This document includes:

1. Revision (text and table and figures)

2. Comment and response

3. Annotated revision

A non-stationary earthquake probability assessment with Mohr-Coulomb failure criterion: including an application to central Taiwan

Abstract: From theory to experience, earthquake probability associated with an active fault should be gradually increasing with time since the last event. In other words, the process should be non-stationary, rather than being stationary as the Poisson process. In this paper, a new non-stationary earthquake assessment is introduced. Different from other analyses, the new model more clearly defines and calculates two stress states or boundary conditions between two consecutive earthquakes, facilitated with the Mohr-Coulomb failure criterion. In addition to the model development, this paper also presents a model application to evaluate earthquake probability associated with the Meishan fault in central Taiwan. Based on the best-estimate return period of 162 ± 50 years, focal depth of 4 ~ 8 km, etc., there could be a 7.6% probability for the fault to induce a major earthquake in years 2015 ~ 2025, and if the earthquake does not recur by 2025, the earthquake probability will increase to 8% in 2025 ~ 2035, a non-stationary probability depending on the starting dates of a given period of time.

Keyword: non-stationary earthquake probability assessment, Mohr-Coulomb failure criterion

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1. Introduction

Owing to our imperfect understandings and natural randomness of earthquake, several models have been proposed for estimating earthquake probability in a given period of time. Among them, the Poisson model might be the one that is mostly used in many applications (e.g., Weichert, 1980; Ang and Tang, 2007; Ashtari Jafari, 2010). However, it must be noted that the Poisson calculation is a “memory-less” model (Devore, 2008), meaning that the Poissonian probability is only a function of length of time, but irreverent to when the last earthquake was occurring.

However, it seems that the recurrence of a characteristic earthquake associated with a given active fault should not be stationary or memory-less. That is, the earthquake probability should be gradually increasing with time. Taking the recent Nepal earthquake in April 2015 for example, the probability for the very next Nepal earthquake to recur in 2015 ~ 2020 should be lower than that in 2115 ~ 2120, although the two have the same length of time.

The scope of this study is to develop a new non-stationary earthquake probability assessment, mainly from the concepts of the Mohr-Coulomb failure criterion. Meanwhile, this paper provides a comprehensive review on other non-stationary earthquake models (Section Two), followed by our non-stationary analysis (Section Three). Then, the new model is demonstrated with a model application to central Taiwan (Section Four), as well as model improvement and future work (Section Five).

2. An overview of non-stationary earthquake models

In this section, we would like to provide a comprehensive review on non-stationary earthquake analyses and models. Specifically, we characterized the models into two groups, referred to as "statistical models" and "physical model."

2.1 Statistical models

Basically, those statistical models developed are more or less a derivative of the stationary Poisson model. For example, Vere-Jones and Ozaki (1982) proposed the use of a time-variant model parameter for the Poissonian calculation, making their model non-stationary although the calculation is still Poissonian in essence. Similarly, another work suggested the use of adjusted return period (related to current time and original return period) for the Poissonian calculation, in order to modify the Poisson model from stationary to non-stationary (Wang et al., 2013).

Another type of modification is to use non-exponential distributions to model earthquake inter-occurrence time intervals as a random variable. (Note that for an event modeled by a Poisson process, the number of events in a given period of time is a discrete random variable following the Poisson distribution; meanwhile the time when the next event would recur is a continuous variable following the exponential distribution.) For example, the log-normal distribution (Ferráes, 2005), Weibull distribution (Yakovlev et al., 2006), and Gamma distribution (Gómez and Pacheco, 2004) have been suggested for the replacement of the exponential distribution, with them all featuring a non-stationary analysis after such modifications.

Based on given earthquake data, it must be noted that the statistical models are all empirical in a sense. In other words, the models are in no consideration of earthquake mechanisms, such as tectonic stress accumulation under the ground.

2.2 Physical models

In consideration of earthquake mechanics, several non-stationary earthquake analyses have also been proposed from a different perspective. It must be noted that the models are not entirely a “product” of physics, but somehow on the basis of the concepts of physics working together with empirical models. Specifically, we would like to introduce three of them in the following that are more related to our non-stationary earthquake model.

The first one we like to introduce here is the time-predictable model (Shimazaki and Nakata, 1980). Fig. 1 is a schematic diagram illustrating the model basics. Essentially, the model is relying on a best-estimate relationship between co-seismic fault slip (or displacement) and time. For instance, given the last event with fault slips as Points A and B (see Fig. 1), then the next event should recur at the time of Points C and D. In other words, the recent event with a smaller fault slip should accompany a smaller stress drop, and under a constant stress increment with time, it should lead to a shorter time for the stress to re-reach a stress level (or failure stress state) that could induce earthquakes.

The next model of the group is the Brownian model (Ellsworth et al., 1999; Matthews et al., 2002). By contrast to the time-predictable model, the Brownian model is not on the basis of a constant stress increment, while considering the stress increments between two consecutive events should be a stochastic process like Fig. 2. Specifically, the model considers the stress-time series is a combination of a long-term stress increment and a Brownian motion simulating transient stress randomness. With such a function, we can estimate the time of the next earthquake by examining if the stress reaches the failure state within a given period of time.

The third one we like to introduce is the negative binomial model (Tejedor et al., 2015). As the previous analyses, the model is also on the basis of two imaginary stress states. As shown in Fig. 3, the essence of the model is that the stress change in unit time could be modeled by two scenarios: stress does and does not increase. As a result, there are many possible "stress routes" (as shown in Fig. 3) between two consecutive events, and the probability and the total time of each route could be calculated with given earthquake return periods. Finally, the inter-occurrence time interval can be derived as a negative binomial distribution for such a non-stationary probability assessment.

To sum up, the three physical models are all facilitated with two stress states that are part of the earthquake occurrence theories generally accepted. Somehow, we do share this perspective for our model development. However, the biggest difference is that our model defines and calculates the two stress states more clearly, on the basis of the Mohr-Coulomb failure criterion that is well established and used in rock mechanics, structural geology, etc.

3. The new non-stationary earthquake probability assessment

3.1 Overviews of Mohr-Coulomb failure criterion and elastic rebound theory

The Mohr-Coulomb failure criterion is a model describing the response of materials subject to external stresses (Pariseau, 2007), and it is commonly applied to rock mechanics as well as other applications. Fig. 4 is a schematic diagram illustrating the essentials of the model. Basically, as the Mohr circle is below the failure envelope, a shear failure is not expected in the material. By contrast, as long as the Mohr circle is in contact with the failure envelope, a shear failure could occur.

On the other hand, it is generally accepted that the ongoing tectonic activities are the main reason causing rock failures under the ground, resulting in an earthquake with the release of accumulated strain energy. Afterward, the energy re-accumulates and re-releases until the next earthquake, and such a theory is referred to as the elastic rebound theory (Keller, 1996), proposed by Reid in the early twentieth century (Reid, 1910).

3.2 The model basics and the algorithms

The two earthquake theories above were mainly the motivation of the new non-stationary model: 1) based on the Mohr-Coulomb failure criterion, the rock subject to the stress state as Mohr Circle C (see Fig. 4) should fail and cause an earthquake, at which we refer to it as failure state; 2) from the elastic rebound theory, the stress state in the rock right after a characteristic earthquake should be restored to Mohr Circle A, which is called the initial state at time t_0.

As a result, the problem to evaluate the earthquake probability within a given time t^* after the last event (or after t_0) is becoming a problem as follows: What is the chance for the major principle stress at time t^* (denoted as $\sigma_{1\_t^*}$) greater than the major principle stress at the failure state (denoted as $\sigma_{1\_failure}$)? Or the question can be mathematically expressed by the following equation:

$$\Pr(earthquake\ within\ t^*\ after\ t_0) = \Pr(\sigma_{1\_t^*} > \sigma_{1\_failure}) \tag{1}$$

Clearly, the problem now is governed by two variables $\sigma_{1\_t^*}$ and $\sigma_{1\_failure}$, and their relationships with other parameters will be detailed later. Note that those notations used in the following derivations are summarized in the end of the paper.

- The major principle stress at failure state, $\sigma_{1\_failure}$

Based on the Mohr-Coulomb failure criterion, the major principal stress at failure state (Point C in Fig. 4) can be expressed as a function of the minor principal stress at failure ($\sigma_{3\_failure}$), and two strength parameters of the shearing plane, i.e., cohesion *c* and friction angle ϕ (Pariseau, 2007):

$$\sigma_{1\_failure} = \sigma_{3\_failure}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2c\times\cos\phi}{1-\sin\phi} \tag{2}$$

- The minor principal stress at failure state, $\sigma_{3\_failure}$

The minor principal stress at failure is attributed to the overburden earth pressure above the focal depth *d*, which can be estimated with the following formula based on rock mechanics:

$$\sigma_{3\_failure} = \gamma \times d \quad (3)$$

where γ is rock unit weight. It must be noted that this model considers $\sigma_{3\_failure}$ as time-invariant (more discussion is given later), and the case shown in Fig. 4 and Eq. 3 is for a thrust-fault earthquake. As for the strike-slip fault, the Mohr circles of the initial state and the failure state are shown in Fig. 5, indicating $\sigma_{3\_failure}$ is equal to $\gamma \times d \times K$ for this case, where K is the coefficient of lateral earth pressure in rock. More discussion over model improvements is given in Section 5.2.

- The major principle stress at time t*, σ_{1_t*}

With tectonic stress increasing with time, the key task of the new analysis is to estimate the major principle stress at time t* after the last event. For thrust-fault earthquakes as those Mohr circles shown in Fig. 4, the major principle stress at time t* can be formulated as follows:

$$\sigma_{1\_t*} = \sigma_{3\_initial} + t* \times ASI \quad (4)$$

where $\sigma_{3\_initial}$ is the minor principal stress at the initial state (or at t_0) and *ASI* is called annual stress increment. Note that from Fig. 4 (the thrust fault) and Fig. 5 (the strike-slip fault), $\sigma_{3\_initial}$ is equal to $\gamma \times d \times K$ for the two cases of the non-stationary analysis.

3.3 The return period $\tilde{t}$ and its relationship with σ_{1_t*}

In addition to $\gamma, d, K,$ etc., the return period $\tilde{t}$ of characteristic earthquakes is another input data of the non-stationary analysis. Moreover, the mean value and standard deviation of σ_{1_t*} can be expressed as a function of $\tilde{t}$, and used for developing its probability density function for the non-stationary probability assessment within the given time *t**.

From the meaning of return period, it is understood that the event will recur when return period $\tilde{t}$ is due. As a result, the major principal stress at return period $\tilde{t}$ (denoted as $\sigma_{1\_\tilde{t}}$) should be equal to $\sigma_{1\_failure}$:

$$\sigma_{1\_\tilde{t}} = \sigma_{3\_initial} + \tilde{t} \times ASI = \sigma_{1\_failure} \tag{5}$$

Therefore, the mean value of *ASI* (denoted as μ_{ASI}) can be derived as follows:

$$\begin{aligned} &E[\sigma_{1\_failure}] = E[\sigma_{3\_initial} + \tilde{t} \times ASI] \\ &=> E[ASI] = \frac{\sigma_{1\_failure} - \sigma_{3\_initial}}{\tilde{t}} = \mu_{ASI} \end{aligned} \tag{6}$$

where *E*[] denotes the mean value of a variable in probability and statistics.

On the other hand, as the variability of annual stress increment is equal to *n* in terms of coefficient of variation (= standard deviation / mean value), its standard deviation (denoted as s_{ASI}) can be derived as follows with its mean value from Eq. 6:

$$n = \frac{s_{ASI}}{\mu_{ASI}} => s_{ASI} = n \times \mu_{ASI} = \frac{n \times (\sigma_{1\_failure} - \sigma_{3\_initial})}{\tilde{t}} \tag{7}$$

With the mean (Eq. 6) of *ASI*, we can continue deriving the mean value of the major principal stress at time *t**:

$$\sigma_{1\_t^*} = \sigma_{3\_initial} + t^* \times ASI$$

$$=> E[\sigma_{1\_t^*}] = E[\sigma_{3\_initial} + t^* \times ASI] = \sigma_{3\_initial} + t^* \times E[ASI] \tag{8}$$

$$= \sigma_{3\_initial} + \frac{t^* \times (\sigma_{1\_failure} - \sigma_{3\_initial})}{\tilde{t}}$$

Similarly, the standard deviation of the major principal stress at time *t** (denoted as $s_{\sigma 1\_t^*}$) can be derived as follows with s_{ASI} in Eq. 7:

$$\sigma_{1\_t^*} = \sigma_{3\_initial} + t^* \times ASI$$

$$=> V[\sigma_{1\_t^*}] = V[\sigma_{3\_initial} + t^* \times ASI] = t^{*2} \times V[ASI] = t^{*2} \times s_{ASI}^2 \tag{9}$$

$$=> s_{\sigma 1\_t^*} = \sqrt{V[\sigma_{1\_t^*}]} = t^* \times s_{ASI} = \frac{t^* \times n \times (\sigma_{1\_failure} - \sigma_{3\_initial})}{\tilde{t}}$$

where *V*[] denotes variance in probability and statistics, and it is the square of standard deviation.

In order to establish the probability density function of σ_{1_t*}, the information about what probability distribution the variable is following is as essential as its mean value and standard deviation. But since the distribution of σ_{1_t*} is unknown (to the best of our knowledge, no study has ever worked on the subject), we suggest the normal distribution for this non-stationary earthquake assessment, as it is usually recommended for a probability analysis when the variables' distribution is unknown (Abramson et al., 2002).

3.4 Summary

Fig. 6 is a schematic diagram illustrating the essentials of the non-stationary assessments. The key to the model is to estimate the probability distribution of the major principal stress at time t* after the last event (or after t_0), and compares it to the stress that could cause rock failures and earthquakes. To sum up, the new non-stationary model is governed by a total of six parameters as follows: return period ($\tilde{t}$), fault-plane strength parameters (c and ϕ), rock unit weight (γ), earthquake focal depth (d), and the variability of annual stress increment in terms of coefficient of variation (n).

3.5 Presumption and limitation

The elastic rebound theory is a plausible explanation to earthquake, but specifically speaking, it is more of a theory about main shocks. As a result, the new non-stationary analysis of the study motivated by such a theory is more applicable to main shocks, a situation similar to other non-stationary models that are also applicable to main shocks rather than dependent shocks (Shimazaki and Nakata, 1980; Ellsworth, 1995; Matthews et al., 2002; Tejedor et al., 2015).

On the other hand, like any other stationary or non-stationary analyses estimating earthquake probability in a given period of time, our model cannot predict the magnitude of the recurring event, either. In other words, the (earthquake) temporal analyses closely related to return period, stress increment, etc. do not further relate the variables to earthquake magnitudes or energy release. Again, such a framework is similar to other stationary or non-stationary temporal analyses only focusing on the earthquake probability in a given period of time, but not on the probability distribution of earthquake magnitude or energy release when the event recurs.

4. A model application

4.1 The Meishan earthquake in central Taiwan

The region around Taiwan is known for high seismicity owing to the location close to the boundaries of tectonic plates. On average, there are around 2,000 earthquakes above M_w 3.0 (moment magnitude) occurring around Taiwan every year, with a catastrophic event, like the M_w 6.4 Meishan earthquake in 1906 and the M_w 7.6 Chi-Chi earthquake in 1999, that could recur in decades.

As a result, we would like to apply the new non-stationary model to Taiwan as a case study. Specifically, we selected the Meishan fault in central Taiwan as the model application, given a few recent studies pointing out the fault should be of "imminent" earthquake risk, for the event's return period being "almost" due (e.g., Wang et al., 2012). By contrast, the reason we did not select the Chelungpu fault as the application is because the active fault should be of lower earthquake risk in next couple decades, given it “just” induced the Chi-Chi earthquake in 1999, and should have a longer return period (about 250 years) than the Meishan earthquake (Cheng et al., 2007). More discussion over this model application is given in Section 5.1.

Fig. 7 shows the location of the Meishan fault in central Taiwan. Accordingly, the fault is very close to a major city (i.e., Chiayi) in central Taiwan, and reportedly the 1906 Meishan earthquake killed around 1,200 people in the area.

4.2 The best-estimate data from the literature

Table 1 summarizes our best-estimate data from the literature for the non-stationary earthquake assessment on the Meishan fault. It must be noted that because the strength parameters of the fault plane are not clear, we used a typical range (see Table 1) from rock mechanics as our best estimates. Similarly, a probable range of 0.2 ~ 0.5 was used as our best estimate for the coefficient of lateral earth pressure in rock, given no site-specific studies and data have been reported. As for the earthquake focal depth, we considered the depth should be close to 6 km as the last Meishan earthquake (Ng et al., 2009). However, in order to account for the focal-depth uncertainty in the analysis, we used a best-estimate range as 4 ~ 8 km.

A similar situation was encountered in the determination of the best-estimate return period. On the basis of 162 years used in recent studies (e.g., Wang et al., 2012), two best-estimate ranges were determined as 162 ± 50 years and 162 ± 100 years. Understandably, the uncertainties (i.e., ± 50 and ± 100) are from our best judgments, given such information is not clear from the literature. As for the variability of annual stress increment, to the best of our knowledge, there is no any research so far that can really answer the question. As a result, the range of 0.25 ~ 1 was used as our best estimate characterizing the variability of annual stress increment in terms of coefficient of variation.

More discussion about the input data characterizations and the model application is given in Section 5.1 in the following.

4.3 Monte Carlo Simulation

Because our input data were characterized by a range rather than a single value, it is difficult to solve the governing equation (Eq. 1) of the non-stationary probability with analytical approaches. Therefore, we used Monte Carlo Simulation (MCS) to solve the problem as many MCS applications. For more details about Monte Carlo Simulation, readers can refer to the textbooks of Ang and Tang (2007), Abramson et al. (2002), among many others.

4.4 The result

With the best-estimate input data summarized in Table 1, Fig. 8 shows the average probabilities for three 10-year periods from Monte Carlo Simulation with a sample size of 5,000. For example, the non-stationary model shows a 7.6% probability for the Meishan fault in central Taiwan to induce a major earthquake in years 2015 ~ 2025, under the return period of 162 ± 50 years. Then, if the event does not recur by 2025, the earthquake probability in 2025 ~ 2035 will increase to 8%; similarly, if the event does not recur by 2035, the probability will further increase to 8.4%. Note that the standard deviations of the three probability estimates are all close to 3.3%, which is a reflection to the input data that were characterized by a range. In other words, if the input data were all characterized by single values, the standard deviation of the probability estimates cannot be calculated and reported.

In addition, the Poissonian probabilities for the same problem are also shown in Fig. 8. It shows that for a 10-year period of time, the earthquake probability is about 6% for the three different periods (i.e., 2015 ~ 2025, 2025 ~ 2035, and 2035 ~ 2045), or the probability is irrelevant to the starting dates of the time period.

Fig. 9 shows the result for the other scenario under return period as 162 ± 100 years. Interestingly, the average probabilities for the three 10-year periods become relatively close to one another, and they are smaller than those estimates subject to the return period of 162 ± 50 years. Our explanation to this is as follows: As shown in Fig. 10, the relationship between return period and earthquake probability could be highly non-linear. Therefore, the average probability subject to a bigger range of return period would be lower than that subject to a smaller range. Nevertheless, with the non-stationary model, the probability estimates do vary with the starting dates of the time, or the probabilities are indeed non-stationary, in contrast to the stationary Poisson process.

5. Discussions

5.1 Input data characterizations

As many analyses, input data characterizations are equally challenging as the model development. As a result, for improving the model estimates, we hope to see more studies focusing on site characterizations with more laboratory works or field instrumentation assessing stress increment variability, fault-plane strength parameters, lateral earth pressure in rock, etc. However, this is beyond the scope of the study focusing on a new non-stationary model development.

On the other hand, one could argue why not choose a geologically well-investigated fault (e.g., the Chelungpu fault) in Taiwan as the model application to reduce uncertainty, and here is our response: The “rock-mechanics” parameters of the model (such as lateral pressure coefficients, variability of stress increment, and the strength parameters of fault planes) are not clear either, even for those so-called well-investigated faults in Taiwan. As a result, no matter which fault was selected as the model application, engineering judgment must involve in the determinations of those “rock-mechanics” parameters, more or less creating the same level of uncertainty when it comes to site characterizations on stress increment variability, lateral earth pressure in rock, etc.

Besides, as mentioned previously the key reason of using the Meishan fault as a case study is owing to its imminent earthquake risk, not to mention the well-characterized return period of 162 years from the Central Geological Survey Taiwan (Lin et al., 2008) could somewhat help increase the reliability of the estimate, in a comparison to other cases without a well-characterized earthquake return period, or at least not yet reported.

5.2 Model improvements

Certainly, the non-stationary analysis of the study can be further modified. For example, in addition to the algorithms for thrust faults and strike-slip faults that have been derived, the model is also applicable to normal faults as those Mohr circles shown in Fig. 11. Accordingly, the minor principal stress at initial state ($\sigma_{3\_initial}$) is equal to

$\gamma \times d \times K$ for this case, with the minor principal stress at time t^* (denoted as $\sigma_{3\_t^*}$) that can be expressed as $\sigma_{3\_initial} - t^* \times ASI$. Next, the same probability calculation is applicable by comparing the minor principal stress at t^* to the minor principal stress at failure ($\sigma_{3\_failure}$), for such a normal-fault earthquake subject to tectonic extension.

Further improvements can be conducted with the consideration of the direction of stress increment, or the direction of tectonic compression/extension. Under the circumstances, the Mohr circles of the initial state and failure state are shown in Fig. 12, with major and minor principal stresses both varying with time.

Nevertheless, no matter how the non-stationary model will evolve, such type of non-stationary analysis from the Mohr-Coulomb failure criterion is as novel, robust and transparent as its counterparts, providing a new alternative to non-stationary earthquake assessment related to a given active fault.

5.3. Earthquake should be stationary or non-stationary?

Although characteristic earthquakes related to a given active fault should be non-stationary, in the 1970s a study has provided statistical evidence to the opposite: earthquake is stationary (Gardner and Knopoff, 1974). However, it must be noted that the study was not focusing on characteristic earthquakes, but based on the regional seismicity in California.

Fig. 13 is a schematic diagram that helps explain the difference between the two problems. For each fault, the recurring earthquake should be a non-stationary process, and the non-stationary earthquake probability would be reset at the last event and gradually increase with time. By contrast, the seismicity in a region would become stationary with so many non-stationary processes present. For example, at $T = t_0$ (see Fig. 13), the sum of that many stationary probabilities should be close to that at $T = t_1$ (or at any moment), although the earthquake probability induced by Fault D should be very low at $T = t_0$, while others are higher.

The relationship can be simply explained with the patron-and-bank analogy. For each patron (analogy to each fault), going to the bank is obviously a non-stationary process, with the probability increasing with time since the very last visit. But for the banks (analogy to the seismicity), it is a stationary process for them, as dealing with so many patrons or so many non-stationary processes at one time.

6. Summary and conclusion

Given the earthquake recurrence associated with an active fault that should be a non-stationary process, this paper introduces a new non-stationary analysis to evaluate earthquake probability within a given period of time. Different from previous models, the new analysis more clearly defines and calculates two earthquake stress states, on the basis of the well-established, Mohr-Coulomb failure criterion.

In addition, this paper also presents a model application to evaluate earthquake probability associated with the Meishan fault in central Taiwan. With the best-estimate return period of 162 ± 50 years, focal depth of 4 ~ 8 km, etc., the active fault has a 7.6% probability (standard deviation equal to 3.3%) of inducing the next Meishan earthquake in 2015 ~ 2025, and if the earthquake does not recur by 2025, then the non-stationary probability will increase to 8% in 2025 ~ 2035, rather than being unchanged, stationary, or independent of the starting dates of a given period of time.

Notations

t_0	The time when the last event occurs
t^*	The time interval after the last event
$\sigma_{1\_t^*}$	Major principle stress at time t^*
$\sigma_{1\_failure}$	Major principle stress at failure state
$\sigma_{3\_failure}$	Minor principal stress at failure state
c	Cohesion of the fault plane
ϕ	Friction angle of the fault plane
d	Earthquake focal depth
γ	Rock unit weight
K	Coefficient of lateral earth pressure in rock
ASI	Annual stress increment
$\tilde{t}$	Earthquake return period
$\sigma_{1\_\tilde{t}}$	Major principal stress at return period
E	Expected value or mean value
V	Variance
μ_{ASI}	Mean value of ASI
n	Coefficient of variation for ASI
s_{ASI}	Standard deviation of ASI
$s_{\sigma 1\_t^*}$	Standard deviation of $\sigma_{1\_t^*}$
$\sigma_{1\_initial}$	Major principal stress at initial state
$\sigma_{3\_initial}$	Minor principal stress at initial state
$\sigma_{3\_t^*}$	Minor principal stress at t^*

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References

Abramson, L. W., Lee, T. S., Sharma, S., and Boyce, G. M.: Slope Stability and Stabilization Methods, John Wiley & Sons, Inc., NJ, 2002.

Ang, A. H. S., and Tang, W. H.: Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering, 2nd Edn., John Wiley & Sons, Inc., NJ, 2007.

Ashtari Jafari, M.: Statistical prediction of the next great earthquake around Tehran, J. Geodyn., 49, 14-18, 2010.

Cheng, C. T., Chiou, S. J., Lee, C. T., and Tsai, Y. B.: Study on probabilistic seismic hazard maps of Taiwan after Chi-Chi earthquake, J. GeoEng., 2, 19-28, 2007.

Devore, J. L.: Probability and Statistics for Engineering and the Sciences, Brooks/Cole, Boston, Massachusetts, 2008.

Ellsworth, W. L., Matthews, M. V., Nadeau, R. M., Nishenko, S. P., Reasenberg, P. A., and Simpson, R. W.: A physically based earthquake recurrence model for estimation of long-term earthquake probabilities, US Department of the Interior, US Geological Survey, 1999.

Ferráes, S. G.: A probabilistic prediction of the next strong earthquake in the Acapulco-San Marcos segment, Mexico, Geofisica internacional, 44, 347-353, 2005.

Gardner, J. K., and Knopoff, L.: Is the sequence of earthquakes in southern California, with aftershocks removed, Poissonian? Bull. Seismol. Soc. Am., 64, 1363-1367, 1974.

Gómez, J. B., and Pacheco, A. F.: The minimalist model of characteristic earthquakes as a useful tool for description of the recurrence of large earthquakes, Bull. Seismol. Soc. Am., 94, 1960-1967, 2004.

Keller, E. A.: Environmental Geology, 7th Edn., Prentice Hall, Inc., NJ, 1996.

Lin, C. W., Lu, S. T., Shih, T. S., Lin, W. H., Liu, Y. C., and Chen, P. T.: Active faults of central Taiwan, Special Publication of Central Geological Survey 21,148 (In Chinese with English Abstract), 2008.

Matthews, M. V., Ellsworth, W. L., and Reasenberg, P. A.: A Brownian model for recurrent earthquakes, Bull. Seismol. Soc. Am., 92, 2233-2250, 2002.

Ng, S. M., Angelier, J., and Chang, C. P.: Earthquake cycle in Western Taiwan: Insights from historical seismicity, Geophys. J. Int., 178, 753-774, 2009.

Pariseau, W. G.: Design Analysis in Rock Mechanics, Taylor & Francis/Balkema, 2007.

Reid, H. F.: The mechanics of the earthquake, the California Earthquake of April 18, 1906, Report of the State Investigation Commission, Carnegie Institution of Washington, Washington, D.C., 2, 1910.

Shimazaki, K., and Nakata, T.: Time-predictable recurrence model for large earthquakes, Geophys. Res. Lett., 7, 279-282, 1980.

Tejedor, A., Gómez, J. B., and Pacheco, A. F.: The negative binomial distribution as a renewal model for the recurrence of large earthquakes, Pure Appl. Geophys., 172, 23-32, 2015.

Vere-Jones, D., and Ozaki, T.: Some examples of statistical estimation applied to earthquake data, Annals of the Institute of Statistical Mathematics, 34, 189-207, 1982.

Wang, J. P., Huang, D., and Chang, S. C.: Assessment of seismic hazard associated with the Meishan fault in Central Taiwan, Bull. Eng. Geol. Environ., 72, 249-256, 2013.

Wang, J. P., Lin, C. W., Taheri, H., and Chen, W. S.: Impact of fault parameter uncertainties on earthquake recurrence probability by Monte Carlo simulation – an example in central Taiwan, Eng. Geol., 126, 67-74, 2012.

Weichert, D. H.: Estimation of the earthquake recurrence parameters for unequal observation periods for different magnitudes, Bull. Eng. Geol. Environ., 70, 1337-1346, 1980.

Yakovlev, G., Turcotte, D. L., Rundle, J. B., and Rundle, P. B.: Simulation-based distributions of earthquake recurrence times on the San Andreas fault system, Bull. Seismol. Soc. Am., 96, 1995-2007, 2006.

Table 1. Summary of the model parameters used in the analyses

Parameters	Focal depth (km)	Unit Weight (kN / m^3)	Cohesion (MN / m^2)	Friction angle (degrees)	Return period (years)	*K**	*n***
Range	4 ~ 8	25 ~ 30	3.6 ~ 22.7	22 ~ 46	112 ~ 212 (162 ± 50) 62 ~ 262 (162 ± 100)	0.2 ~ 0.5	0.25 ~ 1.0
Average	6	27.5	13.2	34	162	0.35	0.63

* *K* = the coefficient of lateral earth pressure in rock; ** *n* = the coefficient of variation for annual stress increment

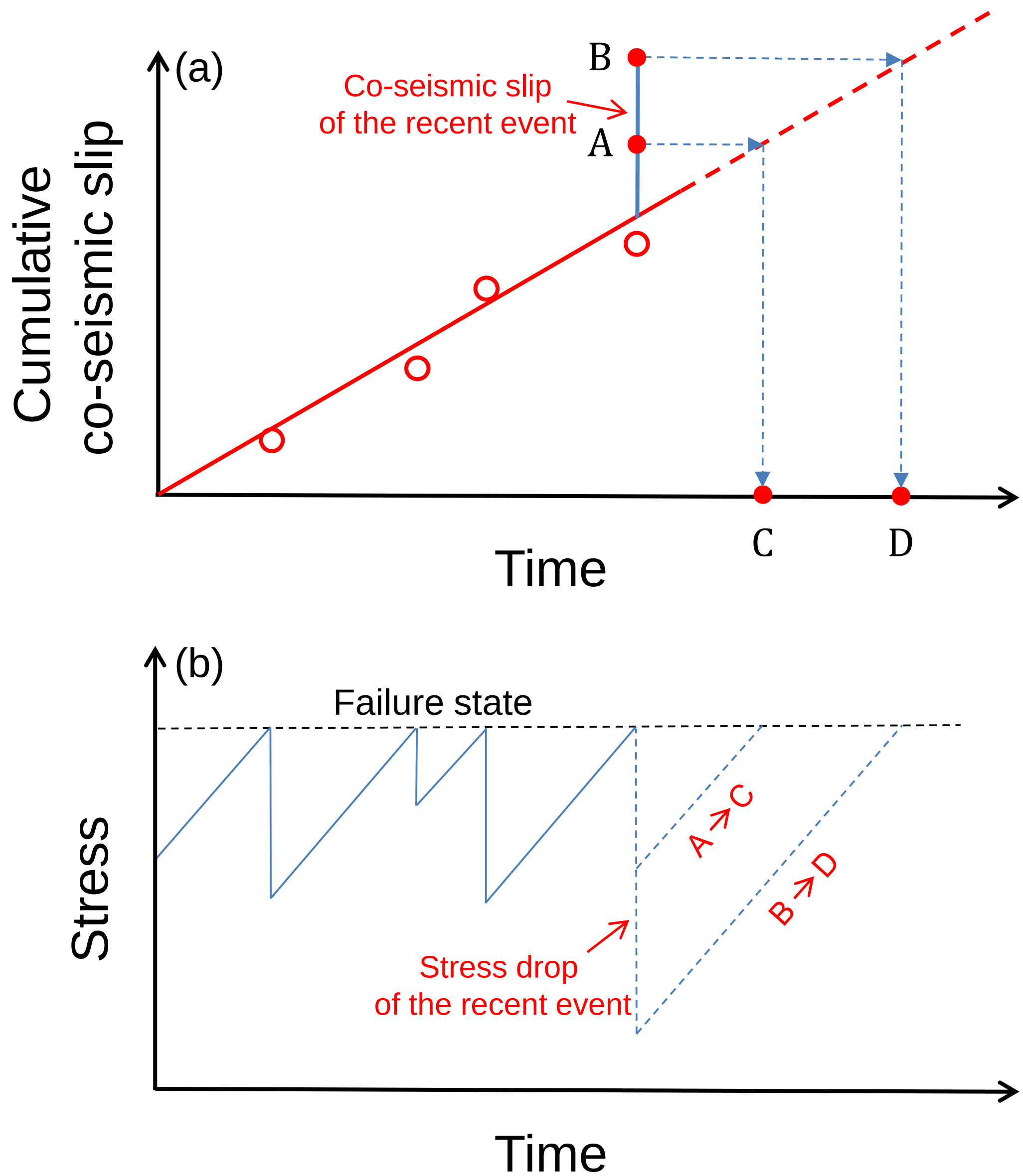

Fig. 1 Schematic diagram for the time-predictable model: a) best-estimate relationship between cumulative co-seismic slips and time, and b) the earthquake-time prediction facilitated with a failure state and a constant stress increment

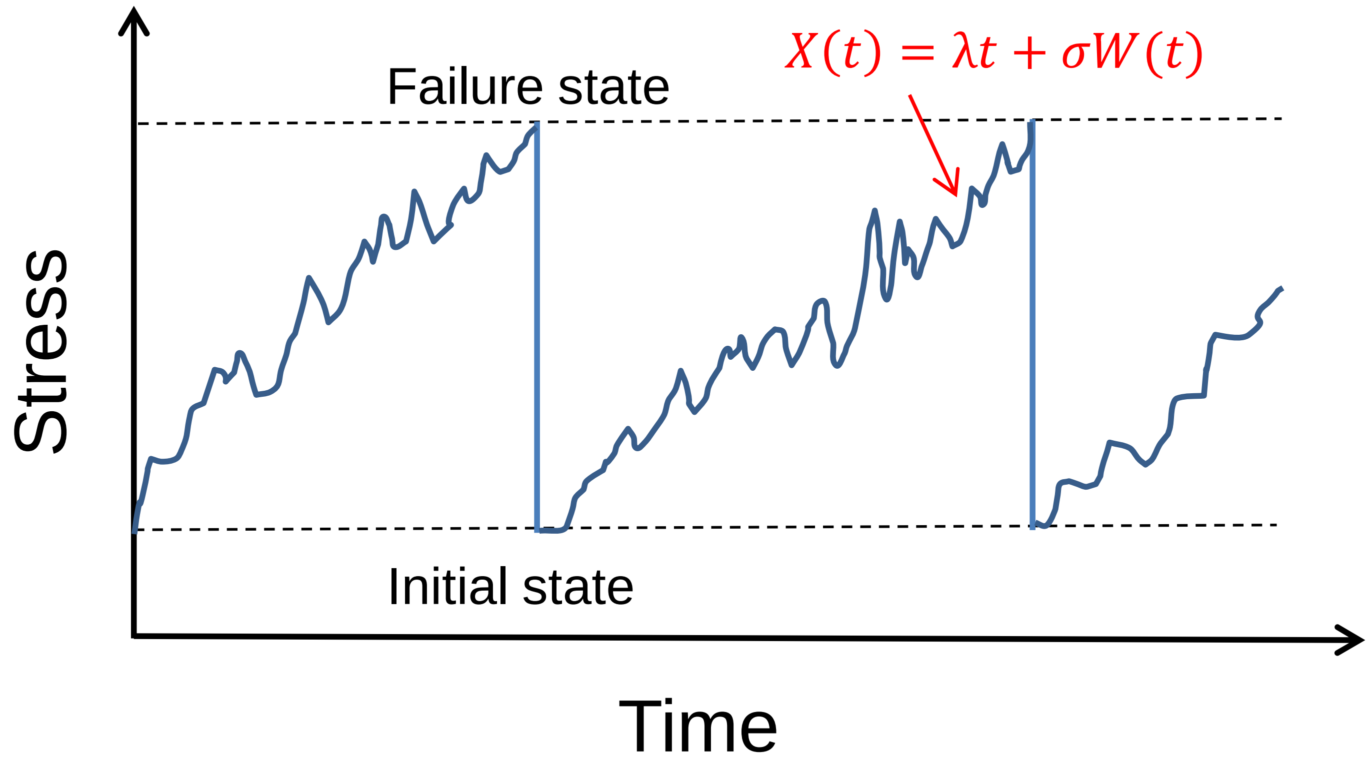

Fig. 2 Schematic diagram showing the essential of the Brownian model; within the two imaginary stress states, the model considers the stress-time series should be random and could be modeled by a long-term stress increment and a Brownian motion as $X(t) = \lambda t + \sigma W(t)$, where $X(t)$ is the stress at time t, λ is long-term stress increment rate, σ is the magnitude of a Brownian motion $W(t)$.

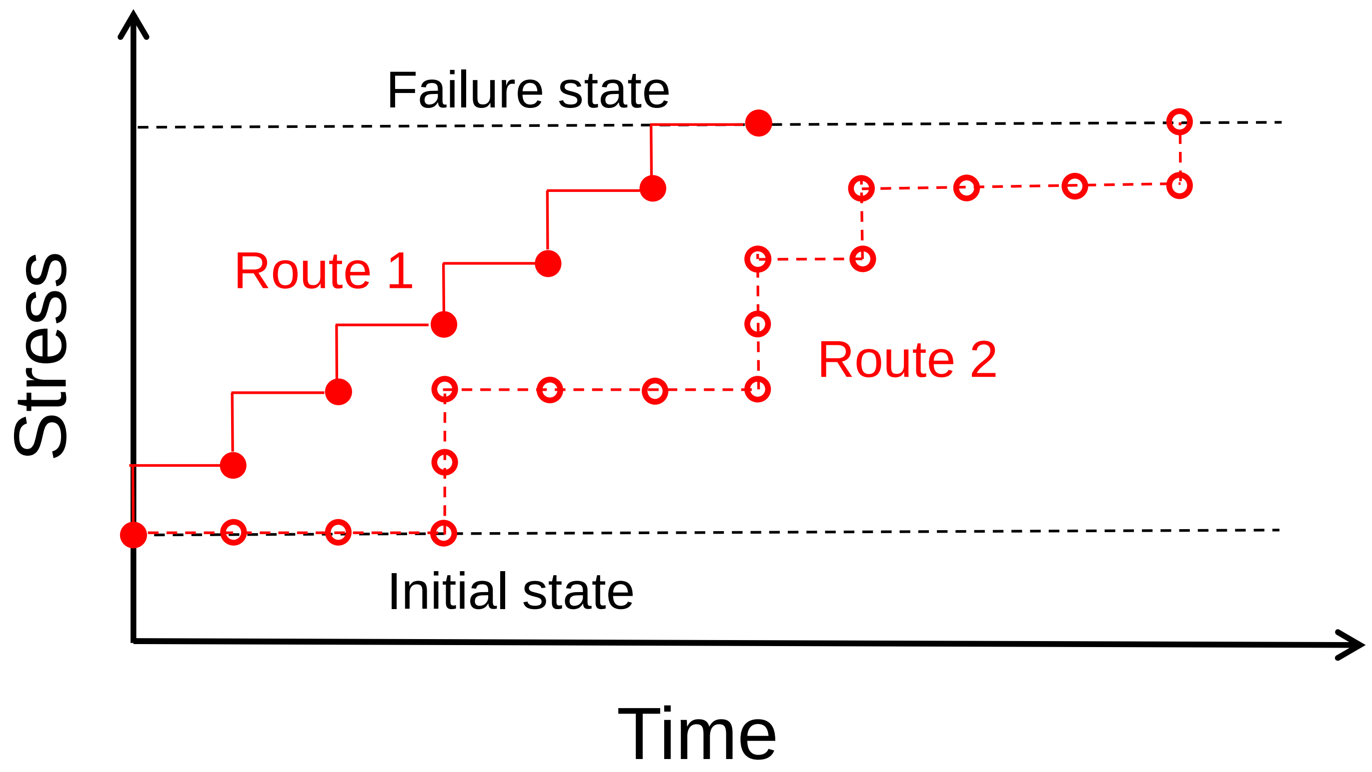

Fig. 3 Schematic diagram illustrating the negative binomial model; between the two stress states, many "stress routes" can be present, and the probability of each route can be estimated with the model, then developing the probability distribution for the interval between two consecutive events

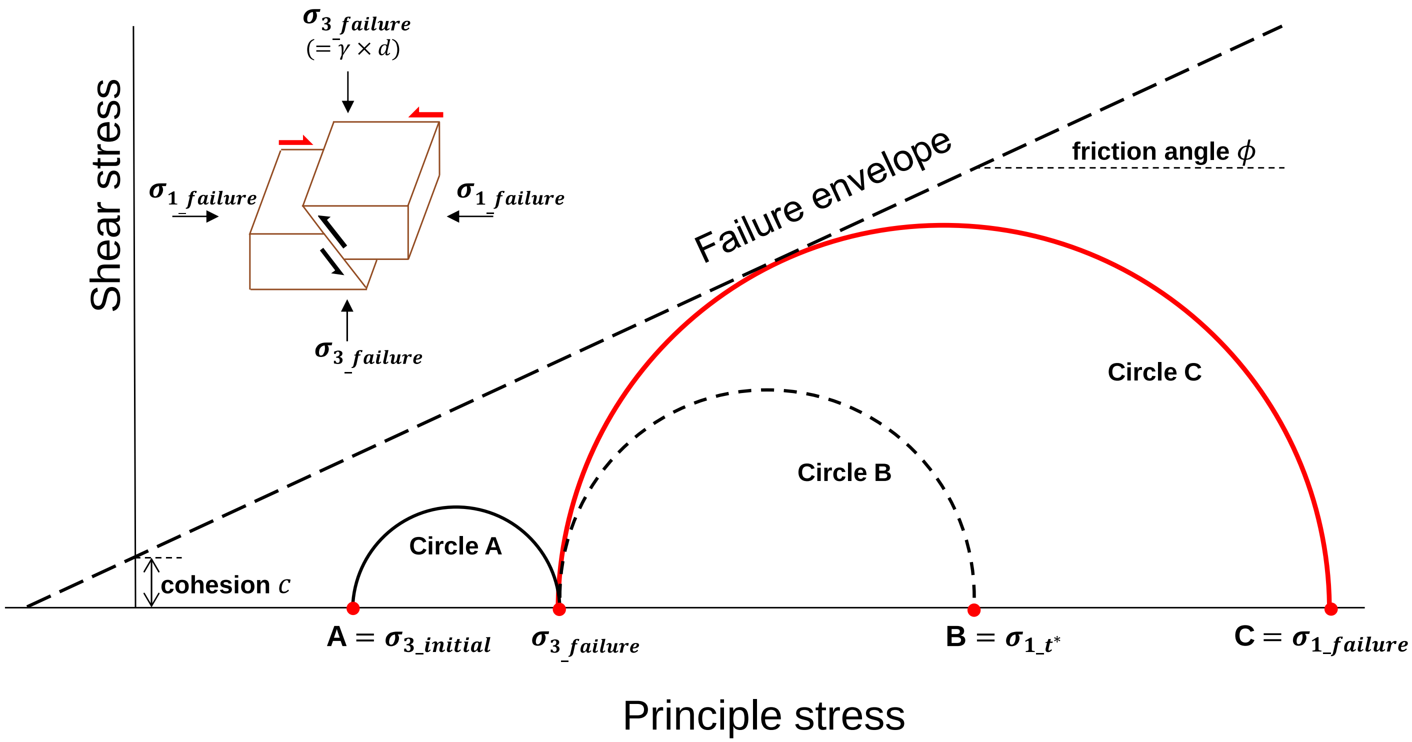

Fig. 4 Schematic diagram illustrating Mohr-Coulomb failure criterion; Circle A represents the initial state after a thrust-fault earthquake or at t_0, Circle B denotes stress states at t^* after t_0, and Circle C is the stress state corresponding to the failure state that causes rock failure and earthquake

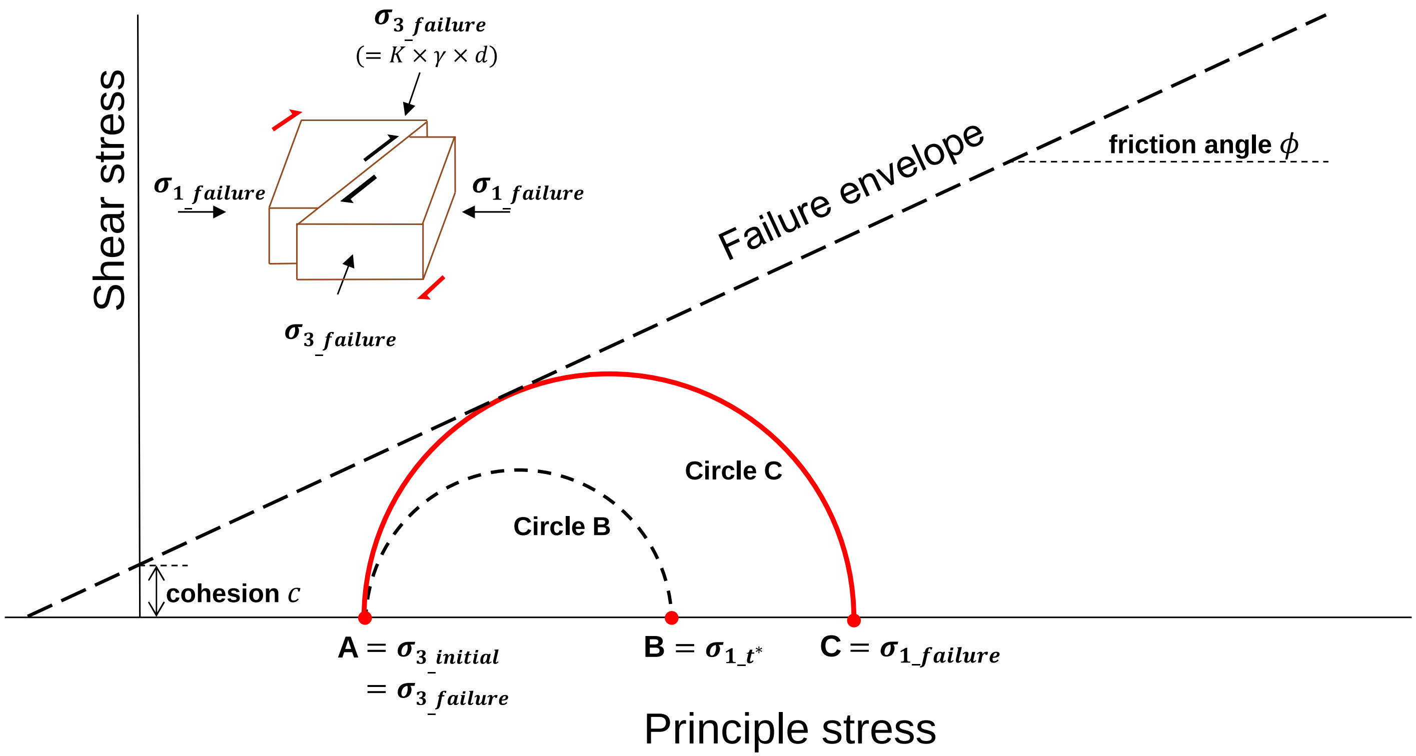

Fig. 5 The Mohr circles for evaluating the non-stationary earthquake probability for strike-skip earthquakes

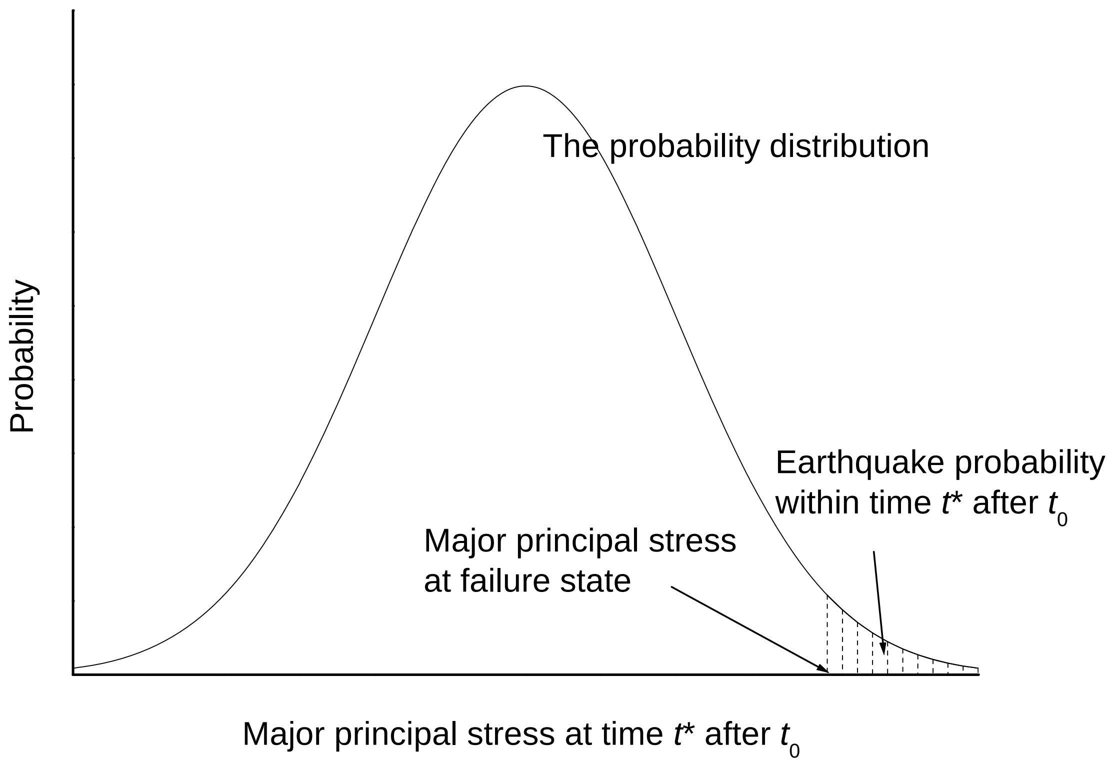

Fig. 6 The essentials of the new non-stationary model: Developing the probability distribution of the major principal stress at time t* (i.e., σ_{1_t*}) after the last event or after t_0

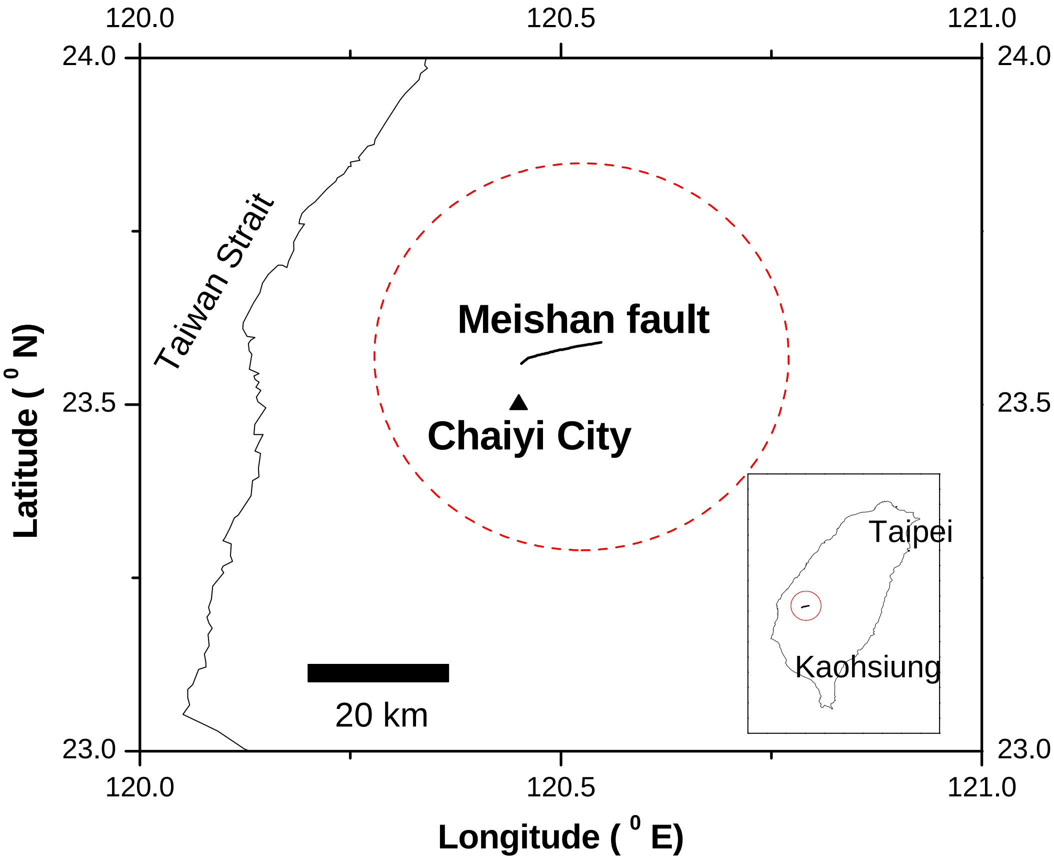

Fig. 7 The location of the Meishan fault in central Taiwan

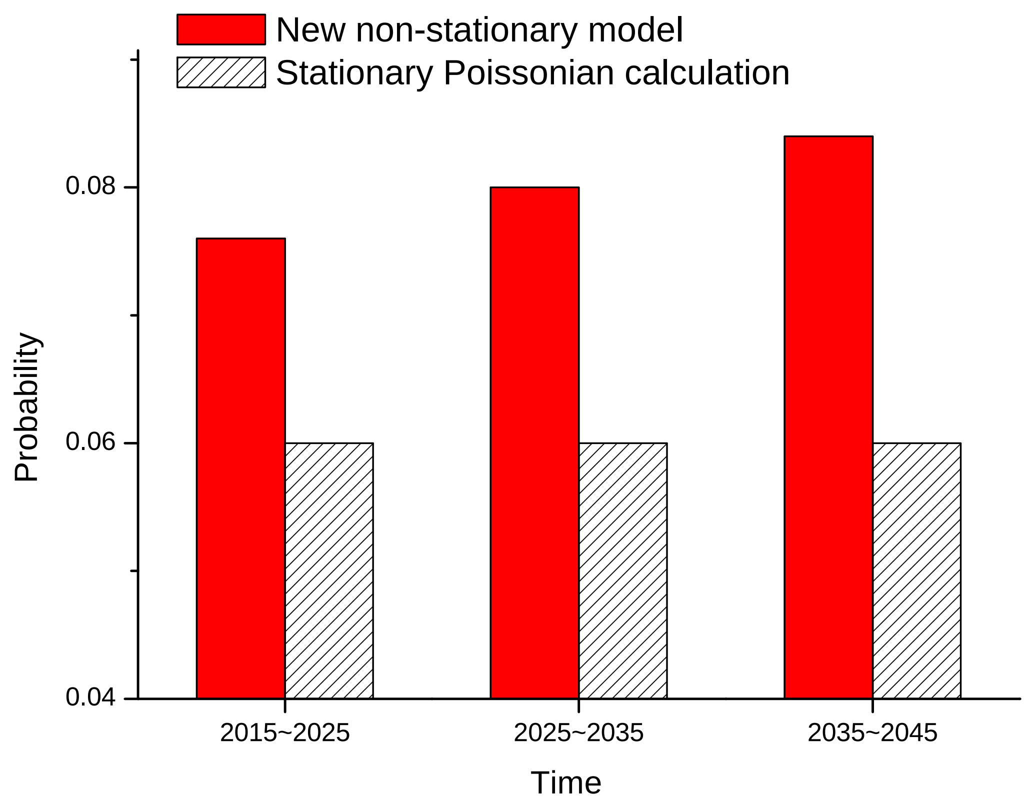

Fig. 8 The earthquake probability associated with the Meishan fault in three 10-year periods subject to the best-estimate return period of 162 ± 50 years (other input data are summarized in Table 1)

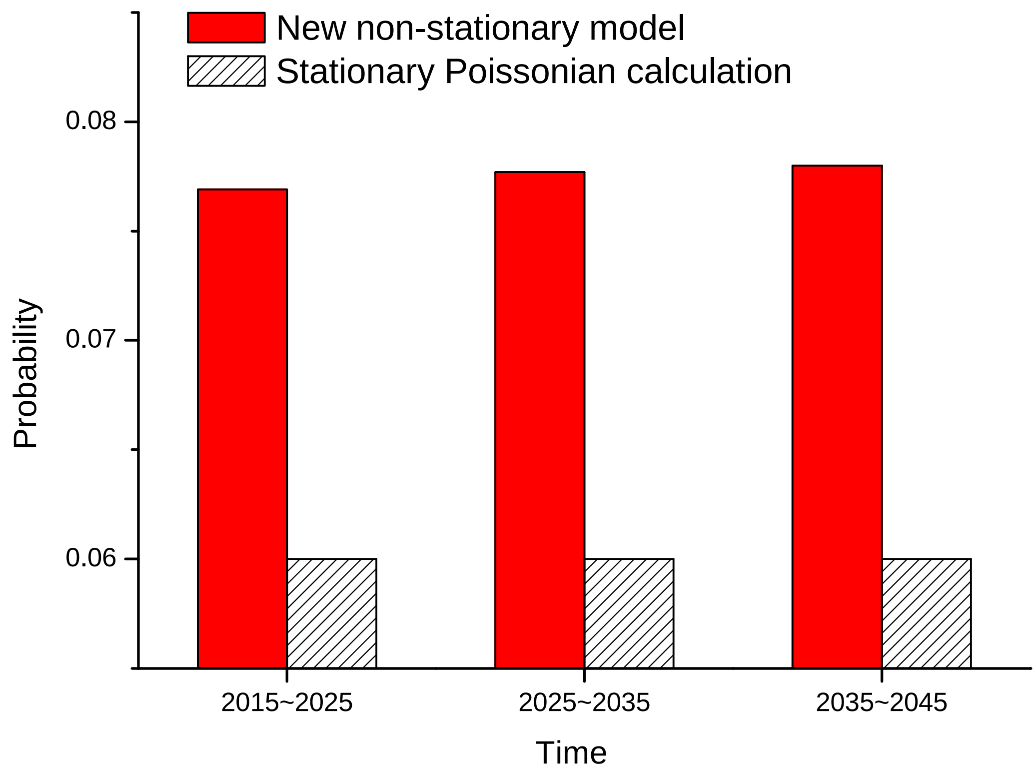

Fig. 9 The earthquake probability associated with the Meishan fault in three 10-year periods subject to the best-estimate return period of 162 ± 100 years (other input data are summarized in Table 1)

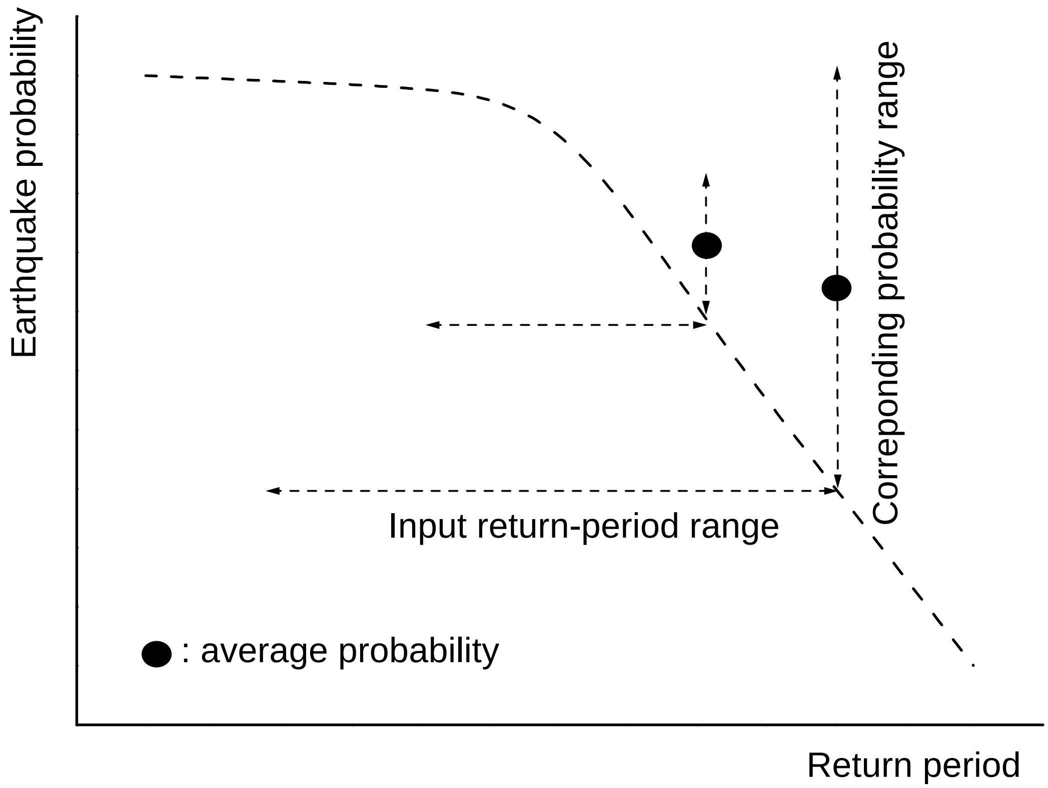

Fig. 10 A schematic graph explaining the average earthquake probability for the model application is decreased with a bigger range of return period, owing to the non-linear relationship between earthquake probability and return period

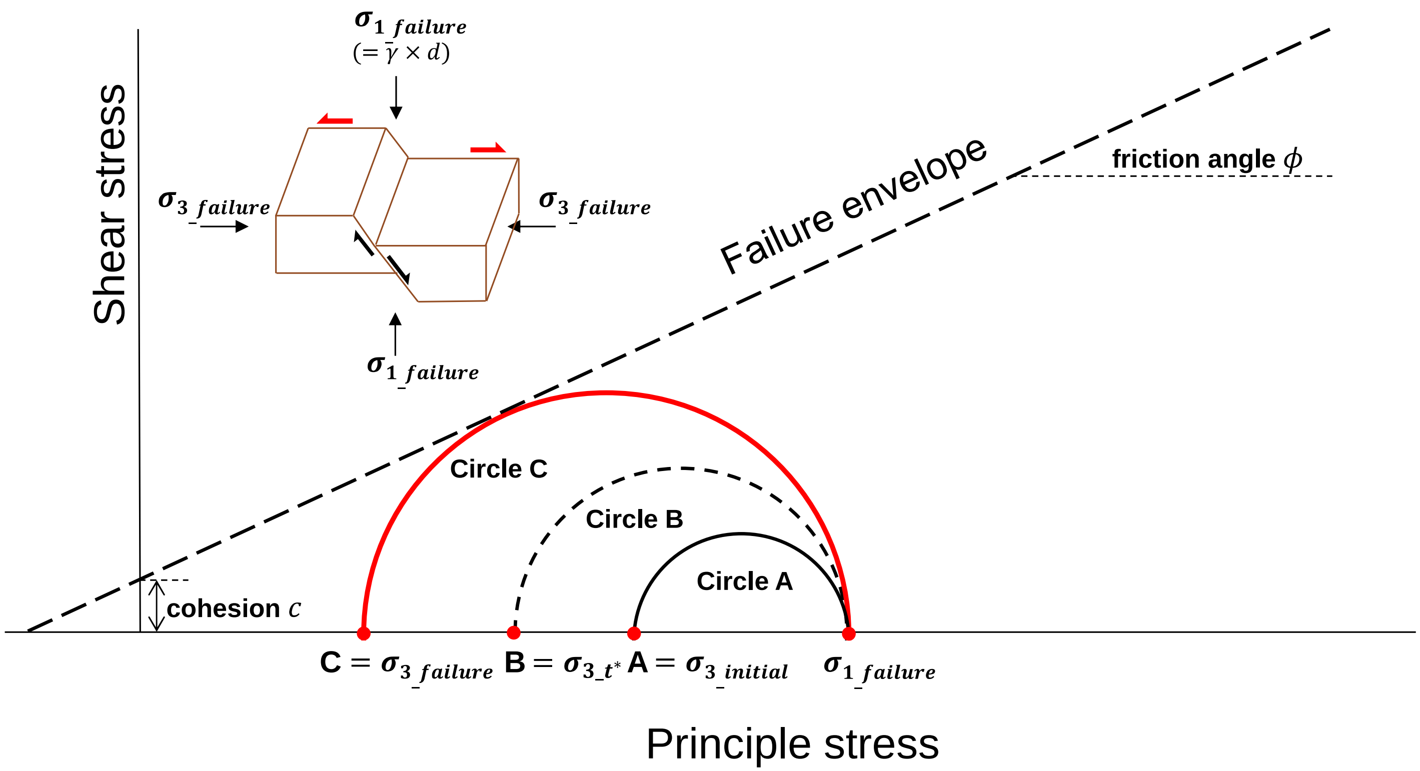

Fig. 11 The Mohr circles for evaluating the non-stationary earthquake probability for normal-fault earthquakes

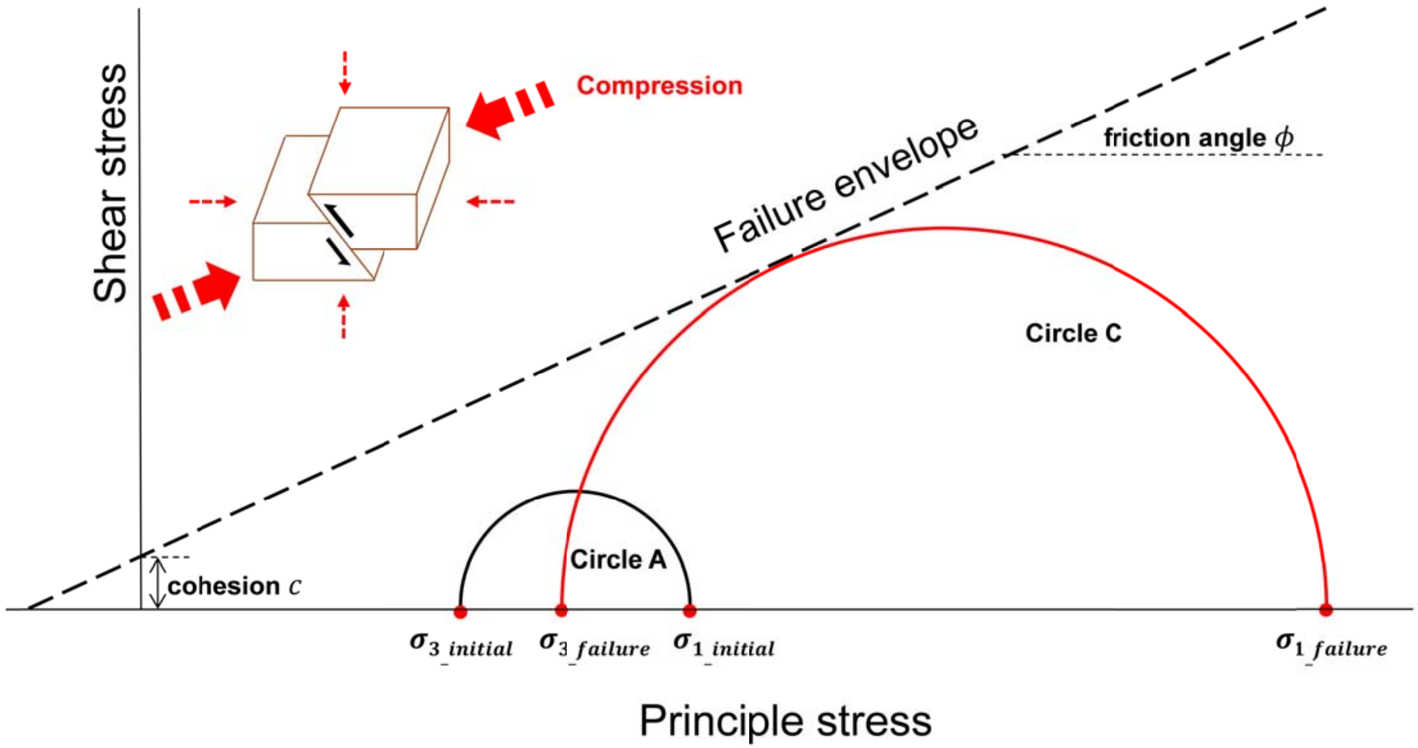

Fig. 12 The Mohr circles for evaluating the non-stationary earthquake probability subject to an oblique tectonic compression

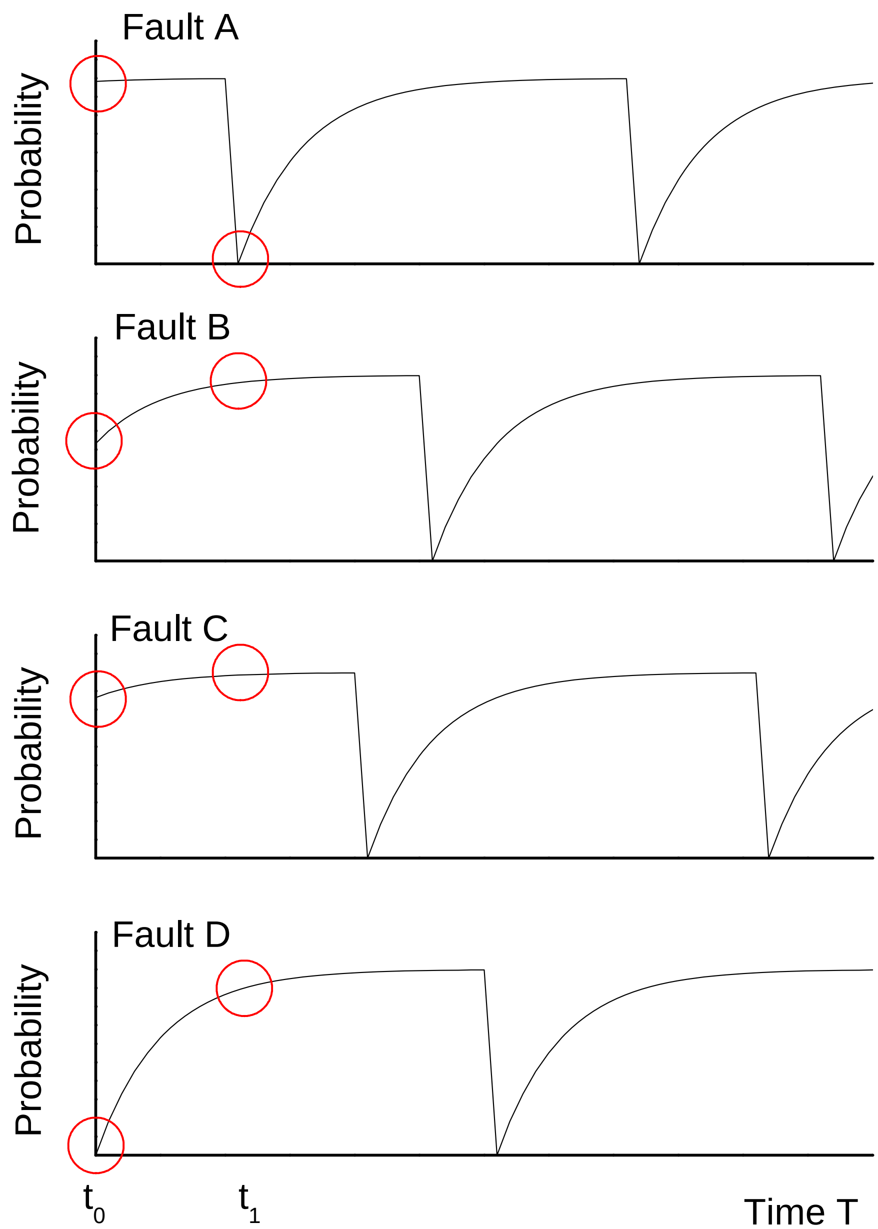

Fig. 13 Schematic diagram illustrating the stationary process after combining many non-stationary processes; taking $T = t_0$ and $T = t_1$ for example, the sum of that many non-stationary probabilities will be close to each other, although the probability is very low for Fault D at $T = t_0$, and it is very low for Fault A at $T = t_1$

Dear Editor Dr. Malamud, the anonymous reviewer, and reviewer Dr. Chan of NTU:

We thank you so much for providing us the valuable comments and suggestions, making our work much improved in so many aspects after revising. The point-by-point comment and response are given in the following. Moreover, for a better trace of our changes and responses in the revision, an annotated revision was attached at the end of this document.

Here, we would like to highlight the novelty of our study as follows: <u>**the new non-stationary earthquake probability assessment from the concepts of Mohr-Coulomb failure criterion**</u>. As receiving only few comments on the model basics and derivations, this novel idea seems agreeable with the Editor and Reviewers. Nevertheless, we endeavored to address other comments as best as we can, such as a more comprehensive literature review and the site-characterization issues in the model application.

<u>**Part I: Response to the comments of the anonymous reviewer**</u>

Comment 1.1

Discussion of the previous literature on non-stationary earthquake models. The manuscript does a poor job of building on previous non-stationary EQ models, with the latest one mentioned (in the introduction) from 1984. I would expect a much more thorough mention of non-stationary EQ models developed in the last 30 years, so it is clear that the present manuscript is BUILDING on these models, and proposing something different, rather than stating it has not been done.

Response:

The comment is highly appreciated and followed.

In the revision, a more comprehensive review on non-stationary models was given in Section Two. Please see <u>**lines 45-108 and Figs. 1-3**</u> of the annotated revision attached in the end.

Comment 1.2

Size of earthquake considered. Throughout the manuscript, there are words like "EQ after t years since last occurrence" or other such language. I realize that the actual model uses other parameters to give an idea of energy released, but can the entire manuscript be gone over to put ’size’ of the earthquake in context in the language used (or energy released, or other measure).

Response:

We followed the comment in the revision as much as possible, with our best understanding and interpretation on the comment “whether the new model can somehow predict earthquake magnitude or energy release when the event recurs.”

Our responses to “the comment” were added in **lines 236-243** of the revision. Basically, like other temporal earthquake analyses, our non-stationary assessment is to estimate the earthquake probability in a given period of time, while it cannot estimate earthquake magnitude or energy release when the event recurs.

(We apologize if the response went to a wrong direction. In case that happens, we would like the reviewer to elaborate the comment a little bit more, so that we can, and will, re-address it in the next revision.)

Comment 1.3

Aftershocks, foreshocks, main shock. Please include brief discussion of how these are included/not included in the model.

Response:

Comment followed. The discussion was given in **lines 230-235** of the revision.

Comment 1.4

[Minor] It would be beneficial to add a figure of the Meishan fault and its surroundings.

Response:

Comment followed. Please see **lines 261-263 and Fig. 7** of the revision.

Comment 1.5

[Minor] Please be clear in symbols, of ML vs. MW vs. other types of magnitude. I was actually surprised to see ML (local magnitude) being used for the earthquake in question.

Response:

Comment followed. Moment magnitude (M_w) was adopted while preparing the revision, such as in **lines 249 - 251.**

Comment 1.6

[Minor] Please add a table of variables used, and where they are introduced, as there are a lot of them.

Response:

Comment followed. A section of Notation was added in the revision. Please see **lines 140 and 396** of the revision.

- **The summary of response Part I**

Comment	Locations in the revision
1.1	**Lines 45-108, Figs. 1-3**
1.2	**Lines 236-243**
1.3	**Lines 235-240**
1.4	**Lines 261-263, Fig. 7**
1.5	**Lines 249-251**
1.6	**Lines 140 and 396**

** End of Part I

Part II: response to the comments of Dr. Chan

Comment 2.1

In this manuscript, the authors developed a new physics-based approach for earthquake forecasting and implemented to the Meishan Fault. The approach might be beneficial for subsequent studies on seismic hazard assessment. This work is interesting and the manuscript is well written. I have some comments, which are detailed in the attached file.

Response

The support is highly acknowledged.

Comment 2.2

Table 1, I am very surprised that there is no uncertainty for return period. Thus, the authors assumed that each earthquake is characteristic with identical stress release. However, many of theoretical models and observations disagree the assumptions. For example, after the 2011 Tohoku earthquake, the occurrence of events with normal mechanism suggests coseismic stress drop are larger than the accumulated stress loading. In addition, the return period along the Meishan Fault between the last two events (1792 and 1904) is 113 year, which is not consistent with the assumption.

Response

The comment is highly appreciated, and the summary of the response is as follows:

a) The reason we used the return period as 162 years in the manuscript is based on the literature. In the revision, we followed the suggestion then using the return periods as 162 ± 50 years and 162 ± 100 years in the model application. However, it must be noted that the "uncertainties" (i.e., ±50 and ±100) are from our best judgments.

b) We could not find the sources of "1792 and 1904 years" of the comment. As a result, our best-estimate return periods as 162 ± 50 and 162 ± 100 years were still based on an "average" value of 162 years from the literature.

c) Please see **lines 275-279 and Table 1** of the annotated revisions attached in the end.

Comment 2.3

The assumed parameters listed in Table 1 are mainly based on the references for general description of the parameters. It is desired to obtain specific parameters for the Meishan Fault and neighboring regional tectonic regime so that the uncertainty of the result might be minimized. Alternatively, application to other fault system with better investigation, e.g., the Chelungpu Fault, might provide a better demonstration for this approach.

Reponses:

Comment highly appreciated; the summary of the response is as follows:

a) The reason we used the Meishan earthquake as the application of the new non-stationary model is owing to its imminent earthquake risk. Moreover, the return period of the Meishan fault should be better characterized than other characteristic earthquakes in Taiwan, as used by a few recent studies.

b) Although the Chelungpu Fault is geologically well-investigated as commented, the fault's "rock mechanics" properties, such as the fault-plane strength parameters, the coefficient of lateral earth pressure in rock, and variability in stress annual increment, are not clear either, even for those geologically, well-investigated faults.

c) As mentioned in the beginning of the document, the novelty of the study is the new non-stationery earthquake probability assessment. Certainly, we also hope to see more studies focusing on site characterizations that could improve the reliability of an application as the model is used. However, this "huge" task is not within the scope of the study, aiming to develop a new non-stationary earthquake analysis.

d) Please **lines 252-260, 266-285, and 322-342** of the revision for the response.

Comment 2.4

It is known that the Meishan Fault as well as the 1906 earthquake is with strike-slip mechanism, i.e., both maximum (σ_1) and minimum (σ_3) principal axes are horizontal. I am not quite sure if the fault with strike-slip mechanism also fulfils the assumption of equation (7).

Responses

Comment followed. More statements were added in the revision to clarify that the non-stationary analysis is applicable to the three types of earthquake. Please see **lines 158-163, 345-352, Figs. 5 and 11** of the revision.

Comment 2.5

In ‘3 The Poissson process and earthquake probability’, I agree that the Poisson model is a stationary function. However, I expect the equation (2) and (3) are unnecessary since they are identical to equation (1).

Response:

Comment followed. The two equations were deleted in the revision.

Comment 2.6

Table 1, earthquake depth is a crucial parameter for the approach. However, I am confused if it is defined by hypocentral depth or rupture depth. In addition, according to field survey, the Meishan Fault obtains surface rupture, i.e., the range should be as shallow as 0 km.

Response:

The depth refers to the focal depth, and it was more clearly stated throughout the revision, such as in **lines 13, 226, 390, 396, etc…**

Comment 2.7

Table 1, I expect the authors want to express ‘Median value’ instead of ‘Central value’.

Response:

Comment followed. The central value was changed to “average.”

- **Summary of Reponses Part II**

Comment	Locations in the revision
2.1	**The support is acknowledged.**
2.2	**Lines 275-279**
2.3	**Lines 252-260, 266-285, and 322-342**
2.4	**Lines 158-163, 345-352, Figs. 5 and 11**
2.5	**Deleted in the revision**
2.6	**Lines 13, 226, 390, 396, etc…**
2.7	**Table 1**

*** End of Response Part II

Finally, we would like to thank you again for the valuable comments on the submission. We hope the responses satisfactorily address your concerns. If not, we are more than glad to make more revisions and explanations in the next round of revision.

Sincerely,

J.P. Wang
&
Yun Xu

Attachment: the annotated revision

Annotated Revision

A non-stationary earthquake probability assessment with Mohr-Coulomb failure criterion: including an application to central Taiwan

Abstract: From theory to experience, earthquake probability associated with an active fault should be gradually increasing with time since the last event. In other words, the process should be non-stationary, rather than being stationary as the Poisson process. In this paper, a new non-stationary earthquake assessment is introduced. Different from other analyses, the new model more clearly defines and calculates two stress states or boundary conditions between two consecutive earthquakes, facilitated with the Mohr-Coulomb failure criterion. In addition to the model development, this paper also presents a model application to evaluate earthquake probability associated with the Meishan fault in central Taiwan. Based on the best-estimate return period of 162 ± 50 years, focal depth of 4 ~ 8 km, etc., there could be a 7.6% probability for the fault to induce a major earthquake in years 2015 ~ 2025, and if the earthquake does not recur by 2025, the earthquake probability will increase to 8% in 2025 ~ 2035, a non-stationary probability depending on the starting dates of a given period of time. R2.6

Keyword: non-stationary earthquake probability assessment, Mohr-Coulomb failure criterion

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1. Introduction

Owing to our imperfect understandings and natural randomness of earthquake, several models have been proposed for estimating earthquake probability in a given period of time. Among them, the Poisson model might be the one that is mostly used in many applications (e.g., Weichert, 1980; Ang and Tang, 2007; Ashtari Jafari, 2010). However, it must be noted that the Poisson calculation is a "memory-less" model (Devore, 2008), meaning that the Poissonian probability is only a function of length of time, but irreverent to when the last earthquake was occurring.

However, it seems that the recurrence of a characteristic earthquake associated with a given active fault should not be stationary or memory-less. That is, the earthquake probability should be gradually increasing with time. Taking the recent Nepal earthquake in April 2015 for example, the probability for the very next Nepal earthquake to recur in 2015 ~ 2020 should be lower than that in 2115 ~ 2120, although the two have the same length of time.

The scope of this study is to develop a new non-stationary earthquake probability assessment, mainly from the concepts of the Mohr-Coulomb failure criterion. Meanwhile, this paper provides a comprehensive review on other non-stationary earthquake models (Section Two), followed by our non-stationary analysis (Section Three). Then, the new model was demonstrated with a model application to central Taiwan (Section Four), as well as model improvement and future work (Section Five).

2. An overview of non-stationary earthquake models

R1.1

In this section, we would like to provide a comprehensive review on non-stationary earthquake analyses and models. Specifically, we characterized the models into two groups, referred to as "statistical models" and "physical model." R1.1

2.1 Statistical models

Basically, those statistical models developed are more or less a derivative of the stationary Poisson model. For example, Vere-Jones and Ozaki (1982) proposed the use of a time-variant model parameter for the Poissonian calculation, making their model non-stationary although the calculation is still Poissonian in essence. Similarly, another work suggested the use of adjusted return period (related to current time and original return period) for the Poissonian calculation, in order to modify the Poisson model from stationary to non-stationary (Wang et al., 2013).

Another type of modification is to use non-exponential distributions to model earthquake inter-occurrence time intervals as a random variable. (Note that for an event modeled by a Poisson process, the number of events in a given period of time is a discrete random variable following the Poisson distribution; meanwhile the time when the next event would recur is a continuous variable following the exponential distribution.) For example, the log-normal distribution (Ferráes, 2005), Weibull distribution (Yakovlev et al., 2006), and Gamma distribution (Gómez and Pacheco, 2004) have been suggested for the replacement of the exponential distribution, with them all featuring a non-stationary analysis after such modifications.

Based on given earthquake data, it must be noted that the statistical models are all empirical in a sense. In other words, the models are in no consideration of earthquake mechanisms, such as tectonic stress accumulation under the ground. R1.1

2.2 Physical models

In consideration of earthquake mechanics, several non-stationary earthquake analyses have also been proposed from a different perspective. It must be noted that the models are not entirely a "product" of physics, but somehow on the basis of the concepts of physics working together with empirical models. Specifically, we would like to introduce three of them in the following that are more related to our non-stationary earthquake model.

The first one we like to introduce here is the time-predictable model (Shimazaki and Nakata, 1980). Fig. 1 is a schematic diagram illustrating the model basics. Essentially, the model is relying on a best-estimate relationship between co-seismic fault slip (or displacement) and time. For instance, given the last event with fault slips as Points A and B (see Fig. 1), then the next event should recur at the time of Points C and D. In other words, the recent event with a smaller fault slip should accompany a smaller stress drop, and under a constant stress increment with time, it should lead to a shorter time for the stress to re-reach a stress level (or failure stress state) that could induce earthquakes.

The next model of the group is the Brownian model (Ellsworth, 1995; Matthews et al., 2002). By contrast to the time-predictable model, the Brownian model is not on the basis of a constant stress increment, while considering the stress increments between two consecutive events should be a stochastic process like Fig. 2. Specifically, the model considers the stress-time series is a combination of a long-term stress increment and a Brownian motion simulating transient stress randomness. With such a function, we can estimate the time of the next earthquake by examining if the stress reaches the failure state within a given period of time. R1.1

The third one we like to introduce is the negative binomial model (Tejedor et al., 2015). As the previous analyses, the model is also on the basis of two imaginary stress states. As shown in Fig. 3, the essence of the model is that the stress change in unit time could be modeled by two scenarios: stress does and does not increase. As a result, there are many possible "stress routes" (as shown in Fig. 3) between two consecutive events, and the probability and the total time of each route could be calculated with given earthquake return periods. Finally, the inter-occurrence time interval can be derived as a negative binomial distribution for such a non-stationary probability assessment.

To sum up, the three physical models are all facilitated with two stress states that are part of the earthquake occurrence theories generally accepted. Somehow, we do share this perspective for our model development. However, the biggest difference is that our model defines and calculates the two stress states more clearly, on the basis of the Mohr-Coulomb failure criterion that is well established and used in rock mechanics, structural geology, etc.

3. The new non-stationary earthquake probability assessment

3.1 Overviews of Mohr-Coulomb failure criterion and elastic rebound theory

The Mohr-Coulomb failure criterion is a model describing the response of materials subject to external stresses (Pariseau, 2007), and it was commonly applied to rock mechanics as well as other applications. Fig. 4 is a schematic diagram illustrating the essentials of the model. Basically, as the Mohr circle is below the failure envelope, a shear failure is not expected in the material. By contrast, as long as the Mohr circle is in contact with the failure envelope, a shear failure could occur.

On the other hand, it is generally accepted that the ongoing tectonic activities are the main reason causing rock failures under the ground, resulting in an earthquake with the release of accumulated strain energy. Afterward, the energy re-accumulates and re-releases until the next earthquake, and such a theory is referred to as the elastic rebound theory (Keller, 1996), proposed by Reid in the early twentieth century (Reid, 1910).

3.2 The model basics and the algorithms

The two earthquake theories above were mainly the motivation of the new non-stationary model: 1) based on the Mohr-Coulomb failure criterion, the rock subject to the stress state as Mohr Circle C (see Fig. 4) should fail and cause an earthquake, at which we refer to it as failure state; 2) from the elastic rebound theory, the stress state in the rock right after a characteristic earthquake should be restored to Mohr Circle A, which is called the initial state at time t_0.

As a result, the problem to evaluate the earthquake probability within a given time t^* after the last event (or after t_0) is becoming a problem as follows: What is the chance for the major principle stress at time t^* (denoted as $\sigma_{1\_t^*}$) greater than the major principle stress at the failure state (denoted as $\sigma_{1\_failure}$)? Or the question can be mathematically expressed by the following equation:

$$\Pr(earthquake\ within\ t^*\ after\ t_0) = \Pr(\sigma_{1\_t^*} > \sigma_{1\_failure}) \quad (1)$$

Clearly, the problem now is governed by two variables $\sigma_{1\_t^*}$ and $\sigma_{1\_failure}$, and their relationships with other parameters will be detailed later. Note that those notations used in the following derivations are summarized in the end of the paper. R1.6

- The major principle stress at failure state, $\sigma_{1\_failure}$

Based on the Mohr-Coulomb failure criterion, the major principal stress at failure state (Point C in Fig. 4) can be expressed as a function of the minor principal stress at failure ($\sigma_{3\_failure}$), and two strength parameters of the shearing plane, i.e., cohesion c and friction angle ϕ (Pariseau, 2007):

$$\sigma_{1\_failure} = \sigma_{3\_failure}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2c \times \cos\phi}{1-\sin\phi} \quad (2)$$

- The minor principal stress at failure state, $\sigma_{3\_failure}$

The minor principal stress at failure is attributed to the overburden earth pressure above the focal depth d, which can be estimated with the following formula based on rock mechanics:

$$\sigma_{3\_failure} = \gamma \times d \quad (3)$$

where γ is rock unit weight. It must be noted that this model considers $\sigma_{3\_failure}$ as time-invariant (more discussion is given later), and the case shown in Fig. 4 and Eq. 3 is for a thrust-fault earthquake. As for the strike-slip fault, the Mohr circles of the initial state and the failure state are shown in Fig. 5, indicating $\sigma_{3\_failure}$ is equal to $\gamma \times d \times K$ for this case, where K is the coefficient of lateral earth pressure in rock. More discussion over model improvements is given in Section 5.2. R2.4

- The major principle stress at time t^*, $\sigma_{1\_t^*}$

With tectonic stress increasing with time, the key task of the new analysis is to estimate the major principle stress at time t^* after the last event. For thrust-fault earthquakes as those Mohr circles shown in Fig. 4, the major principle stress at time t^* can be formulated as follows:

$$\sigma_{1\_t^*} = \sigma_{3\_initial} + t^* \times ASI \quad (4)$$

where $\sigma_{3\_initial}$ is the minor principal stress at the initial state (or at t_0) and ASI is called annual stress increment. Note that from Fig. 4 (the thrust fault) and Fig. 5 (the strike-slip fault), $\sigma_{3\_initial}$ is equal to $\gamma \times d \times K$ for the two cases of the non-stationary analysis.

3.3 The return period $\tilde{t}$ and its relationship with σ_{1_t*}

In addition to γ, d, K, etc., the return period $\tilde{t}$ of characteristic earthquakes is another input data of the non-stationary analysis. Moreover, the mean value and standard deviation of σ_{1_t*} can be expressed as a function of $\tilde{t}$, and used for developing its probability density function for the non-stationary probability assessment within the given time $t*$.

From the meaning of return period, it is understood that the event will recur when return period $\tilde{t}$ is due. As a result, the major principal stress at return period $\tilde{t}$ (denoted as $\sigma_{1\_\tilde{t}}$) should be equal to $\sigma_{1\_failure}$:

$$\sigma_{1\_\tilde{t}} = \sigma_{3\_initial} + \tilde{t} \times ASI = \sigma_{1\_failure} \tag{5}$$

Therefore, the mean value of ASI (denoted as μ_{ASI}) can be derived as follows:

$$\begin{aligned} & E[\sigma_{1\_failure}] = E[\sigma_{3\_initial} + \tilde{t} \times ASI] \\ & \Rightarrow E[ASI] = \frac{\sigma_{1\_failure} - \sigma_{3\_initial}}{\tilde{t}} = \mu_{ASI} \end{aligned} \tag{6}$$

where $E[]$ denotes the mean value of a variable in probability and statistics.

On the other hand, as the variability of annual stress increment is equal to n in terms of coefficient of variation (= standard deviation / mean value), its standard deviation (denoted as s_{ASI}) can be derived as follows with its mean value from Eq. 6:

$$n = \frac{s_{ASI}}{\mu_{ASI}} \Rightarrow s_{ASI} = n \times \mu_{ASI} = \frac{n \times \left(\sigma_{1\_failure} - \sigma_{3\_initial}\right)}{\tilde{t}} \tag{7}$$

With the mean (Eq. 6) of *ASI*, we can continue deriving the mean value of the major principal stress at time t^*:

$$\begin{aligned} &\sigma_{1\_t^*} = \sigma_{3\_initial} + t^* \times ASI \\ &\Rightarrow E[\sigma_{1\_t^*}] = E[\sigma_{3\_initial} + t^* \times ASI] = \sigma_{3\_initial} + t^* \times E[ASI] \\ &\qquad = \sigma_{3\_initial} + \frac{t^* \times \left(\sigma_{1\_failure} - \sigma_{3\_initial}\right)}{\tilde{t}} \end{aligned} \tag{8}$$

Similarly, the standard deviation of the major principal stress at time t^* (denoted as $s_{\sigma1\_t^*}$) can be derived as follows with s_{ASI} in Eq. 7:

$$\begin{aligned} &\sigma_{1\_t^*} = \sigma_{3\_initial} + t^* \times ASI \\ &\Rightarrow V[\sigma_{1\_t^*}] = V[\sigma_{3\_initial} + t^* \times ASI] = t^{*2} \times V[ASI] = t^{*2} \times s_{ASI}^2 \\ &\Rightarrow s_{\sigma1\_t^*} = \sqrt{V[\sigma_{1\_t^*}]} = t^* \times s_{ASI} = \frac{t^* \times n \times \left(\sigma_{1\_failure} - \sigma_{3\_initial}\right)}{\tilde{t}} \end{aligned} \tag{9}$$

where *V*[] denotes variance in probability and statistics, and it is the square of standard deviation.

In order to establish the probability density function of $\sigma_{1\_t^*}$, the information about what probability distribution the variable is following is as essential as its mean value and standard deviation. But since the distribution of $\sigma_{1\_t^*}$ is unknown (to the best of our knowledge, no study has ever worked on the subject), we suggest the normal distribution for this non-stationary earthquake assessment, as it is usually recommended for a probability analysis when the variables' distribution is unknown (Abramson et al., 2002).

3.4 Summary

Fig. 6 is a schematic diagram illustrating the essentials of the non-stationary assessments. The key to the model is to estimate the probability distribution of the major principal stress at time t^* after the last event (or after t_0), and compares it to the stress that could cause rock failures and earthquakes. To sum up, the new non-stationary model is governed by a total of six parameters as follows: return period ($\tilde{t}$), fault-plane strength parameters (c and ϕ), rock unit weight (γ), earthquake focal depth (d), and the variability of annual stress increment in terms of coefficient of variation (n).

R2.6

3.5 Presumption and limitation

R1.3

The elastic rebound theory is a plausible explanation to earthquake, but specifically speaking, it is more of a theory about main shocks. As a result, the new non-stationary analysis of the study motivated by such a theory is more applicable to main shocks, a situation similar to other non-stationary models that are also applicable to main shocks rather than dependent shocks (Shimazaki and Nakata, 1980; Ellsworth, 1995; Matthews et al., 2002; Tejedor et al., 2015).

On the other hand, like any other stationary or non-stationary analyses estimating earthquake probability in a given period of time, our model cannot predict the magnitude of the recurring event, either. In other words, the (earthquake) temporal analyses closely related to return period, stress increment, etc. do not further relate the variables to earthquake magnitudes or energy release. Again, such a framework is similar to other stationary or non-stationary temporal analyses only focusing on the earthquake probability in a given period of time, but not on the probability distribution of earthquake magnitude or energy release when the event recurs. R1.2

4. A model application

4.1 The Meishan earthquake in central Taiwan

The region around Taiwan is known for high seismicity owing to the location close to the boundaries of tectonic plates. On average, there are around 2,000 earthquakes above M_w 3.0 (moment magnitude) occurring around Taiwan every year, with a catastrophic event, like the M_w 6.4 Meishan earthquake in 1906 and the M_w 7.6 Chi-Chi earthquake in 1999, that could recur in decades. R1.5

As a result, we would like to apply the new non-stationary model to Taiwan as a case study. Specifically, we selected the Meishan fault in central Taiwan as the model application, given a few recent studies pointing out the fault should be of "imminent" earthquake risk, for the event's return period being "almost" due (e.g., Wang et al., 2012). By contrast, the reason we did not select the Chelungpu fault as the application is because R2.3

the active fault should be of lower earthquake risk in next couple decades, given it "just" induced the Chi-Chi earthquake in 1999, and should have a longer return period (about 250 years) than the Meishan earthquake (Cheng et al., 2007). More discussion over this model application is given in Section 5.1.

Fig. 7 shows the location of the Meishan fault in central Taiwan. Accordingly, R1.4 the fault is very close to a major city (i.e., Chiayi) in central Taiwan, and reportedly the 1906 Meishan earthquake killed around 1,200 people in the area.

4.2 The best-estimate data from the literature

Table 1 summarizes our best-estimate data from the literature for the non- R2.3 stationary earthquake assessment on the Meishan fault. It must be noted that because the strength parameters of the fault plane are not clear, we used a typical range (see Table 1) from rock mechanics as our best estimates. Similarly, a probable range of 0.2 ~ 0.5 was used as our best estimate for the coefficient of lateral earth pressure in rock, given no site-specific studies and data have been reported. As for the earthquake focal depth, we considered the depth should be close to 6 km as the last Meishan earthquake (Ng et al., 2009). However, in order to account for the focal-depth uncertainty in the analysis, we used a best-estimate range as 4 ~ 8 km.

A similar situation was encountered in the determination of the best-estimate R2.2 return period. On the basis of 162 years used in recent studies (e.g., Wang et al., 2012), two best-estimate ranges were determined as 162 ± 50 years and 162 ± 100 years. Understandably, the uncertainties (i.e., ± 50 and ± 100) are from our best judgments, given such information is not clear from the literature. As for the variability of annual stress increment, to the best of our knowledge, there is no any research so far that can really answer the question. As a result, the range of 0.25 ~ 1 was used as our best estimate characterizing the variability of annual stress increment in terms of coefficient of variation. R2-3

More discussion about the input data characterizations and the model application is given in Section 5.1 in the following.

4.3 Monte Carlo Simulation

Because our input data were characterized by a range rather than a single value, it is difficult to solve the governing equation (Eq. 1) of the non-stationary probability with analytical approaches. Therefore, we used Monte Carlo Simulation (MCS) to solve the problem as many MCS applications. For more details about Monte Carlo Simulation, readers can refer to the textbooks of Ang and Tang (2007), Abramson et al. (2002), among many others.

4.4 The result

With the best-estimate input data summarized in Table 1, Fig. 8 shows the average probabilities for three 10-year periods from Monte Carlo Simulation with a sample size of 5,000. For example, the non-stationary model shows a 7.6% probability for the Meishan fault in central Taiwan to induce a major earthquake in years 2015 ~ 2025, under the return period of 162 ± 50 years. Then, if the event does not recur by 2025, the earthquake probability in 2025 ~ 2035 will increase to 8%; similarly, if the event does not recur by 2035, the probability will further increase to 8.4%. Note that the standard deviations of the three probability estimates are all close to 3.3%, which is a reflection to the input data that were characterized by a range. In other words, if the input data were all characterized by single values, the standard deviation of the probability estimates cannot be calculated and reported.

In addition, the Poissonian probabilities for the same problem are also shown in Fig. 8. It shows that for a 10-year period of time, the earthquake probability is about 6% for the three different periods (i.e., 2015 ~ 2025, 2025 ~ 2035, and 2035 ~ 2045), or the probability is irrelevant to the starting dates of the time period.

Fig. 9 shows the result for the other scenario under return period as 162 ± 100 years. Interestingly, the average probabilities for the three 10-year periods become relatively close to one another, and they are smaller than those estimates subject to the return period of 162 ± 50 years. Our explanation to this is as follows: As shown in Fig. 10, the relationship between return period and earthquake probability could be highly non-linear. Therefore, the average probability subject to a bigger range of return period would be lower than that subject to a smaller range. Nevertheless, with the non-stationary model, the probability estimates do vary with the starting dates of the time, or the probabilities are indeed non-stationary, in contrast to the stationary Poisson process.

5. Discussions

5.1 Input data characterizations

R2.3

As many analyses, input data characterizations are equally challenging as the model development. As a result, for improving the model estimates, we hope to see more studies focusing on site characterizations with more laboratory works or field instrumentation assessing stress increment variability, fault-plane strength parameters, lateral earth pressure in rock, etc. However, this is beyond the scope of the study focusing on a new non-stationary model development. R2-3

On the other hand, one could argue why not choose a geologically well-investigated fault (e.g., the Chelungpu fault) in Taiwan as the model application to reduce uncertainty, and here is our response: The "rock-mechanics" parameters of the model (such as lateral pressure coefficients, variability of stress increment, and the strength parameters of fault planes) are not clear either, even for those so-called well-investigated faults in Taiwan. As a result, no matter which fault was selected as the model application, engineering judgment must involve in the determinations of those "rock-mechanics" parameters, more or less creating the same level of uncertainty when it comes to site characterizations on stress increment variability, lateral earth pressure in rock, etc.

Besides, as mentioned previously the key reason of using the Meishan fault as a case study is owing to its imminent earthquake risk, not to mention the well-characterized return period of 162 years from the Central Geological Survey Taiwan (Lin et al., 2008) could somewhat help increase the reliability of the estimate, in a comparison to other cases without a well-characterized earthquake return period, or at least not yet reported.

5.2 Model improvements

R2.4

Certainty, the non-stationary analysis of the study can be further modified. For example, in addition to the algorithms for thrust faults and strike-slip faults that have been derived, the model is also applicable to normal faults as those Mohr circles shown in Fig. 11. Accordingly, the minor principal stress at initial state ($\sigma_{3\_initial}$) is equal to

$\gamma \times d \times K$ for this case, with the minor principal stress at time t* (denoted as $\sigma_{3\_t^*}$) that can be expressed as $\sigma_{3\_initial} - t^* \times ASI$. Next, the same probability calculation is applicable by comparing the minor principal stress at t* to the minor principal stress at failure ($\sigma_{3\_failure}$), for such a normal-fault earthquake subject to tectonic extension.

Further improvements can be conducted with the consideration of the direction of stress increment, or the direction of tectonic compression/extension. Under the circumstances, the Mohr circles of the initial state and failure state are shown in Fig. 12, with major and minor principal stresses both varying with time.

Nevertheless, no matter how the non-stationary model will evolve, such type of non-stationary analysis from the Mohr-Coulomb failure criterion is as novel, robust and transparent as its counterparts, providing a new alternative to non-stationary earthquake assessment related to a given active fault.

5.3. Earthquake should be stationary or non-stationary?

Although characteristic earthquakes related to a given active fault should be non-stationary, in the 1970s a study has provided statistical evidence to the opposite: earthquake is stationary (Gardner and Knopoff, 1974). However, it must be noted that the study was not focusing on characteristic earthquakes, but based on the regional seismicity in California.

Fig. 13 is a schematic diagram that helps explain the difference between the two problems. For each fault, the recurring earthquake should be a non-stationary process, and the non-stationary earthquake probability would be reset at the last event and gradually increase with time. By contrast, the seismicity in a region would become stationary with so many non-stationary processes present. For example, at $T = t_0$ (see Fig. 13), the sum of that many stationary probabilities should be close to that at $T = t_1$ (or at any moment), although the earthquake probability induced by Fault D should be very low at $T = t_0$, while others are higher.

The relationship can be simply explained with the patron-and-bank analogy. For each patron (analogy to each fault), going to the bank is obviously a non-stationary process, with the probability increasing with time since the very last visit. But for the banks (analogy to the seismicity), it is a stationary process for them, as dealing with so many patrons or so many non-stationary processes at one time.

6. Summary and conclusion

Given the earthquake recurrence associated with an active fault that should be a non-stationary process, this paper introduces a new non-stationary analysis to evaluate earthquake probability within a given period of time. Different from previous models, the new analysis more clearly defines and calculates two earthquake stress states, on the basis of the well-established, Mohr-Coulomb failure criterion.

In addition, this paper also presents a model application to evaluate earthquake probability associated with the Meishan fault in central Taiwan. With the best-estimate return period of 162 ± 50 years, focal depth of 4 ~ 8 km, etc., the active fault has a 7.6% R2.6 probability (standard deviation equal to 3.3%) of inducing the next Meishan earthquake in 2015 ~ 2025, and if the earthquake does not recur by 2025, then the non-stationary probability will increase to 8% in 2025 ~ 2035, rather than being unchanged, stationary, or independent of the starting dates of a given period of time.

Notations

R1.6

t_0	The time when the last event occurs
t^*	The time interval after the last event
$\sigma_{1\_t^*}$	Major principle stress at time t^*
$\sigma_{1\_failure}$	Major principle stress at failure state
$\sigma_{3\_failure}$	Minor principal stress at failure state
c	Cohesion of the fault plane
ϕ	Friction angle of the fault plane
d	Earthquake focal depth R2.6
γ	Rock unit weight
K	Coefficient of lateral earth pressure in rock
ASI	Annual stress increment
$\tilde{t}$	Earthquake return period
$\sigma_{1\_\tilde{t}}$	Major principal stress at return period
E	Expected value or mean value
V	Variance
μ_{ASI}	Mean value of ASI
n	Coefficient of variation for ASI
s_{ASI}	Standard deviation of ASI
$s_{\sigma 1\_t^*}$	Standard deviation of $\sigma_{1\_t^*}$
$\sigma_{1\_initial}$	Major principal stress at initial state
$\sigma_{3\_initial}$	Minor principal stress at initial state
$\sigma_{3\_t^*}$	Minor principal stress at t^*

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R.2.2

Table 1. Summary of the model parameters used in the analyses

Parameters	Focal depth (km)	Unit Weight (kN / m^3)	Cohesion (MN / m^2)	Friction angle (degrees)	Return period (years)	K*	n**
Range	4 ~ 8	25 ~ 30	3.6 ~ 22.7	22 ~ 46	112 ~ 212 (162 ± 50) 62 ~ 262 (162 ± 100)	0.2 ~ 0.5	0.25 ~ 1.0
Average	6	27.5	13.2	34	162	0.35	0.63

R 2.7

* K = the coefficient of lateral earth pressure in rock; ** n = the coefficient of variation for annual stress increment

R1.1

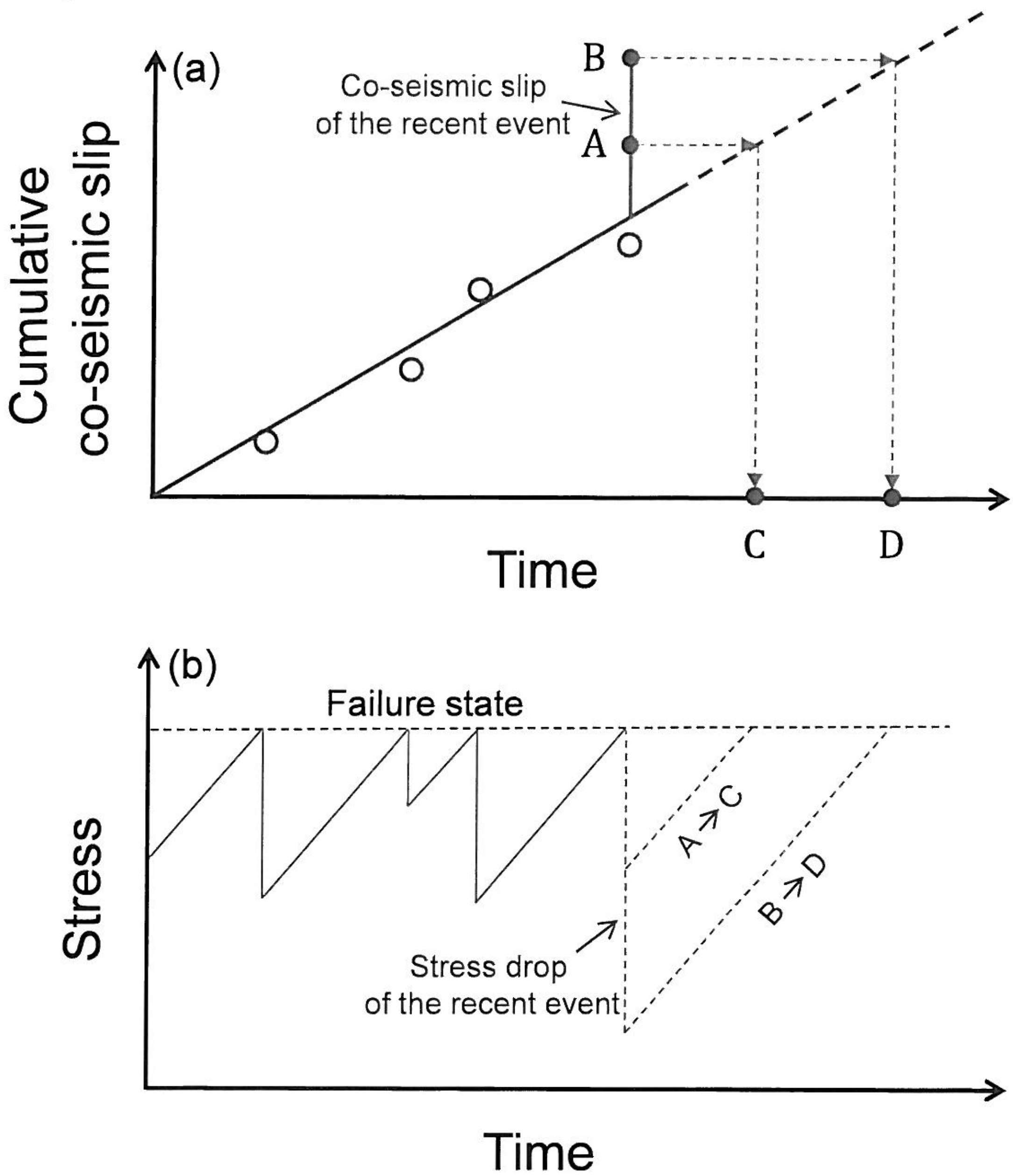

Fig. 1 Schematic diagram for the time-predictable model: a) best-estimate relationship between cumulative co-seismic slips and time, and b) the earthquake-time prediction facilitated with a failure state and a constant stress increment

R1.1

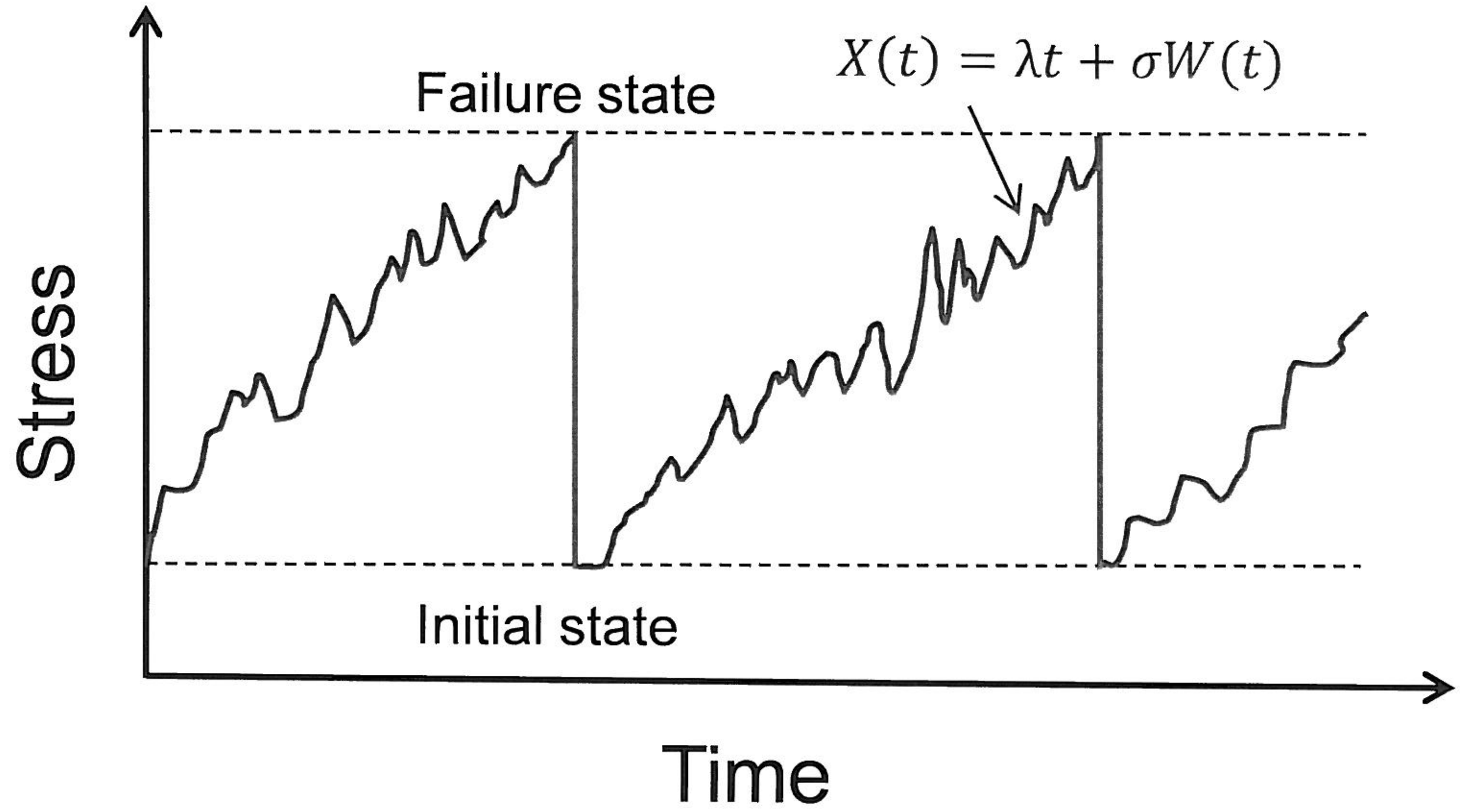

Fig. 2 Schematic diagram showing the essential of the Brownian model; within the two imaginary stress states, the model considers the stress-time series should be random and could be modeled by a long-term stress increment and a Brownian motion as $X(t) = \lambda t + \sigma W(t)$, where $X(t)$ is the stress at time t, λ is long-term stress increment rate, σ is the magnitude of a Brownian motion $W(t)$.

R1.1

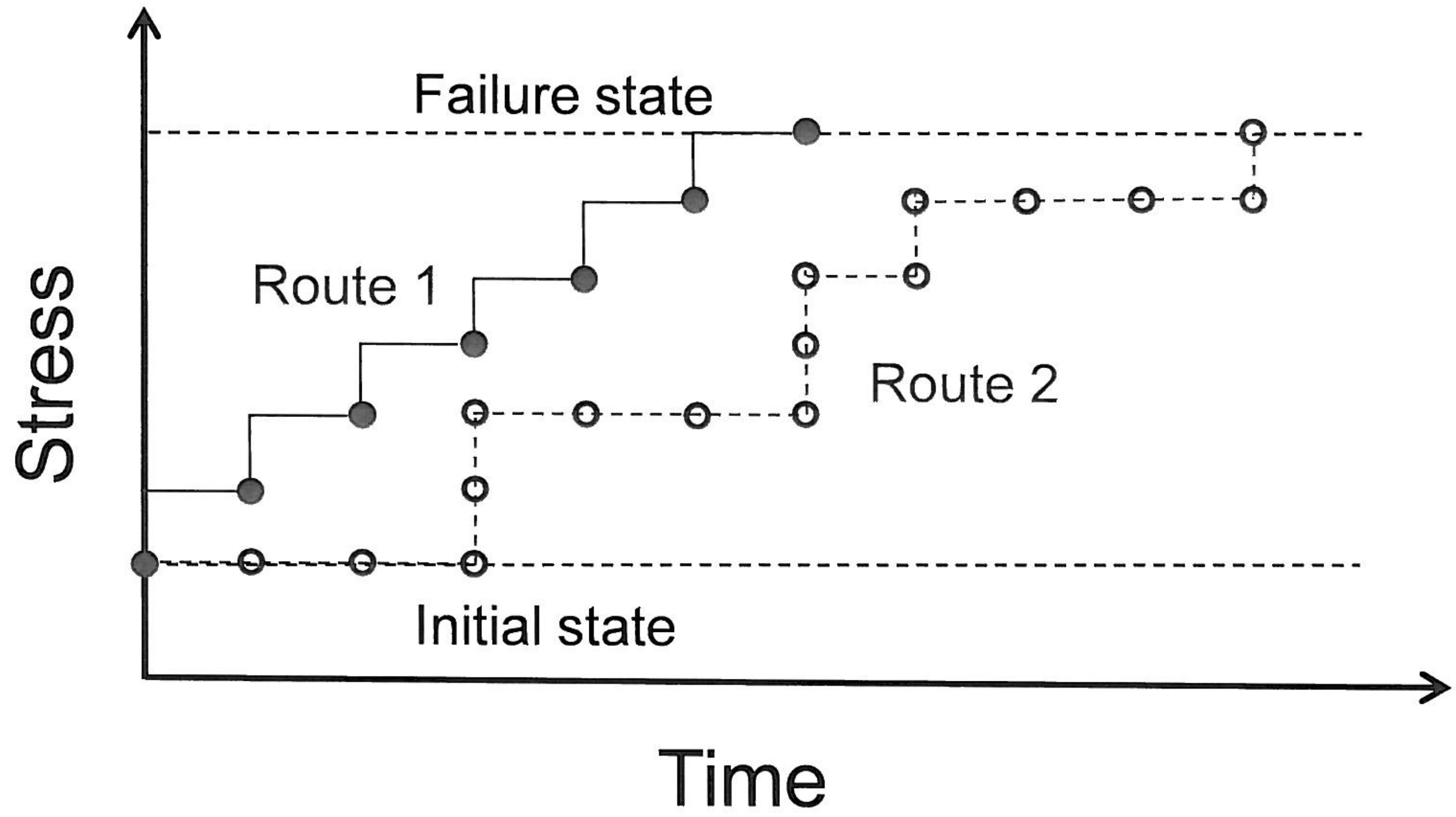

Fig. 3 Schematic diagram illustrating the negative binomial model; between the two stress states, many "stress routes" can be present, and the probability of each route can be estimated with the model, then developing the probability distribution for the interval between two consecutive events

R2.4

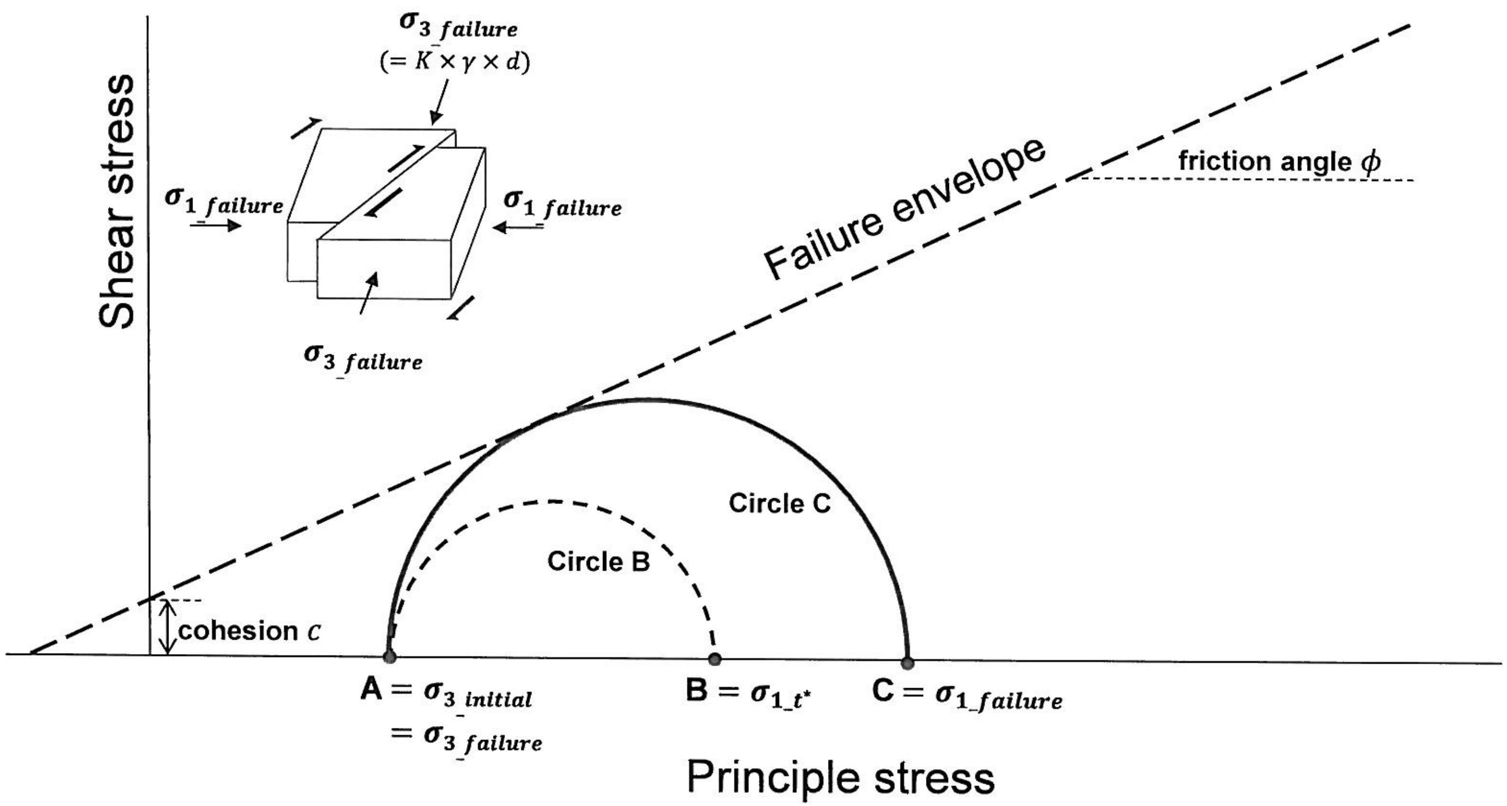

Fig. 5 The Mohr circles for evaluating the non-stationary earthquake probability for strike-skip earthquakes

R1.4

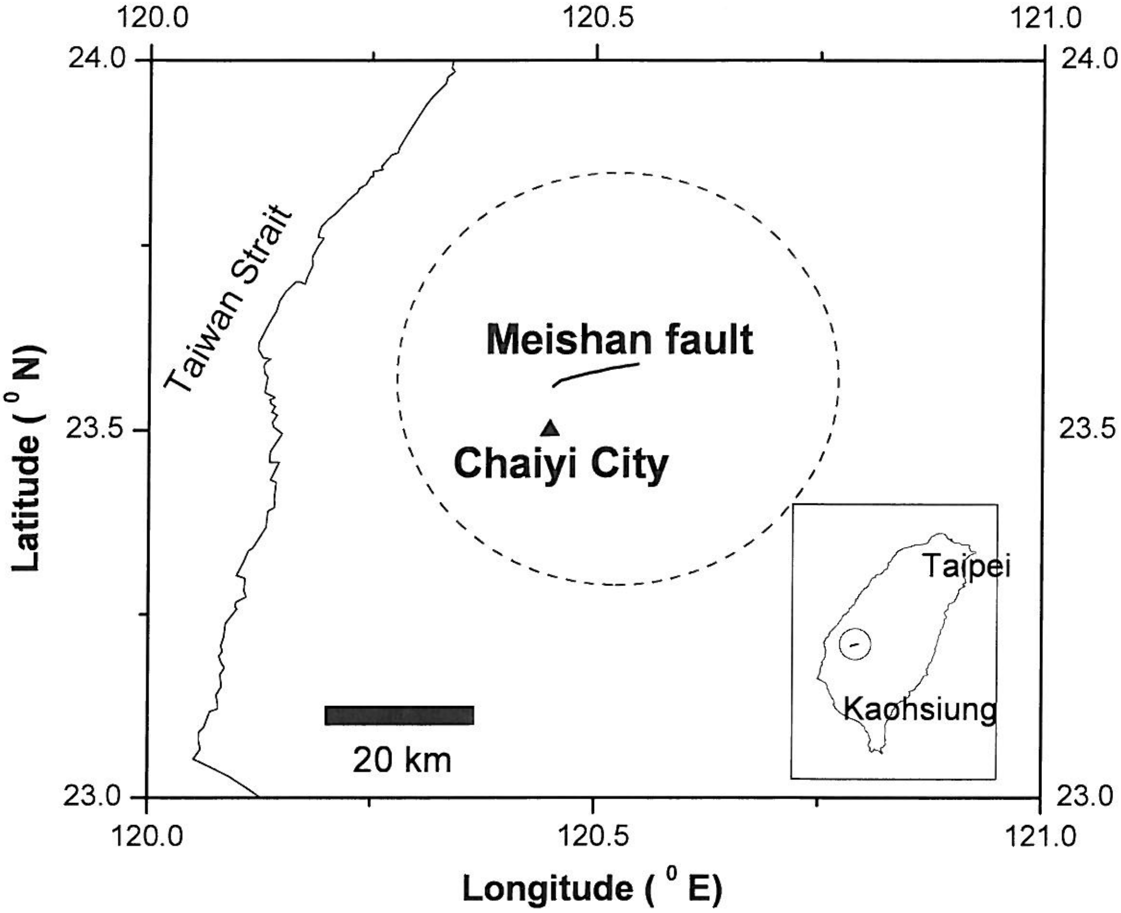

Fig. 7 The location of the Meishan fault in central Taiwan

R 2.4

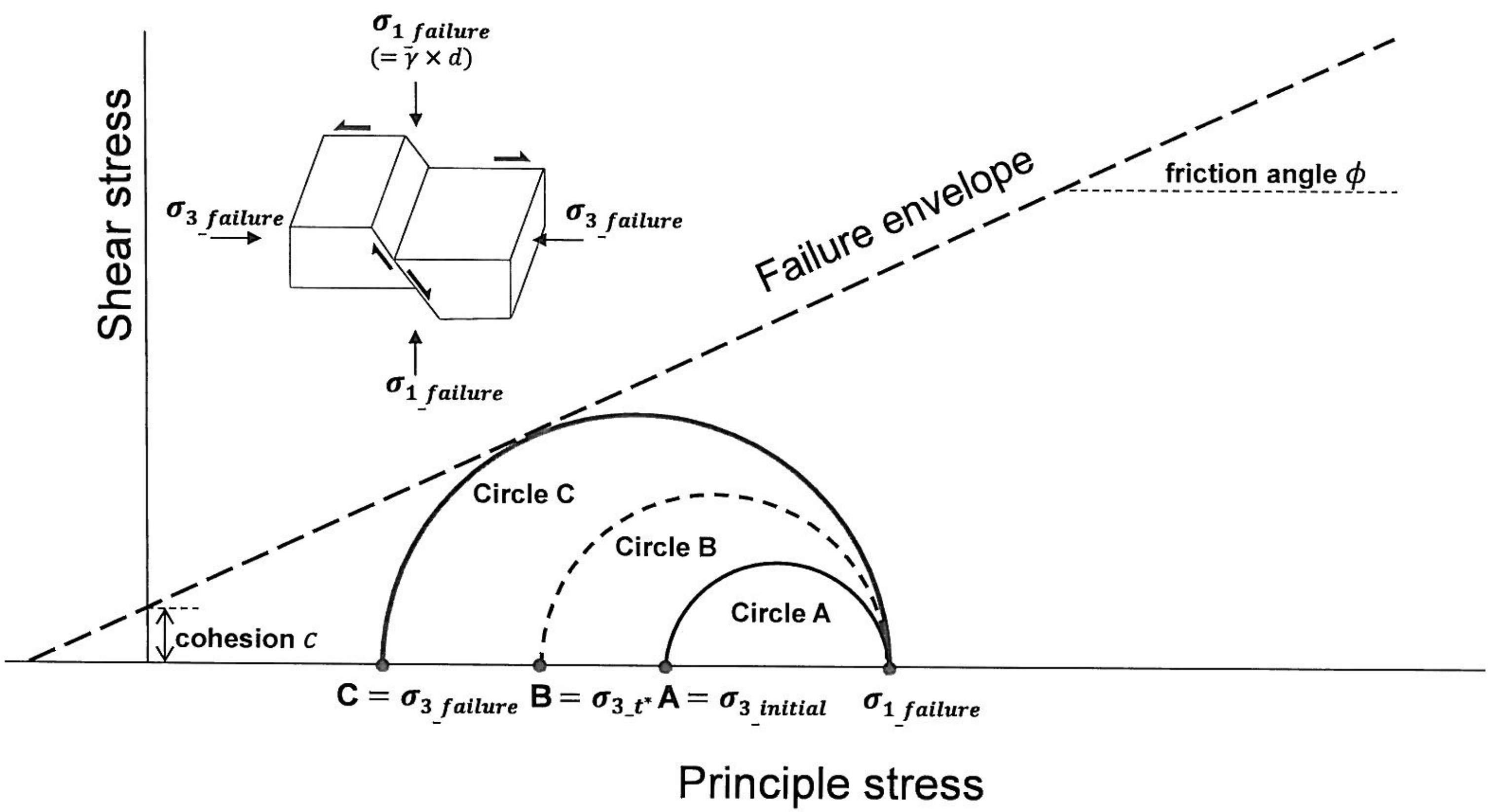

Fig. 11 The Mohr circles for evaluating the non-stationary earthquake probability for normal-fault earthquakes