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Natural Hazards and Earth System Sciences
Discussions

Interactive comment on "Numerical investigation of stability of breather-type solutions of the nonlinear Schrödinger equation" *by* A. Calini and C. M. Schober

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The manuscript describes stability questions of spatially periodic breathers (SPB) in the frame of the one-dimensional focusing NLS. The broad interest to exact NLS solutions on the unstable plane wave background is explained by existence of intriguing models of modulation instability and freak wave phenomena such as the Akhmediev and the Peregrine breathers. The problem of its stability is a hot topic of many scientific discussions. The main point of this debate is the fact that a particular solution could be badly unstable with respect to perturbations. The great progress in this area has been achieved due to experiments in hydrodynamics (A. Chabchoub et al. PRL 2011)

and in optics (B. Kibler et al. Nature phys. 2010). They have experimentally proved that the Peregrine breather could be precisely reproduced. I guess the Authors could cite them and discuss possible applications of the theory described in the manuscript in the frame of hydrodynamic and optical experimental setups.

The manuscript is an extension of the paper "Observable and reproducible rogue waves" by A. Calini and C. M. Schober where they have introduced the main idea of the NLS breathers stability analysis. By the use of explicit linear stability analysis and numerical simulations they have shown, that in the case of N unstable modes (UMs) only the N-mode SPB (so called "maximal breather") is robust with respect to perturbations. The numerical experiments were carried out only for the random phase initial perturbations. In the present paper the Authors have considered the other five cases such as random amplitude perturbations, gaussian perturbations and so on. I believe the cases studied are more than enough to prove the concept of maximal breather stability. Another improvement of the work is the averaging of the numerical results over the ensemble of 100 simulations.

The most interesting fact found by the authors that the coalesced maximal breather is the most robust two-mode SPB. This is absolutely new result and could be considered as the main message of the paper. I believe that the manuscript could be interesting for a general reader as well as for the specialists in this area, especially for experimenters who may try to generate more stable and reproducible breathers following the recommendations given in the article. Meanwhile I have several questions that could help to clarify the text for a reader.

1. Parameters ρ and τ in Eq. 8,9 have been selected so that initial signals are not too close to the unstable plane wave. What is the order of magnitude for the difference between an initial signal and the plane wave? It is interesting to compare it with the perturbation parameter $\epsilon = 10^{-4}$.

2. As I understand you have started from the SPB with some ρ_0, τ_0 + perturbation and

then approximated the result of its evolution by a SPB with other parameters ρ^*, τ^*. Where ρ^*, τ^* have been found by minimization of H_{max} (norm of the difference).

In the experiment we will expect to generate SPB(ρ_0, τ_0) but following your results we will measure SPB(ρ^*, τ^*) where ρ^* and τ^* depend on experimental noise. It is important to know what values of shifts $|\rho_0 - \rho^*|$ and $|\tau_0 - \tau^*|$ could we expect? You have provided an example on Fig. 8 with τ_0 and τ^* obtained in the numerical simulations. But can we estimate the shifts in a general case if we know the initial SPB and perturbation amplitudes? Is it possible to observe, because of shifts, (in the worst case) a coalesced variant (Fig. 2b) instead of SPB with distinct modes as on Fig. 2a (or vice-versa)?

3. Actually, I was surprised by the fact that perturbations lead only to shifts of a solution in time (instead of chaotic dynamics, for example). Even in the case of one-mode SPB in two UMs case we can see from Fig. 5a that the emergent mode of SPB is very close to the exact solution $U^{(1)}$. Only later we observe the appearance of the second mode of unstable background, which takes the form of the exact solution $U^{(2)}$ (that is also seems nontrivial). Is it because you choose relatively small perturbations? Could you comment it?

Best wishes, Andrey Gelash

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