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Discussions

Interactive comment on "Numerical investigation of stability of breather-type solutions of the nonlinear Schrödinger equation" *by* A. Calini and C. M. Schober

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We would like to thank the Referee for carefully reading our paper and for the helpful comments.

Questions (Q) and Responses(R):

(Q1) Parameters ρ and τ in Eq. 8,9 have been selected so that initial signals are not too close to the unstable plane wave. What is the order of magnitude for the difference between an initial signal and the plane wave? It is interesting to compare it with the perturbation parameter $\epsilon = 10^{-4}$.

(R1) The parameters ρ and τ were selected so that the difference of $U^{(j)}(x,0;\rho)$ and the plane wave was $\mathcal{O}(10^{-3})$ and the difference of $U^{(1,2)}(x,0;\rho,\tau)$ and the plane wave was $\mathcal{O}(10^{-2})$ in order to avoid exciting any of the instabilities of the plane wave.

(Q2) As I understand you have started from the SPB with some ρ_0, τ_0 + perturbation and then approximated the result of its evolution by a SPB with the parameters ρ^*, τ^*. Where ρ^*, τ^* have been found by minimization of Hmax (norm of the difference). In the experiment we will expect to generate SPB(ρ_0, τ_0) but following your results we will measure SPB(ρ^*, τ^*) where ρ^*, τ^* depend on experimental noise. It is important to know what values of shifts $\rho^* - \rho_0$ and $\tau^* - \tau_0$ could we expect? You have provided an example on Fig. 8 with τ_0 and τ^* obtained in the numerical simulations. But can we estimate the shifts in a general case if we know the initial SPB and perturbation amplitudes? Is it possible to observe, because of shifts, (in the worst case) a coalesced variant (Fig. 2b) instead of SPB with distinct modes as on Fig. 2a (or vice-versa)?

(R2) Thank you for asking for the values of τ_0 and ρ_0 used in the numerical experiments. It was an oversight not to have explicitly mentioned their values. For the experiments of a one-mode SPB $U^{(1)}(x,t;\rho))$ over a plane wave with one unstable mode, $\rho_0 = 5$. For the one-mode SPB $U^{(j)}(x,t;\rho))$ over a plane wave with two unstable modes, $\rho_0 = 0$. For the case of a plane wave with two unstable modes, the uncoaelsced two-mode SPB $U^{(1,2)}(x,t;\rho,\tau)$ experiments used $\rho_0 = -2$, $\tau_0 = -10$, while the coalesced two-mode SPB experiments used $\rho_0 = -2$, $\tau_0 = -3$. Although the initial random perturbation is $\mathcal{O}(10^{-4})$, for the one-mode neutrally stable SPB, the shift in ρ is $\rho^* - \rho_0 \approx \mathcal{O}(10^{-2})$. The shifts in the parameters for the two-mode SPB are $h = \rho^* - \rho_0 \approx \mathcal{O}(10^{-2})$ and $k = \tau^* - \tau_0 \approx \mathcal{O}(10^{-1})$. These values of the shifts are consistent with a formal determination of h and k by equating a Taylor expansion of $U^{(1,2)}(x,t;\rho_0+h,\tau_0+k)$ with $U^{(1,2)}_{numerical}$. Assuming you started with initial data for an SPB with distinct modes, but with ρ_0 and τ_0 very close to the parameter values for a coalesced SPB, it would be possible to observe the coalesced case because of the parameter shifts – it would depend on the relation between the strength of the perturbation and how close the ρ_0 and τ_0 were to the coalesced case.

(Q3) Actually, I was surprised by the fact that perturbations lead only to shifts of a solution in time (instead of chaotic dynamics, for example). Even in the case of one-mode SPB in two UMs case we can see from Fig. 5a that the emergent mode of SPB is very close to the exact solution U^1 Only later we observe the appearance of the second mode of unstable background, which takes the form of the exact solution U^2 (that is also seems nontrivial). Is it because you choose relatively small perturbations? Could you comment it?

(R3) The linear analysis is relevant only for short times and does not address the potential development of chaotic behavior on a longer time frame (which would be due to perturbations to the equation arising from the numerical scheme or from the experimental setup). Yes, due to the small perturbations in the initial data we are picking up quasiperiodic solutions of the NLS that are nearby to the SPBs (and that looks like a superposition of U^1 and U^2 on this timeframe).

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