[1]E. A.Podolskiy [1]G.Chambon [1]M.Naaim [2]J.Gaume
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Evaluating snow weak-layer failure parameters through inverse Finite Element modeling of shaking-platform experiments

IRSTEA (UR ETGR) – Centre de Grenoble, 2 rue de la Papeterie, BP 76, 38402 St.-Martin-d'Hères CEDEX, France

WSL/SLF, Swiss Federal Institute of Snow and Avalanche Research, 7260 Davos Dorf, Switzerland

Correspondence to: E. A. Podolskiy (evgeniy.podolskiy@gmail.com)

Abstract

abstr Snowpack weak layers may fail due to excess stresses of various natures, caused by snowfall, skiers, explosions or strong ground motion due to earthquakes, and lead to snow avalanches. This research presents a numerical model describing the ~~behavior~~ failure of "sandwich" snow samples subjected to shaking. The Finite Element model treats weak layers as interfaces with variable ~~constitutive behavior~~ mechanical parameters. This approach is validated by reproducing cyclic loading snow fracture experiments. The model evaluation revealed that the Mohr–Coulomb failure criterion, governed by cohesion and friction angle, was adequate to describe the experiments. The ~~"best fit" cohesion and friction angle were ≈1.6kPa and 22.5–60°, indicating that the cohesion mainly determines the outcome of tests. The model showed~~ model showed the complex, non-homogeneous stress evolution within the snow samples and especially the ~~significance of tension for~~ importance of tension on fracture initiation at the edges of the weak layer, caused by dynamic stresses due to shaking. Accordingly, ~~the previously used~~ simplified analytical solution, ignoring the inhomogeneity of tangential and normal stresses along the failure plane, may incorrectly estimate the shear strength of the weak layers. The ~~obtained parameters~~ "best fit" cohesion and friction angle were ≈ 1.6 kPa and 22.5–60°. The values may constitute valuable ~~elements~~ first approximations in mechanical models used for avalanche forecasting.

1 Introduction

intro

Dry snow avalanche release mechanics presents a key research question. Various mechanical models have been used to address the dry snow slab avalanche release problem focused on weak layer failure: e.g. crack models inspired by the over-consolidated clay theory (**?**), cellular-automata models (**?**), fiber-bundle model (**?**), physical-statistical models (**?**), ~~and~~ multiple Finite Element Method, FEM (**??**), and analytical and empirical models (**?**). Recent studies, based on FEM with interfacial constitutive laws for weak layers, have shown that one of the key uncertainties in avalanche forecasting, spatial heterogeneity of weak layers, can be treated by statistical methods and that its importance is reduced for greater snow slab depths (**???**). Moreover, merging of FEM with terrestrial laser scanning input data (e.g. **??**) and the growth of computer performance promise that this decade will see the possibility of precise estimation in terms of statistical distributions of potentially unstable snow masses for feeding into models of avalanche dynamics (**?**). Accordingly, further investigation ~~of the key research question about the~~ concerning the weak layer mechanical behavior and constitutive law and their implementation ~~by~~ in FEM are certainly needed for better quantitative understanding of the avalanche formation process.

For studying dry snow slab avalanches, various approaches have emerged and have been employed in FEM models to represent a snow weak layer under a cohesive slab; for detailed review refer to **?**. Previous studies were mainly designed to investigate~~the following~~: (1) the stress state of a snow slab on a slope (**????**), (2) snow deformation (**?**), (3) skier loading (**?????**), (4) weak layer heterogeneity, super weak zone length and stress concentration, as well as avalanche release slope angles (**???????**), (5) fracture propagation properties (energy release or crack propagation velocity) (**????**), (6) coupled stress-energy model (**?**); anticrack energy release from slope-normal (vertical) collapse (**?**), (7) structural size effect law (**?**), (8) evaluation of field shear frame experiments (**?**); and, finally, (9) snowpack response to explosive air blasts (**?**). To the best of our knowledge, no study has attempted to predict critical inertial loads for failure of snow weak layers in the case of cyclic loading, which presents a basis for model validation for an assessment of the effect of earthquakes on slope failure (**?**).

Previous FEM studies may ~~be roughly~~ roughly be classified into three principally different numerical approaches ~~for consideration of weak layers~~in terms of representation of weak layers (or potential failure surfaces): (1) a thin isotropic (or anisotropic) continuum (**???**), (2) an interface with zero thickness~~and zero volume~~, which may be vertically “collapsible” or not (**???**) or (3) a combination of the first two methods as a thin collapsible/non-collapsible layer with interfaces at the bottom and the top of it (**???**). The above-mentioned ~~constitutive models and~~ approaches are chosen based on the objectives of ~~a~~the study, and at the same time it may be noted that there is no universal, generally accepted framework for treatment of the “slab – ~~weak~~

~~layer~~failure surface" system. ~~On the other hand, if we consider real weak layers, a few types can be distinguished in the field: non-persistent layers (precipitation crystals, or horizontally deposited plate-like snow crystals), persistent weak layers (buried surface hoar, depth hoar, faceted crystals and graupel) and different kinds of interfaces like ice lenses; sun, rain and wind crusts; or just interfaces between two layers of different densities (**??**) . Differences in fracture properties of each of these approximately ten types of layers are poorly understood (**?**) , and application of one particular approach from those listed above is unlikely to be physically relevant for all weak layer types (due to variable microstructural and fracture properties, thickness and residual friction of different types of crystals and interfaces). Moreover, due to~~ Due to computational difficulties related to the size of avalanche release zone, it ~~is~~ generally appears preferable to represent the weak layer by an interface, since its thickness is significantly smaller that the total snow height. ~~By~~ Furthermore, by referring to volumetric layers, the FEM mesh size in the weak layer would have to be smaller than the size of crystals and thus may put the validity of the continuous approach into question. More importantly, it is known from fracture line studies that poor bonding between layers may be a more significant cause of avalanching than low strength within weak layers (**?**). Accordingly, ~~since~~ the idea of treating weak layers as interfaces ~~is~~ appears attractive for large-scale applications ~~(because of the discussion above), more studies are certainly needed~~and will be explored further in this paper.

2 Objectives and scope of the study

The aims of the present work are twofold. ~~Firstly, we study~~ First, we revisit the mechanical behavior of weak layers ~~under accelerated cyclic loading in order to investigate the applicability of an assumed interfacial constitutive law to the analysis of~~ through FEM simulations of previous experiments on failure of layered snow by **?**. ~~Secondly, we analyse the experiments. These experiments were one of the first cold laboratory tests with snow "sandwich" samples, allowing study of the mechanics of weak layer dynamic failure. Complex variation of stresses and normal pressure in particular provided a unique dataset for investigating performance of the assumed failure law under highly variable conditions. In particular, as in **?** , we are interested to test the~~

~~importance of including normal stress dependence in the failure criterion. We hypothesized and~~ Second, we show that the well known Mohr–Coulomb failure criterion with cohesion, which includes normal pressure effects and tensile strength (one of the most common approaches in mechanics of granular materials), may be used as ~~the~~ a first approximation to reproduce ~~the~~ these dynamic experiments. Accordingly, this paper reports on an evaluation of the performance of this failure criterion as well as an evaluation of associated parameters (cohesion and angle of internal friction), through ~~an analysis of tests and a comparison between analytical and FEM solutions, are the main objectives of the paper.~~ a detailed numerical-experimental cross-comparison.

~~In snow science, the Mohr–Coulomb criterion is~~ The idea of describing the failure of snow according to the Mohr–Coulomb failure criterion emerged since the pioneering studies of **?** , **?** , and **?** . According to Mellor's review (**?**) cohesion can be associated with time-dependent intercrystalline bonding (sintering) while the angle of internal friction can be imagined as initial or residual strength of snow with broken bonds. Recently, this criterion was used, for example, for modeling snow erosion by flowing avalanches (**?**), for predicting critical inertial loads for failure of weak layers in seismically active regions (**??**), or for analyzing the packing of snow against sensor surfaces caused by wet avalanche (**?**). ~~However, it is known that the rupture criterion alone is not sufficient for describing the full range of phenomena associated with snow weak layer failure or the release of snow slabs. It is one ingredient among others, needed for complete slab avalanche modeling (e.g. tensile slab failure, stauchwall effects, heterogeneity, post-peak softening, fracture propagation or possible normal collapse; **?**).~~

~~For the scale of our tests, which are not focused on the process of fracture propagation starting from weak zones or imperfections and leading to avalanche release, self-propagating crack is not directly relevant. This is so because our experiments, similarly to work by **?** , are related to failure initiation and larger field experiments are needed to study fracture propagation (and due to other reasons further explained below) . Also, for the small scale of our case, the crack propagation is not relevant because the critical size of the weakness is known to be larger than our samples (**?**) . Furthermore, the rate of high-speed video records taken during the experiments (i.e. frequency 250 Hz; **?**) did not allow us to investigate the fracture propagation process in~~

~~detail. However, we note that the crack always occurred between two consecutive video frames (thus it did not last longer than 4). A study of crack propagation, a topic that has received a lot of attention recently (?) , at such speeds should be based on computationally costly dynamic fracture mechanics, and is beyond the focus and scope of the present experiments and the paper.~~

~~In the present work we consider a weak layer as an interface. The experiments referred to in this paper are well suited for this objective and the context. We remain mindful that our simple approach, including an interface with Mohr–Coulomb failure criterion, may serve as suitable and computationally effective basis for the ultimate purpose of upscaling the method for large-scale simulations. For example, every dynamic snow avalanche simulation starts with the prescription of avalanche release height and volume. Together with entrainment of new snow down-slope these initial conditions strongly determine the outcome of modeling in terms of run-out distance and impact pressures.~~

~~To that end, this paper is organized according to the following structure. Sections 3.1–3.2 explain methods of previous cold laboratory experiments performed by (?) and Sect. ?? presents some background about the Mohr–Coulomb criterion. Section ?? introduces the methods of this paper: 2-D model, including the details of weak layer representation adapted for our FEM analysis. It also describes a simulation of accelerated cyclic loading on a 2-D sample and the procedure of numerical optimization. Finally, Sects. ??, ?? and ?? present the obtained results, sensitivity tests , discussion and conclusions~~One important prediction of Mohr-Coulomb failure criterion is the dependence of shear strength on the normal stress imposed to the sample. Mellor (1975) suggested that this criterion could be problematic for snow due to changes of the material state under pressure. Since then, numerous experimental studies investigated the effects of normal load on shear strength of snow and snow weak layers, mainly through shear frame or shear vane tests. Results, showing an influence of normal stress on various snow types, were reported by ? , ?? , ? , ? and ? . ? reported similar influence on non-persistent weak layers, but found no significant effect on persistent weak layers, thus proposing $\phi = 0°$. Recently ? conducted tests with artificial precipitation snow to investigate temporal variation of the shear strength and concluded that the influence of normal load on the strength was more significant than temperature. Overall, most of these studies investigated the influence of normal pressure using shear-frames. Results obtained with alternative methods, like shaking platform tests (**??**) , may provide valuable new insights on these issues of normal stress influence and applicability of Mohr-Coulomb criterion, and thus remain to be analyzed in that context.

3 Experimental and theoretical background

3.1 Shaking platform experiments

The paper ~~take into account~~ is based on a series of ~~snow samples which were tested~~ experiments using the shaking platform described by **?** and **?**. The procedure ~~could~~ can be briefly summarized as follows: snow samples were frozen to the platform and loaded via inertia ~~due to initiation of the platform's horizontal oscillations from right to left with limited amplitude , but~~ through horizontal oscillations with a constant amplitude of 1.65 cm and with growing frequency ~~of oscillations.The latter~~ (see Sec. **??** for details). The frequency increase caused increases in velocity, acceleration, and thus stresses within the samples; at the point when the ~~increasing~~ stress exceeded the strength of snow, the sample failed. High-speed video records, accelerometers and measurement of the fractured mass revealed the instant of failure and the corresponding peak acceleration (in the range of 2.23–6.36 g) (**??**). Originally, this dynamic experimental approach was developed for studying the shear strength properties of snow and their relationship to vibrations (**?????**). These previously reported tests were performed on homogeneous blocks of snow. ~~Due to this snow structure and configuration, cracks could not be localized in one 2-D failure plane and had complex 3-D geometries that were different from case to case, thus inhibiting straightforward interpretations.~~ **?** introduced a weak layer into the blocks and the possibility to incline the platform at 0, 25 and 35°; these two points make the study more relevant to dry snow slab avalanche release. Nevertheless, free surfaces on five sides of the sample and the ~~probability of edge effects in response of a snow block to loading~~ probable occurrence of stress heterogeneities in the snow block due to edge effects restrict the possibility of simple stress assessments and relating the experimental results to a real snowpack at slope scales. ~~The question of a normal stress effect on the failure of weak layers during experiments is particularly interesting. Without FEM analysis it is hard to estimate its non-homogeneous spatial and temporal evolution within the sample (the same should be noted about shear stresses).~~ For example, an attempt by **?** to calculate dependence of shear strength on presumably constant overburden pressure produced surprisingly high values of internal friction angle (73.4–83.1°) with zero cohesion, thus exemplifying the importance of understanding normal stress ~~oscillations~~ variations in the experiments for reliable interpretations.

The experiments, ~~reproduced~~ considered in this study, were performed in a cold laboratory (with an ambient air temperature of –10 °C) on artificial "sandwich" snow samples ~~(~~constituted by two blocks of snow with a weak layer made of low density snow placed approximately at mid height~~). In total, 19 individual tests with varying properties were modeled. Most relevant parameters and results of experiments are indicated in Table 1; for more details refer to **?** .~~. The samples were prepared by sieving artificial precipitation snow over a cohesive slab with a density around 234 $\mathrm{kg\,m^{-3}}$, covering it with another slab, leaving for 74 hours of sintering, and later cutting vertically the resulting structure into smaller blocks. The resulting weak layer density was around 100 $\mathrm{kg\,m^{-3}}$, and its thickness was around 1–2 cm. If we attempt to identify the closest type of natural weak layer to the artificially created horizons in the middle of snow samples, it would be a non-persistent precipitation layer, made of low density, partly decomposed dendrite crystals, or DFdc according to classification by **?**. The length, width and height of specimens were 0.3, 0.2, and 0.2–0.45 m, respectively. The masses overlaying the weak layers ranged between 1.3 and 4.6 kg. This difference in mass was created by varying the height of the upper block~~by the additional snow frozen immediately before each test to create larger normal pressures. The samples, once frozen to the platform, were vibrated by shaking-platform horizontal oscillations with an amplitude of 1.65until fracture along the weak layer. The latter was documented with high-speed video camera. Additionally, by varying inclination of the platform we produced tests with several slope angles (0, 25 and 35). For these .~~ . For the inclined tests geometry of the sample side cuts was always kept vertical. ~~The critical peak accelerations (in the range of 2.23–6.36) corresponding to failure of the samples were recorded during each~~

~~experiment by a horizontally installed acceleration transducer and were used for estimation of shear strength values of the weak layer (as discussed later).~~

From the different types of tests performed by **?**, we ~~select~~ selected here only the weak layer fracture tests made with horizontal single-degree-of-freedom oscillations (at the same time we ~~emphasize~~ recall that a sample ~~may~~ can have various inclinations: 0, 25 or 35°).

In total, 19 individual tests with varying properties were modeled. Most relevant parameters and results of experiments are indicated in Table 1; for more details refer to **?** .

3.2 Some further experimental conditions relevant to construction of the model

~~In the~~ Four specific points, relevant to the construction of the numerical model, need to be highlighted. First, in the majority of experiments weak layer fractures were observed at the lower interface (between the weak layer – and the lower block). No significant vertical collapse within the weak layer could be recognized during tests (based on video quality we could only restrict the maximum possible collapse as less than $1\,\mathrm{mm}$) (**?**) . Moreover, ~~due to the absence of a crystalline structure that could be associated with any significant volumetric collapse~~with the particular inertial loading considered in this study, we do not expect ~~to have it on a large scale (for example, an order of 3–40was documented by **?** ; **?**).~~

~~Furthermore, we omit the bottom block from modeling in order to reduce computational time based on the following logic. The lower block can be considered as a rigid base of the interface and moves together with the boundary, the overall mechanical behavior of~~ vertical collapse to play a major role in the ~~system may be reproduced by only the upper block and the interface (Fig. **??**). This statement can be supported by observational constraints for shear strains made during the experiments before failures. Analysis~~ failure process. Hence, for the purpose of simplification, the possibility of vertical collapse was not considered in the modeling and the weak layer was represented as a non-collapsible interface.

Second, analysis of video records shows no noticeable horizontal strains in the ~~blocks; due to limitations of the~~ two snow blocks surrounding the weak layer; with the available video quality, the ~~maximum estimate~~ upper bound for strain is less than 0.33 % (**?**) . This means that the whole ~~block is a rigid oscillator, thus allowing us to omit the lower block. Moreover, such assumption~~

~~is valid considering the fact~~ blocks can be regarded, as a first approximation, as rigid bodies. Such assumption amounts to considering that most of the possible deformation is concentrated within the weak layer~~(e. g. loading experiments by ? reported that 90of the sample's global deformation was concentrated in the weak layer).~~ , in agreement with previous studies (???). For the purpose reducing computing costs, we will thus omit the lower block in the model and only consider a system made of an upper block and an interface (Fig. 1). Furthermore, in agreement with this discussion, it will be shown (Sec. 5.3) that the elastic properties of the upper block do not influence failure properties.

~~These facts allow simplification of the model assumptions and remove the necessity of considering vertical collapse, which is still an argued question in the literature (? ; (); ? ; (10001000e. g.BirkSchwJam2009, McClung2011a, McClungBorstad2012 , and the lower block. Thus, for the sake of simplicity, we reduce our problem to a single block with an interface at its lower part and boundary conditions~~Third, we note that the size the samples used in the experiments is much smaller than the critical crack length required for failure self-propagation (??) . In other words, weak layer failure in the experiments is driven only by the applied loading, and not by stress redistributions which remain negligible at the scale considered. Global sample failure occurs when the inertial stresses induced by the oscillations reach the failure criterion in the whole weak layer. Therefore, there is no need of considering the post-failure behaviour of this layer or the progressive accumulation of damage during the successive loading cycles. In this sense, these experiments appear particularly well-suited to focusing on the weak layer failure criterion, independently of post-failure propagation phenomena.

~~Since~~ Lastly, since the experiments had high rates of loading ~~to failure (within a~~ (failure occurred within a second; strain rates were higher than $10^{-3}\,\mathrm{s}^{-1}$), we do not ~~refer to viscous behavior and assume a purely elastic constitutive model for~~ consider viscous behavior of snow. High loading rates guarantee a brittle range for all observed fractures. Such high ~~rate loadings, discussed in this paper, are relevant for any brittle fractures in snow, which can be induced due to natural~~ loading rates are representative of stress variations involved in natural avalanche releases, loading produced by skiers/snowmobilers, explosive air blasts, as well as strong ground motion due to earthquakes or mine blasting (**?**).

3.3 ~~**Mohr–Coulomb failure criterion for snow and scope of this study**~~

~~The idea of describing the failure of snow according to the Mohr–Coulomb failure criterion has been an attractive idea for some purposes since **?** , **?** , **?** , and has been used recently (**???**) . According to Mellor's review (**?**) cohesion can be associated with time-dependent intercrystalline bonding (sintering) while the angle of internal friction can be imagined as initial or residual strength of snow with broken bonds.~~

~~Many experimental studies investigated the effects of normal load on shear strength of snow and snow weak layers, mainly through shear frame or shear vane tests. Experiments, showing an influence of normal stress on various snow types, were performed by **?** , **??** , **?** , **?** and **?** . **?** reported similar influence on non-persistent weak layers, but found no significant effect on persistent weak layers, thus proposing $\phi = 0°$. **?** suggested the idea of using Mohr–Coulomb failure criterion could be problematic, for example, due to changes of the state of the material under pressure (which also depends on temperature). Recently **?** made tests with artificial precipitation snow to investigate temporal variation of the shear strength and concluded that the influence of normal load on the strength was more significant than temperature. Overall, most studies investigated the influence of normal pressure using shear-frames, while results obtained with alternative methods, like shaking platform tests (**??**) , may also provide new valuable insights and thus remain to be understood.~~

~~The experiments shown here suppose that no changes in cohesion had taken place during experiments, since no significant changes of vertical dimensions of samples before and after failure could be observed, and thus provide an opportunity to explore the applicability of the Mohr–Coulomb criterion to some degree. High-speed video analysis of extended column tests by **?** , which involve repeated tapping on a snow column containing a weak layer, indirectly support this assumption (they confirmed that no accumulation of damage could be seen within the weak layer, similarly to the experiments discussed here).~~

~~Furthermore, considering micro-scale, **?** noted that the “formation of snow avalanches and the origin of fractures begins at a scale which is at least 100 times the grain size within the weak layer so that individual grain bonds don’t matter much even if they could be properly dealt with”. For the sake of simplicity we neglect bond-scale processes responsible for development of micro-flaws as well as fatigue. The latter may be refuted as a possible alternative explanation of the experimental results due to no observations indicating that samples subjected to oscillations of longer duration failed at lower accelerations (**?**) .~~

~~Accordingly, the experiments presented here are above the micro-scale, but below the avalanche release scale, and allow to focus only on the failure criterion of snow (i.e. strength). In this light, progressive failure in this study will be driven only by inertial loading (due to oscillations of the shaking platform). Thus we are testing the Mohr–Coulomb failure envelope, which plays a role of failure threshold, and do not investigate post failure phase. We are mindful that in follow-up studies the criterion could be complemented or refined by other effects, like post-peak softening (**?**) for larger scales.~~

4 ~~Methods~~FEM modeling

4.1 ~~FEM model~~

~~We perform FEM analysis~~ Our FEM computations are performed using Cast3M open-source software (http://www-cast3m.cea.fr), a code developed by the French Atomic Research Center (**?**), and employed in previous studies on snow avalanche release (**????**). The code (Education and Research Release, 2010) employs an implicit time integration scheme; governing equations are solved incrementally, thus enabling non-linear computations, and taking into account dynamic effects. ~~In regard to the differences with other available programs (**?**) , we note that Cast3M is open-source software, which allows modifications to be made to the source code.~~

4.1 Model description

4.1.1 Model geometry

~~In order to reproduce the geometry and parameters of the experiments (?) , the initial 2-D geometry for a slab is presented~~ The upper block is represented by a rectangle or parallelogram (for inclined tests), which is 0.3 m long and 0.14–0.36 m high. A 1 cm × 1 cm quadrilateral element shape with four nodes is used for the mesh (QUA4); there are about 14–36 elements in the vertical dimension (depending on the sample height) and 30 in the horizontal dimension~~(1by 1each). The chosen mesh shape is the most common type of mesh used by previous FEM studies on snow (?) , especially since we deal with non-curved geometry and no large strains.~~ . We note, that sensitivity tests with twice ~~higher number of~~ as many elements produced similar, but much more computationally costly results.

~~For representing the weak layer of the "sandwich" samples we treat it~~ The weak layer is treated as an interface~~. The interface is~~ , modeled by joint elements with four nodes (JOI2) ~~but~~ and zero thickness, i.e. an element is created between two segments of two points (Fig. **??**c). There are 30 joint elements (each 1 cm long). The "lower" part of the joint ($_1$A′–$_2$B′; Fig. **??**c) is fixed to the bottom boundary, meaning that there are no vertical and horizontal displacements ~~of this part of the joint are forbidden~~ relative to the boundary. ~~However, the lateral and surface boundaries of the rest of the system~~ Displacements on the upper part ($_4$A–$_3$B) are not restricted, thus allowing free deformation. ~~Therefore, these conditions are both comparable to those of a snow block frozen to the platform.~~

~~We note that the simulated geometry requires half as much computational time as it do if the lower block is included. Furthermore, as it will be shown (Sects. **??** and **??**), by introducing interface stiffness (which may be seen as equivalent to putting the sample on an elastic cushion instead of a rigid plate) and making sensitivity to a wide range of values, it is possible to verify if our assumption is reasonable. The stiffness was found as not playing any important role in the key quantities controlling interface failure process (Sect. **??**). In view of this simple observation it is quite obvious that the assumed model geometry does not control failure.~~

4.1.2 Constitutive laws of the block~~and the interface~~

The upper block is considered as a uniform and isotropic elastic material similarly to many slab models presented in literature (**????**). Accordingly, its behavior is controlled by Young's modulus, E, and Poisson ratio, υ. We use Young's modulus values varying with density after **?** , in the range of 1.2–1.5 MPa. We follow the study by **?** and select a Poisson's ratio to be equal to 0.04 (for temperature −10 °C). Also we note that since the problem deals with dynamics and vibration, non-physical viscosity of the block, η, is introduced into the damping matrix of the model for numerical stability reasons. ~~A choice of material properties of the block (i.e. Young's modulus, Poisson ratio) will be considered below (Sect. **??**).~~ Sensitivity tests to Young's modulus, E, Poisson ratio, υ, and viscosity, η, ~~will be shown in~~ showed that they have a negligible influence on failure properties (see Sect. **??**).

~~The assumed behavior of the interface is that of a joint model based on the~~

4.1.3 Constitutive laws of the interface

The interface is governed by a Mohr–Coulomb failure criterion, ~~which is controlled by the~~ with an angle of internal friction, ϕ, and a cohesion, c:

$$\tau = \sigma \tan(\phi) + c, \tag{1}$$

where τ is shear stress and σ is normal stress (Fig. **??**d). The cohesion is ~~defined in the model through the~~ related to the tensile strength, σ_{st}, as follows (Fig. **??**d):

$$c = \sigma_{st} \tan(\phi). \tag{2}$$

Accordingly, we may refer in the following text to both~~of them (tensile strength , ,~~ tensile strength (σ_{st}~~, and cohesion ,~~) and cohesion (c), depending on the context. ~~Such substitution implies that the failure envelope, having a slope equal to the angle of internal friction, ϕ, intercepts the shear axis at c, and the normal stress axis at σ_{st} (Fig. **??**d). This constitutive relationship was chosen because interfaces without any tensile strength would not be adequate~~

~~for reproducing the tests, which may have~~ We note that considering an interface law which includes a tensile strength is crucial to reproduce our experimental tests, since these may involve significant tension stresses~~(as it will be illustrated later)~~. Additionally to failure criterion, for joint elements we also specify values of shear and normal stiffness, K_s and K_n~~, which control strains of the interface (more details are provided in Sect. ??)~~. To the best of our knowledge, there are hardly any experimental data for weak layer elastic properties (**??**) . After conducting sensitivity tests for different couples of K_s and K_n (within the range 10^5–$10^8\,\mathrm{N\,m^{-3}}$) for a full set of experiments, the shear and normal interface stiffnesses were set to $10^8\,\mathrm{N\,m^{-3}}$. We found negligible effects of K_s and K_n on failure as it will be discussed later in Sect. **??**.

4.1.4 ~~Definition of interface failure~~

~~We define the occurrence of total sample failure as the first instant when all nodes of the interface, N, satisfy the Mohr–Coulomb failure criterion:~~

$$[\text{nodal failure}] \equiv \frac{|\tau| - \sigma \tan\phi}{c} = 0.99999,$$

$$[\text{total failure}] \equiv N_\mathrm{f} = N,$$

~~where N_f is a number of failed nodes. The instant when this condition is satisfied ($N_\mathrm{f} = N$) is treated as the moment of total sample failure, t_m. We just note that for prescribed c and ϕ, and for dynamically changing shear and normal stresses, τ and σ, Eq. (**??**) simply corresponds to the failure criterion (Eq. **??**) rewritten in a form that allows identification of when it is satisfied within the model. Against the above-mentioned background and the size of specimens (Sect. **??**), the implemented approach means that a local node meeting the criterion leaves the system unchanged (i.e. there is no loss of strength leading to a non-linear behaviour), and that system failure can only occur if all interface nodes simultaneously satisfy the failure criterion.~~

4.1.4 Cyclic displacements, inertial loadings and gravity

Before ~~the simulation of inertial loading can be initiated~~ initiating inertial loading for a particular set of parameters, ~~first, we subject our domain~~ we subject the system to its actual weight. ~~For this the matrix of mass is multiplied by a field of gravitational acceleration in the vertical direction. Here, the gravity is applied (to nodes) gradually (i.e.~~ Gravity is applied gradually at a rate of $2.45\,\mathrm{g\,s^{-1}}$ ~~)~~ during 4 s until reaching its 100 % value~~within the first phase of simulation~~, in order to avoid ~~any possible vibration of stresses (within the first 4s followed by another 0.4s for stabilization of the system). Initially the gravity is imposed on a material model with a Poisson ratio~~ numerical instabilities. Furthermore, the Poisson ratio of the block, υ, ~~of zero, for obtaining~~ is set to 0 during this initial phase in order to obtain homogeneous normal stresses within the sample, i.e. without any stress concentrations at the edges. In the next procedural step the ~~material model is replaced by a model with a new Poisson's ratio~~ value $\upsilon = 0.04$ ~~(more details are shown in the next Sect. **??**).~~ is introduced and the system is allowed to stabilize during 0.4 s.

Next, we reproduce horizontal shaking of the platform ~~and, accordingly, introduce inertial forces within the sample~~ by imposing displacements onto the ~~boundary. To recreate the dynamics of our experimental problem we define cyclic basal displacements in the model with an amplitude similar to the one produced by the motor during experiments. Thus, the block moves horizontally a distance $s(t)$ according~~ lower boundary. Consistently with the experimental conditions, the system base is subjected to the following ~~trajectory~~cyclic displacement:

$$s(t) = 0.0165(1 - \cos(\omega(t)t)), \tag{3}$$

where 0.0165 is ~~a~~the displacement amplitude in meters~~(it corresponds to the amplitude of horizontal oscillation of the shaking platform used in experiments)~~. The angular frequency~~coefficient~~, ω, ~~starts to evolve after the initial preparation of the sample (described~~

~~earlier) and~~ increases linearly as a function of time (after initial sample preparation):

$$\omega(t) = \begin{cases} 0 & \text{if } 0 < t < 4.4\,\mathrm{s}, \\ k_\omega \pi (t - 4.4) & \text{if } 4.4 \le t \le 25.0\,\mathrm{s}. \end{cases} \tag{4}$$

~~This angular frequency, controlled by~~ where the coefficient k_ω ~~(varying~~ varies between 0.44 ~~-~~and 1.43 s^{-2} ~~and explained further below),~~ (see below). This angular frequency increase introduces the gradual growth of velocities and accelerations, and thus, stresses, with every oscillation (Fig. **??**). ~~Accordingly, almost all stresses in our system (except gravitational) are driven solely by the horizontal oscillations of the boundary.~~

Since sample's failure always occurs at an instant when acceleration reaches a peak (caused by a change of the platform's direction of movement), and since the corresponding peak acceleration is known from the experimental measurements, we individually adjusted the coefficient k_ω for each test in order to recover the right value of peak acceleration at the instant of failure. An example of k_ω adjustment for one test is provided in Fig. **??**a and b. Values of k_ω obtained for all tests are listed in Table 1.

Here, it is also appropriate to ~~provide a~~recall the simplified analytical evaluation of the shear force evolution (τ_{ex}) ~~used~~ previously used, in the horizontal case, to estimate weak layer shear strength during experiments (**??**)~~in order to see differences with the FEM solution~~:

$$\tau_{\text{ex}}(t) = \frac{m_{\text{f}}\, a(t)}{A}, \tag{5}$$

where m_{f} is ~~a~~the mass of the upper block, $a(t)$ is block acceleration (second derivative of $s(t)$ with respect to time), and A is the area of the failure plane. This analytical ~~solution could also be called a~~ approximation corresponds to a purely static model, since it does not account for dynamic stress inhomogeneities caused by inertia and geometry. Since ~~our simulation is in 2-D and since~~ $m_{\text{f}} = h_{\text{s}} A \rho_{\text{s}}$, where h_{s} is the height of the block and ρ_{s} is its density, Eq. (**??**) can be rewritten for ~~feeding simulation data into it and for~~ further comparisons as~~(horizontal case)~~:

$$\tau_{\text{ex}}(t) = h_{\text{s}} \rho_{\text{s}} a(t). \tag{6}$$

~~Similarly,~~ In the inclined case~~may be expressed as~~, gravity effect should also be taken into account such that:

$$\tau_{\text{ex}}(t) = h_{\text{s}}\rho_{\text{s}}a(t)\cos\alpha + h_{\text{s}}\rho_{\text{s}}g\sin\alpha, \tag{7}$$

where α is the inclination of the boundary~~and the right term corresponds to shear stress due to gravity~~.

4.1.5 ~~Choice~~ Definition of ~~constitutive parameters~~sample failure

~~**Young's modulus**~~

~~For experimental snow densities of upper blocks (212–226), corresponding values of Young's modulus, E, vary depending on the literature source (? ; (); ? ; (10001000e.g.Mellor1975,Stoffel2005,Habermann2008. For simulating upper blocks we use modulus values as a function of density after **?** of 1.2–1.5.~~

~~In regard to the high strain-rates of the experiments considered here (10^{-3}–$10^{-1}$for the block **??**), we should note that there is one possible effect of the rate on the elastic properties of snow. A possible increase of Young's modulus with higher strain-rates was estimated to correspond up to a factor of 3 (? ; (); ? ; (10001000e.g.Kry1975. For our tests this illustrates that the order of magnitude of E remains the same and our assumptions are still consistent with Mellor's review (which provides the most comprehensive summary of static and dynamic E and has not been improved much by any recent studies; for more examples see **?** ; **?** ; **?**). Furthermore, for similarly high strain rates, **?** as well as **?** used values of Young's modulus after **?** as a function of density. Therefore, there are no strong limitations to following them and using the modulus from Mellor's paper for the purposes of the present study. Sensitivity tests for higher Young's Modulus, E, are presented in Sect. **??**. However, we note that the block~~ We define sample failure as the first instant when all nodes of the interface, N, satisfy the Mohr–Coulomb failure criterion:

$$[\text{nodal failure}] \equiv \frac{|\tau| - \sigma \tan\phi}{c} = 0.99999, \tag{8}$$

$$[\text{total failure}] \equiv N_\mathrm{f} = N, \tag{9}$$

where N_f is a ~~quasi-rigid object and higher values of the modulus are not expected to produce significant changes in the model.~~

~~**Poisson ratio**~~

~~For a comparable range of densities, Poisson's ratio of snow, υ, is usually chosen in other FEM studies to be around 0.25–0.3 and thus corresponds to Poisson solid (**????**) . However, considering experimental studies (**???**) , a speculative Mellor's envelope for the Poisson's ratio as a function of density (**?**) and the most recent effort by **?** to refine the Poisson's ratio (which he calls viscous, as a function of density and temperature), we follow the latter study and select a Poisson's ratio for the block to be equal to 0.04 (for temperature −10C and density 212). Furthermore, the Teufelsbauer approximation covers the comparable range of values of those appearing in reviews on snow slab avalanches (? ; (); ? ; (10001000e.g.Schweizer1999, and in high strain-rate measurements (**?**) . Thus the chosen value is consistent with available experimental data. However, sensitivity tests with a higher value of this parameter (0.23) showed that it is not important for the failure results (see Sect. **??**~~number of failed nodes. This instant is denoted t_m. We emphasize here that, when the stresses decrease during the loading cycles, the interface nodes recover their initial elastic behaviour and strength, and that no progressive accumulation of damage is considered (see Sec. 3.2).

~~**Shear and normal stiffness of the interface**~~

4.2 Search for optimal failure parameters

~~Shear and normal stiffness,~~ ~~K_s~~ Systematic numerical simulations were run to find values of cohesion c and friction angle ϕ that minimize the time difference between instants of failure predicted by the model (t_m) and ~~K_n, control the resistance of the joint to vertical and horizontal deformations in response to an applied load. Usually assumed shear stiffness is taken as being equal to a half of normal; moreover, anisotropic layers of buried surface hoar were shown to be softer in shear than in compression (**?**) . Nevertheless, to the best of our knowledge, there are no reliable experimental data for weak layer elastic properties. After conducting sensitivity tests for different couples of~~ ~~K_s~~ ~~and~~ ~~K_n~~ ~~(within the range 10^5–10^8) for a full set of experiments, the shear and normal interface stiffnesses were set to $10^8$to reduce peak elastic displacements of the whole system to realistic values on the order of 10^{-3} m. Some studies assume smaller values, e.g. 10^{-4} m in weak layers, but note that such precise measurements are not available for alpine snow (**?**) . In our case we take it as simply an experimentally verified fact, and note that even if an order of magnitude reduction of stiffness (to 10^7 or 10^6) resulted in larger displacements ($\sim 10^{-2}$ m or higher), no difference in terms of failure was observed. We notice that different shear and normal stiffness values, e.g. 5×10^7 and 10^8respectively, gave same results in terms of failure as when K_s was set equal to K_n.~~

~~For comparison with other studies we may find an equivalent value of Young's modulus for the stiffness values considered in numerical optimization of our study (Sect. **??**). Such equivalent is defined as $E_\mathrm{wl} = K_\mathrm{s,\,n} h_\mathrm{w}$, where h_w is the typical thickness of the weak layer. For example, stiffness values 5×10^7 and $10^8$are equivalent to Young's modulus 0.5 and 1. We also note that the similar and even higher magnitudes of Young's modulus equivalents to those used in our study were employed for modeling snow weak layers in other FEM studies (**???**) . Sensitivity tests with lower stiffness, for example, equivalent to a softer Young's modulus of 0.1, correspond to larger interface horizontal displacements at peak stresses, like 0.5, and thus are considered unrealistically high. More importantly, we found negligible effects of K_s and K_n on failure as it will be discussed later in Sect. **??** .~~

4.3 ~~Computational approach~~

~~A few other issues exist that require additional consideration: (1) the masses of the snow blocks above the weak layers were different from one case to another (and thus, also the sample height), (2) the angular frequency increment was not constant for all experiments due to manual control of its rate. Accordingly, this means that for our FE model the former and the latter factors require individual selection of the height (h_s) , as well as an appropriate rate for frequency growth (k_ω), respectively.~~

~~First, we prescribe to each sample an h_s – equivalent derived from the recorded mass and density (such that $h_\mathrm{s} = m_\mathrm{f}/(A\rho_\mathrm{s})$; Table 1). Next, since any sample's failure occurs at an instant when a particular critical acceleration reaches a peak (caused by a change of the platform's direction of movement), and since the moment of failure and the corresponding peak acceleration are known from measurements, we individually adjust the coefficient k_ω for each test so that the instant of the observed failure (t_e) is reached at the right value of the peak acceleration. The latter allows us to achieve similar acceleration conditions of the model to those of the experiment at the instant of failure, t_e. In order to reach the measured peak accelerations, k_ω should vary between 0.44 and 1.43(Fig. **??**). An example of k_ω adjustment for one test is provided in Fig. **??**a and b. The experimental range for accelerations and time of fracture~~ those measured in the experiments (t_e)~~, where modeling results should fall into, is shown in Table 1. Values of k_ω and material/mechanical properties are listed in Table 1 and 2.~~

~~Finally, by adjusting the two remaining degrees-of-freedom (~~. By adjusting these two degrees of freedom c and ~~$\tan\phi$)~~ ϕ, we investigate the ability of the assumed ~~constitutive law to predict failure time~~weak layer failure threshold to predict correct failure time values. The adjustment is performed on all tests, involving a variety of experimental conditions (different masses and inclination angles), ~~t_e. Accordingly, fitting valuesof cohesion and angle of internal friction, c and ϕ, is the main objective of the study. If the law is valid and may reproduce the variety of presented conditions, we expect to obtain a pair of c and ϕ valid for all the tests,~~ since all ~~experimental procedures were aimed at producing~~ used samples are expected to be characterized by similar weak layer properties~~, which were made of the same snow type. The underlying numerical procedure is described below. After computations, we also compare the analytically obtained stresses (Eqs. **??** and **??**) with those from the FE analysis.~~

4.3 ~~Search for optimal failure parameters~~

~~We assume that~~ . Accordingly, we defined a numerical optimization search procedure based on the following constrained single-objective cost function, C_{FEM}:

$$C_{\mathrm{FEM}}(c,\phi) = \sqrt{\frac{\sum_{i=1}^{n} |t_{\mathrm{m},i} - t_{\mathrm{e},i}|^2}{n}} \tag{10}$$

for a ~~given set of prescribed parameters (i.e. given material and mechanical properties) and experimental criteria (e.g. acceleration at failur the best to reproduce the experiments). Our goal is to find a set of c and ϕ that minimizes the time difference between the model predicted failure (t_{m}) and the test failure (t_{e}), i.e. $|t_{\mathrm{m}} - t_{\mathrm{e}}|$ for all tests~~ number of simulated tests n. We consider a parameter space where cohesion, c, is limited to a range between 0.5 and 2.8 kPa (**?**), and the internal friction coefficient range is to 0.18–3.73, or 10–75° (e.g. **??**); more detailed discussion is provided in Sect. **??**. ~~Accordingly, to move through our parameter space, we chose a numerical optimization search procedure based on the following constrained single-objective cost function, C_{FEM}:~~

$$C_{\mathrm{FEM}}(c,\phi) = \sqrt{\frac{\sum_{i=1}^{n} |t_{\mathrm{m}i} - t_{\mathrm{e}i}|}{n}}$$

~~for a number of simulated tests n. C_{FEM} is basically the Root Mean Square Error (or RMSE).~~

~~Numerical optimization is performed for the whole set of simulations (or for the sample set, as discussed below); for minimizing the $|t_{\mathrm{m}} - t_{\mathrm{e}}|$, we repeat FE simulation with adjusted input parameters.~~

In order to reduce computational costs, instead of covering ~~our~~ the c–ϕ parameter space by all possible discrete combinations ~~, after the first simulations introduced the overall response of the~~ and for the whole ensemble of ~~tests (~~19 ~~in total) to seven different parametric set-ups (combinations of fixed values of c and ϕ, K_{s} and K_{n}) and the identification of possible outliers,~~ tests, we followed cost function gradients manually by selecting a ~~small~~ smaller representative sample of experiments~~for calibration. Thus we reduced the total population of tests to~~

~~a representative~~ . This reduced "calibration" sample ~~, consisting~~ consisted of 5 (or 9) individual tests ~~(selected with varying~~ selected with different inclinations, masses and sizes to avoid possible biases~~)~~. The results obtained with the "calibration" sample ~~, in their turn, will be verified by using the same parameters for the~~ will then be verified on the complete "validation" sample (as will be explained in Sect. **??**). ~~It is important to note that such a validation procedure allows confirmation that the optimal parameters also work for other tests. Thus, a segregation of tests into "calibration" and "validation" samples presents an additional way to verify the results.~~

5 Results

5.1 Mechanical behavior of samples and failure

~~For realistic values of Young's modulus assumed in the model, FEM results support the argument of Sect. 3.2 saying that the block is a stiff oscillator.~~ Figure **??** provides examples of stress ~~inhomogeneities~~ fields within the blocks caused by motion and the geometry of the system. ~~In this regard, two principal observations may~~ Two principal observations can be made for all ~~types of inclinations. (i)~~ inclinations. First, as the block changes its direction of movement and thus experiences high accelerations, we observe the expected emergence of maximum shear stress (see instant t_2 at Fig. **??**). These stresses then decay as the block moves backward and passes through the central position of its trajectory (t_3). At the opposite side of the oscillation (t_4) shear stresses ~~re-emerge with higher amplitude and~~ re-peak with an opposite sign (Fig. **??**). ~~(ii)~~ Second, we see that at the critical points (t_2 and t_4), normal stress remains quasi-constant in the middle of the block, but ~~may have~~ shows important variations of opposite signs at the edges. ~~Meaning that due~~ Due to the inertia of the mass~~(which is fixed to the boundary)~~, one side will ~~have an increase of~~ experience an increase in normal stress, while the other a decrease. With higher accelerations, these decreasing normal stresses ~~may~~ progressively turn into tension. As the block leaves the point t_2 and reaches the opposite critical point (t_4), signs of normal pressure ~~flip~~reverse.

Similarly, in the interface the imposed oscillations ~~gave~~ produce shear stresses with changing directions and ~~produced~~ strong oscillations of normal stress at the edges of the joint (Figs. **??** and **??**). Tensile stresses appearing at the edges ~~of the joint after the start of oscillations~~ clearly illustrate that tensile strength of the weak layer needs to be taken into account for realistic representation of tests (Fig. **??**). Figure **??** also shows the differences between the analytical (Eqs. 8 and 9) and FEM solutions for shear stresses. ~~For example, for the assumed parameters the~~ In general, the FEM gives larger shear stresses~~(~~, by about 20 % in the middle of the horizontally inclined joint~~)~~, thus clearly indicating the limitations of the analytical approach for samples of limited length. For the inclined tests (25 and 35°), the differences between the analytically and FE derived shear stresses in the middle of the interface ~~derived shear stresses~~ are slightly smaller (Fig. **??**b and c). However, edge effects are more significant for these inclined tests (Figs. **??** and **??**)~~due to geometrical effects, thus clearly indicating limitations of the analytical approach (Eq. ??) for samples of limited length.~~.

Figure **??** shows the growth of the number of nodes, N_f, that ~~had~~ have reached failure criterion with time. ~~Here we note that the propagation of the failure condition is not self-induced, but is a load driven process. And as stresses are removed, there is no flaw remaining. As~~ As expected, as the block passes the critical point ~~of its trajectory (where it has a full stop and thus experiences the highest accelerations) and~~ and reverses its direction~~(Fig. ??)~~, the stresses start dropping so that ~~no nodes remain under failure ($N_\mathrm{f} = 0$)~~the number of failed nodes N_f progressively diminishes and is always zero at the central position of the trajectory. The next peak is then larger than the previous one because ~~at the next oscillation the accelerations are larger by some increment, as are the stresses (meaning that with each oscillation we produce a larger loading with higher magnitude), and thus more nodes satisfy the failure criterion. Thus there is no cumulative accumulation of failed nodes.~~ of the progressive acceleration increase. Accordingly, we observe progressive enlargement of the failure zone~~with higher stresses~~, purely driven by the external loading, during the successive oscillations (Fig. **??**)~~, but not crack propagation, therefore we could call it a numerical indication about how close the system is to failure.~~

~~By definition in our model the failure is the first instant when the interface experience stresses which none of its nodes is able to sustain.~~ . The time difference between the instant of “total failure” (t_{m}) and experimental failure (t_{e}) is also indicated ~~at~~ in Fig. **??**. ~~The behavior of this difference and the process of reducing it is discussed below for all experiments.~~

5.2 Mohr–Coulomb parameter optimization

The ~~overall response of all tests to various parameters is provided~~ failure time delay $t_{\mathrm{m}} - t_{\mathrm{e}}$ obtained for all tests and for different pairs (c,ϕ) is indicated in Fig. **??**. For the considered range of parameters~~(see Sect. **??** and the figure's legend),~~ , experimental time-to-failure, t_{e}, is reproduced within ~~±2.5accuracy~~ 20% for the majority of tests with only a few outliers~~larger than that. This~~ . The figure shows that ~~, for example,~~ if the modeled joint has a cohesion that is too high, failure will be delayed compared to t_{e}; on the contrary, if it is too low, failure will occur earlier~~than the observed one (Fig. **??**). In this light~~. In general, the responses of ~~individual tests look similarfor all tests with the same parameters, suggesting that instead of using all of them, we may select a~~ all individual tests to changes in c and ϕ appear similar, thus justifying the choice of a smaller sample for ~~calibration of cohesion c and angle of internal friction ϕ~~adjusting these parameters.

~~Figure~~More specifically, figure **??** shows cost function, C_{FEM}, sensitivity to a selection of a different number of tests and illustrates that ~~such “downscaling” is reasonable and efficient~~ earlier introduced sub-sampling (Sect. **??**) is reasonable for the optimal parameter search. ~~This is true because for~~ For particular variations in parameters ~~a~~the sample's C_{FEM5} (where subscript 5 indicates ~~a~~the number of tests considered) responds similarly to C_{FEM} computed for the complete population of tests. ~~Later, in order to check C_{FEM} sensitivity to number of tests taken into account, results obtained with the “calibration” sample (tests :~~ In the following, two validations samples will be considered, corresponding to C_{FEM5} (tests 27, 30, 33, 35, 41) ~~were verified with a larger number of tests. And in addition the results were also verified with another “validation” sample, presented by the remaining tests (shown below~~and C_{FEM9} (idem with also tests 23, 26, 32, 39).

The ~~evolution of the sample's C_{FEM} with the prescribed cohesion and angle of internal friction~~ variations of C_{FEM5} with cohesion and friction angle, c and ϕ, ~~is~~ are shown in Fig. **??** (and in Table 3). The figure shows all tested combinations of c and ϕ together with some sensitivity tests. ~~All exact values are provided in Table 3.~~ The most important ~~feature~~ outcome of the parameter optimization (Fig. **??**) is ~~a~~the lack of ~~one clear global minima. In Fig. **??** this tendency is expressed as an area~~a clear global minimum in C_{FEM5}. Instead we can observe a valley, which is narrow in cohesion c, but wide in ϕ, and which ~~has~~ is characterized by very close values of ~~C_{FEM}~~ C_{FEM5} (this is more clearly seen in the color contours based on a cubic interpolation). Accordingly, ~~it is evident that simulation results are~~ simulation results appear more sensitive to the cohesion than to the angle of friction. ~~A more detailed interpretation of the significance of this region in terms of the Mohr-Coulomb failure envelope will follow in the discussion Sect. **??**, together with comparison to other studies.~~

Following ~~the finding that some~~ this finding that simulations with different pairs of $c-\phi$ resulted in comparable values of cost function~~C_{FEM} (Table~~ , a similar study was performed on C_{FEM9} (Fig. **??**, Tab. 3)~~, we attempted to increase the total number of tests in the sample for each of these runs with low C_{FEM} from 5 individual tests to 9 (include tests: 23, 26, 32, 39)~~. However, even with additional tests ~~(Fig. **??**; Table 3)~~ a minimum in ϕ did not become evident. We ~~found three pairs (~~could nevertheless identify the three pairs of $c-\phi$ corresponding to the lowest computed cost function values: $1.57\,\mathrm{kPa} - 30°$, $1.57\,\mathrm{kPa} - 35°$, $1.6\,\mathrm{kPa} - 30°$ ~~)~~ ~~that may represent the minimum (~~with $C_{\mathrm{FEM9}} = 0.365\,\mathrm{s}$, $0.373\,\mathrm{s}$ and $0.385\,\mathrm{s}$, respectively~~), but nevertheless we cannot clearly distinguish it from the overall variability~~.

~~Additional numerical experiments with fixed values of cohesion (1.25, 1.57 and 1.8) were made in order to determine the sensitivity of C_{FEM} results solely to values of angle of internal friction, ϕ (Table 3). For, example, the obtained values of C_{FEM9} (for $c = 1.57$, $\phi = 30$ or 40) were equal to 0.365 and 0.477, respectively (Fig. **??**). Compared to the value of 0.373(for the pair 1.57– 35), it is obvious that the response of results to ϕ is negligible and therefore we are still unable to name a single optimal value of the friction angle. Some further discussion of the obtained C_{FEM} profiles along ϕ (with $c =$ constant) will follow in the subsequent Sect. **??**.~~

~~Owing to the fact that for the search through the parameter space we used the "calibration" sample, we ran three "~~Three "validation" sample simulations (including the remaining tests: 25, 31, 37, 40, 42, 43) ~~for verification of the parameters that were responsible for the lowest C_{FEM} (1.57–30, 1.57–35and 1.6–30)~~were run with these parameters resulting from the optimization of the calibration sample. Excluding test 25, which ~~presented~~ behaved as an outlier for the three ~~simulation sets~~cases, the "validation" samples produced similarly low C_{FEM} values to those that were ~~made~~ obtained with the "calibration" sample (Table 3; ~~$C_{\mathrm{FEM5}} = 0.406$~~$C_{\mathrm{FEM5v}} = 0.406$, 0.377 and 0.394 s, respectively). For example, for simulations with $c = 1.57\,\mathrm{kPa}$ and $\phi = 35°$, the time difference between modeled and observed failures correspond, on average, to 5 % of the total duration of each individual test.

5.3 Sensitivity ~~tests~~analysis

In ~~the following~~ this section we briefly describe the sensitivity tests ~~,~~ which were performed in order to confirm that none of the results provided above are affected by other parameters of the model. These tests were performed during different stages of the model development and testing, therefore here we just summarize the main conclusions.

The ranges of values used for ~~the~~ this sensitivity analysis are specified in Table 2. ~~The most important point to highlight is that none of these parameters had effects comparable to the impact of the parameters of the failure criteria, c and ϕ.~~

Sensitivity tests with a higher Young's modulus, E, of the block ~~(2 or 3 times higher; in line with the discussion about E variation due to strain-rates in Sect. ??)~~ have shown negligible increase in the magnitude of stresses within the joint (about 1–2 %), and negligible effects on computed time-to-failure (t_{m}). Numerical experiments (s6y; for 9 tests) with the same cohesion, c, and angle of internal friction ϕ as in the s6 simulations (Table 3), but with a Young's modulus twice ~~as high as in the control simulation,~~ or thrice as high produced similar C_{FEM9} values (0.383 and 0.380 s compared to 0.373 s ~~of the control~~ in the s6 simulations; see Fig. **??** with details also shown in Table 3). ~~Similarly, a threefold increase of Young's modulus (s6yy; for 9 tests) also did not produce significantly different C_{FEM9} (0.380).~~

~~Our sensitivity~~ Sensitivity calculations with respect to Poisson's ratio, υ, in the block showed that a ~~selection~~ value of 0.23 instead of 0.04 produces slightly higher normal stresses within the joint (1.1 % at the largest), and thus may delay timing of total failure but for only one time step ~~(Δt). The effect on normal stress echoes conclusions of **?** .~~ at the most.

~~An increase of the~~ Variations in the numerical viscosity of the block, ~~υ~~η, by two or four orders of magnitude (from $10^4\,\mathrm{Pa\,s}$ up to 10^6 or $10^8\,\mathrm{Pa\,s}$ ~~) has a negligible effect on failure time. Similarly, a decrease of viscosity by two orders of magnitude (from 10^4~~ or down to $10^2\,\mathrm{Pa\,s}$) ~~also has no effect on the discussed results~~have a negligible effect on failure time. The baseline value of $10^4\,\mathrm{Pa\,s}$ was found to be optimal for the overall ~~behavior~~ stability of the model (~~by optimal we mean no artifacts like~~ no artificial high-frequency ~~oscillations or lag of stresses behind displacements~~stress oscillations).

Finally, a relatively low sensitivity of ~~failure properties of the model~~ model results to different combinations of joint stiffness K_s and K_n was found. For example, three sensitivity ~~sets of simulations~~ simulation sets performed for all 19 tests with the same $c = 1.6\,\mathrm{kPa}$ and $\phi = 45°$, but ~~varied~~ varying stiffness values (between 10^3 and $10^8\,\mathrm{N\,m^{-3}}$), produced very similar ~~results in terms of the cost function . In other words, the difference in terms of failure time is not comparable with the magnitude produced by changes in cohesion,~~ cost function values.

Thus, in short, none of the parameters tested in this sensitivity analysis have effects on the computed failure time comparable to the impact of the failure criterion parameters c ~~, and angle of internal friction,~~ and ϕ.

6 Discussion

~~The~~

6.1 Interpretation of observed behaviour

The main objective of the study was to investigate the applicability of the Mohr–Coulomb failure criterion, which is one of the most common ~~criterions~~ failure criteria in mechanics of granular material~~(Sect. ??). The previous section (??) has shown~~ , in relatively complex experiments performed on sandwich snow samples. A first important result is that FEM modeling appeared necessary to capture the stress inhomogeneities arising within the sample that were disregarded by previous analytical analyses (?) . Our results also show that even with a simple set of model assumptions, it ~~could be~~ was possible to reproduce ~~very different~~ correct failure times for very different experimental cases (i.e. with various inclinations, masses and sizes)~~observed during relatively complex experiments. The fact that we find parameters of c and ϕ that also give low C_{FEM} for other tests provides another justification that the approach with the Mohr–Coulomb failure criterion , used in this study, is appropriate for modeling failure in the experiments. In our approach, as elsewhere, the criterion plays a role of a failure threshold. The fulfillment of the failure condition is a load-driven process due to stress inhomogeneity, which is caused by inertial and geometrical effects. Because of the latter,~~ . Importantly, it appears that the normal stress dependence of the failure criterion is an important ingredient that should be taken into account. In particular, criteria involving only a cohesion cannot reproduce as well the considered set of experiments. We also recall that the occurrence of normal stress oscillations ~~in particular~~ impose a requirement of the interface to have tensile strength, σ_{st}~~, in addition to the cohesion, c~~. This means that the weak layer ~~is dependent on the friction angle, and~~ cannot be described by a purely cohesive form of the Mohr–Coulomb failure criterion.

To ~~highlight~~ illustrate the meaning of the cost function C_{FEM} results indicated in Fig. **??**~~in terms of the Mohr–Coulomb failure criterion, we plotted all numerical tests against the most “successful” simulations (i.e. those that have minimal C_{FEM})~~ , all corresponding Mohr-Coulomb failure envelopes are represented in Fig. **??**. ~~On the Fig~~In this figure. **??** we have used green shading and red lines ~~only for results in~~ to highlight results for which the C_{FEM} is lower than 0.5 s (for both ~~types of~~ sample sizes, i.e. with 5 or 9 tests). ~~Strong constraints for~~ It clearly appears that these optimal simulations provide strong constraints on the value of cohesion ~~are evident (~~c, which lies in the range 1.6–1.8 kPa~~) (Fig. ??). The cohesion values obtained~~ . These cohesion values derived through our inverse simulations fall well within the range of measurements reported for weak layers composed ~~from~~ of precipitation particles or interfaces (**?**).

~~Additionally, for comparison, previous analytically derived experimental values (?) are plotted over the modeling results (Fig. **??**). These analytical results indicated a dependence on normal load, and Fig. **??** clearly illustrates that normal stress oscillations and their variability between the tests make the analytical solution hard to interpret without FEM modeling. This also means that the normal stress dependence is an important ingredient of the model which should be accounted for.~~

~~As Fig. **??** shows, the global minima could not be clearly resolved for some particular pairs and thus the cost function is presented by a minima "valley"~~ Unlike for cohesion, the simulations do not provide strong constraints on the values of friction angle ϕ (Figs. **??** and **??**). ~~The latter corresponds to a narrow bottleneck of limited cohesion values, c. Thus modeling suggests that~~ Thus the overall behavior of the observed failures ~~is~~ in the considered experiments appears mostly controlled by ~~a value of~~ cohesion, c~~(Fig. **??**). For the same cohesion a variation of the angle of friction (within~~ , while friction angle ϕ plays only a secondary role (in the range 20 to 60°)~~did not have significant effect on the reproduction of failures (as described in more details below)~~. It is probable that ~~the obtained minima "landscape" is~~ this behaviour is partly due to a limited range of sample heights ~~, inclinationsand thus experimental normal stresses, which may be insufficient for further clarification of angle of internal friction,~~ and inclinations, and thus to insufficient variations of the normal stresses between the different experiments. Slight variability between the tests may further enhance the poor localization of the minimum in ϕ~~.~~ ~~Another explanation for the poorly localized minima may stem from a slight variability between tests~~.

~~Additional interpretation of the performance of the Mohr–Coulomb failure criterion, employed in this study, could be stated as follows (it corresponds to the classical graphical meaning of the criterion)~~An additional effect may also contribute to relatively poor resolution in friction angle provided by our numerical optimization. For a fixed value of cohesion, c, ~~which is considered as the shear strength at zero normal stress (where sign simply depends on a direction of shearing), the angle of internal friction~~ the friction angle, ϕ, ~~corresponds to the slope of the envelope and controls , on one hand,~~ controls both the value of the tensile strength, σ_{st}, and ~~on the other hand the~~ the linear "strengthening" of the interface with higher ~~compression~~ normal stress (e.g. Fig. **??**). ~~Meaning that, for example, with the angle of internal friction higher than 45,~~ Hence, with a higher (resp. lower) friction angle tensile strength of the interface becomes smaller (resp. higher) than the cohesion, and at the same time, the compressive part of the criterion ~~steepens~~ has a steeper (resp. lower) inclination and requires higher (resp. lower) shear stress for failure. ~~On the contrary, a lower angle of internal friction ($<45°$) increases the tensile strength of the interface compared to the cohesion, and at the same time gives a lower inclination of the envelope in compressive mode.~~ Due to stress inhomogeneity ~~caused by inertia, inclination and geometry, the~~ along the interface the above described dual effects are always superimposed ~~onto each other~~ in simulations. Thus, for an instance of high ϕ, if some edge nodes easily "failed" in tension at a given oscillation, the rest of nodes will be stronger in compression. ~~Such dual effect due to mixed failure conditions in the interface highlights the importance of accounting for the angle of internal friction, and explains reasons for comparable time~~ This could explain the comparable times of model failure obtained for some tests computed with fixed cohesion, but different values of ϕ (e.g. Fig. **??**~~; tests: 23, 26, 30, 39, 40).~~

~~Nevertheless, if we assume 90inclination of the platform, we may expect that the angle of internal friction will become a more important factor due to~~). In this respect, we can expect the ~~higher~~ friction angle to play a stronger role for the inclined tests, in which the tensile component of stress ~~, and thus high angles of internal friction (i.e. $>45°$) would correspond to highervalues of the cost function. For tests with fixed cohesion (~~is higher. As shown in Fig. **??**~~)~~, this suggestion ~~can be supported by C_{FEM} shown only for~~ is supported by computations of the cost function performed only on inclined tests (~~27, 33,~~ 26, 27, 32~~), which becomes smaller for 30–35and increases for higher angles of internal friction (Fig. **??**). Meaning that this~~, 33). Values of C_{FEM} display more pronounced variations with ϕ in this case, with a clear minimum for $\phi = 30-35°$. This range may be considered as the ~~potential global minima~~optimal friction angles resulting from our simulations.

~~Previous experimental~~ Experimental data on the angle of internal friction ~~is~~ of snow are very scarce. ~~However, it is worthwhile to note that by plotting the homogeneous snow tensile strength, σ_{st}, against the shear strength one may identify the inclination of the Mohr–Coulomb failure envelope for snow (and thus obtain the failure ϕ). For this we attempt to plot experimental~~

~~measurements (from 14 different studies assembled by ?) of the two against snow density in Fig. ??a (for more tensile measurements see ?). If we try to deduce values of the angle of friction, as $\arctan(\tau_{st}/\sigma_{st})$, by assuming exponential fits for shear and tension strengths for all available data (Fig. ??b), such values will remain approximately within a range of 10 to 40(Fig. ??c), meaning that the tensile strength is higher than the shear strength. This is in agreement with ? and ? , however, since natural variation of strength measurements is high, the considered data includes various types of snow, and exponential fits have low R^2 (power fits yielded similar values), the conclusions remain to be verified. By comparing results of the modeling (Fig. ??) with the literature (Figs. ?? and ??), there is overall consistency with most of the considered data sources, including those in Fig. ??b.~~

~~Nevertheless, previously~~ Previously published values of ϕ vary strongly depending on the literature source (Fig. ~~??~~??a and b). Approximately thirty degrees is commonly ~~used (??) . But~~ reported (??) , but the value may range from 5.7 to 57.7° in experimental data ~~(???) (for example, ? measured the residual friction angle in the field and obtained a result of about 30), or deviate to 45for avalanche fracture line analyses (?) , or even to 73.4 and 83.1for shaking platform tests (?) . Thus, this clearly indicates that the question still remains open to~~ or fracture line analysis (????) . This wide range probably indicates that further clarification and distinction between different ~~types of friction angles~~snow types will be necessary. Nevertheless, ~~keeping in mind the previous remark, if we consider a combination of the above mentioned observation on the ratio between the shear and tensile strength (Figs. ?? and ??) and our numerically obtained results (Fig. ??), we could suggest that the~~ the value around 30–35° ~~may be~~ derived from our analysis appears consistent with these previous experimental data, and we argue that it represents the most physically realistic value for ~~snow that is similar in terms of its~~ weak layers of the type and density used in our experiments.

Finally, we note that while our results stress the importance of accounting for a normal stress dependence of snow failure criterion, the ~~linear shape of~~ linear shape assumed with Mohr-Coulomb criterion is just an approximation at this stage. Refinements of the ~~Mohr-Coulomb failure envelope, which is responsible for the effects mentioned above, was just assumed in this study. Furthermore, the likely limitation of our approach is a non-obvious~~

~~projection of the results onto the domain of higher compressions, in particular due to size effects. Such interesting questions and refinement of the law by other effects (as discussed in Sect. **??**) remain open for future work, which for instance, could consider a closure of the envelope for compression, as well as incorporate other shapes of the envelope; see~~ criterion by testing more complex envelope shapes (? ; (); ? ; (10001000[e.g. ~~**?** .~~

~~The simple model used here provides a reasonable match to the experiments. In the present form our results may not be applicable for failure in a plane perpendicular to weak layers when no shear is applied, and we do not make any claim that the model is universal in relation to fracture propagation. Accordingly the presented constitutive behaviour should be used only for predicting the sample scale shear stress at which snow is expected to fail in a brittle manner~~][]Haefeli1963 or including a closure in compression remain open issues for future work.

7 Conclusions

conclusions

This paper presents a FEM study to simulate snow weak layer failure under cyclic acceleration loading and to analyze the performance of the Mohr–Coulomb failure criterion. The model is tested by comparison with previous cold-laboratory results for shaking platform experiments (**?**). An ensemble of individual experiments is simulated and analyzed for overall sensitivity to the adjustment of the constitutive parameters. Based on more than 500 simulations, we found that the ~~linear elasticity of snow blocks and the~~ Mohr–Coulomb failure criterion for the ~~interface with zero thickness representing the weak layer are~~ weak layer is sufficient and adequate for ~~snow failure~~ the analysis of the experiments. ~~Best-fit~~ Best couples of cohesion and ~~angle of internal friction~~ friction angle, c and ϕ, were found to be [1.6 kPa, 22.5–60°]. The wide range of ϕ highlights the fact that the reproduction of experiments is largely controlled by ~~an absolute~~ the value of cohesion and has relatively low sensitivity to ~~the angle of internal friction~~ friction angle (within the limit shown above). ~~Basing~~ Based on values of the cost function for a limited sample of inclined tests (Fig. **??**) and on previous experimental evidence~~(Fig. **??**)~~, we could suggest that ϕ around 30–35° is the most optimal value, which may be further clarified with follow-up studies. ~~Nevertheless~~In addition, the requirements to consider effects of normal stress on failure, and to include the tensile strength of the interface, were evident, meaning that a purely cohesive form of the Mohr–Coulomb failure criterion is not applicable. The tensile strength could be limited to a range between 0.9 and 3.8 kPa (Table 3), which is comparable to previously reported results ~~(see Fig. **??**a).~~ (**?**) .

The FE results ~~are~~ were also compared with the previously used analytical solution (**??**), which was found to be inadequate for estimating shear stresses along the failure plane, in particular, for cases with ~~any~~ an inclination of the platform. Shear stresses produced during the inclined tests (25 or 35°) were found to be highly non-homogeneous and thus poorly represented by the analytical approach. Accordingly, the interpretation of experiments through the previously used analytical (or “static”) solution is limited~~, due to substantial edge effects (originating from non-uniform normal stress oscillations from compression into tension and caused by interplay between inertial and geometrical effects)~~.

Finally, we are aware that our model with the weak layer representation employed here is only one of many possible approaches, which could have been used to fit the data, and that we confronted the method against only one type of weak layer (composed from precipitation particles) used in previous experiments. Nevertheless, the reasonable results, described in this paper, suggest that our approach may be further verified and developed (for instance, for non-linear shapes of the failure criterion) and may be also applied to other types of loadings and weak layers. Such work along with computationally expensive comparison against other failure criteria could constitute follow-up studies.

~~One of conclusions by **?** was that the application of the Finite Element Method would never be possible in daily avalanche forecasting due to unknown spatial and temporal variation of weak layer properties and uncertainty with the weather. Nevertheless, as our study shows, mechanical application of the method may provide powerful tools for analysis, extraction and validation of theoretical or empirical laws from experimental data for their further usage. Hence, validation of the model and the formulation of an explicit cost function for the optimization of the model create a platform and open perspectives for interpretation of experiments and~~

~~follow-up theoretical studies and analysis. For example, after solving a challenging scientific question of size-effect re-sealing, the obtained values (in combination with other parameters) could be used at larger scales for modeling slope releases or in studies aimed at the impact of seismic loading on snow-covered slopes (?) .~~

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Table 1. List of tests referred for validation of the model, after **?** and prescribed modeling parameters for each test.
table

#	Platform inclination (°)	Mass of fractured snow, m_f (kg)	Peak horizontal acceleration a_p (g)	Total time of vibration until fracture (s)	Estimated shear strength, τ_{ex} (kPa)	Estimated normal pressure at failure, σ (kPa)	Mean density of the block, ($kg\,m^{-3}$)	Frequency coefficient, $k_\omega\,s^{-2}$	h_s – equivalent for FE model (m)	Young's modulus of block (MPa) as function of density, after (**?**)
17	0	2.06	5.56	18.6	1.97	−0.35	226	0.44	0.15	1.5
20	0	2.25	5.72	14.2	2.13	−0.37	226	0.57	0.16	1.5
23	0	2.02	4.96	9.6	1.66	−0.33	226	0.74	0.14	1.5
25	0	2.18	6.36	9.8	2.34	−0.37	218	0.82	0.16	1.3
30	0	2.11	5.05	8.0	1.65	−0.32	218	0.86	0.14	1.3
31	0	2.12	5.33	5.7	1.85	−0.35	218	1.14	0.15	1.3
35	0	2.42	5.91	5.4	2.37	−0.40	212	1.24	0.18	1.2
42	0	2.29	5.55	4.2	2.15	−0.39	212	1.43	0.18	1.2
43	0	2.40	4.41	4.3	1.72	−0.39	212	1.26	0.18	1.2
37	0	3.50	3.51	4.7	1.97	−0.56	212	1.06	0.26	1.2
39	0	4.60	2.70	2.8	2.06	−0.76	212	1.28	0.36	1.2
40	0	4.54	2.80	3.2	2.11	−0.76	212	1.21	0.35	1.2
41	0	4.03	2.63	2.9	1.76	−0.67	212	1.24	0.31	1.2
19	35	1.34	2.23	7.2	0.52	0.10	226	0.62	0.10	1.5
26	35	2.20	3.52	4.8	1.29	0.45	218	1.04	0.17	1.3
27	35	2.22	3.62	8.6	1.28	0.46	218	0.68	0.17	1.3
24	25	1.98	2.53	6.8	0.85	0.05	226	0.69	0.15	1.5
32	25	1.92	4.47	8.7	1.13	0.87	218	0.75	0.15	1.3
33	25	2.04	4.26	8.4	1.15	0.90	218	0.76	0.16	1.3

Table 2. Properties of FEM model (values in square brackets correspond to sensitivity tests).

Object	Property	Value
Block	Length, l	$0.3\,\mathrm{m}$
	Height, h_s	$0.10–0.36\,\mathrm{m}$
	Density, ρ	$212–226\,\mathrm{kg\,m^{-3}}$
	Young's modulus, E	$1.2\times10^6–1.5\times10^6\,\mathrm{Pa}$ [$\times2$ or $\times3$]
	Poisson's ratio, υ	0.04 [0.23]
	Viscosity, η	$10^4\,\mathrm{Pa\,s}$ [$10^2–10^8\,\mathrm{Pa\,s}$]
Interface	Length, l	$0.3\,\mathrm{m}$
	Shear stiffness, K_s	$1\times10^8\,\mathrm{N\,m^{-3}}$ [$10^5–10^8\,\mathrm{N\,m^{-3}}$]
	Normal stiffness, K_n	$1\times10^8\,\mathrm{N\,m^{-3}}$ [$10^5–10^8\,\mathrm{N\,m^{-3}}$]
	Cohesion, c	[$0.5–2.5\,\mathrm{kPa}$, $2.8\,\mathrm{kPa}$]
	Angle of friction, ϕ	[$10–75°$]
Boundary	Inclination	$0°$, $25°$, $35°$
	Oscillations (max amplitude)	Horizontal ($16.5\,\mathrm{mm}$)

Table 3. Sample response to adjustment parameters (see also Fig. **??**)[1].

Run code name	ϕ, °	σ_{st}, Pa	c, Pa	C_{FEM5} for 5 tests (27, 30, 33, 35, 41)	C_{FEM9} for 9 tests (27, 30, 33, 35, 41, 23, 26, 32, 39)	C_{FEM6} for 6 validation tests (25, 31, 37, 40, 42, 43)/C_{FEM5} for 5 validation tests (same without 25)
s1	55	750	1071.1	1.569	–	–
s2	55	1250	1785.2	0.504	0.570	–
s3	45	1000	1000.0	1.746	–	–
s4	45	2000	2000.0	0.875	–	–
s5	35	1250	875.3	2.154	–	–
s6	35	2250	1575.5	0.465	0.373	0.749/0.377
phi	30	2728.8	1575.5	0.463	0.365	0.821/0.406
phi1	40	1877.6	1575.5	0.532	0.477	–
phi2	25	3378.6	1575.5	0.519	0.424	–
s6y[1]	35	2250	1575.5	0.476	0.383	–
s6yy[2]	35	2250	1575.5	0.476	0.380	–
s7	35	3000	2100.6	1.576	–	–
s8	60	500	866.0	2.072	–	–
c3&8	45	1600	1600.0	0.506	0.434	–
c4&9	30	1600	923.8	2.131	–	–
c5&10	60	1600	2771.3	1.722	–	–
c6&11	30	2771.3	1600.0	0.496	0.385	0.794/0.394
c7&12	60	923.7	1600.0	0.454	0.448	–
s9	15	5879.7	1575.5	0.645	0.559	–
s10	75	422.154	1575.5	1.873	1.771	–
s11	22.5	3803.6	1575.5	0.539	0.443	–
s12	67.5	652.6	1575.5	1.017	0.940	–
s15[3]	35	2250	1575.5	0.483	0.404	–
s14[4]	35	2250	1575.5	0.478	0.412	–
s16[5]	35	2250	1575.5	0.446	0.362	–
phi3	50	1322.0	1575.5	0.499	0.513	–
phi4	60	909.6	1575.5	0.501	0.518	–
phi5	30	2684.7	1550	0.476	0.363	–
phi6	20	3434.3	1250	1.047	0.976	–
phi7	30	2165.1	1250	1.033	0.949	–
phi8	40	1489.7	1250	1.049	0.946	–
phi9	50	1048.9	1250	1.096	0.992	–
phi10	60	721.7	1250	1.153	1.118	–
phi11	20	4945.5	1800	0.909	0.869	–
phi12	30	3117.7	1800	0.738	0.744	–
phi13	40	2145.2	1800	0.740	0.723	–
phi14	50	1510.4	1800	0.723	0.750	–
phi15	60	1039.2	1800	0.441	0.485	0.416/0.411
phi16	60	1154.7	2000	0.762	0.810	–
phi17	67.5	517.77	1250	1.428	1.384	–
phi18	67.5	745.58	1800	0.808	0.786	–
phi19	67.5	828.43	2000	0.738	0.776	–
phi20	15	4665.1	1250	1.070	0.997	–
phi21	15	6717.7	1800	0.996	0.964	–
phi22	15	7464.1	2000	1.418	1.399	–
phi23	10	8935.1	1575.5	0.746	0.665	–
phi24	60	1212.8	2100	0.950	0.886	–
phi25	75	482.314	1800	1.565	1.462	–
phi26	75	562.67	2100	1.200	1.150	–
phi27	57.5	1075.2	1687.8	0.467	0.510	–

[1] Sensitivity tests to higher E, $\times 2$; [2] Sensitivity tests to higher E, $\times 3$; [3] Sensitivity tests to higher η, $\times 10^2$; [4] Sensitivity tests to higher η, $\times 10^4$; [5] Sensitivity tests to lower η, $\times 10^{-2}$

Fig. 1. **(a)** 2-D geometry of the discussed experiments and **(b)** an example of corresponding geometry in Finite Element model; **(c)** schematic of the joint element; **(d)** Mohr–Coulomb failure criterion.
figure

Fig. 2. Examples of imposed displacements, $s(t)$, its derivatives and analytical estimation of shear stress. **(a)** Imposed displacements, $s(t)$ (k_ω=0.74 s^{-2}); **(b)** velocity, $s'(t)$; **(c)** acceleration, $s''(t)$; **(d)** analytical shear stress, τ_a (for $h_\mathrm{s} = 0.1\,\mathrm{m}$, $\rho = 200\,\mathrm{kg\,m^{-3}}$).

Fig. 3. Examples of fitting angular frequency by adjusting k_ω: **(a)** $k_\omega = 0.33\,\mathrm{s}^{-2}$ (in black) and $1.43\,\mathrm{s}^{-2}$ (in blue), **(b)** same zoomed; Markers indicate an example of observed peak acceleration reached at observed failure time.

Fig. 4. Shear and normal stress concentrations within blocks (inclined to 0, 25 or 35°) at different consequent phases of oscillations (time increases downward; the inset of the figure shows an example of corresponding instants on the trajectory, i.e. time–displacement plane). (For each inclination left side corresponds to shear, τ, right side – to normal pressure, σ. Note that color intensity is not normalized in order to highlight specific concentrations for each case; in 10^3 Pa).

Fig. 5. Example of evolution of normal stresses in the middle and at edges of the interface (blue corresponds to the middle of the interface; red – to the ~~lower~~ left edge; green – to the ~~upper~~ right edge). **(a)** horizontal test (Test 23); **(b)** and **(c)** – inclined tests (25 and 35°; Tests 33 and 27).

Fig. 6. Examples showing shear stress differences between simple analytical and FEM solutions. **(a)** horizontal test, 0° (Test 23, $h = 0.14\,\mathrm{m}$, $\rho = 226\,\mathrm{kg\,m^{-3}}$); **(b)** inclined test, 25° (Test 33: $h = 0.16\,\mathrm{m}$, $\rho = 218\,\mathrm{kg\,m^{-3}}$); **(c)** inclined test, 35° (Test 27: $h = 0.17\,\mathrm{m}$, $\rho = 218\,\mathrm{kg\,m^{-3}}$). Analytical solutions are shown in blue; FEM – in red (for the middle of the joint), green (left or upper edge), and black (right or lower edge).

Fig. 7. Example of N_f growth with simulation time (for test 30: $c = 1.6\,\mathrm{kPa}$, $\phi = 30°$, $(t_\mathrm{m} - t_\mathrm{e}) = 0.3\,\mathrm{s}$): i.e. instantaneous number of nodes under failure criterion, N_f (t_e is shown by a blue asterisk, t_m by a red circle). Illustrations below indicate which nodes along the length of the interface satisfy failure criterion (i.e. yes – "1", no – "0") at particular instants.

Fig. 8. Example of delays between observed and modeled failures $(t_{\mathrm{m}} - t_{\mathrm{e}})$ for different tests as a function of adjustment parameters (ϕ, c). Blue circles correspond to 30°–1.6 kPa, blue crosses to 30°–0.9 kPa; black triangles to 30°–2.7 kPa, black diamonds to 60°–1.6 kPa.

Fig. 9. Comparison between C_{FEM} obtained for (1) whole population of tests with stiffness K_{s} and $K_{\mathrm{n}} = 10^8\,\mathrm{N\,m^{-3}}$ (C_{FEM19}; 19 tests), (2) for a population excluding outliers and computationally expensive tests (C_{FEM15}; 15 tests: i.e. without 17, 19, 20, 24), and (3) for a sample of the population (C_{FEM5}; 5 tests: only 27, 30, 33, 35, 41).

Fig. 10. Effects of c and ϕ adjustments on time delay between modeled and experimental failures (C_{FEM}, or RMSE; shown for a sample of 5 tests by empty circles, for a sample of 9 tests by crosses, and for Young's modulus sensitivity tests, s6y and s6yy, by pentagrams). Color contours are based on cubic interpolation for generalization of results (C_{FEM5}).

Fig. 11. Illustration of all tested pairs of c and ϕ as parameters of the Mohr–Coulomb failure criterion (blue dashed lines); red ~~curves~~ lines (with green shading) indicate the most successful simulations (i.e. when both C_{FEM}, for the representative sample of 5 or 9 tests, are $\leq 0.5\,\mathrm{s}$). Circles indicate previous analytically derived experimental results (**?**).

Fig. 12. Effect of the angle of friction, ϕ, on C_{FEM} for simulations with the same cohesion $1.57\,\mathrm{kPa}$ (shown for a sample of 5 tests by blue empty circles, for a sample of 9 tests by red crosses, for a sample of 4 inclined tests by black diamonds).

Fig. 13. **(a)**~~Experimental measurements~~ , **(b)** Values of ~~snow shear (in blue) and tensile (in red) strengths as functions~~ the angle of ~~density~~ friction obtained from ~~multiple studies and for~~ different ~~snow types; curves refer to (?) ; for details and full bibliographic references see (?) ; **(b)**~~studies. Y ~~Exponential fits for shear and tensile strengths~~ axis in **a** corresponds to $\tan\phi$ (~~$0.191e^{0.0099\rho}$, $R^2 = 0.57$,~~ it is equal to $c/\sigma_{\rm st}$ and ~~$0.231e^{0.0117\rho}$, R^2=0.62, respectively); dash-dot~~ shown as a blue curve~~shows shear fit divided by tension fit ($0.8251e^{-0.0018\rho}$)~~. ~~**(c)** Corresponding arctangent of the shear to tension proportion~~; ~~dash-dot curves indicate~~ it visualizes the ~~boundaries of the absolute propagated uncertainty (which is $|\Delta f(x_i)| = \sum_{i=1}^{n} |\frac{\partial f}{\partial x_i}||\Delta x_i|$~~ratio between cohesion and tensile strength).

~~**(a), (b)** Examples of values of the angle of friction obtained from different studies (y axis in **a** corresponds to $\tan\phi$, which is equal to $c/\sigma_{\rm st}$ and shown as a blue curve). It is plotted this way in order to visualize the ratio between cohesion and tensile strength).~~