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Estimation of flood design hydrographs using bivariate analysis (copula) and distributed hydrological modelling

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pose, evaluate and select measures to reduce risk. Thus, the European Parliament has adopted the new Directive 2007/60/EC (European Union, 2007) that requires Member State to assess if coastal areas and water courses are at risk from flooding, to carry out maps and take measures to reduce the evaluated risk. The objective of this directive is to establish a framework for the assessment and management of flood risk in Europe, emphasising both the frequency and magnitude of a flood as well as its consequences.

Reliable estimates of the likely magnitude of the extreme floods are essential in order to reduce future flood damage. Despite the occurrence of extreme floods is a problem across Europe, physical mechanisms responsible for the generation of floods will vary between countries and regions. As a result, no standardised European approach to flood frequency estimation exists. Where methods exist, they are often simple and their ability to accurately predict the effect of environmental change (e.g. urbanisation, land-use change, river training and climate change) is unknown.

Moreover, Mediterranean ephemeral streams have specific features compared to other river systems. Mediterranean basins are small, several hundred km², and highly torrential and may generate flash-floods (Camarasa-Belmonte and Soriano, 2012; Koutroulis and Tsanis, 2010). Runoff generation in semiarid zones is the final result of a lot of spatial and temporal complex processes that take place at the hillslope and catchment scale. The complexity of the processes involved derives from great heterogeneity of rainfall inputs, surface and subsurface characteristics, and strong nonlinear dependency on antecedent wetness which controls the infiltration capacity of the soil surface and the connectivity of surface and subsurface runoff pathways (Candela et al., 2005; Nicolau et al., 1996).

The flood frequency analysis (FFA) is the estimation of how often a specified event will occur (Hosking and Wallis, 1997) and aims at estimating the flood event in terms of maximum discharge value corresponding to a given return period and/or relative volume. The probability for future events can be predicted by fitting the past observations to selected probability distributions. The flood event estimation (hydrograph project) requires the use of different methods depending on whether it is enough to know the

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maximum discharge value, or it is necessary to know the full hydrograph. In both cases, the problem can be solved directly, starting from flow measurements available for the basin, or indirectly using rainfall data recorded under the basin using a rainfall-runoff model. This latter approach is the basis of the DDA methods (Derived Distributed Approach) that allows to derive FFC using rainfall-runoff models. Analytical difficulties associated with this approach are, often, overcome by adopting numerical Monte Carlo methods. In these cases, a stochastic rainfall generator is used in order to generate rainfall data for a single event or in continuous (Blöschl and Sivapalan, 1997; Loukas, 2002; Rahman et al., 2002; Aronica and Candela, 2007).

FFC analysis is, usually, based on the derivation of FFC to only define the maximum discharge value corresponding to a given return period. However, for flooding management, it is not enough to know information about flood peak only, but it is also useful to statistically value flood volume and duration. Since flood peaks and corresponding flood volumes are variables of the same phenomenon, they should be correlated and, consequently, multivariate statistical analyses must be applied (Grimaldi and Serinaldi, 2006; Aronica et al., 2012).

In general, multivariate probability models have been limited by mathematical difficulties due to the generation of consistent joint laws with ad hoc marginals. Actually, copulas has overcome many of these problems (Salvadori and De Michele, 2007), as they are able to model the dependence structure independently of the marginal distributions.

Several authors have presented hydrological applications with copulas implementation (Bacchi et al., 1994; Nelsen, 1999; Genest et al., 2007; Gaal et al., 2010; Balistrocchi and Baldassarre, 2011) as complex hydrological phenomena such as floods, storms, and droughts are characterised by correlated random variables. For all these phenomena it is of fundamental importance to be able to link the marginal distributions of different variables (De Michele and Salvadori, 2003; Favre et al., 2004; Salvadori and De Michele, 2004, 2010; Zhang and Singh, 2007; De Michele et al., 2005; Grimaldi and Serinaldi, 2006; Dupuis, 2007; Aronica et al., 2012). Particularly, Favre et al. (2004) re-

3.1 Stochastic generation of rainfall

The hydrological input was derived by using a simple stochastic model to derive single synthetic sub-hourly rainfall events (Brigandí, 2009). Generated rainfall events are totally stochastic but with characteristics in terms of shape, duration and average intensity have to satisfy the parameters derived by statistical analyses of the available historic records.

Once extracted independent rainfall events from the available series of sub-hourly rainfall data, model is based on the two following modules:

- intensity–duration submodel (statistical description and generation of storm characteristics using a multivariate model).
- Temporal pattern submodel (generation of within-storm temporal characteristics as time step intensity variations using simple statistical descriptors).

3.1.1 Intensity–duration submodel

Since storm duration, average intensity or rainfall volumes are variables of the same phenomenon they should be correlated. Consequently, these variables have to be analyzed jointly through multivariate models and, particularly, those based on the theory of copulas.

Copula function C is a function which represents the joint distribution function of two dependent random variables uniformly distributed between 0 and 1:

$$C(u_1, u_2) = \Pr\{U_1 \leq u_1, U_2 \leq u_2\} \quad (1)$$

where u_1 and u_2 denote realizations. Let two random variables, X and Y , with their marginal distribution functions, $F_x(x)$ and $F_y(y)$, through a change of variables:

$$F_x(x) = U_1; \quad F_y(y) = U_2 \quad (2)$$

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it is possible to obtain the multivariate distribution function:

$$C(F_x(x), F_y(y)) = F(x, y) = F(X \leq x; Y \leq y) \quad (3)$$

The advantage of the copula method is that no assumption is needed for the variables to be independent or have the same type of marginal distributions. More information and applications about copulas can be found in Nelsen (1999) and Genest and Favre (2007).

The Archimedean copula family is widely used for hydrological applications, given its easy construction and, particularly, the Frank's copula can be applied when the correlation amongst hydrologic variables is negative (Favre et al., 2004; Zhang and Singh, 2007). De Michele and Salvadori (2003) found how the Frank's copula is the best candidate to model the dependence between average rainfall intensity and storm duration in comparison with other families of copulas, and also to reproduce the asymptotic statistical distribution of the storm depth.

In the present study, Frank's family class of 2-copulas has been considered. Frank copula is a one parameter Archimedean copula:

$$C(u_1, u_2) = -\frac{1}{\theta} \ln \left[1 + \frac{(\exp(-\theta u_1) - 1)(\exp(-\theta u_2) - 1)}{\exp(-\theta) - 1} \right] \quad (4)$$

with generation function:

$$\phi(t) = \ln \left[\frac{\exp(\theta t) - 1}{\exp(\theta) - 1} \right] \quad \text{and} \quad t = u_1 \quad \text{or} \quad u_2 \quad (5)$$

where θ is the parameter of copula function that is related to the Kendall's coefficient of correlation τ between X and Y through the expression:

$$\tau = 1 - 4 \frac{\Delta_1(-\theta) - 1}{\theta} \quad (6)$$

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where Δ_1 is the first order Debye function defined as:

$$\Delta_1(\theta) = \frac{1}{\theta} \int_0^{\theta} \frac{t}{\exp(t) - 1} dt \quad (7)$$

and:

$$\Delta_1(-\theta) = \Delta_1(\theta) + \frac{\theta}{2} \quad (8)$$

5 According to the nonparametric method, the first step in determining a copula is to obtain its generating function from bivariate observations. The procedure to calculate the generating function and the resulting copula followed in this study was described by Genest and Rivest (1993). It assumes that for a random sample of bivariate observations $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$ the underlying distribution function $H_{X,Y}(x, y)$ has an associated Archimedean copula $C(h)$ which also can be regarded as an alternative expression of the joint cumulative distribution function (CDF). The procedure involves the following steps:

1. Determine Kendall's τ from observations.

2. Determine the copula parameter θ from the above value of τ using Eqs. (5–7). For the Frank copula families introduced above, the θ parameter needs to be determined numerically, since there are no closed-form relations between τ and θ .

3. Obtain the copula having calculated the copula parameter θ . One can also obtain the generating function of each copula, since the generating function is expressed in terms of the copula parameter.

20 Use of copula requires the determination of marginal distributions based on univariate data. The marginal distributions here used were: exponential, gamma, Weibull and

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lognormal:

$$F(x) = 1 - e^{-\frac{x}{a}} \quad (9a)$$

$$F(x) = \frac{1}{b^a \Gamma(a)} \int_0^x t^{a-1} e^{-\frac{t}{b}} dt \quad (9b)$$

$$F(x) = 1 - e^{-\left(\frac{x}{a}\right)^b} \quad (9c)$$

$$F(x) = \frac{1}{b\sqrt{2\pi}} \int_0^x \frac{e^{-\frac{[\ln(t)-a]^2}{2b^2}}}{t} dt \quad (9d)$$

3.1.2 Rainfall temporal pattern submodel

In order to define the temporal patterns of rainfall for each event, we used here the idea of mass curves followed by various authors (Huff, 1967; Garcia-Guzman and Aranda-Oliver, 1993; Chow et al., 1988). Following this kind of approach the variability of precipitation within a rainy period is represented by a dimensionless hyetograph $H(d)$, defined as follow:

$$H(d) = \frac{1}{I \cdot D} \int_0^t h(t) dt \quad (10)$$

that identifies the fraction of rainfall accumulated over the time interval $[0, d]$. In Eq. (10), $t(0 \leq t \leq D)$ is a fraction of the total duration D of the considered event and, consequently, $d = t/D(0 \leq d \leq 1)$ is the correspondent dimensionless duration, $h(t)$ is the rainfall depth at time $t(0 \leq h \leq V)$, $V = I \cdot D$ is the total storm volume and D the storm duration for the event.

The normalized events obtained are the input for selecting an appropriate probability function for the hyetograph shape. Any continuous density function could be appropriate to represents the shape for every analyzed time-step between 0–1, but here the

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choice has been orientated towards the Beta distribution because it is a very simple model that fits reasonably well the rainfall data.

The Beta cumulative distribution function for a given value, x , and given pair of parameters a and b is:

$$F(x) = \frac{1}{B(a,b)} \cdot \int_0^x t^{a-1} (1-t)^{b-1} dt \quad (11)$$

where $B(a,b)$ is the Beta function:

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt = \frac{\Gamma(a) \cdot \Gamma(b)}{\Gamma(a+b)} \quad (12)$$

and $\Gamma(*)$ represents the gamma function.

3.2 Rainfall–runoff modelling

In Mediterranean areas, intense weather phenomena, responsible for the flood events, are often characterised by high spatial variability. As consequence, here, starting from a semi-distributed model presented by Di Lorenzo (1993), a conceptual fully distributed model with climatic dependencies for the flow routing is used.

The proposed model is based on the representation in the form of linear kinematic mechanism of transfer of the full outflows coming from different contributing areas of the basin through the definition of a distributed hydrological response array with climatic characteristics.

Rainfall inputs are, also, distributed in space and time-varying. They are represented using a three-dimensional matrix, \mathbf{P} , of order (A, B, N) where A and B are the number of cells in which the basin is divided in the direction x and y . N represents intervals number in which the rainfall event of duration Ω (with $N = \Omega/\Delta t$) is divided for each

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The matrix of runoff Q is obtained by multiplying hydrological response matrix, H with the effective rainfall matrix, P_e :

$$Q_{(\Theta,N)} = H_{(\Theta,A,B)} \times P_{e(A,B,N)} = \frac{1}{\Delta t} \cdot \begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,N} \\ & & Q_{i,j} & \\ \vdots & & & \vdots \\ Q_{\Theta,1} & Q_{\Theta,2} & \cdots & Q_{\Theta,N} \end{bmatrix} \quad (19)$$

in which $Q_{i,j}$ represents the available runoff for the ϑ isochrone zone at time t .

4 Application of the proposed methodology

4.1 Calibration of rainfall model

The calibration of the rainfall generation module was conducted using a sample of rainfall annual maximal rainfall events, extracted from the series of 10 min rainfall data recorded at raingauges above mentioned. Following De Michele and Salvadori (2003), two events are judged independent if they are separated by a dry period of at least 7 h. As consequence an inter-event time equal to 7 h was adopted to separate the single rainfall events and derive their average intensity and duration which represent the correlated variables modelled by Frank copula.

Kao and Govindaraju (2007) stated how the definition of annual maximal events for multivariate problems is somewhat ambiguous. As matter of fact, extreme events could be defined as those storms that have both high volume and peak intensity. Therefore here the definition of extreme rainfall based on events with annual maximum joint cumulative probability has been considered, where the joint cumulative probabilities of samples can be estimated directly via the empirical copula C_n as introduced by Yue (1999).

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Moreover, as all raingauges are in a hydrologically homogeneous area (Cannarozzo et al., 1995) statistical analysis was performed by aggregating all selected events obtaining a final sample of 80 rainfall events whose characteristics are reported in Table 1.

The procedure described in the previous paragraph has been applied in order to derive θ parameter for the Frank copula and the generating function. The copula so obtained for the analyzed case study, is characterized by a parameter θ to -3.7573 calculated using the Eq. (6) where the Kendall's coefficient of correlation τ is equal to -0.381 .

In order to indicate the goodness of fit of chosen copula, the empirical C_n and best fitted copula joint distribution were reported in a Q-Q plot (Fig. 2, left). To confirm the goodness of the chosen copula to describe the data, the parametric and nonparametric values of the function $K(z)$, as defined by Genest and Rivest (1993), are shown in Fig. 2 (right). These comparisons confirm how Frank 2-Copula is well suited to describe the dependence structure between the available intensity-duration data.

The parameters of the marginal distributions considered were estimated by applying the Maximum Likelihood method and the best fitted distribution was selected using various criteria. More in particular the AIC criterion, the Relative Root Mean Square Error (RRMSE), and the Anderson–Darling test were applied to verify the goodness of the fitting. The results, as shown in Tables 2 and 3, returned lognormal probability distribution as best marginal distribution for average storm intensity, and Weibull distribution as best marginal distribution for storm duration. Further, Fig. 3 show the two marginal distributions defined, respectively by Eqs. (9c–d) and empirical exceedances probabilities computed using Gringorten formula of the observed data.

Finally, the Beta distribution has been fitted to adimensional shape sample of each events. Model parameters are estimated by maximum likelihood (ML), while the goodness of fit was verified by the Pearson test. The historical normalized mass curves derived for all the events selected are plotted in Fig. 4. Then, these curves were sampled in 11 equal time-step (0; 0.1; 0.2; ...; 0.9; 1) and, for each time-step considered,

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also for the calibration of previous events because any significant changes of land uses in the Imera catchment were recognised.

Similarly, the Strickler roughness coefficients (Fig. 7) have been rescaled according to some weights, $w_{k_{i,j \rightarrow out}}$, allowing for roughness variability into the catchment, with regard to a reference roughness value \bar{k} , i.e. the spatially-averaged value of $k_{i,j \rightarrow out}$.

$$k_{i,j \rightarrow out} = w_{k_{i,j \rightarrow out}} \cdot \bar{k} \quad (21)$$

The spatially-averaged value of $k_{i,j \rightarrow out}$ can be easily calculated starting from its effective spatial distribution of CN, in relation of soil type and land use by the modified Engmann's table (Engmann, 1986; Candela et al., 2005) (Table 5), by using standard GIS tools. Its value is equal to $20.5 \text{ m}^{-1/3} \text{ s}^{-1}$.

The calibration of the three model parameters has been carried out comparing observed and predicted variables in terms of discharges for the event of 21 December 1976, registered at Imera meridionale at Drasi station (Fig. 8).

Particularly, in calibration it is not difficult to get optimal fittings to the observations by adjusting parameter values but rather that there are many sets of parameter values that will give acceptable fits to the data (Beven, 1993). Often, there are no techniques available for estimating or measuring the values of effective parameters required at the grid element or catchment scale. These values will therefore be subject to some uncertainty, especially in semiarid areas for which data are not always adequate and there is an extreme variability in space and time of all factors controlling the runoff processes (Yair and Lavee, 1982).

In this study the uncertainty in the identification of model parameters has been assessed using the Generalised Likelihood Uncertainty Estimation (GLUE) procedure of Beven and Binley (1992). GLUE is a Monte Carlo based technique that allows for the concept of equifinality of parameter sets in the evaluation of modelling uncertainty. It transforms the problem of searching for an optimum parameter set into a search for the sets of parameter values, which would give reliable simulations for a wide range of

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plot, as defined by Genest and Rivest (1993), are shown for the three copula families considered. For a best detection of modelling the correlation structure, the normalised scatter plot of the empirical and theoretical 5000 pairs are reported in Fig. 13. In addition, more rigorous tests base on statistical analisys have been performed. Specifically, the AIC criterion and the Log-Likelihood test were applied to verify the goodness of the fitting. The graphical tools and the statistical test returns how Gumbel–Hougard copula family is the best choice for describing the dependence structure between the flood peaks and volume data.

The parameters of the marginal distributions here used (exponential, gamma, Weibull, lognormal, and GEV) these distributions were estimated by applying the Maximum Likelihood method and the best fitted distribution was selected using various criteria. Again, the AIC and Anderson–Darling test (Kottegoda and Rosso, 1997) have been applied to verify the goodness of the fitting (Tables 8 and 9). The goodness-of-fit criteria return gamma distribution as best marginal distribution for both univariate variables. Further, Fig. 14 shows the marginal distribution defined by Eq. (9b) compared with the exceedances probabilities computed using Gringorten formula of the empirical data.

A comparison between a sample generated from the Gumbel–Hougard copula and the empirical ($Q_{\max} - V$) pairs is plotted in Fig. 8 (left). Also, contours of the fitted copula that represent the events with the same probability of occurrence are shown (Fig. 8, right).

The second step of the procedure is devoted to the derivation of the shape of the FDH generated via cluster analysis with Ward method (1963) using the procedure proposed by Aronica et al. (2012). The procedure consists in normalising the empirical hydrographs (5000 in this study) that a unit peak flow and a unit flood volume have been resulted. The normalised hydrographs are then grouped in various clusters according with Ward method (1963) (minimum variance algorithm that minimizes increments in sums of squares of distances of any two clusters that can be formed at each step). The results of this cluster analysis are the three shapes of hydrograph showed in Fig. 16. In

relation to the number of hydrographs belonging to each cluster, a probability of about 0.11 (Shape 1), 0.5 (Shape 2) and 0.39 (Shape 3) has been assigned to these shapes.

The final step of the procedure allows to obtain the FDH for any return time by merging the non-dimensional hydrographs (for specific probability) and the generated peak-volume pairs derived using copula. The choice of a joint (bivariate) return period (JRP) is the core problem of the final step. Several authors found (see for instance, Requena et al., 2013; Vandenberghe et al., 2012) how different methods can be considered for defining joint return periods estimated by fitted copulas. Here, following the approach proposed by Vandenberghe et al., (2012) the joint return period can be easily calculated by means of a bivariate copula $C(u_1, u_2)$ as:

$$T = \frac{1}{1 - C(u_1, u_2)} = \frac{1}{1 - C(F_x(x), F_y(y))} \quad (23)$$

As matter of fact, this method is an extension of the classical definition of a univariate return period. All couples (u_1, u_2) that are on the same contour (corresponding with an isoline p) of the copula C will have the same bivariate return period.

Hence, for a given design return period T , the corresponding level $p = C(u_1, u_2)$ can easily be calculated using the Eq. (23) and all the pairs (u_1, u_2) on the isoline p have the same return period. In order to select the single design point $(u_{1,T}, u_{2,T})$ Vandenberghe et al. (2012) suggest the select the point with the largest joint probability:

$$(u_{1,T}, u_{2,T}) = \underset{C(u_1, u_2) = p}{\operatorname{argmax}} f_{XY} \left(F_X^{-1}, F_Y^{-1} \right) \quad (24)$$

The corresponding design values $Q_{\max, T}$ and V_T can be easily calculated by inverting the marginal CDF:

$$Q_{\max, T} = F_{Q_{\max}}^{-1}(u_{1, T}); \quad V_T = F_V^{-1}(u_{2, T}) \quad (25)$$

As example the FDH with a design return period of $T = 100$ yr is calculated. For $T = 100$ yr, the corresponding copula level p equals 0.99 and corresponds with an isoline (Fig. 17).

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The Eq. (24) can be solved to find the single design point $(u_{1,T}, u_{2,T})$ with the largest joint probability, i.e. (0.9949, 0.990). Using the inverse marginal CDFs the design event pair is obtained: $(Q_{\max,T}, V_T) = (4929.3 \text{ m}^3 \text{ s}^{-1}, 162.2 \text{ Mm}^{-3})$. Finally, the design hydrograph can be obtained using the shape 3 and de-normalising the time and the discharge by multiplying for the values of the pair (Fig. 18).

6 Conclusions

In this study a procedure to derive Flood Design Hydrographs (FDH) using a bivariate representation of rainfall forcing (rainfall duration and intensity) described by copulas coupled with a distributed rainfall-runoff model is presented. In order to estimate the return period of the FDH which give the probability of occurrence of a hydrograph flood peaks and flow volumes obtained through R-R modeling has been statistically treated via copulas. The choice of copulas is motivated by the strong capability to describe the statistical correlation between variables which allows to obtain the return period related to the FDH and not only to a single variable (usually the peak flow) as in the univariate analysis. This circumstance have a significant importance in all those case where all the hydrological variables (flood volume, flood peaks, etc.) included in a Design Hydrograph plays an important role (i.e, flood propagation modeling for the delimitation of inundate areas with hazard/risk classification).

In addition, a statistical label (in terms of probability of occurrence) has been give also to the hydrograph shape through the cluster analysis of the R-R model generated hydrographs. This completes the statistical definition of the FDH which can be identified with a specific return period (Joint Return Period, JRP).

The procedure described above applied to the case study of Imera catchment in Sicily, Italy, shows how this approach methodology allows a reliable and estimation of the Design Flood Hydrograph in a way which can be easily implemented also in the practical situations.

These results, hence, underline the necessity of considering JRP estimation methods in the definition of design events for all practical purposes.

Further research efforts will be devoted to move from one-design-event methods to ensemble-design-event methods by considering uncertainty via a complete application of GLUE procedure.

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Table 1. Information and statistics of the rainfall data for 80 events registered from 2010 to 2011.

	Intensity (mm h^{-1})	Volume (mm)	Duration (min)
Max	26.02	156.2	2060.0
Min	1.62	18.0	170.0
Mean	5.04	66.9	975.0
Standard deviation	3.76	27.6	453.0

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Table 2. Marginal distribution parameters and goodness of fit results for average intensity (*I*).

	<i>a</i>	<i>b</i>	AIC	RRMSE	<i>A – D</i> *
Exponential	5.03644	–	420.672	13.890	10.557
Gamma	3.31078	1.52123	376.565	7.91708	3.099
Weibull	5.67845	1.57408	394.866	10.3672	4.928
Lognormal	1.45814	0.51516	357.209	5.84947	1.411

* Critical value for 95 % significance level = 2.492.

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Table 3. Marginal distribution parameters and goodness of fit results for duration (D).

	a	b	AIC	RRMSE	$A - D^*$
Exponential	16.2542	–	608.136	56.130	9.588
Gamma	3.84882	4.22315	554.563	10.590	0.808
Weibull	18.3596	2.32434	549.195	6.842	0.451
Lognormal	2.65285	0.56908	564.288	18.451	1.522

* Critical value for 95 % significance level = 2.492.

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Table 4. Parameters estimation of the Beta distribution.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
α	1.070	1.369	1.706	2.029	2.029	3.629	4.643	5.600	9.558
β	8.127	4.727	3.456	2.790	2.790	2.543	2.151	1.649	1.198



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Table 5. Engmann modified table reported Strickler's coefficient values related to Imera catchment land use.

Land use	Urban	Bare rocks	Arable land	Untilled	Vineyard	Clear forest
Strickler coefficient	100.0	50.0	22.0	20.0	7.69	6.67

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Table 6. Ranges of parameters considered for the calibration.

Parameter	Lower limit	Upper limit	Max eff value
c	0	1	0.68
\overline{CN}	70	90	87
\overline{k}	50	100	75.8

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Table 7. Goodness of fit results for copula (Q_{\max}, V).

Copula family	θ	LL	AIC
Gumbel	14.792	9581.66	19 163.32
Frank	57.476	10 354.17	20 708.35
Clayton	27.585	10 140.84	20 281.68

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Table 8. Marginal distribution parameters and goodness of fit results for flood peaks (Q_{\max}).

	a	b	c	AIC	$A - D^*$
Exponential	985.84	–	–	7.895	1.110
Gamma	1.04	944.78	–	7.895	1.089
Weibull	982.33	0.99	–	7.895	1.130
Lognormal	6.34	1.16	–	7.913	2.819
GEV	425.29	438.01	0.51	7.916	1.204

* The critical values for the Anderson–Darling test is 2.492.

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Table 9. Marginal distribution parameters and goodness of fit results for flood volumes (V).

	a	b	c	AIC	$A - D^*$
Exponential	39.66	–	–	4.6816	3.745
Gamma	1.27	31.18	–	4.6649	0.680
Weibull	41.41	1.12	–	4.6716	1.199
Lognormal	3.24	1.03	–	4.6856	2.804
GEV	17.8733	20.327	0.39014	4.6824	0.944

* The critical values for the Anderson–Darling test is 2.492.

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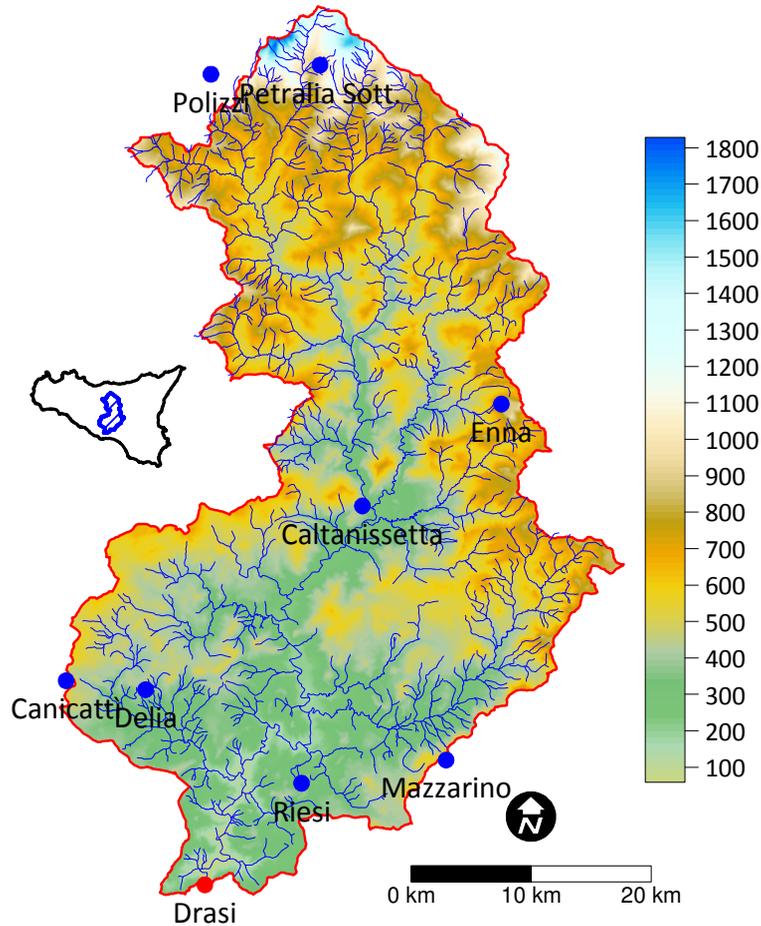


Fig. 1. Imera catchment layout.

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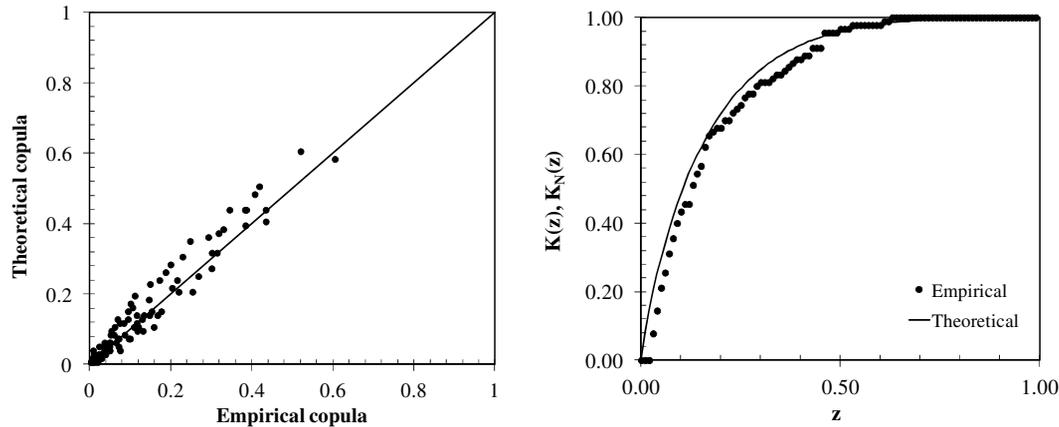


Fig. 2. Comparison between empirical joint distribution and best fitted copula. Goodness test for Frank's copula.

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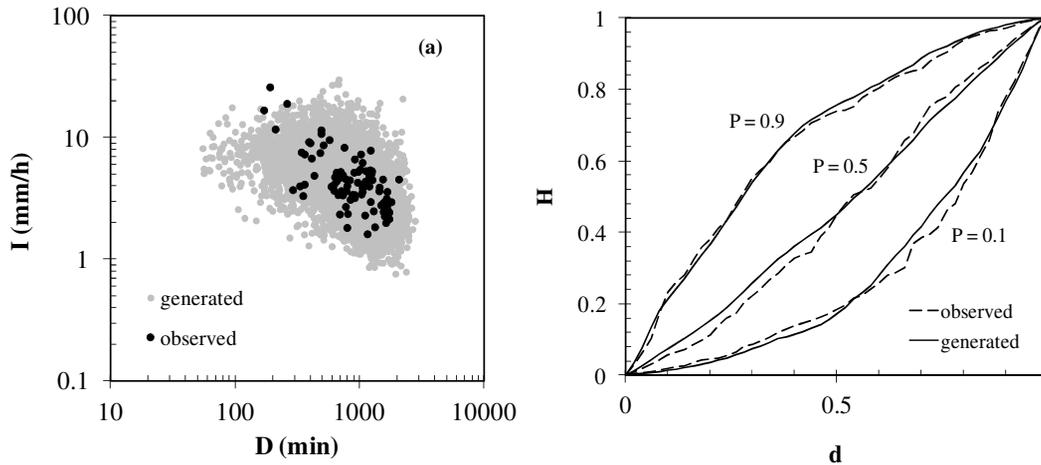


Fig. 5. Duration-intensity correlation (left) and adimensional shapes (right).

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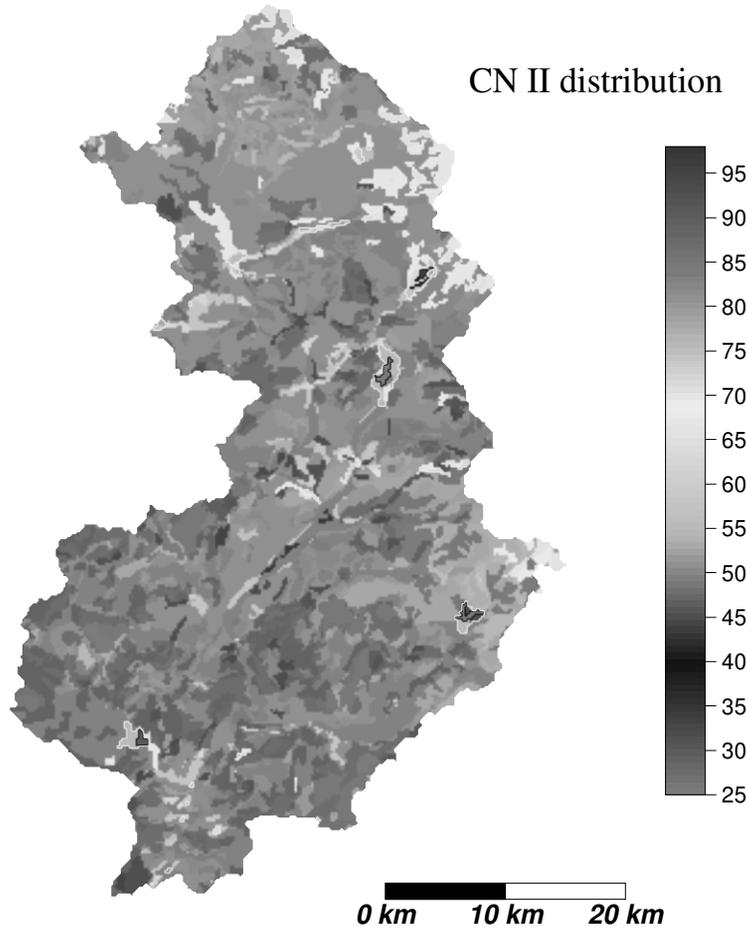


Fig. 6. Spatial distribution of CN_{II} for the Imera catchment.

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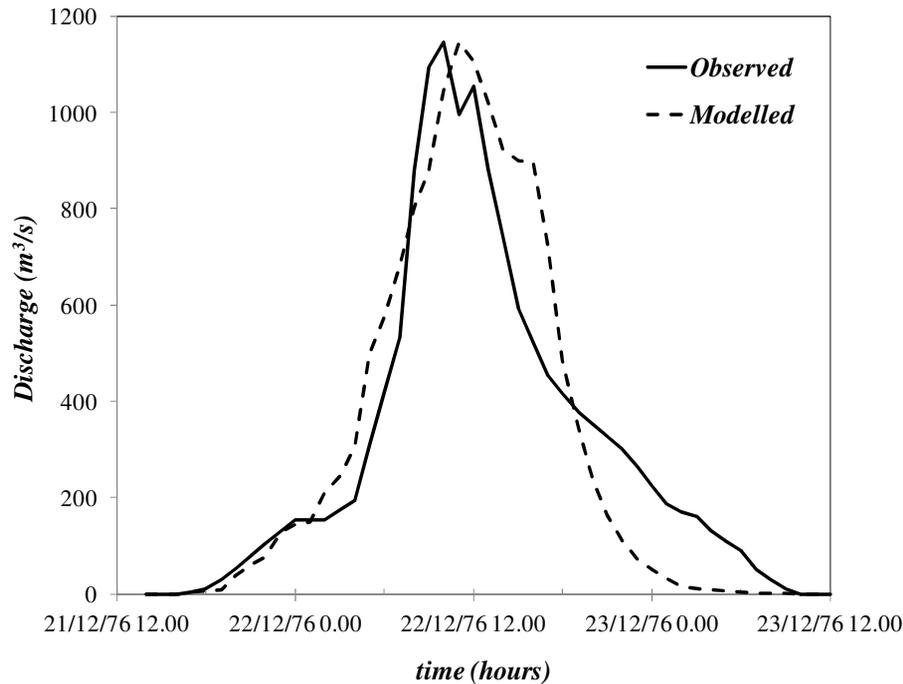


Fig. 10. Comparison between observed and modelled hydrographs for the event of 21 December 1976.

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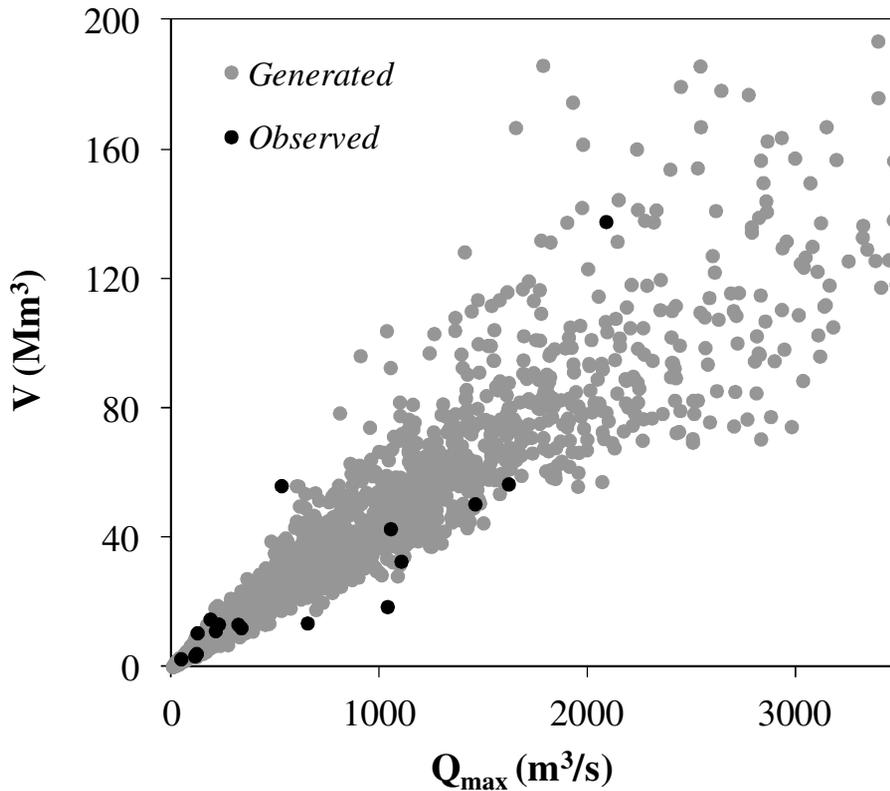


Fig. 11. Comparison between scatter plot of the observed and generated pairs (Q_{\max} , V).

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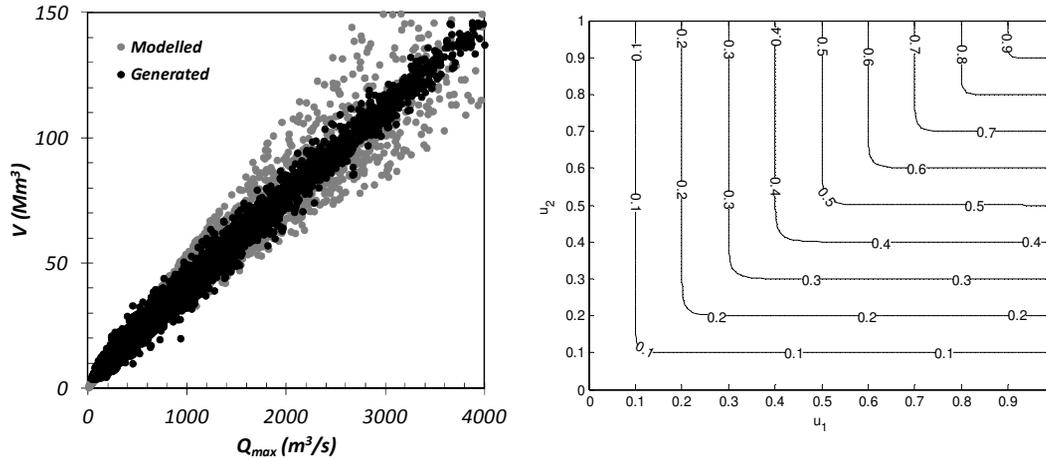


Fig. 15. Comparison between a sample generated from the Gumbel copula and the observed data (left) with the copula contours (right).

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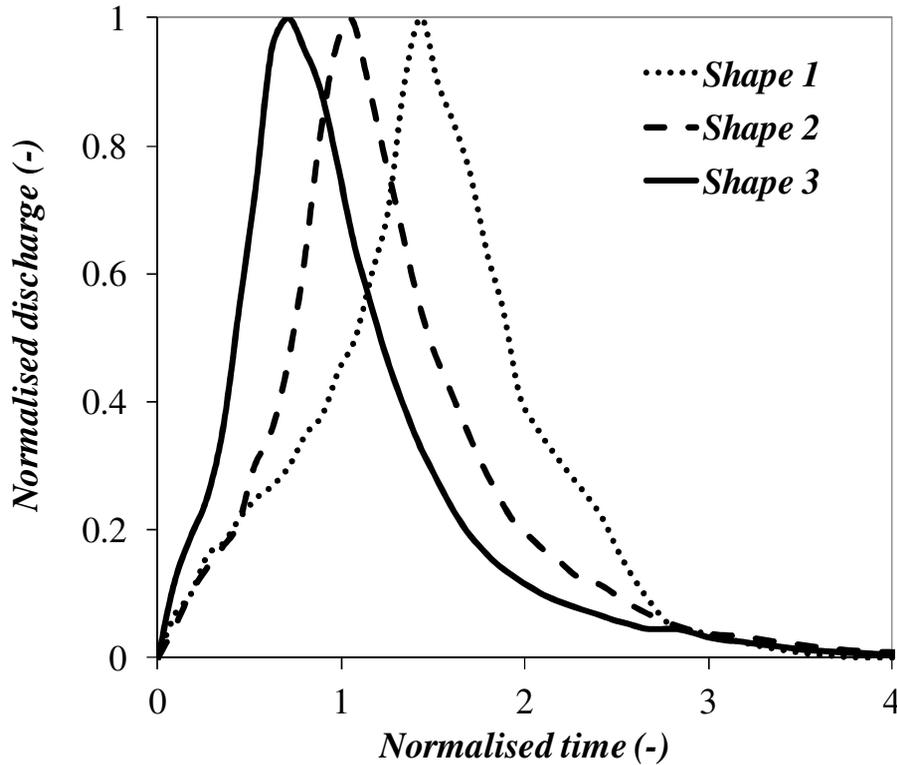


Fig. 16. Non-dimensional clustered hydrographs.

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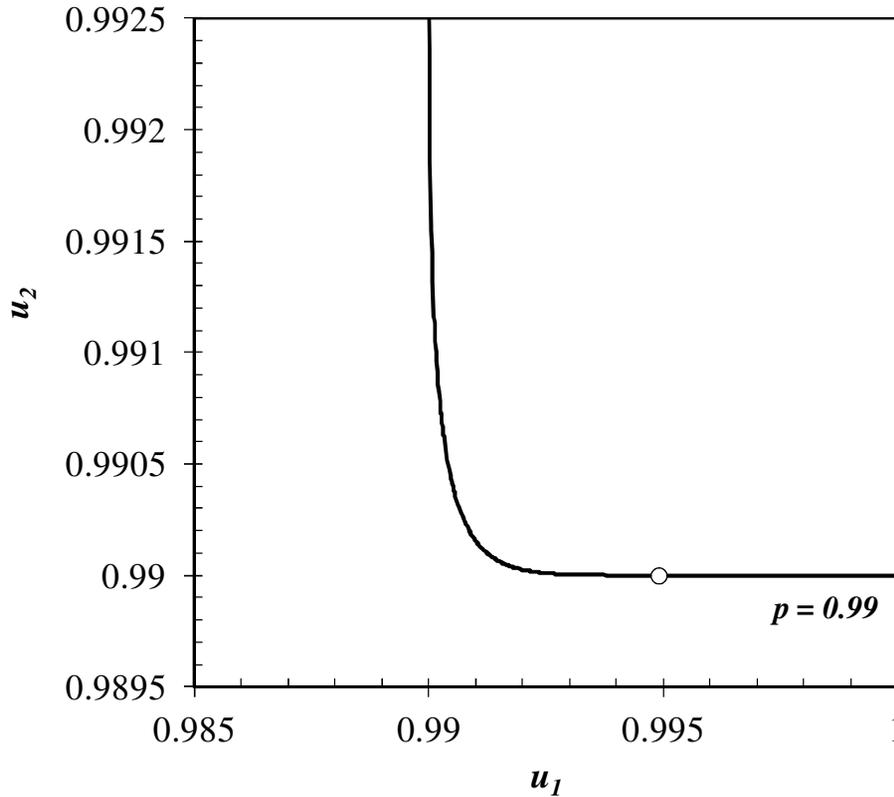


Fig. 17. p level of the copula $C(u_1, u_2)$ corresponding to a JRP = 100 yr, with indication of the single design point (white dot).

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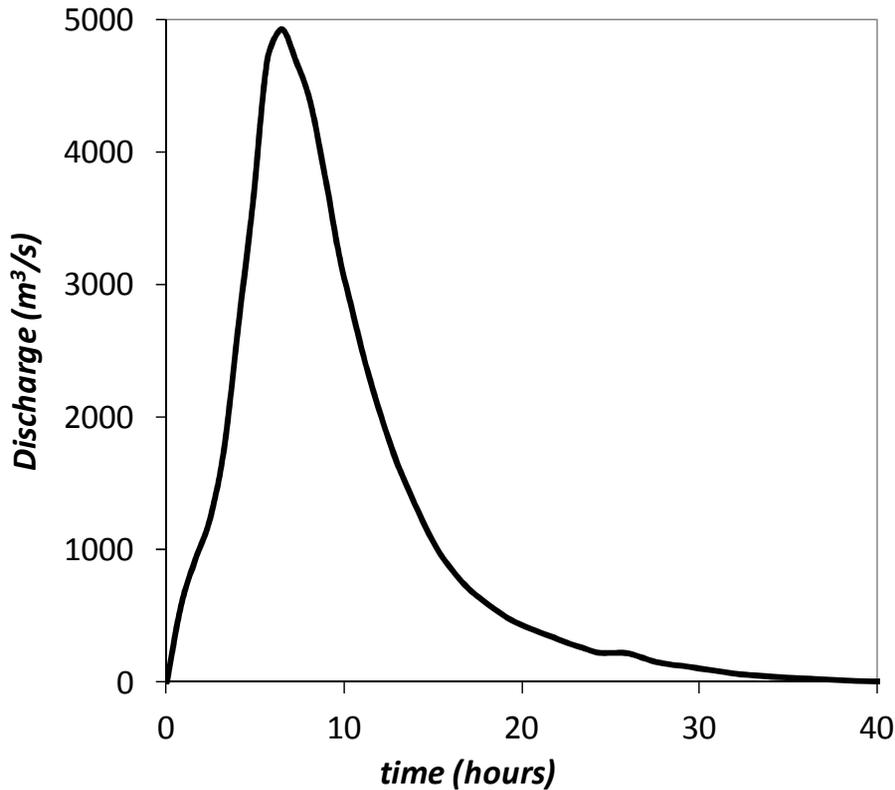


Fig. 18. Flood Design Hydrograph corresponding to a JRP = 100 yr.

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