

# Resonance phenomena at the long wave run-up on the coast

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## Abstract

Run-up of long wave on a beach consisting of three pieces of constant but different slopes is studied. Linear shallow-water theory is used for incoming impulse evolution and non-linear corrections are obtained for the run-up stage. It is demonstrated that bottom profile influences the run-up characteristics and can lead to the resonance effects: increasing of wave height, particle velocity, and number of oscillations. Simple parameterization of tsunami source through an earthquake magnitude is used to calculate the run-up height versus earthquake magnitude. It is shown that resonance effects lead to the sufficient increasing of run-up heights for weakest earthquakes and tsunami wave does not break on chosen bottom relief if the earthquake magnitude does not exceed 7.8.

**Keywords:** Tsunami, Nonlinear shallow-water theory, Resonance phenomena

## 1. Introduction

The resonance phenomena play significant role in the run-up amplification and lead to different physical effects for waves in coastal zone: long duration of water oscillations, later arrival of wave with maximal amplitude comparing with leading waves, group structure of waves. Meanwhile, usually these effects are neglected when the run-up processes are studied. The most part of theoretical results for run-up stage are based on rigorous analytical solutions of the shallow-water theory for waves climbing on the beach of constant slope. This approach was suggested in the pioneer work by Carrier and Greenspan (1958). They applied the hodograph

transformation to the nonlinear system of shallow-water equations and obtained the linear wave equation for an auxiliary function; all physical variables (free surface displacement, depth-averaged velocity, offshore coordinate and time) were explicitly expressed using this function and its partial derivatives. The main advantage of wave equation for an auxiliary function having a form of cylindrical wave equation is that it has to be solved on semi-axis with given boundary conditions while the initial equations have to be solved in domain with unknown moving boundary (shoreline). Meanwhile, the explicit form of the analytical solution generally requires the numerical manipulations to present physical variables in the wave field. That is why various shapes of the incident solitary wave have been specially analyzed: soliton (Pedersen et Gjevik, 1983; Synolakis, 1987), sine pulse (Mazova *et al.*, 1991), Lorentz pulse (Pelinovsky et Mazova, 1992), Gaussian pulse (Carrier *et al.*, 2003; Kânoğlu, 2004; Kânoğlu et Synolakis, 2006), *N*-waves (Tadepalli et Synolakis, 1994; Kânoğlu, 2004), some specific localized disturbances (Tinti et Tonini, 2005; Pritchard et Dickinson, 2007; Dobrokhotov et Tirozzi, 2010). It should be noted that different formulas for maximum run-up of solitary waves of various shapes can be provided in terms of wave amplitude and significant wave length describing practically important cases with good accuracy (Didenkulova *et al.*, 2008; Didenkulova & Pelinovsky, 2008, Antuono & Brocchini, 2010). Various shapes of the periodic incident wave trains such as the sine wave (Carrier et Greenspan, 1958; Madsen et Fuhrman, 2008), cnoidal wave (Synolakis *et al.*, 1988; Synolakis, 1991) and nonlinear deformed periodic wave (Didenkulova *et al.*, 2006; 2007) have been also studied to obtain the run-up characteristics. It is important to mention that the run-up height is higher if periodic incident wave is cnoidal or nonlinear deformed wave compared with a simple sine wave of the same amplitude and period. Some results are obtained for irregular incident waves modeled by the Fourier superposition of the sine waves with random phases (Didenkulova *et al.*, 2010; 2011) or the random set of solitons (Brocchini et Gentile, 2001).

In all studies mentioned above the rigorous analytical solutions are obtained if the wave propagates on a plane beach of constant slope. Really, such plane can approximate the face-shore bathymetry only, and then it has to be matched with horizontal bottom profile. In fact, the rigorous analytical solutions can be obtained here in the linear theory only (Synolakis, 1987; Pelinovsky, 1996; 2006; Madsen

et Fuhrman, 2008). If the bottom slope in face-shore area is small, the extreme run-up characteristics weakly differ from a case when the bottom has constant slope everywhere. Non-linearity leads to the correction of obtained results. First of all, there is nonlinear wave deformation in the region where inclination of bottom changes. This effect is thoroughly investigated with use of boundary value approach for pulse-like and periodic input including waves propagating over different kinds of non-planar bathymetries in (Brocchini, 2001), (Antuono et Brocchini, 2007; 2008; 2010). In this case the wave evolution seems to be unaffected by the bottom perturbations. The second one is that a wave moving on horizontal bottom nonlinearly deformed within nonlinear shallow-water equations as Riemann wave and its shape in the entry of plane beach differs from an initial shape (Didenkulova *et al.*, 2006; 2007). It should be noted that both factors amplify the run-up heights. A new effect appeared for the wave run-up on a plane beach matched with horizontal bottom is the influence of the bottom slope on the shape of water oscillations on the shore. If the incident wave has a bell-shape, the water oscillations on the shore repeat its shape if the bottom slope is big (limiting case is a vertical wall), and accompanied by the negative second oscillation if the bottom slope is small. Such behavior is explained by the resonance effects which are weak for such geometry – from physical point of view it is an open resonator<sup>1</sup> (Pelinovsky, 1996; 2006; Madsen et Fuhrman, 2008). If the bottom slope differs relatively small from the uniform value, the changes of run-up height are also small (Soldini *et al.*, 2013).

For more complicated geometry of coastal zone consisting of several pieces with different slopes, the solutions for each region of constant slope are matched (Kânoğlu and Synolakis, 1998, Didenkulova, 2009). Simplified solutions in the form of a product of such elementary solutions can be given if the incident wave length is less than a bottom piece length. For general ratio between these different lengths, as it is known, the resonances appear due to multi-reflection from matching points and interference between such waves. Some allusion on possible resonances for wave run-up can be found in (Kajiura, 1977; Mazova, 1985). They investigated linear approximation of the run-up characteristics due to sine incident wave. The resonance phenomena are important for tsunami waves (LeBlond and

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<sup>1</sup> If the wave maker located near the shore, of course, the resonant effects are big (Stefanakis *et al.*, 2011; Ezersky *et al.*, 2013).

Mysak, 1981; Massel, 1989; Mei, 1983; Neu and Shaw, 1987 ). Photos of tsunami wave trains in different coastal locations are well-known after the 2004 Indonesian and 2011 Japanese tsunamis. Usually appearance of resonance effects is connected with a complicated two-dimensional bathymetry of bays and jagged coastal line. In the present paper we aim to investigate run-up resonance phenomena for one dimensional case of wave propagation. We intend to show that for certain frequencies depending on bottom profile, run-up amplification may be high even for very simple bathymetry and it influences the shape of the water oscillations on the coast.

The paper is organized as follows. In the second section we describe our model of bottom profile and present the results for run-up amplification versus the frequency of linear harmonic incident wave. In the section 3 results of calculations of the run-up characteristics caused by Gaussian impulse and N-wave impulse are presented. Section 4 is devoted to non-linear effects appearing in run-up. Discussion of result applicability for natural hazard description and some conclusions are given in section 5.

## 2. Theoretical model and run-up due to linear harmonic wave

Long wave run-up on a long beach is described by 1D nonlinear shallow water equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [u(h + \eta)] = 0 \quad (2)$$

where  $u$  is the depth-averaged velocity,  $h=h(x)$  is the unperturbed water depth,  $\eta=\eta(x,t)$  is the free surface displacement,  $g$  is the acceleration of gravity. In the linear approximation the system (1)-(2) is transformed into one equation

$$\frac{\partial^2 \eta}{\partial t^2} - g \frac{\partial}{\partial x} \left[ h \frac{\partial \eta}{\partial x} \right] = 0 \quad (3)$$

To demonstrate the resonance effects in the run-up characteristics, we use three pieces-wise profile of unperturbed depth which are typical for real ocean bottom: (A) continental shelf  $0 \leq x \leq x_0$ , (B) continental slope  $x_2 < x < 0$ , (C) constant

depth ocean  $x < x_2$  (see **Fig. 1**). Such topography was used in numerous papers on tsunami run-up. We would like to emphasize that in some cases such simple model of bottom profile describes very precisely natural conditions. For instance, exactly such model was used to prepare numerical simulations of tsunami near the Indian coast (Neetu *et al.*, 2011).

The wave field in zone of constant depth (C) is presented as a sum of incident and reflected harmonic waves with constant amplitude  $A_i$  and  $A_r$  :

$$\eta = \left( A_i e^{-ik(x_0-x)} + A_r e^{ik(x_0-x)} \right) e^{-i\omega t} \quad (4)$$

In zone (B) with a constant bottom slope  $\tan \beta$  (continental slope), we seek a harmonic solution of the form:

$$\eta = A(x) e^{-i\omega t} \quad (5)$$

where  $\omega$  is the frequency and  $A(x)$  is an amplitude function. By inserting (5) in (3), the amplitude equation for harmonic wave is represented as:

$$g \tan(\beta)(x_1 - x) \frac{\partial^2 A}{\partial x^2} - g \tan(\beta) \frac{\partial A}{\partial x} + \omega^2 A = 0 \quad (6)$$

After introducing the variable transformation

$$\bar{\sigma} = 2\omega \sqrt{\frac{x_1 - x}{g \tan \beta}} \quad (7)$$

the equation (6) can be simplified to the Bessel equation of the first kind

$$\bar{\sigma} \frac{\partial^2 A}{\partial \bar{\sigma}^2} + \frac{\partial A}{\partial \bar{\sigma}} + \bar{\sigma} A = 0 \quad (8)$$

Its solution may be expressed as a sum of the **zeroth order** Bessel functions of the first  $J_0$  and second  $Y_0$  kinds with two constants  $C_1$  and  $C_2$ :

$$A = C_1 J_0(\bar{\sigma}) + C_2 Y_0(\bar{\sigma}), \quad \bar{\sigma} = 2\omega \sqrt{\frac{x_1 - x}{g \tan \beta}} \quad (9)$$

In the near-shore zone (A) the solution for wave amplitude is also presented in Bessel functions. Taking into account that the wave field should be limited at the shore ( $x = x_0$ ), the wave amplitude is described by:

$$A = R J_0(\sigma), \quad \sigma = 2\omega \sqrt{\frac{x_0 - x}{g \tan \alpha}} \quad (10)$$

where  $R$  is also constant (in general, complex constant). It is evident that  $R$  describes the amplitude of the water level oscillation on unmoved shoreline

(linear run-up height). If the bottom has the constant slope everywhere the value  $f$   $|R|$  computed in the linear theory coincides with run-up height in nonlinear theory (Synolakis, 1987; Pelinovsky et Mazova, 1992). For more complicated geometry this statement is not proved and we will discuss this later.

Using continuity conditions for horizontal velocity and free surface displacement for  $x = 0$  and  $x = x_2$ , one can match solutions in different segments at this transition points and obtain the following system:

For  $x = 0$ :

$$RJ_0(\sigma_0) = C_1J_0(\bar{\sigma}_0) + C_2Y_0(\bar{\sigma}_0) \quad (11)$$

$$RJ_1(\sigma_0) = C_1J_1(\bar{\sigma}_0) + C_2Y_1(\bar{\sigma}_0) \quad (12)$$

For  $x = x_2$ :

$$C_1J_0(\bar{\sigma}_1) + C_2Y_0(\bar{\sigma}_1) = A_i e^{-ik(x_0-x_2)} + A_r e^{ik(x_0-x_2)} \quad (13)$$

$$C_1J_1(\bar{\sigma}_1) + C_2Y_1(\bar{\sigma}_1) = -iA_i e^{-ik(x_0-x_2)} - iA_r e^{ik(x_0-x_2)} \quad (14)$$

where  $\sigma_0 = \sigma(x=0)$ ,  $\bar{\sigma}_0 = \bar{\sigma}(x=0)$ ,  $\bar{\sigma}_1 = \bar{\sigma}(x=x_2)$ ,  $J_1$  and  $Y_1$  the first order

Bessel functions of first and second kinds.

If the incident wave amplitude  $A_i$  is known, the linear run-up height  $R$  can be evaluated by solving the last system (11)-(14):

$$R = K(\omega)A_i, \quad K(\omega) = \frac{2(J_0(\bar{\sigma}_0)Y_1(\bar{\sigma}_0) - J_1(\bar{\sigma}_0)Y_0(\bar{\sigma}_0))}{W(Y_0(\bar{\sigma}_1) - iY_1(\bar{\sigma}_1)) - N(J_0(\bar{\sigma}_1) - iJ_1(\bar{\sigma}_1))} e^{-ik(x_0-x_2)} \quad (15)$$

where

$$W = J_0(\bar{\sigma}_0)J_1(\sigma_0) - J_0(\sigma_0)J_1(\bar{\sigma}_0), \quad N = Y_0(\bar{\sigma}_0)J_1(\sigma_0) - J_0(\sigma_0)Y_1(\bar{\sigma}_0).$$

Term in numerator of (15) may be simplified using Wronskian (Abramovich & Stegun, 1964) as presented in Kânoglu & Synolakis 1998:

$$J_0(\bar{\sigma}_0)Y_1(\bar{\sigma}_0) - J_1(\bar{\sigma}_0)Y_0(\bar{\sigma}_0) = -\frac{2}{\pi\bar{\sigma}_0}.$$

**Fig. 2** represents run-up amplification  $|R|/A_i$  for three different sets of bottom slopes characterized roughly for the Indian coast bathymetry (Neetu *et al.*, 2011) where the Makran tsunami was observed on 27 November 1945. If the bottom slopes in zones A and B are the same the resonance effects are very weak (dash-point curve in **Fig. 2**), and this coincides with known results (Pelinovsky, 1996;

2006; Madsen et Fuhrman, 2008). But in the case of different bottom slopes in zones A and B resonance effects are clearly visible (solid and dashed lines in **Fig. 2**). Several resonant modes with frequencies  $(\omega_1, \omega_2, \dots)$  may be excited in the coastal zone and the amplification coefficient can reach values of 10-20 times.

Characteristic period of the first resonant peak  $T=2\pi/\omega_1$  is roughly 2 hr that coincide with observed tsunami record in this area (1.5–3 hr) according to (Neetu *et al.*, 2011).

### 3. Run-up due of solitary bell and N impulses

Resonance curves given in the previous section show a substantial increase of run-up heights for certain frequencies of harmonic incident waves. Whereas run-up height for harmonic wave is given by (15), the oscillations of water level on the shore (linear run-up) generated by solitary tsunami wave may be presented using the Fourier transformations:

$$R(t) = \frac{1}{2\pi} \int K(\omega) S(\omega) e^{(-i\omega t)} d\omega \quad (16)$$

where  $S$  is the Fourier transformation of incident wave

$$S(\omega) = \int \eta(t) e^{(i\omega t)} dt \quad (17)$$

Usually the shape of the incident tsunami wave is unknown and it is characterized by different functions, see for instance (Didenkulova *et al.*, 2008). We chose here two characteristic and qualitatively different cases: the initial displacement of the free surface of one sign and alternating displacement with zero averaged value. In the first case it is a Gaussian pulse, and in the second one so-called N pulse:

$$\eta_{in\ G} = \eta_0 e^{(-(t/\tau_0)^2)} \quad (18)$$

$$\eta_{in\ N} = \eta_0 \left( \frac{t}{\tau_0} \right) e^{(-(t/\tau_0)^2)} \quad (19)$$

Both pulses are characterized by two parameters: the duration and amplitude. Within linear theory, the value of wave amplitude is not important and can be used for scaling of run-up characteristics. The second parameter, wave duration  $\tau_0$  plays an important role due to resonance effects.

Meanwhile, in tsunami practice, both parameters (amplitude and duration) of the incident wave are not independent and are determined by the parameters of

tsunami source. Here we apply our theoretical results to tsunamis generated by the underwater earthquakes. Now, the characteristics of tsunami source are calculated using the Okada solution (Okada, 1985), and they depend on the several fault parameters. For simplified estimates it is more convenient to have the relations between tsunami source parameters and earthquake magnitude. Such relations are known in seismology (Sato, 1979; Wells et Coppersmith, 1994). Similar relations are given for parameters of tsunami source (Pelinovsky, 1996; 2006; Bolshakova et Nosov, 2011). Here we will use the following relations between the displacement amplitude of the free water surface  $\eta_0$  and the characteristic size of tsunami source  $L$  with the earthquake magnitude  $M$  (Pelinovsky, 1996; 2006).

$$\log(\eta_0) = 0.8M - 5.6 \quad (20)$$

$$\log(L) = 0.5M - 2.2 \quad (21)$$

In the shallow-water approximation the duration of tsunami waves going out the source is  $\tau_0 = L / \sqrt{gh}$ , where  $h$  is water depth in the tsunami source. Of course, the formulas (20) and (21) are very approximated and may use only for simplified estimations.

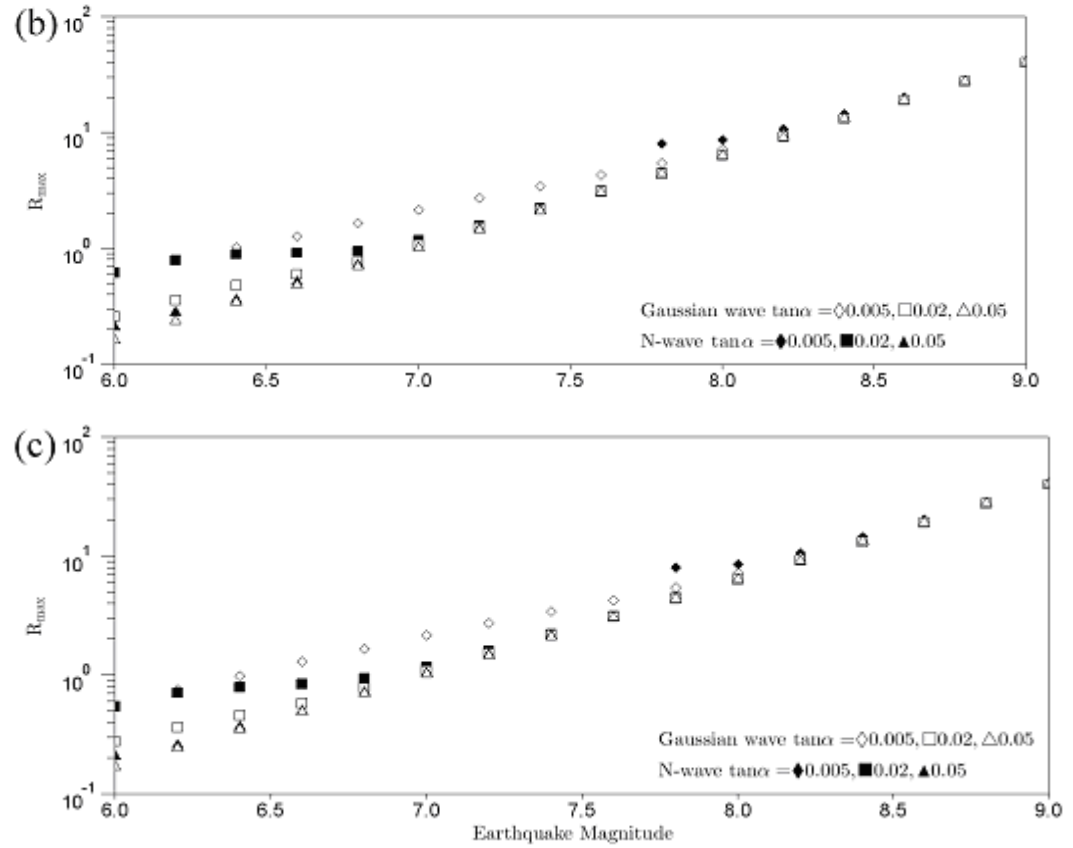
Thus, we can use magnitude of earthquake to describe solitary bell, or N wave. The results of calculations of linear run-up function  $R(t)$  for various values of the earthquake magnitude are presented below.

values of the earthquake magnitude: 7.7 and 8.5. Duration of the incident tsunami waves decreases then magnitude decreases, and its spectrum width increases. It means that weakest earthquake induces more resonant modes in coastal zone than strongest earthquake. As a result, the number of water oscillations on the shore increases with magnitude decreasing. If the initial shape is N-wave, the number of oscillations is higher than for Gaussian input because its spectrum is narrower. It is important to mention that run-up height of N-wave is higher than for bell-wave and this was obtained firstly in (Tadepalli et Synolakis, 1994).

The run-up amplification factor ( $R/A_i$ ) decreases with increasing of earthquake magnitude up to  $M = 7$ , and then it remains almost constant (**Fig. 4**). Tsunami generated by strong earthquake has large wavelength and in this case as indicated above, the resonance effects are very weak. The weakest tsunamis having the shortest wavelength are amplified more due to resonance effects. Increasing of the bottom slope in zone B ( $\tan\beta$ ) reduces the run-up height as it might be expected.



Maximal run-up height grows with earthquake magnitude increasing as it is shown in



**Fig. 5.** It should be noted that resonance effects “lift up” the values of run-up height for weakest earthquake, where as run-up height weakly depends on the values the bottom slopes in given ranges under consideration. It is important to note that curves in **Fig. 4** and 5 are obtained in the linear approximation. Criteria for applicability of the linear solution will be discussed in the next section.

## 4. Estimations of non-linear effects

Calculations of maximal run-up heights in sections 2 and 3 were done in linear approximation. As it is indicated above it is difficult to solve the nonlinear shallow-water equations for piece-wise bottom profile. Taking into account that bottom slope is changed on depth in 4000 m and 200 m, and wave amplitude does not exceed a few meters we may assume that all nonlinear effects are manifested on the last run-up stage. In this case we may use the rigorous solution of the nonlinear shallow-water equations for the long wave run-up on a beach of constant slope, which is very well developed, see references in Introduction. Here

follow to (Pelinovsky et Mazova, 1992) we convert obtained linear solution into “nonlinear” solution. According to this procedure, we should firstly find “linear” expression for horizontal velocity on the unmoved shoreline ( $x = 0$ ) which is followed from kinematics

$$U(t) = \frac{1}{\tan \alpha} \frac{dR}{dt} \quad (22)$$

where as earlier  $\tan \alpha$  is bottom face-slope. “Nonlinear” velocity of the moving shoreline,  $u(t)$  can be obtained from linear function  $U(t)$  by the Riemann transformation (Pelinovsky et Mazova, 1992)

$$u(t) = U\left(t + \frac{u}{g \tan \alpha}\right) \quad (23)$$

It is evident that maximal values of “nonlinear” and “linear” velocities coincide.

Vertical displacement of the moving shoreline,  $r(t)$  can be found from kinematic condition

$$r(t) = \frac{\int u(t) dt}{\tan \alpha}$$

And after substitution of (16) can be reduced to

$$r(t) = R\left(t + \frac{u}{\tan \alpha g}\right) - \frac{u^2(t)}{2g} \quad (24)$$

It should be noted that this is only true for the analytical structure of the solution, but the solution itself also depends on the data assignment as an initial value or a boundary value problem (Antuono & Brocchini, 2007).

The important conclusion from Eq. (24) is that extremes of the vertical displacement in the linear and nonlinear theories coincide (in this moment the horizontal velocity  $u=0$ ), confirming the use of linear theory to predict extreme values. Therefore, the linear theory adequately describes the run-up height.

Simple formulas of Riemann transformation from linear to nonlinear solutions allow us to obtain the wave breaking criterion. Strictly speaking, this criterion is found from zero condition for Jacobian of hodograph (Legendre) transformation. Note that this transformation was used to obtain eq. (15) - (17). On the other hand, the solution for velocity (16) resembles the well know Riemann wave in nonlinear acoustics and hydrodynamics (the role of coordinate plays the inverse value of the  $g \tan \alpha$ ). Such wave would overturn with increasing of amplitude. Exactly this fact

has been used in (Pelinovsky, 1966, 2006; Didenkulova, 2009) to find wave breaking criteria on the shore. From equation (16) it is easy to calculate the time derivative of the velocity in incident wave.

$$\frac{du}{dt} = \frac{dU / dt}{1 - \frac{dU / dt}{g \tan \alpha}} \quad (25)$$

tends to the infinity when the denominator approaches zero. As follows from the theory of hyperbolic equations it leads to the gradient catastrophe identified and to the plunging breaking of the long water waves. In this case a water displacement contains the jump of its first derivative. This implies the condition of the first wave breaking

$$Br = \frac{\max(dU / dt)}{\tan \alpha g} = \frac{\max(d^2 R / dt^2)}{\tan^2 \alpha g} = 1 \quad (26)$$

where the parameter  $Br$  has the sense of breaking parameters. Fig. 6 shows the temporal evolution of the breaking parameter, linear and nonlinear water level oscillations on the shore and shoreline velocities versus time for solitary impulse and N-wave impulse for magnitude  $M=7.7$ . It is clearly seen the difference between linear and nonlinear solutions for moving shoreline. It is important to mention that breaking parameter is less 1, so tsunami wave should climb on the shore without -breaking for chosen bottom geometry. It should be emphasized that for all results presented in **Fig. 4** and 5 criterion  $Br < 1$  is satisfied.

## 5. Discussion and conclusions.

The run-up of tsunami waves on the coast is studied for following bottom geometry: ocean of constant depth, steep continental slope, beach of gentle constant slope. It is demonstrated that run-up characteristics are strongly depend on the frequency of the incident wave due to resonance effects. They are studied for conditions of the Indian coast where the 1945 Makran tsunami has been recorded. Amplification ratio can be higher in 10 times then for a case of the uniform averaged slope. The run-ups of solitary wave of bell or N shape are studied in details. It is found that run-up of N waves is higher than for solitary wave. It is due to stronger manifestation of the resonance effects for N-wave than for bell shape wave. Using simple parameterization of tsunami source through an

earthquake magnitude the run-up heights are calculated versus earthquake magnitude. It is shown that the resonance effects can “lift up” the values of run-up heights for weakest magnitudes due to resonance amplification of shortest waves generated by weakest earthquake. Nonlinear correction of obtained results is given. It is shown that for typical conditions of the Indian coast where the 1945 Makran tsunami was observed the breaking parameter is less than 1 and tsunami waves climb on a coast with no breaking.

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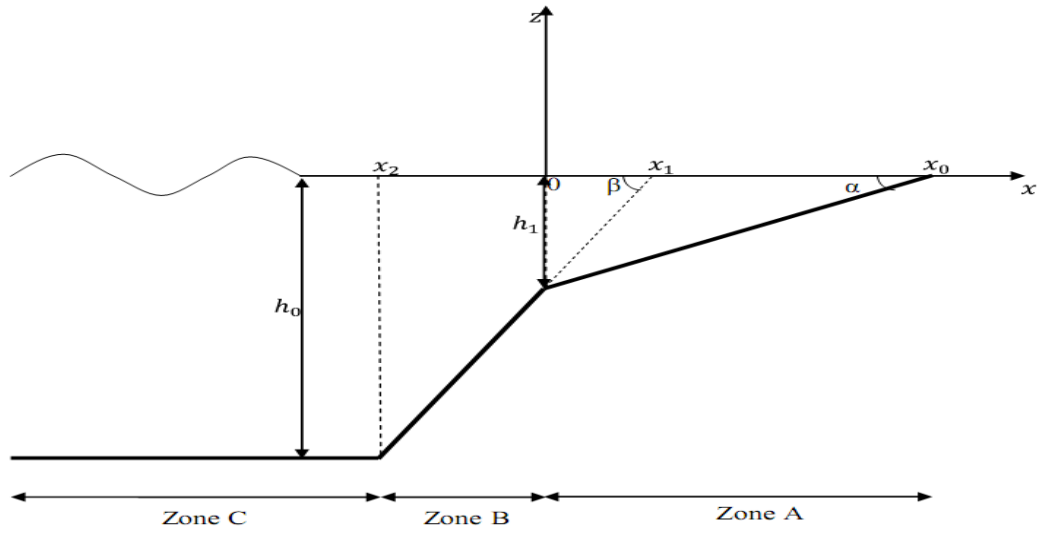
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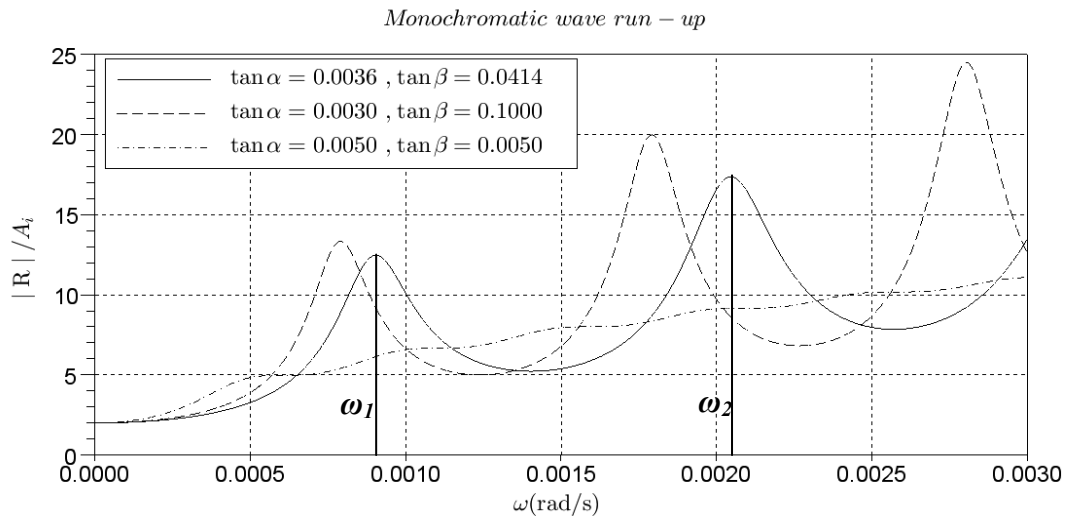
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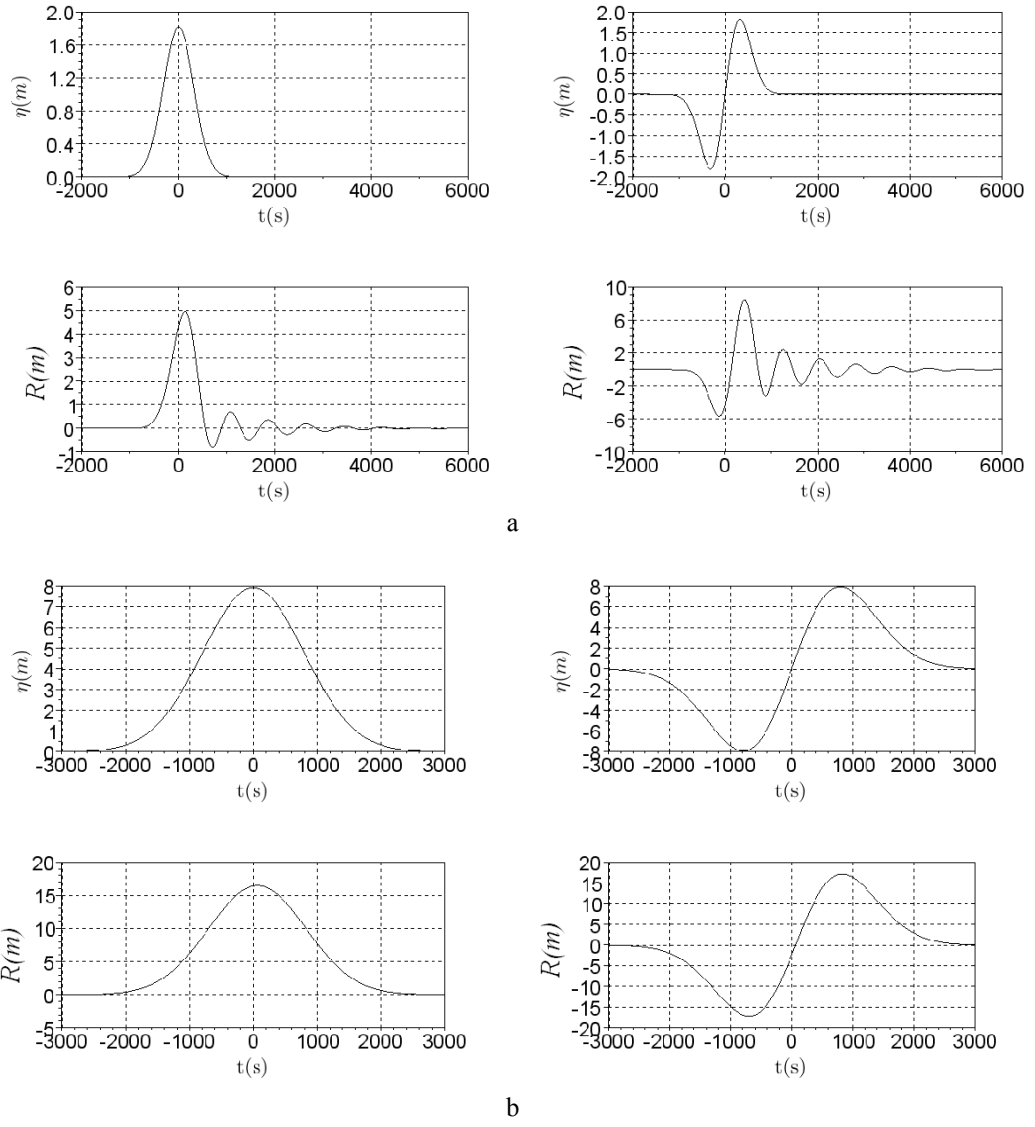


**Fig. 1** Schema of bottom profile.

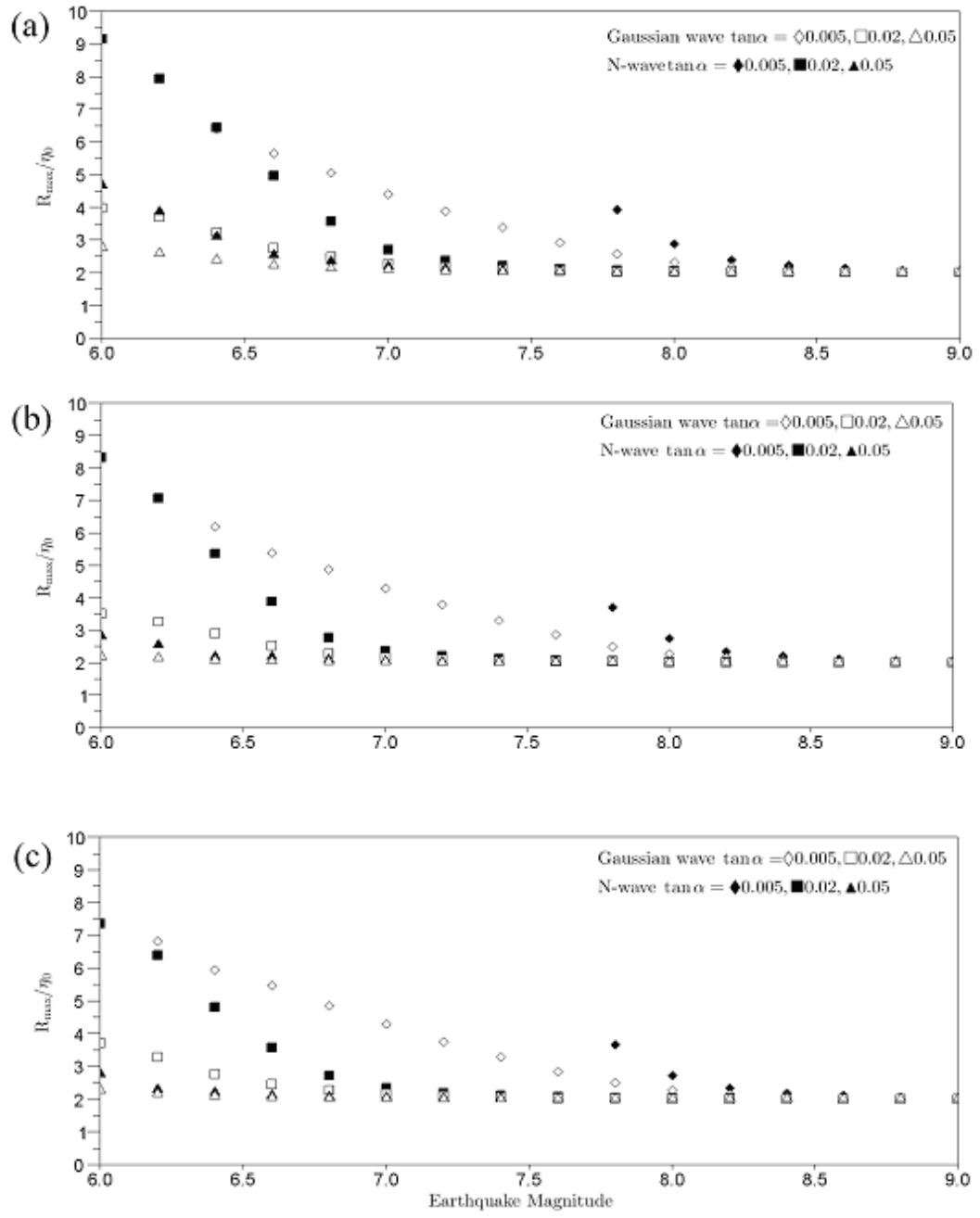


**Fig. 2** Run-up amplifications for three sets of bottom slopes

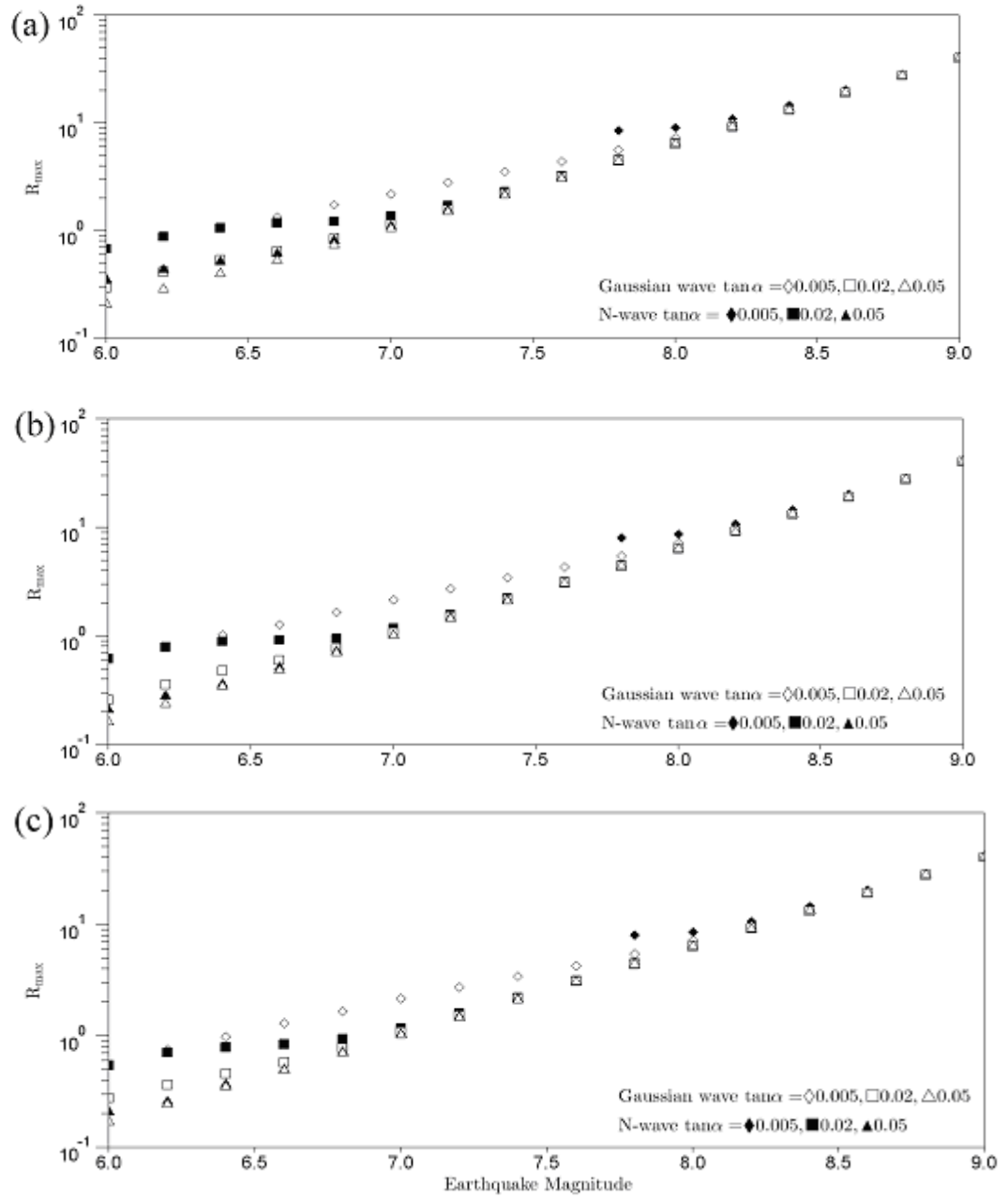




**Fig. 3** Incident wave and water oscillations on the shore for  $h_0=4000$  m,  $h_I=200$  m,  $\tan\alpha=0.005$ ,  $\tan\beta=0.1$ , and different earthquake magnitude: a)  $M = 7.7$  b)  $M = 8.5$ .



**Fig. 4** Run-up amplification factor for Gaussian impulses and N-wave impulse ( $h_0=4000\text{m}$ ,  $h_l=200\text{m}$ ) for different inclinations of continental shelf and continental slope: (a)  $\tan\beta=0.1$ , (b)  $\tan\beta=0.5$ , (c)  $\tan\beta=3$ .



**Fig. 5** Maximal run-up height versus the earthquake magnitude for Gaussian impulses and N-wave impulse, ( $h_0=4000$  m,  $h_l=200$  m) for different inclinations of continental shelf and continental slope: (a)  $\tan \beta=0.1$ , (b)  $\tan \beta=0.5$ , (c)  $\tan \beta=3$ .

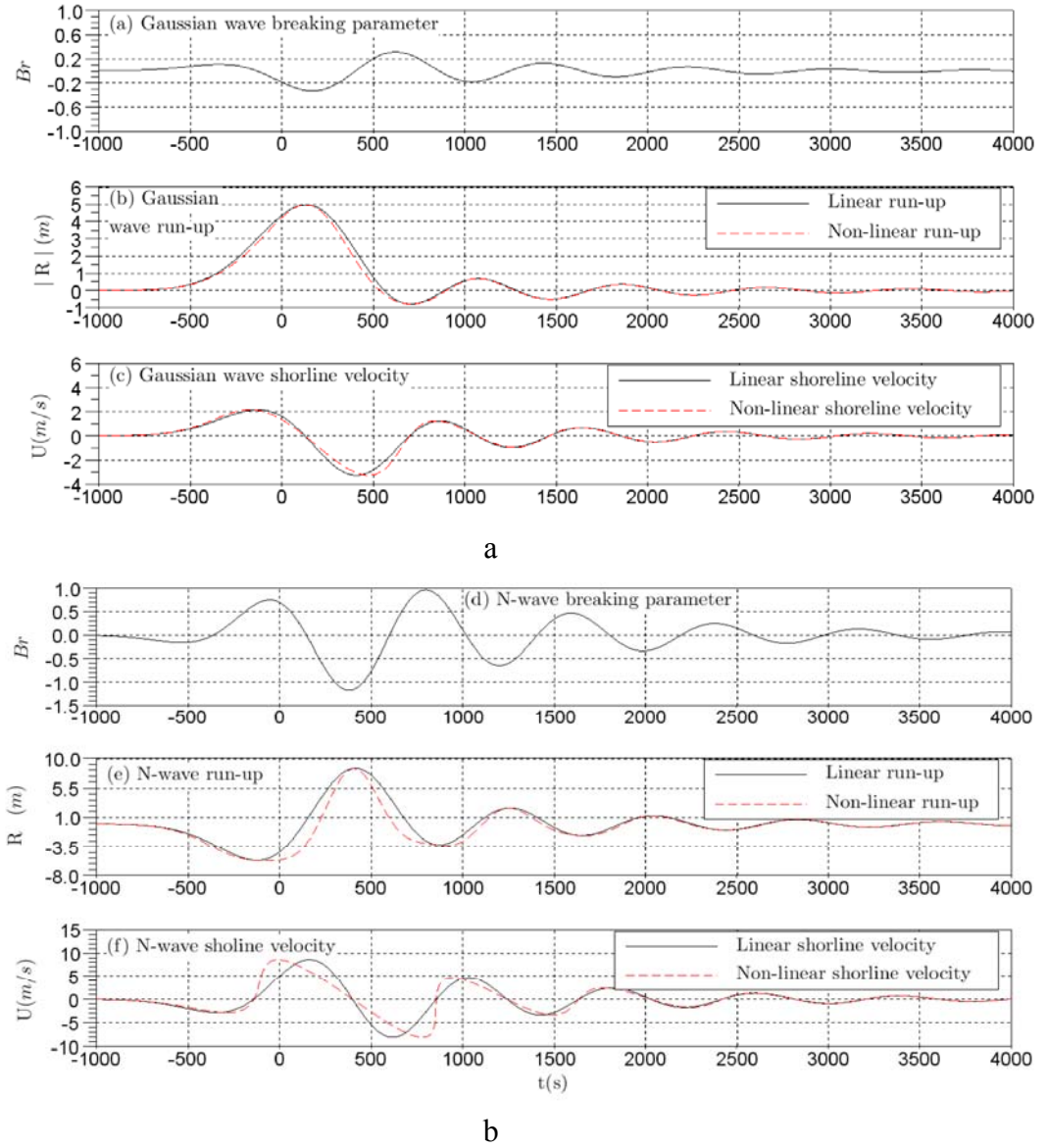


Fig. 6 Breaking parameter, linear and nonlinear variations of water level on shore and shoreline velocities versus time for  $h_0=4000$  m,  $h_l=200$  m,  $\tan\alpha=0.005$ ,  $\tan\beta=0.1$  and  $M=7.7$ : a) Gaussian wave, b) N-wave