

Authors' Reply

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Title: Modelling wildland fire propagation by tracking random fronts

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We are very grateful to the Referee whose comments allow us to better clarify some important features of our approach. Below we reply to Referee and we discuss the changes introduced to improve the manuscript.

However, before replies, let us remind that in the proposed approach the instantaneous random fireline position \mathbf{X} is stated to be $\mathbf{X} = \bar{\mathbf{x}} + \mathbf{X}'$ in analogy with the classical Reynolds decomposition. Thus, $\bar{\mathbf{x}}$ is the average position and \mathbf{X}' is the random fluctuation with zero mean. The average value $\bar{\mathbf{x}}$ is understood as ensemble averaged. Hence, we have that the instantaneous random value of the front velocity turns out to be $\mathbf{V}(\mathbf{X}, t) = \bar{\mathbf{V}} + \mathbf{V}'$, where $\bar{\mathbf{V}}$ is the ensemble averaged velocity. We assume that the modulus of $\bar{\mathbf{V}}$ contains the ROS, computed according to anyone of the literature standard models, and also a term due to fire spotting.

In particular, a formula for the ROS with a strong theoretical base was derived by Frandsen (1971). Frandsen's formula is also the starting point for the celebrated Rothermel's formula (1972). Many other determinations of the ROS are given in literature.

In Rothermel's report (1972), Frandsen's formula is stated at the very beginning as equation number (1). By carefully reading the description of the terms, it emerges that the exact Frandsen's ROS is given by the ratio between the heat flux absorbed and the heat required to bring a unit weight of fuel to ignition. With Rothermel's words:

"In one sense, equation (1) shows that the rate of spread during the quasi-steady state is a ratio between the heat flux received from the source in the numerator and the heat required for ignition by the potential fuel in the denominator".

Within this framework, we have proposed a method to model the effects and the dynamics of fluctuations: something apparently not included in the existing models for fire propagation that are based on the level set method. We argue that these fluctuations are due to the several physical processes involved and two of them are here considered, namely turbulence and fire spotting. This connection with physics leads us to establish the desired dynamics of fluctuations.

Below, we reply separately to main remarks.

References

W. H. Frandsen, Fire spread through porous fuels from the conservation of energy. *Combustion and Flame* **16**, 9–16, 1971.

R. C. Rothermel, A Mathematical Model for Predicting Fire Spread in Wildland Fires, Tech. Rep. Research Paper INT-115, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah 84401, 1972.

My first concern is that the paper does not explain in enough detail what is meant by the effect of preheating by hot air. This effect is treated as an additional process that is apparently not included in the baseline level-set formulation but one could argue that this effect is already accounted for in the standard models of the rate of spread (ROS). This important point needs to be clarified.

The point raised up by the Referee is right from the experimental point of view. In fact, since the propagation of the fire is due to many factors, the measured fireline position includes for sure all related processes. For a given measured fireline velocity, it is possible to simulate a fire with a model based on the level set method.

But the measured fireline velocity cannot be obtained by the computation from standard formulae of the ROS because, following Frandsen (1971) and Rothermel (1972), the ROS is given by the ratio between the heat flux absorbed and the heat required to bring a unit weight of fuel to ignition and the dynamics of the full process is not included. Actually, Referee is right because the pre-heating effect is included into the standard models of the ROS, however what is missing in ROS' formulae and previous models is the dynamics of such effect.

Then we are not providing a new formula of the ROS but we are providing a method to take into account such dynamics. In particular, the pre-heating effects can be regarded as an accumulation process that allows ignition as a consequence of a sufficiently prolonged exposition of the fuel to high temperature. This accumulation can be viewed as a "memory effects" (in space and time) and, as such, cannot be properly accounted for by a mere modification of the ROS that can take into account only "local" effects.

Finally, since dynamics and accumulation cannot be provided taking into account solely the standard definition of the ROS, the pre-heating action appears as an additional process.

More in general, we claim that the magnitude of the measured fireline velocity should be compare with the ROS obtained from the motion of some threshold value of the observable proposed by us.

The above explanation is given in a short paragraph that is located at the end of the Introduction, right before the description of the organization of the paper, and it is the following:

"Moreover, since the solution of the reaction-diffusion equation is not zero on an infinite domain, the potential fire ahead the selected frontline can be considered as a long-range action of the fire itself and then generating a pre-heating effect. In particular, the accumulation in time of such potential fire can be associated to an amount of heat and then related to the increasing of the fuel temperature (possibly up to the ignition threshold). Thus ignition is modelled as the consequence of a sufficiently prolonged exposition to high temperatures. This accumulation can be regarded as a memory effect governed by the dynamics of the process that, clearly, cannot be dealt by adding a suitable term to the ROS which only allows to take into account local effects."

My second concern is that the proposed formulation appears to be unnecessarily complex: the proposed randomization process could easily be included in a standard level-set approach by performing an ensemble of simulations that would account for uncertainties in the ROS model parameters. In fact, this stochastic approach based on an ensemble Kalman filter (EnKF) has recently been explored in the literature (see Refs. [1- 4] below). The advantage of an ensemble-based approach is that there is a clear differentiation between the fire physics (represented by the ROS model) and the sources of uncertainties (represented for instance by variations in the input parameters to the ROS model). The authors should compare their proposed method to a more straightforward ensemble-based alternative.

Referee remarks an important question. Actually, the present approach and the ensemble Kalman filter (EnKF), in spite of some mathematical analogies, are deeply different in their meaning.

The EnKF is a statistical operational technique to handle uncertainties in the estimation of the ROS. But uncertainties in measurements are not straightforward related to physical random fluctuations. Our approach is a formulation based on the idea to consider random fluctuations and their dynamics as caused by the physical processes involved. This approach difference generates also a quantitative difference.

In fact, the data error in EnKF is distributed according to an ansatz done by the users and this choice drives the statistics. According to pure statistical arguments, this ansatz is generally Gaussian. In the proposed approach, statistics of fluctuations are described by a proper model of the corresponding physical process. Thus different processes follow different statistics. We have used Gaussian density for turbulence and log-normal distribution for firebrand jumps. Hence, this physical picture allows to consider separately each involved process. The whole fluctuations turn out to be distributed according to the convolution of them. The resulting probability density function is noted by f and its dynamics enters into the description by the term $\frac{\partial f}{\partial t}$, see formula (13).

Furthermore, this approach leads to a physically based correction for the ROS that is due to fire-spotting and it is stated in (28). Actually, we are not able to understand how this can be obtained by the EnKF that in general can consider only fluctuations with zero mean.

However, the two approaches are not in opposition. The EnKF can be used as an improvement to compute the average ROS in the present approach. The coupling of the present model to a data assimilation algorithm based on an EnKF approach is surely an enhancement of the model. This could allow to take into account the uncertainties on the ROS input parameters. We believe that it represents an enhancement of the present model which deserves consideration in the sequel of the work described here.

In the revised version, the EnKF suggested by the Referee has now been properly mentioned in the Introduction and the recommended references have been cited as well:

“It should be stressed that, in the proposed approach, the randomization of the fireline motion is accounted for as due to physical processes, namely the turbulent hot-air transport and the fire spotting phenomenon. If uncertainties on the input data necessary to compute the ROS are to be taken into account, resulting in a ROS treated as a random variable, the model proposed here could be improved by coupling it with a data assimilation algorithm based, for example, on the so-called ensemble Kalman filter (Mandel et al., 2008; Beezley et al., 2008; Mandel et al., 2011; Rochoux et al., 2012, 2013).”

More, the above explanation is included as follows at the beginning of Section 4 after the first paragraph:

“Then also the instantaneous front velocity can be represented by the sum of a deterministic part and random contributions. This formulation has a formal analogy with the so-called ensemble Kalman filter (EnKF) (Mandel et al., 2008; Beezley et al., 2008; Mandel et al., 2011; Rochoux et al., 2012, 2013). The EnKF is a statistical operational technique to handle uncertainties in the estimation of the ROS. But uncertainties in measurements are not straightforward related to physical random fluctuations and data error is generally Gaussian distributed according to pure statistical arguments. In contrast, the proposed approach is based on the idea to consider fluctuations and their dynamics as due to physical processes with random nature. This physical picture allows to consider separately each involved process and statistics of fluctuations are described by specific models. The PDF of fluctuations $f(\mathbf{x}; t|\bar{\mathbf{x}})$ and its dynamics enters into the description through the term $\frac{\partial f}{\partial t}$, see formula (13). This difference between the EnKF and the present approach generates a quantitative difference.”

What concerns mathematical complexity, the EnKF theory is not simpler than our approach. See for example

J. Mandel, L. Cobb and J. D. Beezley, On the convergence of the ensemble Kalman filter. *Applications of Mathematics* **56** (6), 533–541, 2011.

To conclude, all the typographical errors, spelling errors, etc. pointed out by the Referee have been corrected as suggested.

Concerning the clarification requested by the Referee regarding the very high value considered in the paper for the wind velocity, we agree that 17.88 m s^{-1} is surely very high. The main reason for considering such a high velocity is that simulations are performed using fire spotting parameterizations as derived by Sardoy et al. (2008), see (25) and (33-34). Then, in order to correctly use those parameterizations and be confident that they hold, as well as to facilitate the comparison of our results to those obtained by other authors (Sardoy et al., 2008), we have kept the same system configuration.

This is now mentioned in the manuscript (Section 5.1), as Table 2 is introduced, with the text

“It should be noted that, despite the fact that a wind velocity of 17.88 m s^{-1} may appear very high, this value has been chosen as to favour the comparison with results published by other authors (Sardoy et al., 2008).”

and also by inserting in the caption of Table 2 the following:

“These values correspond to the same system configurations considered by Sardoy et al. (2008).”