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Numerical modelling of tsunami wave run-up and breaking within a two-dimensional atmosphere–ocean two-layer model

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Abstract

A numerical model of propagation of internal gravity waves in a stratified medium is applied to the problem of tsunami wave run-up onto a shore. In the model, the ocean and the atmosphere is considered as a united continuum whose the density varies with height with a saltus at a water–air boundary. Correct conditions of join at a water–air interlayer are automatically ensured because the solution is searched for as a generalised one. The density stratification in the ocean and in the atmosphere is supposed to be an exponential one, but in the ocean, a scale of stratification of density is large and the density varies slightly. The wave running to a shore is taken as a long solitary wave. The wave evolution is simulated with consideration of time-varying vertical wave structure. Inshore, the wave breaks down, and intensive turbulent mixing develops in water thickness. The effect of breakdown depends on shape of the bottom. If slope of the bottom is small, and inshore the depth grows slowly with distance from a shore, then mixing happens only in the upper stratum of the fluid, thanks to formation of a dead region near the bottom. If the bottom slope inshore is significant, then the depth of fluid mixing is dipped up to 50 metres. The developed model shows the depth of mixing effects strongly depends on shape of a bottom, and the model may be useful for investigation of influences of strong gales and hurricanes on coastline and beaches and investigation of dependence of stability of coastline and beaches on bottom shape.

1 Introduction

Many monographies (Pelinovsky, 1996, 1982; Levin, 2005; Massel, 2001; Carrier and Greenspan, 1958; Kânoğlu , 2004; Fedotova and Chubarov, 2001; Beizel et al., 1990; Zajtsev et al., 2010; Kurkin, 2004; Zaitsev, 2005), Internet resources (Earthquake, 2012; Nami-Dance-Software, 2014; Tsunami center, 2014) is devoted to modelling of generation and propagation of tsunami waves in the ocean. Successes in studying of tsunami waves are significant. Despite it, the problem of tsunami wave propagation

NHESSD

2, 3397–3421, 2014

remains an interesting subject for scientific investigations. When a tsunami wave approaches to a shore, the wave breaks and causes very intensive turbulent fluid mixing. Mixing processes develop not only in water, but in the atmosphere as well. This stage of development of a tsunami wave characterised by origin and development of turbulent mixing, is difficult for numerical modelling, and successes in its study are less significant. In the present investigation, some numerical model intended for study of this phenomenon is developed and applied.

The up-to-date numerical models of propagation of a tsunami wave are described in particular, in Imamura et al. (2006); Titov et al. (2003); Shi et al. (2012a). A series of TUNAMI programs in Imamura et al. (2006), numerical model MOST in Titov et al. (2003) is described; the refined variant of program FUNWAVE in Shi et al. (2012a) is suggested. The majority of models of tsunami wave propagation are based on some variants of equations of a shallow water. In shallow water equations, a fluid flow average over a vertical variable is applied. This approach simplifies the equations and allows simulation of propagation of waves in the ocean up to long distances with excellent accuracy. However, because of averaging over a vertical variable, shallow water models do not describe mixing effects. Consequently, we cannot take any shallow water model as a base of our approach.

Korteweg-de Vrice models with variable coefficients, also Gardner's equations describe the run up of a tsunami wave onto a shore, with consideration of an interior vertical structure of a wave (see Didenkulova et al., 2012, and the literature listed there). However, these models are asymptotic ones, also they are of restricted field of applicability.

Models with a free surface (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012) possibly may be good base for studying of origin and development of fluid mixing under the influence of an ingoing wave. When one analyses applicability of some numerical models for calculation of processes of mixing he must take in view one important point. The spatial scales of wave phenomena will be changed with many orders after breaking of an incoming wave and development of turbulent mixing. In these conditions, precise fulfillment of fundamental conservation laws for numerical methods is very important. The energy conservation law automatically follows the equations for density and momentum when we consider differential equations. However, finite-difference equations differs from differential ones and conservation of energy does not follow conservation of mass and momentum for finite-difference equations. Therefore, it is necessary to select specially the numerical method in order to ensure not only conservation of mass and momentum, but also conservation of energy. Corresponding examination of conservation laws is not fulfilled for existing numerical models (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012) with a free surface; and computational formulas (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012) are difficult for analysis of conservation laws because they are multiple-stage and contain many corrective terms.

Let us consider methods (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012) in more details. In Chen et al. (1997), the TVD-scheme for modelling of wave propagation over an irregular bottom within the limits of model of a heavy fluid placed in a gravity field, with a free surface was suggested. The numerical method (Chen et al., 1997) has been checked up by comparison of simulations with experiment in a laboratory tank with sizes 40 sm × 20 sm × 4 sm, and good coincidence of simulations with experiment has been demonstrated. The spatial scales of waves in the tank are small, and owing to it, the waves in the tank (Chen et al., 1997) are strongly influenced by viscosity. The strong viscosity flattens the solution, and it significantly simplifies numerical integration of equations. The spatial scales of real oceanic waves running onto a shore are not less than one thousand times exceed the scales of waves in the tank. An ocean wave should overcome many stages of wave breakding before the scales of processes in the ocean wave become comparable with the scales of waves in the tank. Viscosity influence is insignificant at these first phases of breaking of waves. Therefore, model (Chen et al., 1997) is not applied to modelling of oceanic processes, really.

In Shi et al. (2012b); Abbasov (2012) some improvements of numerical scheme (Chen et al., 1997) giving methods applicable to simulation of coastal processes are proposed. Models (Shi et al., 2012b; Abbasov, 2012) are not taking into account viscos-

NHESSD
2, 3397–3421, 2014

ity as the viscosity influence is negligible for processes of considered scales, but they take into account turbulent mixing diffusion or turbulent viscosity, which are significant.

The numerical models which use semiempirical models of turbulence ($k-\varepsilon$ model, like in Shi et al. (2012b), or turbulent viscosity like in Abbasov (2012)) demand empirical selection of turbulent coefficients. Owing to complicated dynamics of waves, turbulent coefficients may depend on coordinates and time. Natural processes unlike experiments in tanks are non-reproducible. It gives us certain problems at adjustment of turbulent models.

In Dambieva and Hakimzjanov (2008); Afanasev and Berezin (2003); Shokin (1990) some problems with a free surface were solved with an assumption that the fluid flow is potential.

The numerical model applied in the present paper is two-dimensional nonhydrostatic. It allows simulation of propagation of internal gravity waves in a stratified fluid. Originally, the given numerical model has been developed for simulation of propagation of internal waves in a medium with a continuous density stratification. However, the model is rather universal, and in the present paper, we apply our numerical model to simulation of waves in a stratification with a density jump. We consider the ocean and the atmosphere as a united continuum. The density of this continuum has a saltus at a water–air boundary.

As the atmosphere is taken into account in the present model, simultaneously with a wave in the ocean. the atmospheric disturbance induced by this wave is simulated. Relation of the present model to models (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012) is the following: if we substitute the expression $\rho(x,z,t) = \rho_{\text{water}} * \eta(-(z-\sigma(x,t)))$ for density into Eq. (1), where η is a unit step function and $\sigma(x,t)$ is the function describing water–air interface, then our Eq. (1) turn into equations similar (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012). In this sense, one may consider our model Eq. (1) as generalisation of models (Chen et al., 1997; Shi et al., 2012b; Abbasov, 2012).

Unlike Chen et al. (1997), viscosity is not taken into account in Eq. (1) because it is negligible for waves of considered scales. Unlike Shi et al. (2012b), we do not apply

NHESSD
2, 3397–3421, 2014

Modelling of a tsunami wave

S. P. Kshevetskii and I. S. Vereschagina

$k-\varepsilon$ model for taking into account turbulent diffusion of mixing, and unlike Abbasov (2012) we do not introduce turbulent viscosity into our model. Origin of turbulent mixing is one of important interests of our investigation, and consequently using of any parameterisation of processes of turbulent mixing contradicts the purposes of our research.

In Kshevetskii (2006), the model has been applied to modelling of processes of mixing in a fluid with continuous stratification, and numerical calculations have been compared to a laboratory experiment. It has been shown that the given numerical method well describes the first phases of development of fluid mixing. Nevertheless, analysing numerical model (Kshevetskii, 2006), one should take into consideration that the vortexes break at development of turbulent mixing, and since some instant the resolution ability of our grid may become insufficient for resolving small vortexes, and viscosity influence can become significant. From this time, turbulent mixing is modelled only qualitatively, and we already cannot give guarantee of exact quantitative describing. Nevertheless, at the first stages of wave breaking, the scales of vortexes being arisen are not yet very small, and the accuracy of our model is quite satisfactory. Within the present work we are interested only in general characteristics of processes: at what distance from a shore the fluid mixing arises; up to what depths this fluid mixing penetrates; and how it depends on shape of a bottom. The suggested model is quite adequate for examination of these parameters.

Universality is an important feature of the present model. The model allows taking into consideration in details a density stratification of oceanic water with height. It is known that internal gravity waves in a stratified fluid can break, with turbulence formation, and they breaks more often than waves on a fluid surface. Therefore, the water density stratification in the ocean may be an important factor boosting turbulent mixing in a fluid when wave come onto a shore, and our model allows to probe it.

However, in the present paper the effect of an ocean water stratification on propagation and breakdown of a tsunami wave is touched only slightly. Historically, the shallow water models are traditionally applied to simulation of tsunami wave propagation; these models are well probed owing to long-term history of these models. Our model is new;

NHESSD
2, 3397–3421, 2014

the model is significantly more difficult mathematically because we resolve non-linear elliptical operators at each time-step. The model is unusual and is insufficiently probed owing to lack of experience of such models. Any new model should overcome some natural trajectory of development, for the purpose of its assimilation, testing, examination of its advantages and lacks. The present paper is devoted to such initial study of the model. At the first investigation phases, even ability of this very complicated model to describe correctly the wave propagation in the ocean demands verification.

This model takes into account interaction of oceanic and atmospheric waves. As the atmospheric gas density is small, the atmosphere feebly influences wave propagation in the ocean. Nevertheless, interaction of an ocean wave with the atmosphere is interesting for study. Simulations show that some atmospheric disturbance induced by an ocean wave propagates together with the wave in the ocean. In the atmosphere, a wind arises over the ocean wave; the direction of this wind is opposite to direction of movement of water in the wave. It causes conditions for occurrence on a water–air interface the secondary, small-scale waves generated due to interaction of the ocean wave with the atmosphere. The atmospheric gas is capable also to influence the breakdown of a wave: though the air density is small, but air possesses elastic properties and can strengthen instabilities. For example, the water stream in vacuum propagates without breaking, but in air, the water stream may be divided into drops owing to instabilities arising on a water–air boundary.

In (Kshevetskii, 2006), the author has shown that simultaneous fulfillment of all fundamental conservation laws and some inequality for density, given in Kshevetskii (2006), is a necessary condition of stability and convergence of a numerical method. Simultaneous fulfillment of all fundamental conservation laws and the inequality for density is proved in Kshevetskii (2006) for our numerical model.

The similar numerical approach has been earlier applied to modelling of turbulent processes in the atmosphere, and has allowed simulation of turbulence formation at turbopause heights in the atmosphere (about 100 km) at the expense of breaking of grav-

NHESSD
2, 3397–3421, 2014

S. P. Kshevetskii and I. S. Vereschagina

ity waves propagated upward from ground sources (Gavrilov and Kshevetskii, 2005; Kshevetskii, 2003).

2 Problem statement

We consider movement of an incompressible fluid placed in a gravity field. The problem is two-dimensional. The fluid flows over an irregular bottom. The fluid behaviour is described by a set of equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho \frac{\partial \Psi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\rho \frac{\partial \Psi}{\partial x}\right) = 0, \qquad \frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}\left(\rho \frac{\partial \Psi}{\partial z}\right) + \frac{\partial}{\partial x}\left(\rho \frac{\partial \Psi}{\partial x}\right)\right] + \frac{\partial^2}{\partial x \partial z}\left[\rho\left(\left(\frac{\partial \Psi}{\partial z}\right)^2 - \left(\frac{\partial \Psi}{\partial x}\right)^2\right)\right] - \left(\frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial^2 \Psi}{\partial x^2}\right)\rho \frac{\partial \Psi}{\partial x}\frac{\partial \Psi}{\partial z} - \frac{\partial}{\partial x}(\rho g) = 0 \tag{1}$$

following Euler's equations for an incompressible fluid. Here ρ is medium density; Ψ is a flow function; t is time; g is the free fall acceleration; x is a horizontal coordinate; and z is a vertical coordinate. $u = \frac{\partial \Psi}{\partial z}$ is a horizontal velocity; $w = -\frac{\partial \Psi}{\partial x}$ is a vertical velocity.

Dynamics of atmospheric parameters is described by the same equations, but with density equal to atmospheric gas density. That is, the atmosphere is described in the model by incompressible fluid equations. Usage of approximation of an incompressible fluid for an atmospheric gas is possible because the atmospheric gas density is small; therefore, the energy of waves in the atmospheric part of our model is small, and high accuracy of an atmospheric part of the model is not required. The characteristic time of the processes taking place at tsunami wave propagation is estimated about 100 s; and it is possible to show that an incompressible fluid approximation is applicable for describing such slow processes even in case of gas.

NHESSD
2, 3397–3421, 2014

At $t = 0$, the density ρ [kg m^{-3}] is given by the formula:

$$\rho(x,z,0) = \begin{cases} 1000 \times \exp\left(-\frac{z}{H_W}\right) & z < 0, \\ 1.2 \times \exp\left(-\frac{z}{H_A}\right) & z > 0, \end{cases} \quad (2)$$

The formula (2) describes real density change with height for the atmosphere ($z > 0$) and some model density behaviour for ocean water ($z < 0$). $H_A = 8$ km is a scale of stratification of the atmospheric air, $H_W = 100$ km is taken as a scale of stratification for water. At $t > 0$, the stratification may be found by solving the set of Eq. (1).

Finite-difference approximation of equations always introduces some error into a model. This error is equivalent to occurrence in the equations of some additional sources of mass, momentum, energy. It is essentially impossible to be saved of discretization errors, but one can select discretization of equations in such a form that appearing additional sources of mass, momentum, energy cancels each other on the average. For this purpose, initial equations are rewritten in an integral shape in terms of fundamental laws of conservation of mass, momentum, and energy. The numerical scheme should be chosen so that fundamental conservation laws in their integral shape have been satisfied. That is, we require that the time-variation of integrals of conservative values over any volumes be equal to flows of these values through surfaces of volumes (in discrete model the integrals are approximated by the corresponding integral sums over grid points). The numerical schemes precisely supporting fundamental laws of conservation of mass, momentum, energy in a summation form, are called a conservative ones. Integral equations do not contain differentiation operations; therefore, the requirement of differentiability of a solution misses. The solutions satisfying to fundamental conservation laws in the integral shape are named generalised solutions or weak solutions. Conservative numerical method has an advantage because though discretization of equations introduces some errors into a model, but nevertheless hydrodynamic conservation laws are fulfilled with high accuracy for volumes with scales being larger than spatial grid step; therefore, the fluid flow is modelled correctly at scales exceeding the grid step.

If some clear boundary between fluids with different densities exists and is described by a differentiable function, then standard conditions of join on the interface follow the definition of a generalised solution. The technique of generalised solutions is applicable also to cases when boundary behaviour is very complicated and is not described by differentiable functions. Such a situation may occur due to wave breaking.

A conservative numerical method of a second order of accuracy in space and time is used for solving equations (1). The finite-difference scheme is an explicit-implicit one. The spased net a cross is used. The computative formulas of a completely conservative numerical method are very cumbersome; the finite-difference equation for the flow function takes more than a page of text; therefore, this equation is not written in the present paper. The numerical method has been calculated and programmed by means of a program of analytical evaluations. The used difference grid, the derivation of finite-difference formulas, the proof of full conservatism of the method, the outcomes of testing of the method is published explicitly in Kshevetskii (2006).

The problem is solved in a rectangular domain Ω with height of 2 km and breadth 30 km (Fig. 1). An atmospheric gas is above, within the interval of heights $0\,\text{km} < z < 1\,\text{km}$, and either an oceanic water or a ground is below, within the interval of heights $-1\,\text{km} < z < 0\,\text{km}$.

The condition of fluid nonpenetrating $\Psi|_{\partial\Omega} = 0$ is imposed along the boundary $\partial\Omega$ of the domain Ω. The condition $\Psi|_{\text{ground}} = 0$ is imposed as well in the all field of a ground. The field of a ground is artificial; this field is introduced into the problem for simplification of working with boundary conditions at programming of calculations.

The ocean coast corresponds to 0.0 marks at a horizontal coordinate axis. More to left of 0.0 mark, that is, at $x < 0$, the fluid is missing and there is only a land and an atmospheric gas over it.

A solitary wave running to a shore is set as an initial condition. The initial solitary wave is constructed as follows. Some exact solution for the solitary gravity wave running

NHESSD
2, 3397–3421, 2014

to a shore is borrowed from the theory of long surface waves. This solitary flow function is applied only for water. Therefore, we determine this flow function in an atmospheric part of our model so that it is continuous at a water–air interface and is equal to zero at the boundary of our domain Ω. We obtain:

$$\Psi(x,z,0) = A\frac{g}{c(x)}\exp\left(-\left(\frac{x-x_0}{\lambda}\right)^2\right)\varphi(z,x), \tag{3}$$

$$\rho(x,z,0) = \rho_0\left(z - A\exp\left(-\left(\frac{x-x_0}{\lambda}\right)^2\right)\right), \qquad \varphi(z,x) = \begin{cases}(z+h_W(x)), & z<0\\ \frac{h_w(x)}{h_A}(h_A - z), & z>0\end{cases}$$

Here $h_W(x)$ is the ocean depth, $c(x) = \sqrt{gh_W(x)}$, $h_A = 1\,\mathrm{km}$ is the upper bound coordinate in the atmospheric model, $A = 10\,\mathrm{m}$. Calculations are fulfilled for two variants: (a) the angle of slope of the bottom near the shore is 45° and (b) the angle of slope of the bottom near the shore is 10°.

In case (a), the parameters are: $x_0 = 14\,\mathrm{km}$ is a distance from the centre of the initial wave to the shore; $\lambda = 5\,\mathrm{km}$ is a wave half-width; the sloping bottom is transformed into an equal horizontal plateau at the depth of 1 km. In case (b), the parameters are: $x_0 = 14\,\mathrm{km}$; $\lambda = 4\,\mathrm{km}$, the bottom is transformed into a horizontal plateau at the depth of 650 m. The case (a) is considered for checking the model and in order to understand what the wave presents by itself in our model.

The constructed initial solitary wave, extended in the ocean and in the atmosphere is a vortex whose part is in the ocean, and another part is in the atmosphere. Though we have determined an atmospheric part of the flow function artificially, the occurrence of united vortex, which is partially in the ocean, and partially in the atmosphere, is naturally. This is a corollary of boundary conditions and the requirement of flow function continuity on the interface. When the wave propagates to a shore, the given vortex is deformed a little, gains the stable shape, and further is stable and moves to a shore (Fig. 3).

3 Outcomes of numerical modelling

One of purposes of this paper is to show that numerical mode (Kshevetskii, 2006) not only describes propagation of internal gravity waves in a continuous stratification, but also describes wave propagation in stratification with a density jump. In particular, the model allows simulating tsunami wave propagation.

The difference grid is variable. In case (a) (the angle of slope of the bottom is equal 45°), the horizontal grid step is up to 400 m at distances far from the shore, and the vertical grid step is up to 10 m at heights far from the ocean surface. The horizontal net is condensed near the shore, and the vertical net is condenced near the ocean surface. Inshore, the horizontal grid step is decreased up to 0.5 m, and near the ocean surface the vertical grid step is decreased up to 0.25 m.

Outcomes of numerical modelling of wave propagation are shown below. In Fig. 2, the flow function for the initial wave is shown. The wave in the atmosphere–ocean system is a vortex, which is strongly drawn out along a horizontal axis. Some vortex part is in the ocean, and another part is in the atmosphere. The fluid flow in the ocean is almost potential. The united vortex is obtained because the flow function is closed through the atmosphere.

In Figs. 3 and 4, the propagating wave in the atmosphere–ocean system, time $t = 60\,s$, is shown. Simulations show that the vortex is stable and moves to a shore without breaking. The speed of wave propagation is approximately equal $100\,\mathrm{m\,s^{-1}}$ that corresponds to the theory of gravity surface waves. Within the approach of small-amplitude long waves propagating over a plain horizontal bottom, one can construct an analytical solution for Eq. (1) for initial conditions Eq. (3). The outcomes of numerical modelling well coincide with this analytical solution, and Fig. 3 may be considered as a test for our numerical model.

At $t = 60\,s$, the wave front already reaches the shore, and the wave starts to come under the shore influence. Inshore, water-level upraise begins. In the ocean, the fluid flow in the wave is directed to the shore, but in the atmosphere, on the contrary, the gas is moving from the shore. At the water–air interface, there is a velocity jump. It causes conditions for development of instability and for generating secondary, small-scale surface waves due to influence of the atmosphere onto the propagating ocean wave.

At $t = 90\,s$, the interaction of the wave with the shore becomes significant. The wave is stopped. Behind a head wave, a pair of vortexes with opposite gyrations has arisen. Water-level upraise inshore reaches 7 m.

At $t = 120\,s$, the small derivative vortex comes off the head vortex and gets on the shore (Fig. 5). The vortex moving to the shore has been strongly deformed, and at some distance from the shore, there are vortexes with gyration opposite to gyration of the incoming vortex. The wave reflected from the shore has been formed.

At $t = 135\,s$, the water level has been lifted up about 16 m inshore. On the density jump, small-scale fluctuations are developed; these are small-scale waves, which arise owing to instability on velocity shift. These waves exist due to the atmosphere influence on the propagating basic wave.

Interaction of the wave with a shore leads to intensive mixing of fluid inshore. In Fig. 6, the fluid horizontal velocity is shown; inshore, we see intensive mixing of ocean fluid and mixing of atmospheric air. Processes of mixing of water extend downwards up to the depth of 50 m. Mixing has a small-scale structure, however it is not visible in Fig. 6, because this drawing demonstrate only an overall picture.

The wave run-up onto a shore with small slope of a bottom (case b) is shown for comparison in Fig. 7. The wave gets on a shoal on the same mechanism, as in a case (a). The initial vortex is deformed when it come close to the shoal. Then some derivative vortex separates from the main vortex and gets on the shoal. However this forward separated vortex is more high-power, than in case (a), also it is drawn out. The main vortex is stopped far from the shore. Centre of the first vortex is lifted up above the quiet ocean level. This effect is ensured at the expense of water-level uprise.

In Fig. 8, the uprise of a water level and the inundation for a case (b) is shown. The water-level upraise in case (b) is approximately identical to the one for case (a); and in both cases, the uprise is approximately equal to 16 m. An uprise depends essentially on momentum and energy of an incoming wave. In both cases, momentums and energies of initial waves approximately coincide. Therefore, the uprise of a water-level in both cases (a) and case (b) is identical.

4 Conclusions

The wave run-up onto a shore for cases of slope of a bottom 45° and 10° is simulated numerically. Within the considered model, it is shown that a tsunami wave is a stable moving vortex, which partially is in the ocean, and partially is in the atmosphere.

The water flow in the ocean is close to a potential flow, and the vortex is organised at the expense of closure of flow lines through the atmosphere.

The speed of wave propagation approximately coincides with the propagation velocity of a long surface gravity wave.

The air movement in the atmosphere is of an opposite direction to the movement of water in the ocean, and velocity shift takes place on the surface of density jump. It causes conditions for development of instability and leads to formating of secondary small-scale waves on the interface over a propagating tsunami wave.

Uprise of the water-level for both mentioned cases are approximately equal to 16 m.

The mechanism of penetration of a wave onto a shore (case a) or onto a shoal (case b) appears uniform: the derivative vortex separates from the basic vortex and gets onto a shore or onto a shoal. The remaining wave is reflected from a shore or from a shoal.

Simulations have allowed doing an estimate of the depth to which effects of intensive small-scale mixing of fluid inshore extend. In case (a), the depth of mixing is about 50 m. In case (b), the depth of fluid mixing does not exceed 20 m. In case (b), decreasing of the depth up to which mixing extends is caused by the effect of arising of a dead region under an ingoing wave when the wave is approaching to a shoal; and the basic movement of the water takes place in the upper stratum, with corresponding increasing of velocities of flows.

NHESSD
2, 3397–3421, 2014

Water-level uprise inshore is appreciably spotted by energy of an ingoing wave. For simulations, the numerical methods supporting fundamental conservation laws are used; and consequently one can expect that inundation near the shore be calculated correctly. Nevertheless, accuracy of the model should be a subject of further examination by means of comparison of outcomes of calculations with laboratory experiments and full-scale observations.

Analysing imperfections of the numerical model, we emphasise the flattening of density in a neighbourhood of transition water–air. This imperfection does not affect the quality of simulations as a whole, because main energy of a wave is spreaded over thickness of water; and the processes in thickness of water determine in the mane the behaviour of transition water–air, while inverse influence of transitive water–air stratum on the wave as a whole is insignificant. Nevertheless, the problem of more detailed modelling of water–air transition deserves further consideration. Possibly, modelling of water–air transition can be easily refined at the expense of using of more fine difference grid, or at the expense of improvement of finite-difference approximation of the equation for density.

The authors consider that the given experience of modelling of propagation and breaking of a tsunami wave within the limits of a two-dimensional united water–air model is successful as a whole.

The developed model can also be of certain interest in connection with the problem of degradation of shores and erasures of beaches due to strong gales and hurricanes. Intensive mixing of water during strong gales and hurricanes can cause lifting up of sand from a bottom, and it creates conditions when the sand can be carried away into a sea or into the ocean. Stability of a coastline strongly depends on a bottom shape inshore. The present investigation shows that if depth inshore increases slowly, then the depth of mixing of fluid is small owing to formation of a dead region near the bottom inshore, and in this case the bottom and coast line should be more stable.

NHESSD
2, 3397–3421, 2014

S. P. Kshevetskii and I. S. Vereschagina

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NHESSD
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NHESSD
2, 3397–3421, 2014

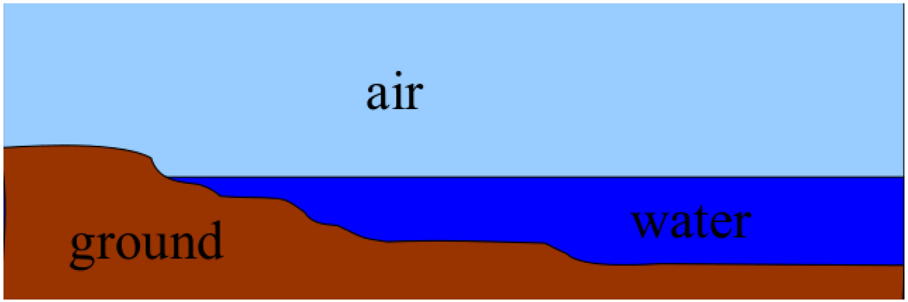

Fig. 1. Rectangular domain in which the equations are solved.

NHESSD
2, 3397–3421, 2014

Modelling of a tsunami wave

S. P. Kshevetskii and I. S. Vereschagina

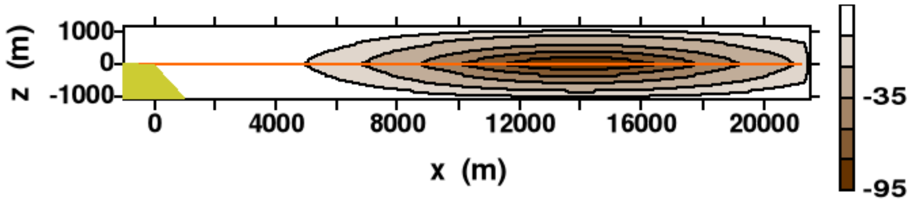

Fig. 2. The flow function Ψ for a wave running to a shore, $t = 0$ s (variant a).

NHESSD
2, 3397–3421, 2014

S. P. Kshevetskii and I. S. Vereschagina

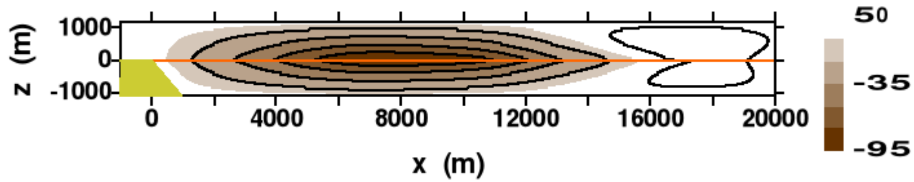

Fig. 3. The flow function Ψ for a wave running to a shore, $t = 60\,\mathrm{s}$ (variant a).

NHESSD
2, 3397–3421, 2014

Modelling of a tsunami wave

S. P. Kshevetskii and I. S. Vereschagina

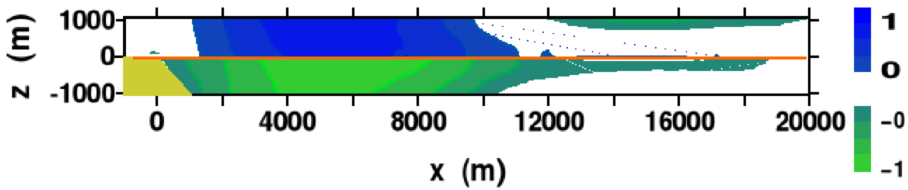

Fig. 4. The horizontal velocity, t = 60 s (variant a) (intensity of colours corresponds to values of velocity).

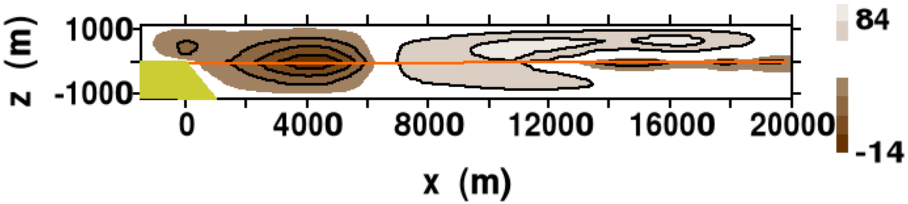

Fig. 5. The flow function Ψ, $t = 135\,s$ (variant a).

NHESSD
2, 3397–3421, 2014

Modelling of a tsunami wave

S. P. Kshevetskii and I. S. Vereschagina

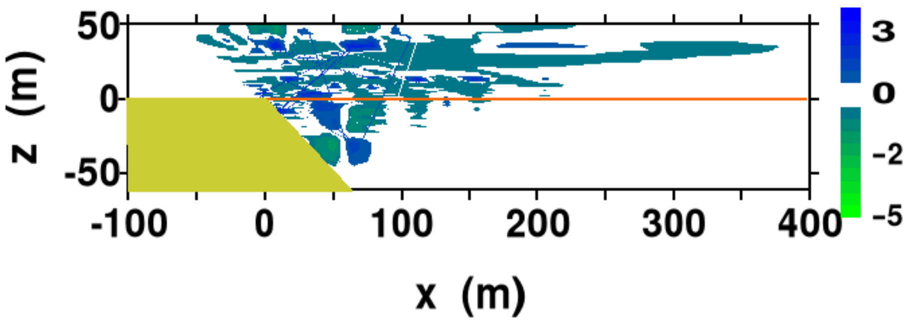

Fig. 6. The horizontal velocity, $t = 135\,\mathrm{s}$ (variant a) (intensity of colours corresponds to values of velocity).

NHESSD
2, 3397–3421, 2014

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S. P. Kshevetskii and I. S. Vereschagina

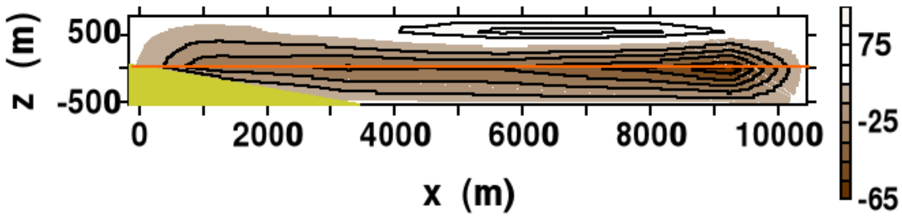

Fig. 7. The flow function, t = 82.5 s (variant b).

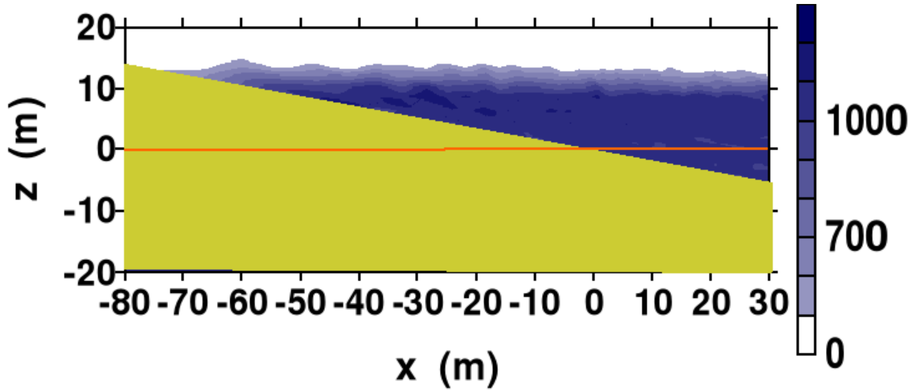

Fig. 8. The coastal flood, $t = 82.5$ s (variant b).

NHESSD
2, 3397–3421, 2014

Modelling of a tsunami wave

S. P. Kshevetskii and I. S. Vereschagina