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Resonance phenomena at the long wave run-up on the coast

A. Ezersky¹, D. Tiguercha¹, and E. Pelinovsky^{2,3,4,5}

 ¹Laboratoire de Morphodynamique Continentale et Côtière (M2C), UMR6143, Université Caen Basse-Normandie, 24 rue des Tilleuls, 14000 Caen, France
 ²Department of Nonlinear Geophysical Processes, Institute of Applied Physics, Nizhny Novgorod, Russia
 ³Department of Applied Mathematics, Nizhny Novgorod State Technical University,

Nizhny Novgorod, Russia

⁴Department of Information Systems, National Research University – Higher School of Economics, Nizhny Novgorod, Russia

⁵Institute for Analysis, Johannes Kepler University Linz, Austria

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Correspondence to: E. Pelinovsky (pelinovsky@hydro.appl.sci-nnov.ru)

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Abstract

Run-up of long wave on a beach consisting of three pieces of constant but different slopes is studied. Linear shallow-water theory is used for incoming impulse evolution and non-linear corrections are obtained for the run-up stage. It is demonstrated that ⁵ bottom profile influences the run-up characteristics and can lead to the resonance effects: increasing of wave height, particle velocity, and number of oscillations. Simple parameterization of tsunami source through an earthquake magnitude is used to calculate the run-up height versus earthquake magnitude. It is shown that resonance effects lead to the sufficient increasing of run-up heights for weakest earthquakes and ¹⁰ tsunami wave does not break on chosen bottom relief if the earthquake magnitude does not exceed 7.8.

1 Introduction

The resonance phenomena play significant role in the run-up amplification and lead to different physical effects for waves in coastal zone: long duration of water oscillations, later arrival of wave with maximal amplitude comparing with leading waves, group structure of waves. Meanwhile, usually these effects are neglected when the run-up processes are studied. The most part of theoretical results for run-up stage are based on rigorous analytical solutions of the shallow-water theory for waves climbing on the beach of constant slope. This approach was suggested in the pioneer work

- ²⁰ by Carrier and Greenspan (1958). They applied the hodograph transformation to the nonlinear system of shallow-water equations and obtained the linear wave equation for an auxiliary function; all physical variables (free surface displacement, depth-averaged velocity, offshore coordinate and time) were explicitly expressed using this function and its partial derivatives. The main advantage of wave equation for an auxiliary function
- having a form of cylindrical wave equation is that it has to be solved on semi-axis with given boundary conditions while the initial equations have to be solved in domain with



unknown moving boundary (shoreline). Meanwhile, the explicit form of the analytical solution generally requires the numerical manipulations to present physical variables in the wave field. That is why various shapes of the incident solitary wave have been specially analyzed: soliton (Pedersen and Gjevik, 1983; Synolakis, 1987), sine pulse (Mazova et al., 1991), Lorentz pulse (Pelinovsky and Mazova, 1992), Gaussian pulse

- Mazova et al., 1991), Lorentz pulse (Pelinovsky and Mazova, 1992), Gaussian pulse (Carrier et al., 2003; Kânoğlu and Synolakis, 2006), N-waves (Tadepalli and Synolakis, 1994), some specific localized disturbances (Tinti and Tonini, 2005; Pritchard and Dickinson, 2007; Dobrokhotov and Tirozzi, 2010). It should be noted that different formulas for maximum run-up of solitary waves of various shapes can be provided in terms
- of wave amplitude and significant wave length describing practically important cases with good accuracy (Didenkulova et al., 2008; Didenkulova and Pelinovsky, 2008). Various shapes of the periodic incident wave trains such as the sine wave (Carrier and Greenspan, 1958; Madsen and Fuhrman, 2008), cnoidal wave (Synolakis et al., 1988; Synolakis, 1991) and nonlinear deformed periodic wave (Didenkulova et al., 2006,
- ¹⁵ 2007) have been also studied to obtain the run-up characteristics. It is important to mention that the run-up height is higher if periodic incident wave is cnoidal or nonlinear deformed wave compared with a simple sine wave of the same amplitude and period. Some results are obtained for irregular incident waves modeled by the Fourier superposition of the sine waves with random phases (Didenkulova et al., 2010, 2011) or the
 ²⁰ random set of solitons (Brocchini and Gentile, 2001).

In all studies mentioned above the rigorous analytical solutions are obtained if the wave propagates on a plane beach of constant slope. Really, such plane can approximate the face-shore bathymetry only, and then it has to be matched with horizontal bottom profile. In fact, the rigorous analytical solutions can be obtained here in the

²⁵ linear theory only (Synolakis, 1987; Pelinovsky, 1996, 2006; Madsen and Fuhrman, 2008). If the bottom slope in face-shore area is small, the extreme run-up characteristics weakly differ from a case when the bottom has constant slope everywhere. Non-linearity leads to the correction of obtained results. First of all, there is nonlinear wave deformation in the region where inclination of bottom changes. This effect is thoroughly



investigated with use of boundary value approach in (Antuono and Brocchini, 2007, 2008, 2010). The second one is that a wave moving on horizontal bottom nonlinearly deformed within nonlinear shallow-water equations as Riemann wave and its shape in the entry of plane beach differs from an initial shape (Didenkulova et al., 2006, 2007).

- ⁵ It should be noted that both factors amplify the run-up heights. A new effect appeared for the wave run-up on a plane beach matched with horizontal bottom is the influence of the bottom slope on the shape of water oscillations on the shore. If the incident wave has a bell-shape, the water oscillations on the shore repeat its shape if the bottom slope is big (limiting case is a vertical wall), and accompanied by the negative second
- oscillation if the bottom slope is small. Such behavior is explained by the resonance effects which are weak for such geometry from physical point of view it is an open resonator¹ (Pelinovsky, 1996, 2006; Madsen and Fuhrman, 2008). If the bottom slope differs relatively small from the uniform value, the changes of run-up height are also small (Soldini et al., 2013).
- For more complicated geometry of coastal zone consisting of several pieces with different slopes, the solutions for each region of constant slope are matched (Kânoğlu, 2004; Didenkulova, 2009). Simplified solutions in the form of a product of such elementary solutions can be given if the incident wave length is less than a bottom piece length. For general ratio between these different lengths, as it is known, the res-
- onances appear due to multi-reflection from matching points and interference between such waves. Some allusion on possible resonances for wave run-up can be found in (Kajiura, 1977; Mazova, 1985). They investigated linear approximation of the run-up characteristics due to sine incident wave. The resonance phenomena are important for tsunami waves (LeBlond and Mysak, 1981; Massel, 1989; Mei, 1983). Photos of
 tsunami wave trains in different coastal locations are well-known after the 2004 Indonesian and 2011 Japanese tsunamis. Usually appearance of resonance effects is
 - connected with a complicated two-dimensional bathymetry of bays and jagged coastal

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¹If the wave maker located near the shore, of course, the resonant effects are big (Stefanakis et al., 2011; Ezersky et al., 2013).

line. In the present paper we aim to investigate run-up resonance phenomena for one dimensional case of wave propagation. We intend to show that for certain frequencies depending on bottom profile, run-up amplification may be high even for very simple bathymetry and it influences the shape of the water oscillations on the coast.

- ⁵ The paper is organized as follows. In the second section we describe our model of bottom profile and present the results for run-up amplification versus the frequency of linear harmonic incident wave. In the Sect. 3 results of calculations of the run-up characteristics caused by Gaussian impulse and N-wave impulse are presented. Section 4 is devoted to non-linear effects appearing in run-up. Discussion of result applicability
- ¹⁰ for natural hazard description and some conclusions are given in Sect. 5.

2 Theoretical model and run-up due to linear harmonic wave

Long wave run-up on a long beach is described by 1-D nonlinear shallow water equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} = 0$$

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$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left[u \left(h + \eta \right) \right] = 0$$

where *u* is the depth-averaged velocity, h = h(x) is the unperturbed water depth, $\eta = \eta(x,t)$ is the free surface displacement, *g* is the acceleration of gravity. In the linear approximation the system (1)–(2) is transformed into one equation

$$\frac{\partial^2 \eta}{\partial t^2} - g \frac{\partial}{\partial x} \left[h \frac{\partial \eta}{\partial x} \right] = 0.$$
(3)

To demonstrate the resonance effects in the run-up characteristics, we use three pieces-wise profile of unperturbed depth which are typical for real ocean bottom:

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(1)

(2)

(A) continental shelf $0 \le x \le x_0$, (B) continental slope $x_2 < x < 0$, (C) constant depth ocean $x < x_2$ (see Fig. 1). Such topography was used in numerous papers on tsunami run-up. We would like to emphasize that in some cases such simple model of bottom profile describes very precisely natural conditions. For instance, exactly such model 5 was used to prepare numerical simulations of tsunami near the Indian coast (Neetu et al., 2011).

The wave field in zone of constant depth (C) is presented as a sum of incident and reflected harmonic waves with constant amplitude A_i and A_r :

$$\eta = \left(A_{i}e^{-ik(x_{0}-x)} + A_{r}e^{ik(x_{0}-x)}\right)e^{-i\omega t}.$$
(4)

In zone of continental slope (B) amplitude equation for harmonic wave with frequency 10 ω is presented as $(\tan\beta$ is a bottom slope)

$$g\tan(\beta)(x_1 - x)\frac{\partial^2 A}{\partial x^2} - g\tan(\beta)\frac{\partial A}{\partial x} + \omega^2 A = 0.$$
 (5)

Its solution may be expressed as a sum of the Bessel functions of first J_0 and second Y_0 kinds with two constants C_1 and C_2 :

¹⁵
$$A = C_1 J_0(\bar{\sigma}) + C_2 Y_0(\bar{\sigma}), \quad \bar{\sigma} = 2\omega \sqrt{\frac{x_1 - x}{g \tan \beta}}$$
 (6)

In the near-shore zone (A) the solution for wave amplitude is also presented in Bessel functions. Taking into account that the wave field should be limited at the shore $(x = x_0)$, the wave amplitude is described by:

$$A = RJ_0(\sigma), \quad \sigma = 2\omega \sqrt{\frac{x_0 - x}{g \tan \alpha}}$$

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(7)

where *R* is also constant (in general, complex constant). It is evident that *R* describes the amplitude of the water level oscillation on unmoved shoreline (linear run-up height). If the bottom has the constant slope everywhere the value of |R| computed in the linear theory coincides with run-up height in nonlinear theory (Synolakis, 1987; Pelinovsky and Mazova, 1992). For more complicated geometry this statement is not proved and

we will discuss it later.

Using continuity conditions for horizontal velocity and free surface displacement for x = 0 and $x = x_2$, one can find the correlation between linear run-up height *R* and amplitude of incident wave A_i :

¹⁰
$$R = |K(\omega)|A_{i}, \quad K(\omega) = \frac{2(J_{0}(\bar{\sigma}_{0})Y_{1}(\bar{\sigma}_{0}) - J_{1}(\bar{\sigma}_{0})Y_{0}(\bar{\sigma}_{0}))}{W(Y_{0}(\bar{\sigma}_{1}) - iY_{1}(\bar{\sigma}_{1})) - N(J_{0}(\bar{\sigma}_{1}) - iJ_{1}(\bar{\sigma}_{1}))}e^{-ik(x_{0} - x_{2})}$$
 (8)

where

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$$W = J_0(\bar{\sigma}_0)J_1(\sigma_0) - J_0(\sigma_0)J_1(\bar{\sigma}_0), \\ N = Y_0(\bar{\sigma}_0)J_1(\sigma_0) - J_0(\sigma_0)Y_1(\bar{\sigma}_0), \\ \sigma_0 = \sigma(x=0),$$

$$\bar{\sigma}_0=\bar{\sigma}(x=0),\quad \bar{\sigma}_1=\bar{\sigma}(x=x_2).$$

- ¹⁵ Figure 2 represents run-up amplification $|R|/A_i$ for three different sets of bottom slopes modelling the Indian coast bathymetry (Neetu et al., 2011) where the Makran tsunami was observed on 27 November 1945. If the bottom slopes in zones A and B are the same the resonance effects are very weak (dash-point curve in Fig. 2), and this coincides with known results (Pelinovsky, 1996, 2006; Madsen and Fuhrman, 2008). But
- ²⁰ in the case of different bottom slopes in zones A and B resonance effects are clearly visible (solid and dashed lines in Fig. 2). Several resonant modes with frequencies $(\omega_1, \omega_2, ...)$ may be excited in the coastal zone and the amplification coefficient can reach values of 10–20 times.

Characteristic period of the first resonant peak $T = 2\pi/\omega_1$ is roughly 2 h that coincide with observed tsunami record in this area (1.5–3 h) according to (Neetu et al., 2011).



3 Run-up due of solitary bell and N-impulses

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Resonance curves given in the previous section show a substantial increase of runup heights for certain frequencies of harmonic incident waves. Whereas run-up height for harmonic wave is given by (8), the oscillations of water level on the shore (linear run-up) generated by solitary tsunami wave may be presented using the Fourier transformations:

$$R(t) = \frac{1}{2\pi} \int \mathcal{K}(\omega) \mathcal{S}(\omega) e^{(-i\omega t)} d\omega$$

where S is the Fourier transformation of incident wave

 $S(\omega) = \int \eta(t) e^{(i\omega t)} dt \,. \tag{10}$

¹⁰ Usually the shape of the incident tsunami wave is unknown and it is characterized by different functions, see for instance (Didenkulova et al., 2008). We chose here two characteristic and qualitatively different cases: the initial displacement of the free surface of one sign and alternating displacement with zero averaged value. In the first case it is a Gaussian pulse, and in the second one so-called N-pulse:

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$$\eta_{\text{in}G} = \eta_0 e^{\left(-(t/\tau_0)^2\right)}$$
 (11)

$$\eta_{\rm inN} = \eta_0 \left(\frac{t}{\tau_0}\right) e^{\left(-\left(t/\tau_0\right)^2\right)}.$$
(12)

Both pulses are characterized by two parameters: the duration and amplitude. Within linear theory, the value of wave amplitude is not important and can be used for scaling

of run-up characteristics. The second parameter, wave duration τ_0 plays an important role due to resonance effects.

(9)

Meanwhile, in tsunami practice, both parameters (amplitude and duration) of the incident wave are not independent and are determined by the parameters of tsunami source. Here we apply our theoretical results to tsunamis generated by the underwater earthquakes. Now, the characteristics of tsunami source are calculated using the Okada solution (Okada, 1985), and they depend on the several fault parameters. For simplified estimates it is more convenient to have the relations between tsunami source parameters and earthquake magnitude. Such relations are known in seismology (Sato, 1979; Wells and Coppersmith, 1994). Similar relations are given for parameters of tsunami source (Pelinovsky, 1996, 2006; Bolshakova and Nosov, 2011). Here we will use the following relations between the displacement amplitude of the free water surface η_0 and the characteristic size of tsunami source *L* with the earthquake magnitude *M* (Pelinovsky, 1996, 2006).

 $\log(\eta_0) = 0.8M - 5.6$

15 $\log(L) = 0.5M - 2.2$.

In the shallow-water approximation the duration of tsunami waves going out the source is $\tau_0 = L/\sqrt{gh}$, where *h* is water depth in the tsunami source. Of course, the formulas (13) and (14) are very approximated and may use only for simplified estimations.

Thus, we can use magnitude of earthquake to describe solitary bell, or N-wave. The results of calculations of linear run-up function R(t) for various values of the earthquake magnitude are presented below.

Figure 3 demonstrates the shapes of water level oscillations on the shore for two values of the earthquake magnitude: 7.7 and 8.5. Duration of the incident tsunami waves decreases then magnitude decreases, and its spectrum width increases. It means that ²⁵ weakest earthquake induces more resonant modes in coastal zone than strongest earthquake. As a result, the number of water oscillations on the shore increases with magnitude decreasing. If the initial shape is N-wave, the number of oscillations is higher



(13)

(14)

than for Gaussian input because its spectrum is narrower. It is important to mention that run-up height of N-wave is higher than for bell-wave and this was obtained firstly in (Tadepalli and Synolakis, 1994).

The run-up amplification factor (R/A_i) decreases with increasing of earthquake mag-⁵ nitude up to M = 7, and then it remains almost constant (Fig. 4). Tsunami generated by strong earthquake has large wavelength and in this case as indicated above, the resonance effects are very weak. The weakest tsunamis having the shortest wavelength are amplified more due to resonance effects. Increasing of the bottom slope in zone B $(\tan \beta)$ reduces the run-up height as it might be expected.

Maximal run-up height growths with earthquake magnitude increasing as it is shown 10 in Fig. 5. It should be noted that resonance effects "lift up" the values of run-up height for weakest earthquake, where as run-up height weakly depends on the values the bottom slopes in given ranges under consideration. It is important to note that curves in Figs. 4 and 5 are obtained in the linear approximation. Criteria for applicability of the linear solution will be discussed in the next section.

Estimations of non-linear effects 4

Calculations of maximal run-up heights in Sects. 2 and 3 were done in linear approximation. As it is indicated above it is difficult to solve the nonlinear shallow-water equations for piece-wise bottom profile. Taking into account that bottom slope is changed on depth in 4000 m and 200 m, and wave amplitude does not exceed a few meters 20 we may assume that all nonlinear effects are manifested on the last run-up stage. In this case we may use the rigorous solution of the nonlinear shallow-water equations for the long wave run-up on a beach of constant slope, which is very well developed, see references in Introduction. Here follow to (Pelinovsky and Mazova, 1992) we convert obtained linear solution into "nonlinear" solution. According to this procedure, we 25 should firstly find "linear" expression for horizontal velocity on the unmoved shoreline



(x = 0) which is followed from kinematics

$$U(t)=\frac{1}{\tan\alpha}\frac{\mathrm{d}R}{\mathrm{d}t}\,,$$

where as earlier $\tan \alpha$ is bottom face-slope. "Nonlinear" velocity of the moving shoreline, u(t) can be obtained from linear function U(t) by the Riemann transformation (Pelinovsky and Mazova, 1992)

$$u(t)=U\left(t+\frac{u}{g\tan\alpha}\right)\,.$$

It is evident that maximal values of "nonlinear" and "linear" velocities coincide.

Vertical displacement of the moving shoreline, r(t) can be found from kinematic condition

10 $r(t) = \frac{\int u(t) dt}{\tan \alpha}$.

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After substitution of (16) into this expression, it can be reduced to

$$r(t) = R\left(t + \frac{u}{\tan\alpha g}\right) - \frac{u^2(t)}{2g}.$$
(17)

The important conclusion from Eq. (17) is that extremes of the vertical displacement in the linear and nonlinear theories coincide (in this moment the horizontal velocity u = 0), confirming the use of linear theory to predict extreme values. Therefore, the linear theory adequately describes the run-up height.

Simple formulas of Riemann transformation from linear to nonlinear solutions allow us to obtain the wave breaking criterion. Strictly speaking, this criterion is found from zero condition for Jacobian of hodograph (Legendre) transformation. Note that this transformation was used to obtain Eqs. (15)–(17). On the other hand, the solution for velocity (16) resembles the well know Riemann wave in nonlinear acoustics



(15)

(16)

and hydrodynamics (the role of coordinate plays the inverse value of the $g \tan \alpha$). Such wave would overturn with increasing of amplitude. Exactly this fact has been used in (Pelinovsky, 1966, 2006; Didenkulova, 2009) to find wave breaking criteria on the shore. From Eq. (16) it is easy to calculate the time derivative of the velocity in incident wave:

$$\int \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}U/\mathrm{d}t}{1 - \frac{\mathrm{d}U/\mathrm{d}t}{g\,\mathrm{tan}\,\alpha}}.$$

10

It tends to the infinity when the denominator approaches zero. As follows from the theory of hyperbolic equations it leads to the gradient catastrophe identified and to the plunging breaking of the long water waves. In this case a water displacement contains the jump of its first derivative. This implies the condition of the first wave breaking

$$Br = \frac{\max(dU/dt)}{\tan \alpha g} = \frac{\max(d^2 R/dt^2)}{\tan^2 \alpha g} = 1,$$
(19)

where the parameter Br has the sense of breaking parameters. Figure 6 shows the temporal evolution of the breaking parameter, linear and nonlinear water level oscillations on the shore and shoreline velocities versus time for solitary impulse and N-wave impulse for magnitude M = 7.7. It is clearly seen the difference between linear and nonlinear solutions for moving shoreline. It is important to mention that breaking parameter is less 1, so tsunami wave should climb on the shore without-breaking for chosen bottom geometry. It should be emphasized that for all results presented in Figs. 4 and 5 criterion Br < 1 is satisfied.

5 Discussion and conclusions

²⁰ The run-up of tsunami waves on the coast is studied for following bottom geometry: ocean of constant depth, steep continental slope, beach of gentle constant slope. It

(18)

is demonstrated that run-up characteristics are strongly depend on the frequency of the incident wave due to resonance effects. They are studied for conditions of the Indian coast where the 1945 Makran tsunami has been recorded. Amplification ratio may be 5 times higher than for the case of the uniform averaged slope. The run-ups

- of solitary wave of bell or N-shape are studied in details. It is found that run-up of N-waves is higher than for solitary wave. It is due to stronger manifestation of the resonance effects for N-wave than for bell shape wave. Using simple parameterization of tsunami source through an earthquake magnitude the run-up height are calculated versus earthquake magnitude. It is shown that the resonance effects can "lift up" the
- values of run-up heights for weakest magnitudes due to resonance amplification of shortest waves generated by weakest earthquake. Nonlinear correction of obtained results is given. It is shown that for typical conditions of the Indian coast where the 1945 Makran tsunami was observed the breaking parameter is less than 1 and tsunami waves climb on a coast with no breaking.
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Fig. 1. Schema of bottom profile.





Fig. 2. Run-up amplifications for three sets of bottom slopes.







Fig. 3. Incident wave and water oscillations on the shore for $h_0 = 4000 \text{ m}$, $h_1 = 200 \text{ m}$, $\tan \alpha = 0.005$, $\tan \beta = 0.1$, and different earthquake magnitude: (a) M = 7.7 and (b) M = 8.5.



Fig. 4. Run-up amplification factor for Gaussian impulses and N-wave impulse ($h_0 = 4000$ m, $h_1 = 200$ m) for different inclinations of continental shelf and continental slope.







Fig. 5. Maximal run-up height versus the earthquake magnitude for Gaussian impulses and N-wave impulse, ($h_0 = 4000 \text{ m}$, $h_1 = 200 \text{ m}$) for different inclinations of continental shelf and continental slope.



Fig. 6. Breaking parameter, linear and nonlinear variations of water level on shore and shoreline velocities versus time for $h_0 = 4000$ m, $h_1 = 200$ m, $\tan \alpha = 0.005$, $\tan \beta = 0.1$ and M = 7.7: (a) Gaussian wave and (b) N-wave.

