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Discussions

Time-frequency analysis of the sea state with the "Andrea" freak wave

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Abstract

The non-linear and non-stationary properties of a special field wave record are analyzed with the Wigner spectrum with the Choi–Williams kernel. The wave time series, which was recorded at the Ekofisk complex in the Central North Sea at 00:40 UTC on 9 November 2007, contains an abnormally high wave known as "Andrea" wave. The ability of the Wigner spectrum to reveal the wave energy distribution in frequency and time is demonstrated. The results are compared with previous investigations for different sea states and also the state with the abnormal Draupner's New Year wave.

1 Introduction

Several events have been reported in the literature during the last twenty years, in which ships have been damaged by giant waves as for example by Kjeldsen (1997) and Faulkner and Buckley (1997). The interest to this type of waves leads to the analysis of data from real sea states, looking for occurrences of abnormally high waves, which have been reported in an increasing number of works. Instrumental records have identified this type of waves in the North Sea (Skourup et al., 1996; Haver and Andersen, 2000; Wolfram et al., 2000; Guedes Soares et al., 2003; Magnusson and Donelan, 2013), from the Japan Sea (Yasuda and Mori, 1997; Mori et al., 2002), from the Gulf of Mexico (Guedes Soares et al., 2004) and even from the Baltic Sea (Didenkulova, 2011). A more comprehensive review of registered huge waves is made in Kharif et al. (2009).

Waves of anomalously large size, called abnormal, freak, or rogue waves are very steep in the last stage of their evolution, propagating as a wall of water. A typical abnormal wave is a single event with a characteristic life time of just few seconds. Before breaking, it has a crest, three to four times higher than the crests of neighbouring waves and appears almost instantly as it has been identified in wave records. There are no

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doubts that such waves are essentially of nonlinear character and can be generated by different mechanisms (Kharif et al., 2009).

Understanding of the nature of abnormal waves can be achieved by a detailed analysis of the time records of free surface elevation using different methods to determine their time-frequency energy distributions.

The standard Fourier analysis is a powerful tool for investigation of waves because it gives opportunity to decompose the series to individual frequency components and to specify their relative intensity. However, the usual frequency spectrum does not give any information when these frequencies bring a great amount of energy. The assumption is that it happens by the same manner during the all period of measurement.

Visual observation of the sea surface shows that the wind waves with various heights are moving in reiterated groups of different length and number of waves. This means that for short time intervals nearly equal to the group duration there exists significant change of the wave energy per unit square surface. Therefore the wind wave for short intervals of time is not stationary process as usually is supposed.

Development of methods for investigation of the spectrum changing in time begins with several classical works (Gabor, 1946; Ville, 1948; Page, 1952) that developed the idea of Wigner (1932) to take into account the quantum corrections for thermodynamic equilibrium. These studies mark the beginning of the fundamental analysis and explanation of the physical and mathematical ideas for understanding of what is changeable in time spectrum. The main idea is to find a join function of frequency and time that describes the energy distribution of the process. In the ideal case such distribution will be used and transformed in a same manner as any joint distribution. Nevertheless, the joint functions in frequency and time of the wave energy are not distributions in a probabilistic sense because the time-frequency spectra give distribution of the energy while probabilistic joint distribution of two variables describes their joint probability. But from historical reasons often the time-frequency spectra of the signals are called also "distributions".

The upper authors lead themselves in their works by a partial mathematical similarity with some problems in quantum mechanics and signal processing. It is necessary to highlight clearly that the analogy between the quantum mechanics and the theory of signals is formal and the physical interpretation in these two branches of knowledge is completely different (Cohen, 1995).

First Gabor (1946) developed the mathematical basis of the time-frequency method and introduced very important concept for analytical signal that later came into radio-physics and theory of signals. Further, (Turner, 1954; Levin, 1967) it is shown that using procedures similar to that in work of Page (1952) is possible to find many other time-frequency distributions. Rihaczek (1968) deduced a new distribution examining problems of physics connected with signals spreading. A lot of ideas about the convolution and filtering of the non-stationary stochastic processes are introduced in detailed work of Mark (1970) and are disseminated now in everyday practice of many scientific fields.

The last two decades of the twentieth century mark a great progress of investigations dedicated to the time-frequency structure of the signals. This revival of the scientific interest (Claasen and Meklenbrauker, 1980a, b, c; Janse and Kaizer, 1983) is accompanied with development of the unique ideas of the spectral distributions and demonstrates their practical application. Something more, begins clearing up of the similarity and difference between the quantum mechanics and the theory of the real signals. In the works of Boashash (Boashash and Whitehouse, 1986; Boles and Boashash, 1988) may be for the first time the ideas developed in Claasen and Meeklenbrauker (1980a, b, c) are implemented to deal with the real problems of geophysics.

In the course of time a new manner of thinking has arised for interpretation and use the time-frequency energy distributions. Different points of view and different scientific interests extend the understanding of what are the non-stationary processes and diversify the techniques of their investigation. A good review of the applied and scientific problems connected with time-frequency analysis one can find in Cohen (1995), Hlawatsch and Boudreaux-Bartels (1992), Boashash (1991), Mecklenbrauker (1985), Hlawatsch and Flandrin (1992) and Huang et al. (1999).

Some of time-frequency energy distributions are already used to study the nature of the ocean wind waves such as wavelet distribution (Liu, 1994, 2000a, b; Massel, 2001) spectrogram distribution (Guedes Soares and Cherneva, 2005; Cherneva and Guedes Soares, 2005), the empirical mode decomposition (Huang et al., 1999; Shurlman, 2001; Velcheva and Guedes Soares, 2004, 2007) and Wigner spectra (Cherneva and Guedes Soares, 2008, 2011, 2012).

The present study analyses sea state wave data from measurement in a single point, in order to characterize the non-stationary and non-linear properties of the large waves registered in the North Sea at Andrea platform. It is a follow-up of series of studies performed (Guedes Soares et al., 2003, 2004; Guedes Soares and Cherneva, 2005; Cherneva and Guedes Soares, 2008, 2011, 2012; Velcheva and Guedes Soares, 2004, 2007). In what follows, Sect. 2 gives a theoretical background of presentation of wind waves as an analytical process, introduces a definition of a Wigner spectrum and briefly describes the Benjamin–Feir instability. Results and discussions are subject of Sect. 3. Conclusions are made in Sect. 4.

2 Theoretical background

2.1 Wind wave as an analytical process

It was already mentioned that the concept for the analytical signal belongs to Gabor (1946). After the pioneering work of Longuet-Higgins (1952) mathematical and physical methods from radio-physics, theory of noise and signal processing are widely introduced in wind wave investigation (Bitner, 1980; Bitner-Gregersen and Gran, 1983; Tayfun and Lo, 1989 for example).

If $\eta(t)$ is the surface elevation and $\acute{\eta}(t)$ is its Hilbert transform, the complex process $x(t) = \eta(t) + j\acute{\eta}(t)$ is an analytical process, corresponding to $\eta(t)$. The process $x(t)$ can be presented also as $x(t) = |A(t)|\exp[j\psi(t)]$ where the envelope $|A(t)|$ and the phase $\psi(t)$ are defined by:

$$|A(t)| = [\eta^2(t) + \acute{\eta}^2(t)]^{1/2} \tag{1}$$

$$\psi(t) = \mathrm{arctg}[\acute{\eta}(t)/\eta(t)] \tag{2}$$

The phase $\psi(t)$ can also be written as:

$$\psi(t) = \omega_0 t + \phi(t) \tag{3}$$

where ω_0 is co-called carrier frequency and $\phi(t)$ is the local phase. The time derivative of the phase function $\psi(t)$ defines a local frequency of the time series:

$$\omega(t) = \mathrm{d}\psi/\mathrm{d}t = \omega_0 + \mathrm{d}\phi(t)/\mathrm{d}t. \tag{4}$$

In this sense, the local frequency $\omega(t)$ is a rate of change of the phase (Van der Pol, 1946). If it is assumed that at each time instant t there exists only one single frequency component, the local frequency is defined also as an average frequency at a particular time. Then the sea state is restricted to a single mean wave characteristic like the local frequency, which changes in time.

In Eq. (3) the unwrapped phase function $\psi(t)$ consists of a linear part $\omega_0 t$ increasing with time and a deviation part $\phi(t)$ superimposed on $\omega_0 t$. Therefore, from the slope of the unwrapped phase function $\psi(t)$ it is possible to find the carrier frequency ω_0. It is obvious that the carrier frequency ω_0 does not coincide with the spectrum peak frequency ω_p. Usually ω_p is less than ω_0 for the real sea state spectra and the fact is not connected with downshifting of the spectrum peak frequency due to Benjamin–Feir instability. Only when the waves have nearly equal phases during a given time interval and the change of the phases in time $\theta = \mathrm{d}\phi(t)/\mathrm{d}t$ is small, than the local frequency is equal to the carrier frequency ω_0. If the phase change has negative slope $\theta = \mathrm{d}\phi(t)/\mathrm{d}t < 0$ than the local frequency is lower than the carrier one. A detailed discussion on the local frequency of the signal can be found in classical work of Cohen (1989).

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2.2 Wigner spectrum

For the analytical signal $x(t)$ Fonolosa and Nikias (1993, 1994) define the Wigner high order time-frequency spectrum as

$$W_{kx}(t, f_1, \ldots, f_k) = \iiint\limits_{\Omega u \tau_1} \ldots \int\limits_{\tau_k} \Phi(\Omega, \tau_1, \ldots, \tau_k) \times R_{ku}(\tau_1, \ldots, \tau_k) \exp(2\pi j u \Omega) \exp(-2\pi j t \Omega) \times \prod_{i=1}^{k} \exp(-j 2\pi f_i \tau_i) \mathrm{d}\tau_i \mathrm{d}u \mathrm{d}\Omega \tag{5}$$

Here $W_{kx}(t, f_1, \ldots, f_k)$ is a k-dimensional Fourier transform of a k-dimensional local moment function $R_{ku}(\tau_1, \ldots, \tau_k)$ and $\Phi(\Omega, \tau_1, \ldots, \tau_k)$ is a kernel introduced to reduce the aliasing. The definition (5) is preferred because it affords an opportunity to estimate the higher order time-frequency spectra of the waves as it was done before for the "New Year wave" (Cherneva and Guedes Soares, 2008).

For $k = 1$ the Eq. (5) leads to definition of the Wigner spectrum. The most applied time-frequency Wigner spectra can be obtained using different kernels $\Phi(\Omega, \tau)$. In particular, the Wigner–Ville distribution corresponds to $\Phi(\Omega, \tau) = 1$ and the Rihachek distribution – to $\Phi(\Omega, \tau) = \exp(j\pi\tau\Omega)$ (Cohen, 1966). Here the Choi–Williams kernel $\Phi(\Omega, \tau) = \exp(-\Omega^2\tau^2/\sigma)$ is used where $\sigma = 0.05$ (Choi and Williams, 1989).

2.3 Benjamin–Feir instability

Ocean waves in their nature are non-linear and dispersive waves. Experimental studies of Benjamin and Feir (1967) show that regular wave trains in deep water are responsible to a number of instabilities known now as Benjamin–Feir instability. The instability is a result of an interaction among three monochromatic wave trains: carrier ω_c, upper ω_+ and lower ω_- sideband waves. According to a perturbation analysis accomplished

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by Benjamin and Feir (1967) based on the Euler equations, if $\delta\omega$ is a small frequency perturbation, than the carrier wave and the sideband waves have to answer the requirements to four wave resonance conditions for infinitesimal waves where

$$\begin{aligned} \omega_{\pm} &= \omega_c \pm \delta\omega \\ 2\omega_c &= \omega_+ + \omega_- \quad , \\ 2k_c &= k_+ + k_- + \Delta k \end{aligned} \tag{6}$$

In Eq. (6) the frequencies and the wave numbers are connected by the linear dispersion relationship in deep water and Δk is a small mismatch. The group of three waves with initial carrier wave amplitude a_c is unstable if the inequality $0 < \hat{\delta} \leq \sqrt{2}$ is satisfied, where $\hat{\delta} = \delta\omega/\varepsilon\omega_c$ and $\varepsilon = a_c k_c$. Then the sideband waves begin to grow in amplitude at the expense of the main wave. The maximum growth in $\boldsymbol{x}$ direction $\beta_x = \mathrm{d}(\ln a)/\mathrm{d}(kx)$ appears when $\hat{\delta} = 1.0$ and the initial phases of the sidebands are $\psi_{\pm} = -\pi/4$. This theory is in a good agreement with observations for wave steepness ε in interval $[0.07, 0.17]$ (Benjamin, 1967).

The evolution of a non-linear wave train without dissipation manifests the so-called "Fermi–Pasta–Ulam" phenomenon: periodically increasing and decreasing of the modulation that cause returning of the wave in its initial form (Lake et al., 1977). Further, Longuet-Higgins (1978) found that sub harmonic instabilities of the Benjamin–Feir type are restricted to waves whose steepness $\varepsilon = ak$ has an upper limit. As ε increases beyond 0.346 the wave modes become stable again.

Tulin and Waseda (1999) have compared their experimental data with the theoretical predictions of Benjamin and Feir (1967) and Krasitskii (1994). They found that Krasitskii's modification of the Zakharov evolution equation (Zakharov, 1968) correctly predicts the major features of the energy increase in the lower sideband relative to the upper sideband. Also they argued that downshifting to a lower sideband of the spectral peak appears as well in the absence of breaking, and showed the significant role of the balance between momentum losses and energy dissipation in the exchange of energy between the sidebands.

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A detailed review on the subject one can find in the works of Yuen and Lake (1980), Hammack and Henderson (1993), Kharif et al. (2009), and Massel (2010).

3 Results and discussion

Here a wave time series measured at 00:40 UTC on 9 November 2007 is analyzed. The series containing a huge wave named "Andrea" rogue wave has been recorded at Ekofisk complex operated by Conoco Phillips in the central North Sea (56°30′ N and 3°12′ E) where the water depth is between 70 m and 80 m. A detailed description of the weather conditions, measuring installations and some comparisons between the Draupner's New Year wave and the Andrea wave can be found in Magnusson and Donelan (2013). Table 1 shows some of the data in that paper with some of the present results.

In Table 1 are presented three parameters characterizing the non-linearity of the series with freak waves measured in the Draupner and the Andrea platforms – $k_p h$, $\varepsilon = a_s k_p$ and a_s/h, where $a_s = H_s/2$, h is the depth, H_s is the significant wave height and k_p is the wave number of the spectrum peak frequency. Similar parameters but for the individual abnormal waves marked by subscript freak are calculated also, where H_{freak} is the height of the freak wave and $a_{\text{freak}} = H_{\text{freak}}/2$; T_{freak} is its individual period and k_{freak} is the wave number corresponding to that period. The steepness of the series is 0.081 for the New Year wave and 0.106 for the Andrea wave. The individual steepness of the rogue waves is $\varepsilon_{\text{freak}} \approx 0.3$ that is less than the upper limit of 0.346 (Longuet-Higgins, 1978). For the freak waves the parameter $k_{\text{freak}} h \approx 2$, $a_{\text{freak}}/h \approx 0.2$ and it can be concluded that the registered two abnormal waves are strongly nonlinear (Kurkin and Pelinovsky, 2004; Cherneva and Guedes Soares, 2008).

The Andrea time series available has 4600 registered ordinates with 5 Hz sampling frequency that gives 15 min duration of registration presented in Fig. 1a. The existence of foam or possible breaking is taken into account as is described in Magnusson and Donelan (2013). The series is too short to derive any conclusion about the distribution

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of the wave heights or crests as done in Cherneva et al. (2009, 2011). Because of that in this work the study is limited to non-linear and non-stationary properties of the waves.

Three high wave groups are interesting and investigated here: the first one is a long group in time interval (560–720) seconds containing a huge wave in its beginning; the second group exists in interval (420–540) seconds; waves in the third group in interval (130–200) seconds are high as the waves in the second group but have not the typical triangle structure as the first two groups. In Fig. 1a the groups are separated from the rest of the series by vertical dashed lines.

In Fig. 1b one can find the phase ϕ development in time. The phase ϕ can be approximated by straight lines with negative slopes during the investigated groups. It is obvious that the local frequency $\omega(t)$ of the largest wave groups is less than the calculated carrier frequency ω_0 of the series because $\mathrm{d}\phi(t)/\mathrm{d}t < 0$. Between the high groups there are time intervals when the amplitudes are very small and a significant positive phase change is registered which is manifested by jumps of ϕ. Such jumps had been already observed in Guedes Soares and Cherneva (2005) for the waves in deep water near the Portuguese coast. First Melville (1983) studding the evolution of nonlinear wave trains suggests that "crest pearing" (Ramamonjiarisoa and Molo-Cristensen, 1979; Molo-Cristensen and Ramamonjiarisoa, 1982) may appear as a result of large positive gradients in phase speed when one crest overtakes the previous.

To the left side of Figs. 2–4 is presented spectrum of the whole series with peak frequency $\omega_\mathrm{p} = 0.475\,\mathrm{rad\,s^{-1}}$ calculated by the well-known Welch method (Welch, 1967). The time-frequency spectra of the three groups from Fig. 1 are drawn by ten different colors to the right sides of the figures respectively. For presentation of the Wigner spectra the surface of the energy distribution is normalized to its maximum value. Ten colors are used to show the Wigner spectra that means that the "resolution" by color is 10 % of the maximum value. For the sake of convenience the waves are given by white color in the same time scale as the time-frequency distribution. Spectrum and time-frequency spectra are calculated with same frequency resolution.

The Wigner spectrum of the group containing "Andrea" freak wave is presented in Fig. 2. During the abnormal wave the energy is distributed in a wide frequency interval. There are several peaks that coincide by corresponding peaks of the usual frequency spectrum. The group has the triangle form typical for the nonlinear waves.

Investigating the nonlinear high waves and the group development along the tank triangle group's form has been registered many times (Cherneva and Guedes Soares, 2012). That form is a result of continuous shrinking of the group in different distances from the wave maker and changing the place of the highest wave from the middle to the front of the group. For the tank produced waves the process is accompanied with frequency downshifting of the energy peak inside the group demonstrated by the time-frequency spectrum. The group is extremely short at the point of focus and has form similar to the New Year wave group. Moving forward the focus the group disintegrates and the time-frequency spectrum shows frequency upshifting of the energy peak. The same character of the group development with distance is noted by Kharif et al. (2008) in their investigation of the wind influence on the wave groups for different wind velocities U (including $U = 0$) and distances.

It has to be remarked that there is no registered downshifting of the energy maximum of the time-frequency spectrum of the group containing the "Andrea" abnormal wave. Making a spectrogram analysis of real sea states it was also indicated the fact that not all the groups demonstrate frequency downshifting of the energy. A similar conclusion was obtained by Su (1982) investigating the gravity wave group evolution with moderate and high steepness.

It is also necessary to mentioned that a frequency upshifting like that one for the groups in tank after the focus is not registered in real sea states.

Frequency downshifting of the energy peak is demonstrated in Fig. 3 which presents the time-frequency spectrum of the series between 440 s and 520 s (see Fig. 1a). Again, during the highest wave of the group the energy spreads in a wide frequency interval with three dominating peaks. The peak that prevail over two other peaks has frequency higher than ω_p of the usual spectrum. The energy peak that towers in further

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waves in the group slowly moves to lower frequencies and reaches the place of ω_p. If the same scenario like that explained before for the generated in tank non-linear waves takes place for the examined wave group, possibly it will transform to a short group similar to that of the New Year wave after several wavelengths.

Figure 4 shows the Wigner spectrum of the group that exists in the time interval (130–235) seconds (see Fig. 1a). The phase rate for that group is not constant and slowly changes in time. The picture of the peaks registered in the time-frequency spectrum of the group is very interesting. First three waves have heights less than 3 m. Their energy peaks are less than 10 % of the highest peak during the group lifetime and are invisible. The fourth and the fifth waves have the peak of energy in same frequency. The energy of that peaks increases in time. It reaches its maximum value during the highest wave of the group and drastically decreases for the next waves. A second peak in lower frequency than the first one during the highest wave appears. It begins to dominate in energy distribution in lifetime of the next waves. Weak downshifting of that second peak is observed since it reaches the frequency ω_p. Similar type of frequency downshifting during the group lifetime had been already observed (Guedes Soares and Cherneva, 2005) for a two wave systems sea state without abnormal waves, broad spectrum and a small angle between the main directions of the systems. The peak in higher frequency of one wave system shifts to low frequency and unites with the peak of the second wave system. It can be suggested that the complicated time-frequency spectrum picture of the group III from the Fig. 1b possibly is a result of interaction of wave components coming from separate directions. As Magnusson and Donelan (2013) note "The Andrea wave is observed just past 00:00 UTC, after a stable period with less wind forcing, but at the start of a new wind increase from a new and slightly different direction".

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4 Conclusions

The example of Andrea series is measured in harsh meteorological conditions and has a higher steepness than previously analyzed waves by Guedes Soares and Cherneva (2005). The abnormal "Andrea" wave exists in a large group of 12 waves and differs from the "New Year wave" that exists in a group similar to well-known "three sisters".

Here, the time-frequency analysis is made by more sophisticated Wigner spectrum but shows a similar character of energy distribution during the group lifetime as that traced out by spectrogram analysis. It is revealed that during the highest wave of the group the energy spreads in a wide frequency interval with several peaks and confirms the previous results for the abnormal Draupner wave (Cherneva and Guedes Soares, 2008).

The abnormally high "Andrea" wave is registered for wave group having constant local frequency. That local frequency $\omega(t)$ is less than the calculated carrier frequency ω_0 of the all series using Eq. (4). In time, the abnormal wave happens exactly in the moment when the series highest peak of energy exists and has frequency equal to the frequency ω_p of the usual stationary spectrum.

Not all groups investigated in real sea states demonstrate a frequency downshifting of the energy in time. For example, the long group that contains the "Andrea" freak wave has not frequency downshifting of that type registered in Marintek tank. Also, frequency upshifting like that one marked after the focus for the groups in tank is not registered in real sea states investigated by the authors.

The energy distribution in time of the groups that have changeable local frequency $\omega(t)$ is more sofisticated. The waves in such groups are not very steep, but in time interval when the highest waves exist a significant downshifting of the local spectrum peak is observed. It is realized by jump during the highest wave. It is suggested that more complicated downshifting is possibly a consequence of interaction of wave components coming from separate directions.

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Table 1. Parameters characterizing the nonlinearity of the series and of the individual freak waves.

quantity	H_s, m	$k_p h$	$\varepsilon = a_s k_p$	a_s/h	H_{freak}, m	T_{freak}, s	ε_{freak}	$k_{freak} h$	a_{freak}/h
Andrea	9.18	1.61	0.106	0.066	21.14	12.0	0.295	1.9543	0.151
Draupner	11.92	1.96	0.081	0.085	25.01	13.1	0.293	1.640	0.179

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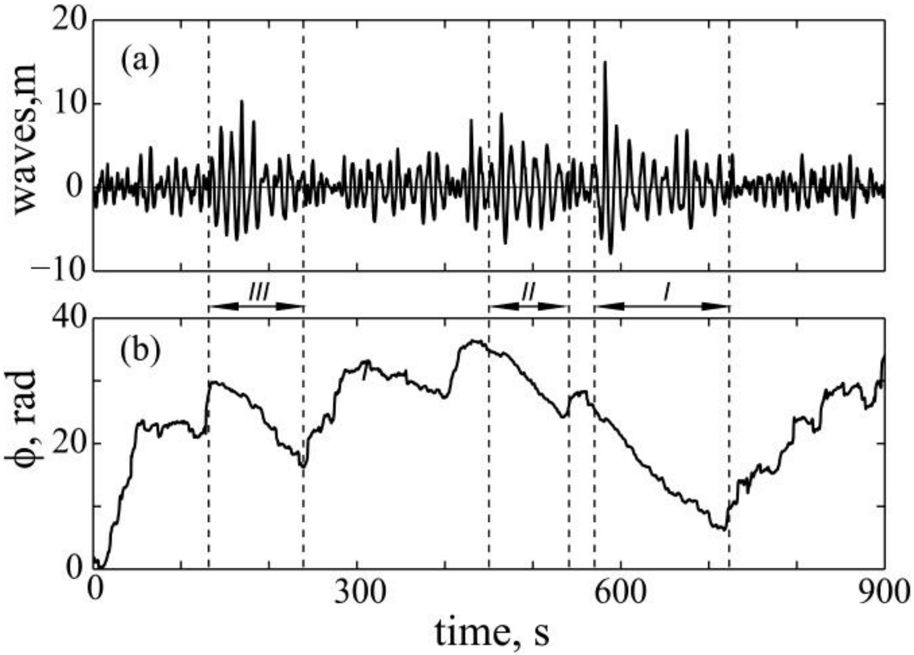

Fig. 1. "Andrea" freak wave. **(a)** Series measured at 00:40 UTC on 9 November 2007; **(b)** phase ϕ in time.

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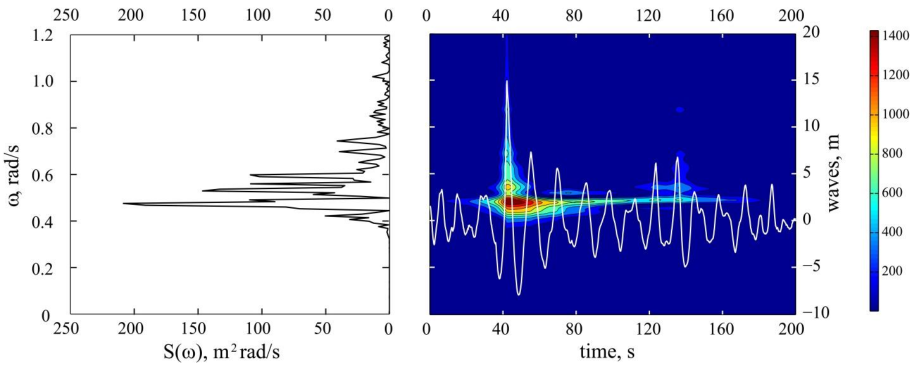

Fig. 2. Time-frequency spectrum of the group I containing the freak wave.

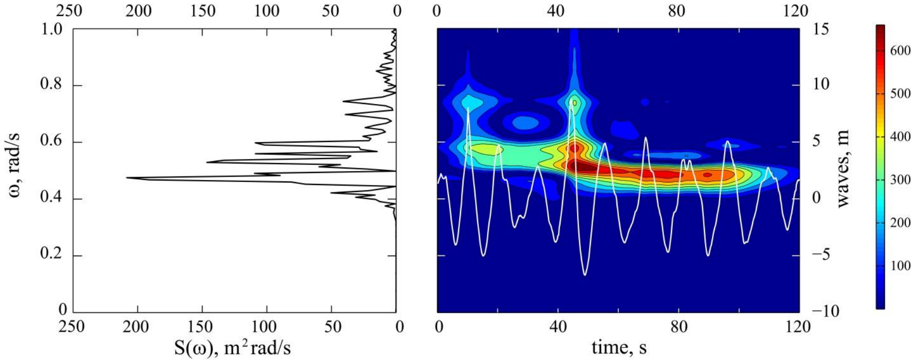

Fig. 3. Time-frequency spectrum of the second group II between 440 s and 520 s.

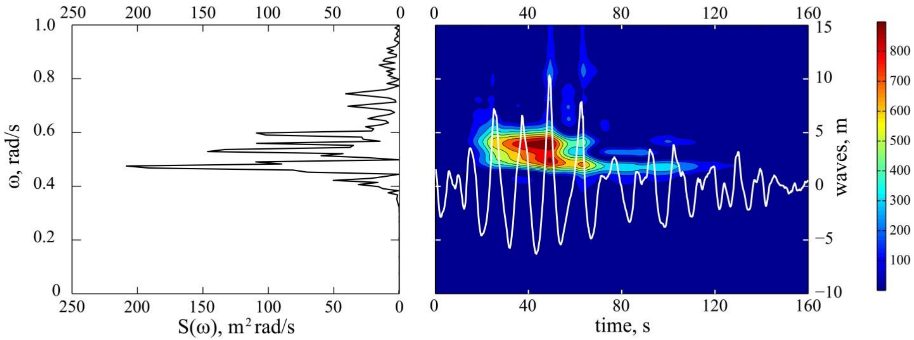

Fig. 4. Time-frequency spectrum of the group III between 130 s and 235 s.

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