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# Non-linear water waves generated by impulsive motion of submerged obstacle

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# Abstract

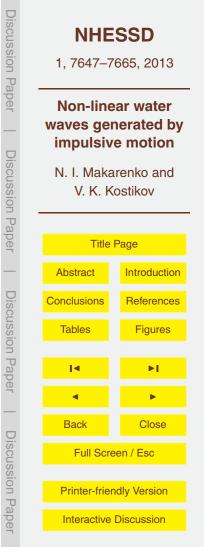
Fully nonlinear problem on unsteady water waves generated by impulsively moving obstacle is studied analytically. Our method involves the reduction of Euler equations to the integral-differential system for the wave elevation together with normal and tan-

<sup>5</sup> gential fluid velocities at the free surface. Exact model equations are derived in explicit form in the case when the isolated obstacle is presented by totally submerged elliptic cylinder. Small-time asymptotic solution is constructed for the cylinder which starts with constant acceleration from rest. It is demonstrated that the leading-order solution terms describe several wave regimes such as the formation of non-stationary splash
 <sup>10</sup> jets by vertical rising or vertical submersion of the obstacle, as well as the generation of diverging waves is observed.

# 1 Introduction

In this paper, we consider non-stationary problem on generation of intense water waves caused by localized underwater disturbances. There are several physical mechanisms of teacement of access floor due to

- of tsunami-type wave formation such as piston displacement of ocean floor due to submarine earthquake, low frequency hydro-elastic seismic oscillations, landslides and avalanches (Levin and Nosov, 2009). Long wave theory is well consistent in the case when the horizontal scale of deformed bottom area is much bigger than the ocean depth. In this context, mathematical models describing the generation, propagation
- and run-up of tsunami waves were considered in many papers (see Pelinovsky, 1982, 1996; Pelinovsky and Mazova, 1992; Liu et al., 2003; Tinti and Tonini, 2005; Dutykh and Dias, 2009). Non-linear energy transfer from the elastic oscillations of fluid layer to long gravitational waves were studied by Novikova and Ostrovsky (1982), Nosov and Skachko (2001), Nosov et al. (2008).
- <sup>25</sup> Theoretical works on water waves generated by compact submerged sources (i.e. obstacles having relatively short horizontal scales) deal mostly with objects of simple



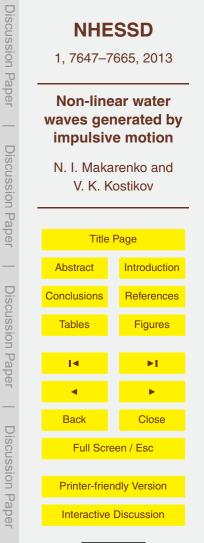


geometric form, such as circular cylinder in 2-D motion and sphere in 3-D case. In addition, we neglect vorticity to simplify the evaluation of pure inertial effects for the fluid flows generated by obstacle. Such viewpoint is a common issue in marine hydrodynamics related to the problem on interaction of submerged and floating bodies with

- <sup>5</sup> water surface (McCormick, 2010). This is due to difficulty of mathematical formulation involving exact nonlinear boundary condition at unknown free surface, as well as exact boundary condition on the body surface or topographic condition on the bottom surface. Another example is the mathematical model of far-field internal waves generated by internal tides over ocean bottom ridges. Simplified statement of this guestion deals with the problem on harmonic oscillations of cylinder or sphere in infinitely deep 10

stratified fluid (Voisin et al., 2011a, b). There are few analytical works related to the problem on nonlinear interaction of submerged body with free surface of ideal fluid. Small-time non-stationary analytic

- solution was obtained by Tyvand and Miloh (1995a, b) and Makarenko (2003) in the presence of circular cylinder, and by Pyatkina (2003) for a submerged sphere. Our 15 analysis uses reduction of Euler equations to the boundary integral-differential system which contains unknown variables on the free surface only. This method was developed by Ovsyannikov et al. (1985) for a non-linear water wave problem without submerged obstacle. Extension of this method exploits the idea suggested firstly by Wehausen
- (see Wehausen and Laitone, 1960) for stationary linear problem with submerged cir-20 cular cylinder. It turns out that the Wehausen method can be generalized by using Milne-Thomson transformation (see Milne-Thomson, 1996) even that the cylinder has non-circular cross-section (Makarenko, 2004). We apply this approach to study fully nonlinear water wave problem in the presence of vertically moving plate modelled by thin elliptic cylinder. 25





# 2 Basic equations

The potential 2-D-flow of infinitely deep ideal fluid is considered on the coordinate system Oxy with a vertical y axis, so the free surface  $\Gamma(t)$  has the form  $y = \eta(x,t)$  with the equilibrium level y = 0. Compact plate-type solid obstacle is modelled here by thin <sup>5</sup> elliptic cylinder having non-dimensional horizontal semi-axis a and vertical semi-axis b ( $a \gg b$ ) (see Fig. 1). This finite-size obstacle is supposed here to be totally submerged under free surface. We suppose also that the cylinder moves without rotation, and the center of its cross-section has known trajectory  $\mathbf{x}_{c}(t) = (x_{c}(t), y_{c}(t))$ . Dimensionless formulation of the problem uses the initial depth of submergence  $h_{0}$  of the center as a length scale, characteristic speed of cylinder  $u_{0}$  as the velocity unit, the quantity  $\rho u_{0}^{2}$  as the pressure unit (here  $\rho$  is the fluid density), and the ratio  $h_{0}/u_{0}$  as the time unit. Basic non-stationary Euler equations for the fluid velocity  $\mathbf{u} = (U, V)$  and the pressure p are

$$U_t + UU_x + VU_y + \rho_x = 0, \tag{1}$$

15 
$$V_t + UV_x + VV_y + \rho_y = -\lambda$$
,

$$U_x + V_y = 0, \quad U_y - V_x = 0.$$

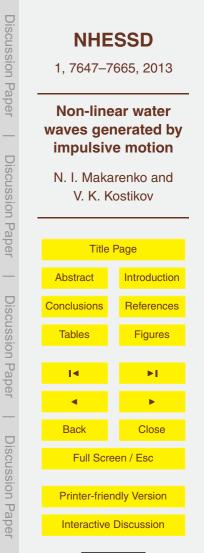
Here the constant  $\lambda = gh_0/u_0^2$  is the square of the inverse Froude number defined by the gravity acceleration *g*. Boundary conditions at unknown free surface  $\Gamma(t)$  are

<sup>20</sup> 
$$\eta_t + U\eta_x = V$$
,  $p = 0$   $(y = \eta(x, t))$ 

and the condition on the surface of obstacle S(t) has the form

$$(\boldsymbol{u}-\boldsymbol{u}_{\mathrm{c}})\cdot\boldsymbol{n}=0.$$

Here *n* is the unit normal to the boundary of elliptic cylinder, and  $u_c = x'_c(t)$  is the speed of submerged body at present time *t*. In general, such an obstacle could be simply



(2) (3)

(4)

(5)



modelled by moving finite-length horizontal plate of vanishing thickness. However, we consider here smooth obstacle in order to avoid singularity of solution at the edge points of the curve S(t). We suppose that the fluid is at rest at infinity

$$U, V \to 0, \eta \to 0$$
  $(|\mathbf{x}| \to \infty).$ 

15

We suppose also that the initial velocity field satisfies compatibility conditions 5

$$U_{0x} + V_{0y} = 0, U_{0y} - V_{0x} = 0,$$

$$(u_0 - u_c(0)) \cdot n_0 = 0.$$
(6)

These matching conditions are fulfilled obviously for initial data  $U_0 = V_0 = 0$  and  $x'_c(0) =$  $y'_{c}(0) = 0$  when the obstacle starts moving from the rest. 10

#### **Boundary integral-differential equations** 3

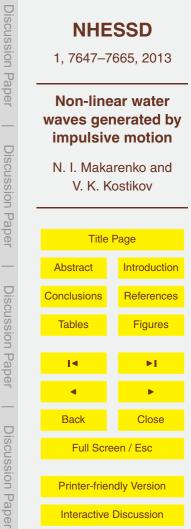
Main difficulty to solve the problem formulated above arises from unknown free surface with non-linear boundary condition. Therefore we reduce Eqs. (1)-(3) to the system of integral-differential equations being one-dimensional with respect to spatial variables. Let

$$u(x,t) = (U + \eta_x V)|_{y = \eta(x,t)}, \quad v(x,t) = (V - \eta_x U)|_{y = \eta(x,t)}$$

be tangential and normal fluid velocities at the free surface  $\Gamma(t)$ . Excluding the pressure from momentum Eqs. (1) and (2) we obtain under both conditions Eq. (4) the non-linear evolution system for  $\eta, u, v$  which has the form

20 
$$\eta_t = v, \qquad u_t + \frac{1}{2} \frac{\partial}{\partial x} \frac{u^2 - 2\eta_x uv - v^2}{1 + \eta_x^2} + \lambda \eta_x = 0.$$
 (8)

Differential Eq. (8) should be complemented by integral equation at the free boundary  $\Gamma(t)$  appearing due to the Cauchy–Riemann Eq. (3) for the fluid velocity components 7651



U, V. Derivation procedure uses the analyticity of complex velocity F = U - iV with respect to complex variable z = x + iy. We refer here to Silverman (1973) for a related facts of complex analysis. Basically, the function F(z, t) satisfies the Cauchy integral formula

$$5 \quad 2\pi i F(z,t) = \int_{\Gamma(t)} \frac{F(\zeta,t) d\zeta}{\zeta - z} + \int_{S(t)} \frac{F(\zeta,t) d\zeta}{\zeta - z}$$
(9)

where z is arbitrary point belonging to the flow domain. Let  $z_c(t) = x_c(t) + i y_c(t)$  be the symmetry center of submerged body bounded by elliptic contour S(t) in a complex z plane. We introduce an auxiliary  $\tau$  plane in order to use conformal map by evaluation of integral on curve S(t) in Eq. (9). Note that the Joukowski function

10 
$$\zeta = Z_{\rm c}(t) + \tau + \frac{a^2 - b^2}{4\tau}$$
 (10)

maps conformally the exterior domain  $|\tau| \ge r$  of the circle having the radius r = (a+b)/2 onto exterior domain of the ellipse S(t). Inverse map is given by analytic branch of the function

$$\tau(\zeta; t, a, b) = \frac{1}{2} \left( \zeta - z_{\rm c}(t) + \sqrt{(\zeta - z_{\rm c}(t))^2 - a^2 + b^2} \right) \tag{11}$$

which conform exterior domain of ellipses with a given semi-axes *a* and *b* onto exterior domain  $|\tau| \ge r$  of the circle of the radius *r*. This function satisfies together with the Cauchy kernel the following useful relation:

$$\frac{1}{\zeta-z}=\frac{\tau'(z)}{\tau(\zeta)-\tau(z)}+\frac{\tau_1'(z)}{\tau(\zeta)-\tau_1(z)},$$



where is denoted  $\tau_1(z) = (a^2 - b^2)/(4\tau(z))$ . In addition, if the complex variable  $\zeta$  belongs to elliptic contour S(t), we have also the relation

$$\frac{1}{\tau(\zeta)-\tau(z)}=-\frac{1}{\tau(z)}\overline{\left(\frac{\tau(\zeta)}{\tau(\zeta)-\tau_*(z)}\right)},$$

where  $\tau_* = r^2/\bar{\tau}$  is the inversion image of  $\tau$  with respect to the circle  $|\tau| = r$ , and bar <sup>5</sup> denotes complex conjugate.

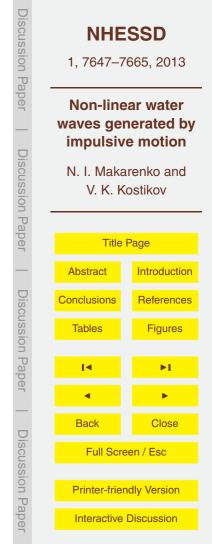
Using these relations enables us to exclude the integration around the obstacle surface S(t) by transforming to integrals on the free surface  $\Gamma(t)$  due to the residue theorem. The derivation is straightforward, but we refer to the paper by Makarenko (2004) which involves calculations of related integrals. By this way, the Eq. (9) takes the form

$$2\pi i F(z,t) = \int_{\Gamma(t)} \frac{F(\zeta,t) \mathrm{d}\zeta}{\zeta-z} + r^2 \left( \int_{\Gamma} \frac{F(\zeta,t) \mathrm{d}\zeta}{\tau(\zeta) - \tau_*(z)} - k \int_{\Gamma(t)} \frac{F(\zeta,t) \mathrm{d}\zeta}{\tau(\zeta) - k \overline{\tau_*(z)}} \right) \frac{\tau'(z)}{\tau^2(z)} + 2\pi i r^2 \left( z_{\mathrm{c}}'(t) - k \overline{z_{\mathrm{c}}'(t)} \right) \frac{\tau'(z)}{\tau^2(z)},$$

where the parameter k is given by the formula

$$k = \frac{a-b}{a+b}.$$

The limit value k = 0 corresponds to the circular obstacle of the radius r = a = b. In this case, the conform map  $\tau(\zeta; t, a, b)$  reduces to the linear function  $\tau = \zeta - z_c(t)$ , so the formula (12) couples the Cauchy integral on  $\Gamma(t)$  together with their Milne-Thomson transformation. This combination of integrals on  $\Gamma(t)$  provides automatically the validity of boundary condition (5) on the curve S(t) due to the Milne-Thomson circle theorem. In that sense, the Eq. (12) with  $a \neq b$  gives the extension of the Milne-Thomson theorem



(12)



for the elliptic contour S(t). Note that the residual term with  $z'_{c}(t)$  appearing in the right-hand side of Eq. (12) is similar to the fluid velocity term for the elliptic cylinder surrounded by infinite flow.

Real-valued form of integral equation arises if we set  $z = x + i\eta(x,t)$  in Eq. (12) taking into account discontinuity jump of the Cauchy integral at the free surface  $\Gamma(t)$ . By that, combination of real and imaginary parts of Eq. (12) yields singular integral equation for the functions  $\eta$ , u, v having the form

$$\pi v(x) + \int_{-\infty}^{\infty} A(x,s)v(s)ds = \int_{-\infty}^{+\infty} B(x,s)u(s)ds + f(x),$$
(13)

where the real-valued kernels A and B can be split by the formulas

<sup>10</sup> 
$$A = A_{\Gamma} + r^2 A_S, \qquad B = B_{\Gamma} + r^2 B_S$$

Here the kernels

$$A_{\Gamma}(x,s) + iB_{\Gamma}(x,s) = \frac{i[1 + i\eta_x(x)]}{x - s + i[\eta(x) - \eta(s)]}$$

correspond to the perfect non-linear water wave problem without any submerged obstacle, and the additional kernels

<sup>15</sup> 
$$A_{S}(x,s) + iB_{S}(x,s) = \left\{ \frac{1}{\overline{\tau(s+i\eta(s))} - \overline{\tau_{*}(x+i\eta(x))}} + \frac{k}{\tau(s+i\eta(s)) - k\overline{\tau_{*}(x+i\eta(x))}} \right\} \left\{ \frac{i}{\tau(x+i\eta(x))} \right\}_{x}$$

appear due to the presence of the obstacle. The extra term f in Eq. (13) has the form

$$f(x) = r^2 \operatorname{Re} \left\{ \frac{2\pi i \left( k \overline{z'_{c}(t)} - z'_{c}(t) \right)}{\tau(x + i \eta(x))} \right\}_{x}.$$
7654



The time variable *t* is omitted everywhere in Eq. (13) because it appears in this integral equation only as a parameter. More precisely, this independent variable is presented here in explicit form only by known functions  $z_c(t)$  and  $z'_c(t)$ . So, the time *t* is coupled implicitely with unknown functions v(x,t), u(x,t),  $\eta(x,t)$ . They should satisfy the boundary integral-differential system Eqs. (8) and (13) which is equivalent to the fully non-linear water wave problem in the presence of submerged elliptic obstacle. This BEq system determines immediately the exact wave elevation and fluid velocity at the free surface.

### 4 Small-time solution

5

We consider here in more details initial stage of the fluid motion beginning from the rest  $\eta(x,0) = u(x,0) = 0$ . It is supposed that the obstacle starts smoothly with zero speed  $z'_{c}(0) = 0$  at the time moment t = 0. Therefore we have also v(x,0) = 0 immediately from the Eq. (13). Thus, we can seek a solution as a power series

$$\eta(x,t) = t\eta_1(x) + t^2\eta_2(x) + t^3\eta_3(x) + \dots,$$
  

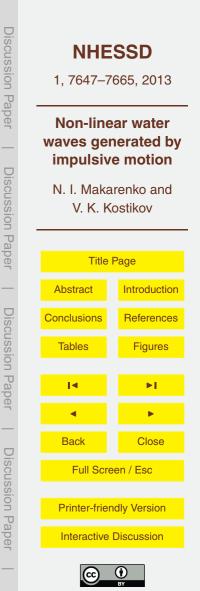
$$u(x,t) = tu_1(x) + t^2u_2(x) + t^3u_3(x) + \dots,$$
  

$$v(x,t) = tv_1(x) + t^2v_2(x) + t^3v_3(x) + \dots$$

Coefficients  $\eta_n$  and  $u_n$  are evaluated via  $v_n$  by recursive formulas following from the evolution Eq. (8). Namely, we have  $\eta_1 = u_1 = u_2 = 0$  and

$$\eta_n = \frac{1}{n} v_{n-1} \qquad (n = 2, 3, ...),$$

$$u_3 = \frac{1}{6} \left( v_1^2 - \lambda v_1 \right)_x, u_4 = \frac{1}{4} (v_1 v_2)_x - \frac{1}{12} \lambda v_{2x}, ...$$



Consequently, integral Eq. (13) for normal velocity v reduces to the set of recursive integral equations for coefficients  $v_n(n = 1, 2, ...)$  having the form

$$\pi v_n(x) + r^2 \int_{-\infty}^{+\infty} A_S^{(0)}(x,s) v_n(s) ds = \varphi_n(x).$$

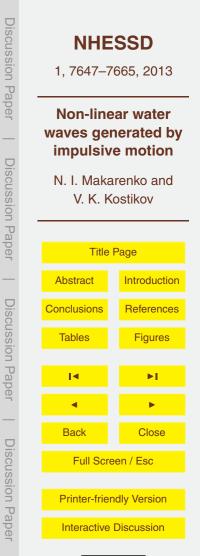
Here the kernel  $A_S^{(0)}$  is a known function collecting lower-order terms of the kernels *A* and *B* from Eq. (13) with substituted power series of  $\eta$ ,

$$A_{S}^{(0)}(x,s) = Re\left\{\left(\frac{1}{\overline{\tau(s)} - \overline{\tau_{*}(x)}} + \frac{k}{\tau(s) - k\overline{\tau_{*}(x)}}\right)\left(\frac{i}{\tau(x)}\right)_{x}\right\}.$$

The same is true for the right-hand side function  $\varphi_1$  which presents lowest-order term of the function *f* from Eq. (13). In contrast, the functions  $\varphi_n$  ( $n \ge 2$ ) depend non-locally and non-linearly on the coefficients  $v_1, \ldots, v_{n-1}$ .

Further simplification is possible by the perturbation procedure involving appropriate small parameters. It was rigorously proved in the paper by Makarenko (2003) that for circular cylinder having small radius r = a = b the solution of Eqs. (8), (13) is of the order  $r^2$ . Subsequently, the terms to be neglected are of the order  $r^4$  by this approximation. This fact explains especially why the leading-order solution is in very good agreement with fully non-linear numerical solutions and laboratory experiments, even the submergence depth is comparable with the radius of the cylinder (up to the bound  $r \sim 0.5$ ). Keeping this in mind, we apply similar perturbation method in the case of thin elliptic obstacle with sufficiently small horizontal semi-axis *a* and vanishing vertical semi-axis *b* = 0. The most simple explicit formula for a symmetric wave solution appears in the case of vertical rising or vertical submersion of obstacle with constant

acceleration *w*. By that, we can originally take  $u_0 = \sqrt{wh_0/2}$  as the velocity unit, so the trajectory of the obstacle takes the dimensionless form as  $x_c(t) = 0$ ,  $y_c(t) = -1 \pm t^2$ .



(14)



As a result, we obtain small-time asymptotic solution of accuracy  $\sim a^4$  which has free surface elevation  $\eta$  as follows:

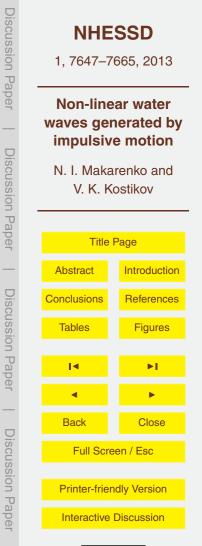
$$\eta(x,t) = Q_x(x;a)t^2 + P_{xx}(x;a)\left(\pm\frac{\lambda}{6} - 1\right)\frac{t^4}{2} + O(t^6).$$
(15)

Here the function P(x;a) is even in x, the function Q(x;a) is odd, and both these elementary functions are defined as  $x \ge 0$  by the formulas

$$P(x;a) = -2 + \sqrt{2}\sqrt{\sqrt{(x^2 - a^2 - 1)^2 + 4x^2} - x^2 + a^2 + 1},$$
  
$$Q(x;a) = 2x - \sqrt{2}\sqrt{\sqrt{(x^2 - a^2 - 1)^2 + 4x^2} + x^2 - a^2 - 1}.$$

Note that only the term with  $\lambda$  in the formula (15) has the sign depending on the direction of motion. Namely, sign "plus" in this formula corresponds to the lift of obstacle, and "minus" should be taken for submersion. In any case, the terms combined in the solution (15) provide the water surface  $y = \eta(x,t)$  to oscillate near the equilibrium level y = 0. Such a simple analytic solution allows easily to compare the wave regimes (at least qualitatively) by variation of basic parameters. Figures 2–5 demonstrate typical wave profiles generated due to the vertical motion of accelerated obstacle. For all the pictures, the center of elliptic body is placed initially at the point x = 0, y = -1 as t = 0. Dashed and solid lines are used to show the water surface corresponding to the position of obstacle at the instant.

Early stages of the wave motion by slow and fast rising of submerged plate are com-<sup>20</sup> pared on Figs. 2 and 3. Rapidity of the motion is regulated here by unique dimensionless parameter  $\lambda$ . In both cases, the obstacles are shown at the same submergence depths reached by the same dimensionless time moments t = 0.6 and t = 0.8 (but not at the same physical time!). It is clear that the wave generator has enough time to produce the system of diverging waves before the moderately accelerated obstacle comes





near the free surface (see Fig. 2). In contrast, fast motion of the obstacle leads to formation of added fluid layer which results in strong displacement of the free surface concentrated over the plate (Fig. 3). The Fig. 4 shows that the volume of this inertial fluid layer depends strongly on horizontal size of obstacle. This effect can be explained <sup>5</sup> by the fact that the inertial fluid layer forms over the rigid obstacle during the time period of the order  $t^2$  (Makarenko, 2003). Finally, the Fig. 5 demonstrates an example of free surface flow generated by vertical submersion of the obstacle. This picture reveals significant distinction of this case from the flow regimes mentioned above. Namely, one can see here that the splash jet forms only after initial depression of free surface to caused by sinking of the obstacle.

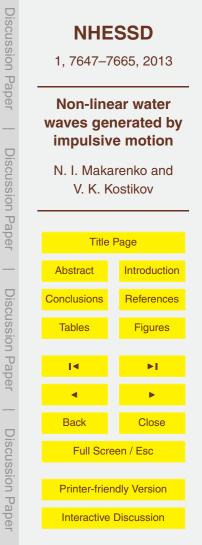
# 5 Conclusions

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In this paper, nonlinear water wave problem is considered in the presence of thin horizontal elliptic cylinder submerged beneath a free surface of deep fluid. An equivalent system of integral-differential equations for the wave elevation and the velocity components at free surface is formulated. In this framework, we study theoretically the piston mechanism of generation of hazardous tsunami-type waves due to fast deformation of compact bottom area. Small-time asymptotic solution is constructed by elementary functions for a vertically accelerated obstacle started from rest. Magnitude of acceleration is involved into the mathematical model as the control parameter, so the initial stage of the flow is simulated with high asymptotic accuracy. Variation of basic parameters demonstrates that constructed approximate solution realistically reproduces main

features of non-stationary wave process.

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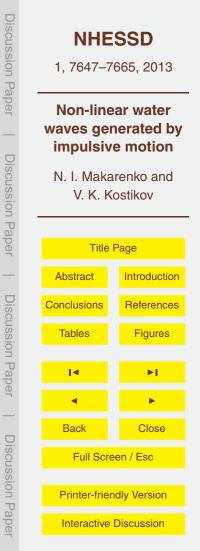
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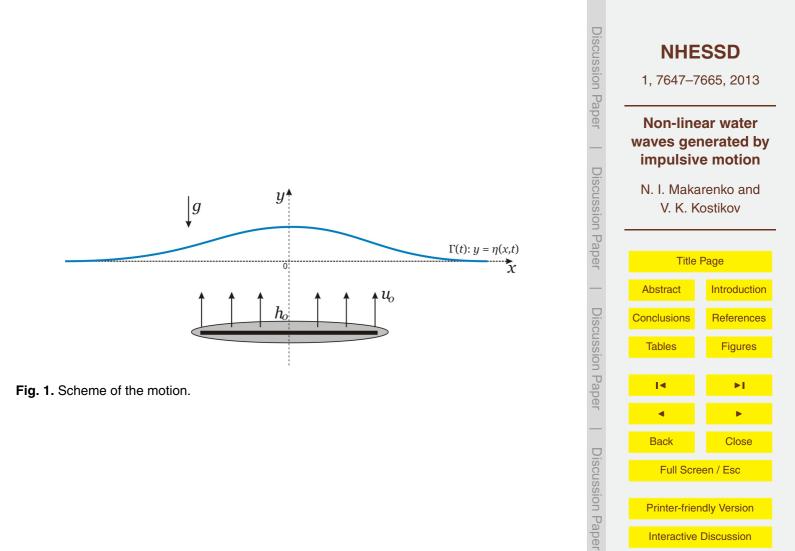


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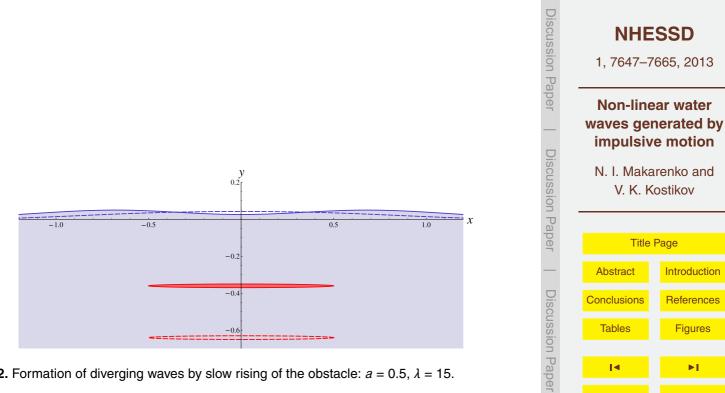






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Interactive Discussion



**Fig. 2.** Formation of diverging waves by slow rising of the obstacle: a = 0.5,  $\lambda = 15$ .



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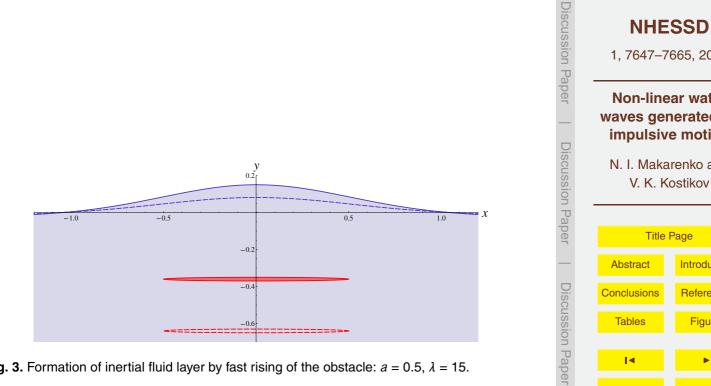
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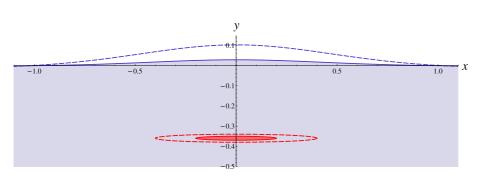
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**Fig. 3.** Formation of inertial fluid layer by fast rising of the obstacle: a = 0.5,  $\lambda = 15$ .







**Fig. 4.** Formation of added fluid mass by the obstacles with different semi-axes: a = 0.2 (solid lines) and a = 0.4 (dashed lines);  $\lambda = 5$ , t = 0.8.

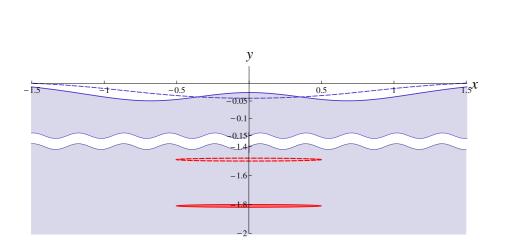
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**Fig. 5.** Formation of splash jet by submersion of the obstacle: a = 0.5,  $\lambda = 3$ . Position at the time t = 0.6 (dashed lines) and t = 0.8 (solid lines).

