. Figure 3 illustrates these interpretations for two exceedance event sizes, fixed $C_x = 0.75$:

Hazard function theory

L. K. Read and R. M. Vogel

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Hazard function theory

L. K. Read and
R. M. Vogel

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



(a) $p_o = 0.002$, showing the number of time periods until $H(\tau) = 1$; and (b) $p_o = 0.001$, showing the total number of events over time. In Fig. 3a, we note that the stationary $M = 1$ line corresponds with the $H(\tau) = 1$ for $\tau = 500$ as expected, and that as M increases, the time until an exceedance event occurs dramatically decreases (note the log x axis scale). Figure 3b depicts a similar story, but illustrates an alternate interpretation: the total number of exceedance events over a time period for the rarer $p_o = 0.001$ event, where $H(\tau)$ ranges from 1 for $\tau = 1000$ as expected under stationary conditions, to $H(\tau) = 10+$ events in under 50 time periods with a large M .

When one wishes to communicate the risk of failure and event likelihood, the cumulative hazard function is a useful metric for describing total risk (or reliability) over a certain planning horizon. While our analysis assumes that the trend would increase over the entire time period, perhaps a “worst case” scenario, our results show that in the presence of an increasing trend in the POT of a natural hazard series, we may experience far more exceedance events than expected under stationary conditions. Ignoring such trends may result in significant increased damages and losses from under-design of infrastructure or insufficient planning in populated areas.

After computing $S_T(t)$ and $h(t)$ we can easily use Eq. (1) to determine the pdf of the time to failure distribution for the nonstationary GP2 model by Eq. (15). Since we are interested in the behavior of $f_T(t)$ due to trends on a range of physical systems and for extreme events, we plot $f_T(t)$ for a fixed $C_x = 0.75$ in Fig. 4, for a range of increasing trends ($M = 1, 1.02, 1.1, 1.25$) and three event sizes ($p_o =$ (a) 0.01, (b) 0.002, (c) 0.001). We note that the shape of $f_T(t)$ evolves from the expected exponential curve under stationary conditions, to a more symmetric or normally distributed shape as M increases. These results complement those by Read and Vogel, (2015a, b) who show similar behavior for a nonstationary 2-parameter lognormal model of an AMS series of floods. Interestingly, Fig. 4b and c shows little difference in timing of the peak, especially for larger M values ($M > 1.1$), suggesting that for systems experiencing large increasing trends, rare events ($p_o = 0.002$) and extremely rare events ($p_o = 0.001$) may exhibit similar probabilistic behavior.

Hazard function theory

L. K. Read and
R. M. Vogel

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Similarly, in Fig. 5 we fix the trend at $M = 1.05$ and explore the behavior of $f_T(t)$ over a realistic range of C_x values (0.5, 0.75, 1.5), for the same three event sizes ($\rho_o =$ (a) 0.01, (b) 0.002, (c) 0.001). As consistent with Fig. 1 showing $h(t)$ for various C_x values, the shape of $f_T(t)$ in less variable systems (lower C_x) is more impacted by a trend than a more variable system, as indicated by the sharp peaks and shift in timing of the peaks (Fig. 5a–c). We again highlight the similar shape and timing of peaks in the $\rho_o = 0.002$ and $\rho_o = 0.001$ events, an unanticipated yet consistent result with Fig. 4.

Our investigation of the behavior of $f_T(t)$ for the nonstationary GP2 model indicates that the shape and timing of the distribution changes with both the magnitude of the trend and variability of natural hazard. We also note that $f_T(t)$ exhibits complex patterns under nonstationary conditions, e.g. $f_T(t)$ is less impacted in shape/timing by M for smaller events (ρ_o), and that the presence of a trend leads to a range of shapes (approaching normal for large positive M) of the time to failure distribution. This more complicated behavior implies that under the premise of nonstationarity, we can no longer assume the failure time distribution is exponential in shape and that the MTTF is equal to $1/\rho$. In fact, the mean of the distribution of T is no longer a sufficient statistic as is the case under stationary conditions. We are the first to document such changes in the context of natural hazards for the GP2 distribution and anticipate that others will continue to do so for specific events that exhibit nonstationarity. Using the derivations of $h(t)$, $S_T(t)$, $H(\tau)$, and $f_T(t)$ that we have provided here, one can use knowledge of the system (M , C_x) and existing design metrics (ρ_o and reliability standards) in combination with HFA to better understand and characterize natural hazards as they change through time.

4 Summary and conclusions

We have presented a general introduction to the probabilistic analysis of nonstationary natural hazards using the well-developed field of hazard function analysis (HFA). We cited numerous sources of evidence which suggests that the magnitudes of a variety

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



of natural hazards are increasing, thus our study should be of considerable interest in the coming years. To the authors' knowledge, the analysis and discussion presented here provides the first formal probabilistic analysis of the link between a natural hazard with design event X , and the corresponding properties of the probability distribution of the time to failure T associated with a structure designed to protect against that design event. Through the lens of HFA, we have investigated the impacts of a positive trend in natural hazard event magnitudes for a random variable X that follows a GP2 distribution on the likelihood of future hazards under nonstationary conditions. We introduce a complete probabilistic analysis useful for re-evaluating event likelihood, reliability (survival) and cumulative hazard under nonstationary conditions which should prove useful for a wide range of natural hazards. Our results are applicable for understanding the probabilistic behavior of the time to occurrence of natural hazards subject to increasing trends.

By explicitly linking properties of the time to failure T with the exceedance probability ρ of a natural hazard (X), we have derived the primary hazard analysis equations: the hazard function $h(t)$, the survival (reliability) function $S_T(t)$, the cumulative hazard function $H(\tau)$, and the pdf of the time to failure distribution $f_T(t)$ corresponding to a POT series of natural hazards which follows the GP2 distribution. We parameterize this GP2 model such that it only depends on the design exceedance probability at time $\tau = 0$, ρ_0 , the known system variability C_x , and the magnification factor M , and use this to explore the impact of positive trends on the reliability, or survival $S_T(t)$ until an exceedance event. Findings of this investigation suggest that under nonstationary conditions, medium and large events could occur with much greater frequency than under stationary conditions (no trend). We find that the total number of hazards as characterized by $H(\tau)$ within a given planning period may substantially increase in the presence of a positive trend in the POT magnitudes of a natural hazard. Perhaps most importantly, under nonstationary conditions, the distribution of the time to the natural hazard is no longer exponentially distributed, and instead takes on a distribution with extremely complex shapes, depending on the variability of the hazard and the magnitude of the

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



trend. As trend magnitudes increase for a range of event sizes (p_o), the shape of the distribution of the survival time approaches normality in shape and exhibits a sharp peak with a heavy upper-tail. We also find that variability impacts the shape and timing of this peak in $f_T(t)$, such that less variable systems (lower C_x) are more affected by larger M values, i.e. produce a more pronounced peak and a greater shift in timing.

The implications of these findings for planning and design for nonstationary natural hazards are significant. Given a historic (or future) increasing trend in the magnitudes of some hazard, we should prepare to experience exceedance events much more frequently. For fields that use reliability as a primary design standard, our analysis suggests that the presence of a positive trend corresponds to a lower system reliability for a given design event (x_o) than under stationary conditions. Through exploration of $S_T(t)$ for various trend factors, we also note that determining the reliability of a system over time is more complicated given uncertainty in the magnitude of the trend and how it will manifest through time. In either case, continuing to assume stationary conditions when computing system reliability for design purposes, when a positive trend in the POT magnitudes has been observed historically, may pose a significant risk to the populations and infrastructure in that region. Thus we recommend that design practices should be reviewed and adapted for cases where nonstationary behavior of natural hazards is evident in order to avoid under-design (also see Vogel et al., 2013).

Overall, we have shown that HFA provides a set of tools for understanding the probabilistic behavior of nonstationary natural hazards for application to a wide range of natural phenomena. Using the well-studied theory of HFA, engineers and planners can use language from HFA – hazard rate, survival, cumulative hazard – to relate to risk and reliability under nonstationarity for natural hazards, advancing risk communication in this field. We intend for this analysis to inform future work on modeling nonstationary natural hazards with HFA, for example by developing other models that may include co-variates, extensions to AMS series, and also exploring the impact of decreasing trends. We expect that additional research on this topic will contribute to the emerging conversation on planning for nonstationary natural hazards and shed light

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



on innovative methods to determine best practices for infrastructure design. Results of this work further support the need for a risk-based decision analysis framework for selecting a design event under nonstationarity (Rosner et al., 2014). Such a framework can provide guidance in choosing infrastructure that minimizes the risk of under-design (protection) and over-design (excess spending) through probabilistic decision trees.

Acknowledgements. The authors are grateful to the US Army Corps of Engineers (USACE) Institute of Water Resources for funding this work through support in part by an appointment to the USACE Research Participation Program administered by the Oak Ridge Institute for Science and Education (ORISE) through an interagency agreement between the US Department of Energy (DOE) and the USACE. ORISE is managed by ORAU under DOE contract number DE-AC05-06OR23100. All opinions expressed in this paper are the author's and do not necessarily reflect the policies and views of USACE, DOE or ORAU/ORISE. All data are freely available and analyzed with the R statistical software package version 3.1.1.

References

- Di Baldassarre, G., Montanari, A., Lins, H., Koutsoyiannis, D., Brandimarte, L., and Blöschl, G.: Flood fatalities in Africa: from diagnosis to mitigation, *Geophys. Res. Lett.*, 37, L22402, doi:10.1029/2010GL045467, 2010.
- Bayazit, M.: Nonstationarity of hydrological records and recent trends in trend analysis: a state-of-the-art review, *Environmental Processes*, 2, 527–542, 2015.
- Beguería, S., Angulo-Martínez, M., Vicente-Serrano, S. M., López-Moreno, J. I., and El-Kenawy, A.: Assessing trends in extreme precipitation events intensity and magnitude using non-stationary peaks-over-threshold analysis: a case study in northeast Spain from 1930 to 2006, *Int. J. Climatol.*, 31, 2102–2114, doi:10.1002/joc.2218, 2011.
- Beirlant, J., Goegebeur, Y., Segers, J., and Teugels, J.: *Statistics of Extremes: Theory and Applications*, John Wiley and Sons, West Sussex, United Kingdom, 2006.
- Blaikie, P., Cannon, T., Davis, I., and Wisner, B.: *At Risk: Natural Hazards, People's Vulnerability and Disasters*, Routledge, London, United Kingdom, 2014.
- Bonnin, G. M., Maitaria, K., and Yekta, M.: Trends in rainfall exceedances in the observed record in selected areas of the United States, *J. Am. Water Resour. As.*, 47, 1173–1182, 2011.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



- Coles, S., Bawa, J., Trenner, L., and Dorazio, P.: An Introduction to Statistical Modeling of Extreme Values, Springer, Bristol, United Kingdom, 2001.
- Cox, D. R.: Regression models and life-tables, *J. Roy. Stat. Soc. B Met.*, 34, 187–220, 1972.
- Davison, A. C. and Smith, R. L.: Models for exceedances over high thresholds, *J. Roy. Stat. Soc. B Met.*, 52, 393–442, 1990.
- de Winter, R. C., Sterl, A., and Ruessink, B. G.: Wind extremes in the North Sea Basin under climate change: an ensemble study of 12 CMIP5 GCMs, *J. Geophys. Res.-Atmos.*, 118, 1601–1612, doi:10.1002/jgrd.50147, 2013.
- Ellsworth, W. L.: Injection-induced earthquakes, *Science*, 341, 1225942, doi:10.1126/science.1225942, 2013.
- Fawcett, L. and Walshaw, D.: Sea-surge and wind speed extremes: optimal estimation strategies for planners and engineers, *Stoch. Env. Res. Risk A.*, 29, 1–18, doi:10.1007/s00477-015-1132-3, 2015.
- Finkelstein, M.: Failure Rate Modelling for Reliability and Risk, Springer Science and Business Media, Bloemfontein, South Africa, 2008.
- Guha-Sapir, D., Hargitt, D., and Hoyois, P.: Thirty Years of Natural Disasters 1974–2003: The Numbers, Presses univ. de Louvain, Louvain-la-Neuve, Belgium, 2004.
- Gumbel, E. J.: Statistics of Extremes, Courier Corporation, Mineola, New York, 2012.
- Gutenberg, B. and Richter, C.: Seismicity of the Earth, Princeton, New Jersey, 1954.
- Hillier, F. S. and Lieberman, G. J.: Introduction to Stochastic Models in Operations Research, McGraw-Hill Companies, New York, 1990.
- Holmes, T. P., Hugget Jr., R. J., and Westerling, A. L.: Statistical analysis of large wildfires, in: The Economics of Forest Disturbances, edited by: Holmes, T. P., Prestemon, J. P., and Abt, K. L., Springer Netherlands, Medford, Massachusetts, 59–77, 2008.
- Hosking, J. R. M. and Wallis, J. R.: Parameter and quantile estimation for the generalized pareto distribution, *Technometrics*, 29, 339–349, doi:10.1080/00401706.1987.10488243, 1987.
- Katz, R. W. and Brown, B. G.: Extreme events in a changing climate: variability is more important than averages, *Climate Change*, 21, 289–302, doi:10.1007/BF00139728, 1992.
- Katz, R. W., Brush, G. S., and Parlange, M. B.: Statistics of extremes: modeling ecological disturbances, *Ecology*, 86, 1124–1134, doi:10.1890/04-0606, 2005.
- Keellings, D. and Waylen, P.: Increased risk of heat waves in Florida: characterizing changes in bivariate heat wave risk using extreme value analysis, *Appl. Geogr.*, 46, 90–97, doi:10.1016/j.apgeog.2013.11.008, 2014.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



- Khaliq, M. N., Ouarda, T. B. M. J., Ondo, J.-C., Gachon, P., and Bobée, B.: Frequency analysis of a sequence of dependent and/or non-stationary hydro-meteorological observations: a review, *J. Hydrol.*, 329, 534–552, doi:10.1016/j.jhydrol.2006.03.004, 2006.
- Kiefer, N. M.: Economic duration data and hazard functions, *J. Econ. Lit.*, 26, 646–679, 1988.
- 5 Kim, Y., Jang, S.-C., and Lim, T.-J.: Hazard analysis of typhoon-related external events using extreme value theory, *Nucl. Eng. Technol.*, 47, 59–65, doi:10.1016/j.net.2014.08.001, 2015.
- Klein, J. P. and Moeschberger, M. L.: *Survival analysis: techniques for censored and truncated data*, Stat. Bio. Health, 2nd ed., Springer, New York, 2003.
- Kleinbaum, D. G. and Klein, M.: *Survival Analysis*, Springer, New York, 1996.
- 10 Lansey, K. E., Duan, N., Mays, L. W., and Tung, Y. K.: Water distribution system design under uncertainties. *J. Water Res. Pl.-ASCE*, 115, 630–645, doi:10.1061/(ASCE)0733-9496(1989)115:5(630), 1989.
- Liu, Y., Stanturf, J., and Goodrick, S.: Trends in global wildfire potential in a changing climate, *Forest Ecol. Manag.*, 259, 685–697, doi:10.1016/j.foreco.2009.09.002, 2010.
- 15 Lopatoukhin, L. J., Rozhkov, V. A., Ryabinin, V. E., Swail, V. R., Boukhanovsky, A. V., and Degtyarev, A. B.: *Estimation of Extreme Wind Wave Heights*, Secretariat of the World Meteorological Organization, 1–73, Geneva, 2000.
- Madsen, H., Rasmussen, P. F., and Rosbjerg, D.: Comparison of annual maximum series and partial duration series methods for modeling extreme hydrologic events: 1, at-site modeling, *Water Resour. Res.*, 33, 747–757, doi:10.1029/96WR03848, 1997.
- 20 Méndez, F. J., Menéndez, M., Luceño, A., and Losada, I. J.: Estimation of the long-term variability of extreme significant wave height using a time-dependent Peak Over Threshold (POT) model, *J. Geophys. Res.-Oceans*, 111, C07024, doi:10.1029/2005JC003344, 2006.
- Obeysekera, J. and Park, J.: Scenario-based projection of extreme sea levels, *J. Coastal Res.*, 29, 1–7, doi:10.2112/JCOASTRES-D-12-00127.1, 2012.
- 25 Palutikof, J. P., Brabson, B. B., Lister, D. H., and Adcock, S. T.: A review of methods to calculate extreme wind speeds, *Meteorol. Appl.*, 6, 119–132, doi:10.1017/S1350482799001103, 1999.
- Pike, M. C.: A method of analysis of a certain class of experiments in carcinogenesis, *Biometrics*, 22, 142–161, doi:10.2307/2528221, 1966.
- 30 Pisarenko, V. F. and Sornette, D.: Characterization of the frequency of extreme earthquake events by the generalized pareto distribution, *Pure Appl. Geophys.*, 160, 2343–2364, doi:10.1007/s00024-003-2397-x, 2003.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



- Prosdocimi, I., Kjeldsen, T. R., and Svensson, C.: Non-stationarity in annual and seasonal series of peak flow and precipitation in the UK, *Nat. Hazards Earth Syst. Sci.*, 14, 1125–1144, doi:10.5194/nhess-14-1125-2014, 2014.
- Read, L. K. and Vogel, R. M.: Reliability, return periods, and risk under nonstationarity, *Water Resour. Res.*, 51, 6381–6398, doi:10.1002/2015WR017089, 2015a.
- Read, L. K. and Vogel, R. M.: Hazard function analysis for flood planning under nonstationarity, *Water Resour. Res.*, under review, 2015b.
- Renard, B., Lang, M., and Bois, P.: Statistical analysis of extreme events in a non-stationary context via a Bayesian framework: case study with peak-over-threshold data, *Stoch. Env. Res. Risk A.*, 21, 97–112, doi:10.1007/s00477-006-0047-4, 2006.
- Rosner, A., Vogel, R. M., and Kirshen, P. H.: A risk-based approach to flood management decisions in a nonstationary world, *Water Resour. Res.*, 50, doi:10.1002/2013WR014561, 2014.
- Roth, M., Buishand, T. A., Jongbloed, G., Klein Tank, A. M. G., and van Zanten, J. H.: Projections of precipitation extremes based on a regional, non-stationary peaks-over-threshold approach: a case study for the Netherlands and north-western Germany, *Weather Clim. Extrem.*, 4, 1–10, doi:10.1016/j.wace.2014.01.001, 2014.
- Ruggiero, P., Komar, P. D., and Allan, J. C.: Increasing wave heights and extreme value projections: the wave climate of the US Pacific Northwest, *Coast. Eng.*, 57, 539–552, doi:10.1016/j.coastaleng.2009.12.005, 2010.
- Salas, J. D. and Obeysekera, J.: Revisiting the concepts of return period and risk for nonstationary hydrologic extreme events, *J. Hydrol. Eng.*, 19, 554–568, doi:10.1061/(ASCE)HE.1943-5584.0000820, 2014.
- Schoenberg, F. P., Peng, R., and Woods, J.: On the distribution of wildfire sizes, *Environmetrics*, 14, 583–592, doi:10.1002/env.605, 2003.
- Smith, R. L.: *Extreme Values in Finance, Telecommunications, and the Environment*, CRC Press, Boca Raton, FL, 2003.
- Stedinger, J. R., Vogel, R. M., and Foufoula-Georgiou, E.: Frequency analysis of extreme events, Chapter 18 in *Handbook of Hydrology*, edited by: Maidment, D. R., McGraw-Hill, New York, 1993.
- Strupczewski, W. G., Singh, V. P., and Feluch, W.: Non-stationary approach to at-site flood frequency modelling I, maximum likelihood estimation, *J. Hydrol.*, 248, 123–142, doi:10.1016/S0022-1694(01)00397-3, 2001.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Sugahara, S., da Rocha, R. P., and Silveira, R.: Non-stationary frequency analysis of extreme daily rainfall in Sao Paulo, Brazil, *Int. J. Climatol.*, 29, 1339–1349, doi:10.1002/joc.1760, 2009.

Thompson, E. M., Baise, L. G., and Vogel, R. M.: A global index earthquake approach to probabilistic assessment of extremes, *J. Geophys. Res. Sol. Ea.*, 112, B06314, doi:10.1029/2006JB004543, 2007.

Todorovic, P.: Stochastic models of floods, *Water Resour. Res.*, 14, 345–356, doi:10.1029/WR014i002p00345, 1978.

Tramblay, Y., Neppel, L., Carreau, J., and Najib, K.: Non-stationary frequency analysis of heavy rainfall events in southern France, *Hydrol. Sci. J.*, 58, 280–294, doi:10.1080/02626667.2012.754988, 2013.

Tung, Y.-K.: Models for evaluating flow conveyance reliability of hydraulic structures, *Water Resour. Res.*, 21, 1463–1468, doi:10.1029/WR021i010p01463, 1985.

Tung, Y.-K., and Mays, L. W.: Risk models for flood levee design, *Water Resour. Res.*, 17, 833–841, doi:10.1029/WR017i004p00833, 1981.

Utsu, T.: Representation and analysis of the earthquake size distribution: a historical review and some new approaches, in: *Seismicity Patterns, their Statistical Significance and Physical Meaning*, Birkhäuser Basel, New York, 509–535, 1999.

Villarini, G., Smith, J. A., Serinaldi, F., Ntelekos, A. A., and Schwarz, U.: Analyses of extreme flooding in Austria over the period 1951–2006, *Int. J. Climatol.*, 32, 1178–1192, doi:10.1002/joc.2331, 2012.

Vogel, R. M., Yaindl, C., and Walter, M.: Nonstationarity: flood magnification and recurrence reduction factors in the United States¹, *J. Am. Water Resour. As.*, 47, 464–474, doi:10.1111/j.1752-1688.2011.00541.x, 2011.

Vogel, R. M., Rosner, A., and Kirshen, P. H.: Brief Communication: Likelihood of societal preparedness for global change: trend detection, *Nat. Hazards Earth Syst. Sci.*, 13, 1773–1778, doi:10.5194/nhess-13-1773-2013, 2013.

Waylen, P. R.: Statistical analysis of freezing temperatures in Central and Southern Florida, *J. Climatol.*, 8, 607–628, 1988.

Wienke, A.: *Frailty Models in Survival Analysis*, CRC Press, Boca Raton, 2010.

Young, I. R., Zieger, S., and Babanin, A. V.: Global trends in wind speed and wave height, *Science*, 332, 451–455, doi:10.1126/science.1197219, 2011.

Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Table 1. Summary of natural hazards employing Poisson-GP model.

Natural Hazard	References	Evidence of nonstationarity in process	Nonstationary POT model formulated?
Floods	Todorovic (1978); Madsen et al. (1997)	Bayazit (2015) for a review	Yes – Strupczewski et al. (2001); Villarini et al. (2012)
Earthquakes	Gutenberg and Richter (1954); Utsu (1999)	Ellsworth (2013) for a review	No
Extreme rainfall	Sugahara et al. (2009); Bonnin et al. (2011)	Beguieria et al. (2011); Trambly et al. (2013); Roth et al. (2014); Sugahara et al. (2009)	Yes, in all references
Wildfires Extreme wind speeds	Holmes et al. (2008) Palutikof et al. (1999) for review; Jagger and Elsner (2006)	Liu et al. (2010) Young et al. (2011); Pryor and Barthelmie (2010) for review	No Young et al. (2011)
Wave height (proxy for storm surge)	Davison and Smith (1990)	Mendez et al. (2006); Young et al. (2011); Ruggerio et al. (2010)	Yes – in Mendez et al. (2006); and Ruggerio et al. (2010)
Daily max and min temperatures	Waylen (1988)	Keellings and Waylen (2014)	Yes, but not derived in text
Ecological extremes	Katz et al. (2005)	Katz et al. (2005) (sediment yield)	Yes

Hazard function theory

L. K. Read and R. M. Vogel

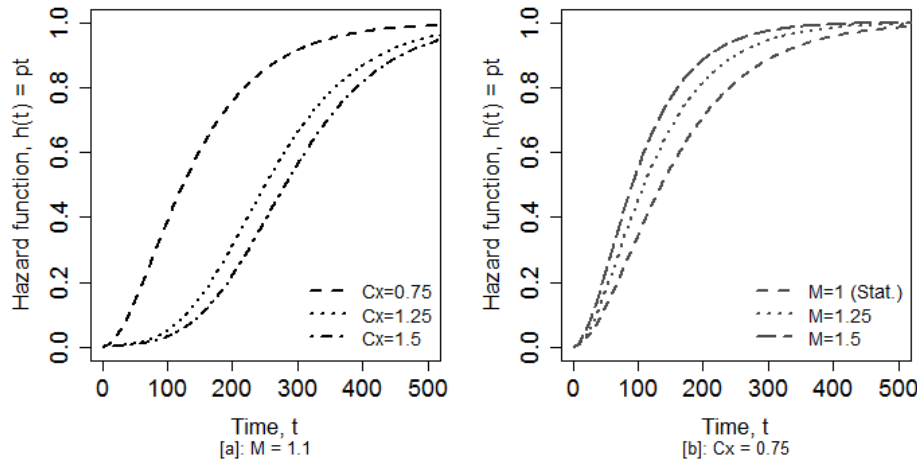


Figure 1. Hazard function $h(t)$ for the nonstationary GP2 model, $p_0 = 0.002$ for **(a)** a range of variability ($C_x = 0.75, 1.25, 1.5$), given $M = 1.1$; **(b)** a range of trend values ($M = 1.1, 1.25, 1.5$), given $C_x = 0.75$.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

⏪

⏩

◀

▶

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)

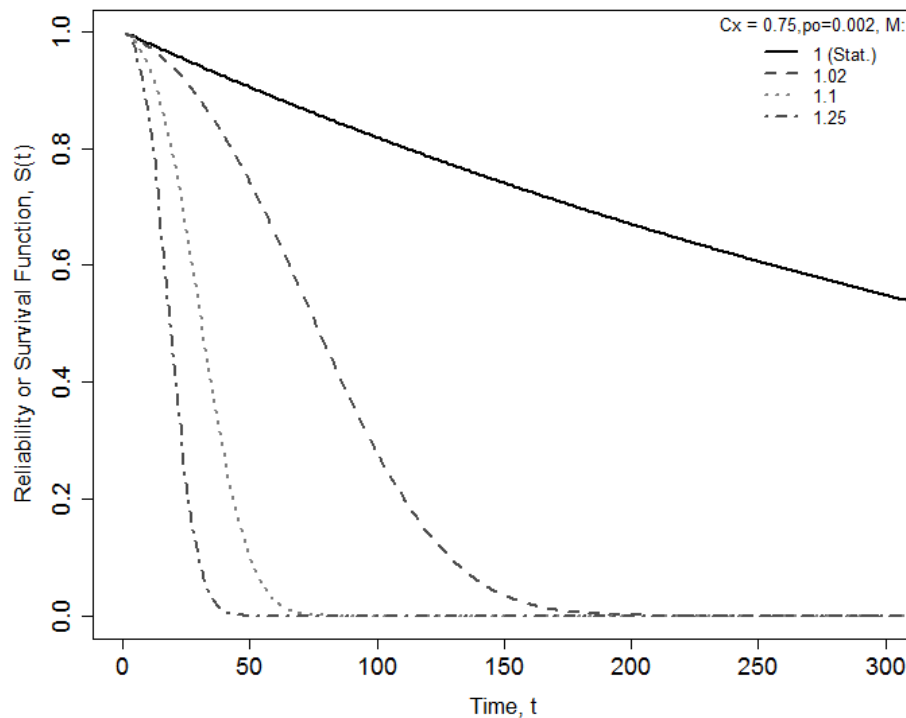


Figure 2. Reliability or Survival function $S_T(t)$ for the nonstationary GP2 model, $p_0 = 0.002$ and $C_x = 0.75$, for a range of trend values ($M = 1, 1.02, 1.1, 1.25$).

Hazard function theory

L. K. Read and
R. M. Vogel

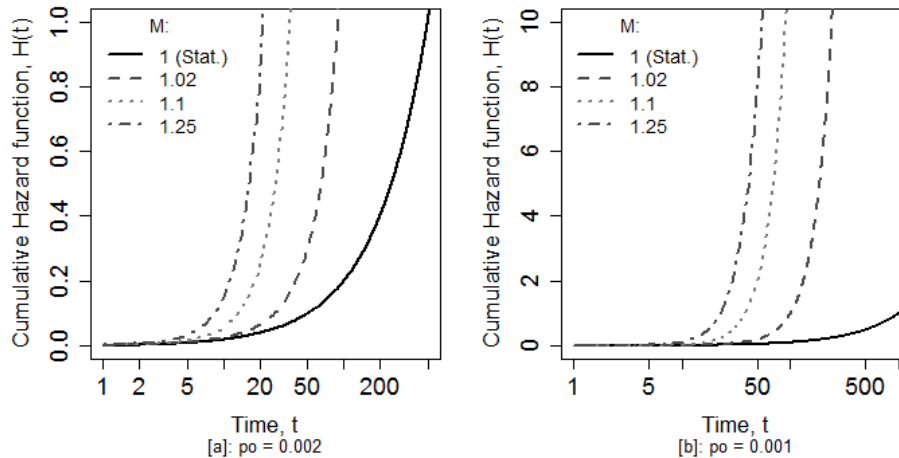


Figure 3. Cumulative hazard function $H_T(t)$ for the nonstationary GP2 model, with a fixed $C_x = 0.75$ for a range of trend values ($M = 1, 1.02, 1.1, 1.25$); panels show two different size exceedance events **(a)** $p_0 = 0.002$ and **(b)** $p_0 = 0.001$.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Hazard function theory

L. K. Read and
R. M. Vogel

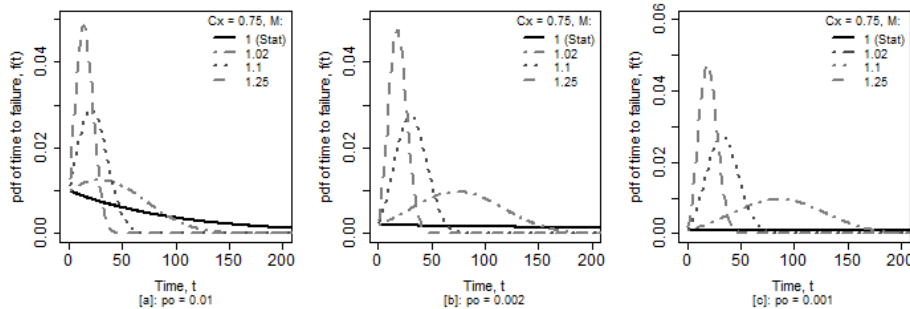


Figure 4. Probability density function (pdf) of the time to failure distribution for the nonstationary GP2 model, with a fixed $C_x = 0.75$ for a range of trend values ($M = 1, 1.02, 1.1, 1.25$); panels show three exceedance event sizes increasing in extremity **(a)** $p_o = 0.01$, **(b)** $p_o = 0.002$ and **(c)** $p_o = 0.001$.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Hazard function theory

L. K. Read and
R. M. Vogel

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)

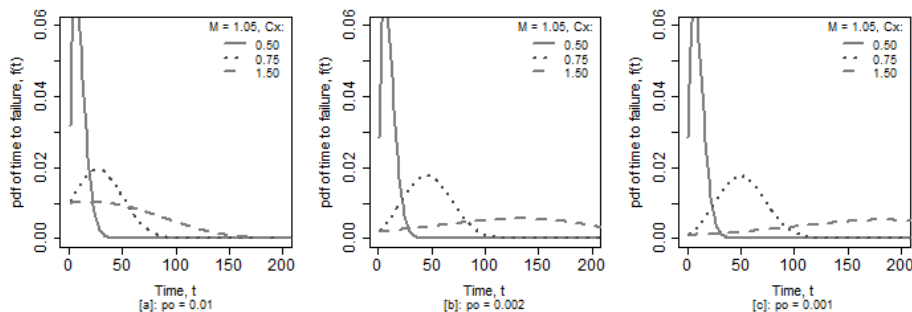


Figure 5. Probability density function (pdf) of the time to failure distribution for the nonstationary GP2 model, with a fixed $M = 1.05$ for a range of trend values ($C_x = 0.5, 0.75, 1.50$); panels show three exceedance event sizes increasing in extremity **(a)** $p_0 = 0.01$, **(b)** $p_0 = 0.002$ and **(c)** $p_0 = 0.001$.