**Natural Hazards and Earth System Sciences**

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Discussions

# A seismic hazard analysis considering uncertainty during earthquake magnitude conversion

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**Earthquake analysis considering magnitude conversion uncertainty**

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## Abstract

The magnitude of earthquakes can be described with different units, such as moment magnitude $M_w$ and local magnitude $M_L$. A few empirical relationships between the two have been suggested, such as the model calibrated with the earthquake data in Taiwan. Understandably, such a conversion relationship through regression analysis is associated with some error because of inevitable data scattering. Therefore, the underlying scope of this study is to conduct a seismic hazard analysis, during which the uncertainty from earthquake magnitude conversion was properly taken into account. With a new analytical framework developed for this task, it was found that there is a 10 % probability in 50 yr that PGA could exceed 0.28 g at the study site in North Taiwan.

## 1 Introduction

The magnitude of earthquakes can be portrayed with a variety of units, such as local magnitude $M_L$ and moment magnitude $M_w$. For example, the 1999 Chi-Chi earthquake in Taiwan was reportedly with a local magnitude and moment magnitude equal to $M_L = 7.3$ and $M_w = 7.6$, respectively. Generally speaking, moment magnitude is usually adopted in a ground motion model (e.g., Cheng et al., 2007; Lin et al., 2011); on the other hand, an earthquake catalog lists the event in local magnitude (e.g., Wang et al., 2011, 2012a).

The correlation between the two units was reported in some studies. For example, based on adequate earthquake samples in Taiwan, Wu et al. (2001) suggested the following empirical relationship:

$$M_L = 4.53 \times \ln(M_w) - 2.09 + \varepsilon \tag{1}$$

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where $\varepsilon$ denotes the model's error. Based on the fundamentals of regression analysis (Ang and Tang, 2007), its mean value is zero, and its standard deviation was equal to 0.14 owing to the scattered data in this pool of samples.

With the earthquake catalog, ground motion models, and $M_L$-to-$M_w$ equation, a few earthquake studies for Taiwan were conducted (Wang et al., 2011, 2012a,b). However, when magnitude conversion was needed during those analyses, the model uncertainty was not taken into account. For example, given $M_L = 6.5$, the moment magnitude is equal to $M_w = 6.66$ with Eq. (1). But note that such a conversion was made regardless of the model's error $\varepsilon$.

Therefore, an underlying scope of this study is to perform a seismic hazard analysis accounting for the uncertainty during magnitude conversion, as well as those because of uncertain earthquake size, location, and motion attenuation. The new analytical framework is to utilize a common probabilistic analysis, i.e., First-Order-Second-Moment (FOSM). The new application was then used for the seismic hazard assessment at a site in North Taiwan.

## 2 Probabilistic analysis and deterministic analysis

Given a function of random variables as $A = B + C$, deterministic analysis can find the mean value of $A$ with those of $B$ and $C$, but the standard deviation of $A$ cannot be determined. In contrast, probabilistic analysis can estimate both, with the mean values and standard deviations of $B$ and $C$ available. As a result, the interpretations made with deterministic analysis are irrelevant to the input's variability, which becomes its shortcoming compared to probabilistic analysis. For example, the soil's friction angles in two slopes are found equal to 20, 30, 40° and 29, 30, 31° with three samples from each slope. In this case, the deterministic analysis will estimate the same safety margins for the two slopes because of the same mean value of the soil property, despite the obvious difference in the variability.

But it must be noted that when the function of variables is complex, the probabilistic analysis becomes hard to solve, or even the analytical solution might not be available. Therefore, a few alternatives, such as FOSM, the Rosenblueth approach, Monte Carlo Simulation, were developed and commonly employed to solve problems on a prob-abilistic basis, such as seismic hazard analysis and slope stability evaluation (e.g., Wang et al., 2012c, 2013a,b).

## 3 Overview of FOSM

FOSM is on the basis of the Taylor expansion (Hahn and Shapiro, 1967). Given a function $Y = g(X_1, X_2, \ldots, X_n)$, the mean value and variance of $Y$, denoted as $E(Y)$ and $\mathrm{Var}(Y)$, can be approximated with only the first-order terms being retained:

$$E(Y) \approx g\left(E(X_1), E(X_1), \ldots, E(X_n)\right) \tag{2}$$

and

$$\mathrm{Var}(Y) \approx \sum_{i=1}^{n}\left[\left(\frac{\partial Y}{\partial X_i}\right)^2 \mathrm{Var}(X_i)\right] + 2\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial Y}{\partial X_i}\frac{\partial Y}{\partial X_j}\mathrm{Cov}(X_i, X_j)\right) \text{ for } i < j \tag{3}$$

where Cov denotes the covariance. When any of two input variables is independent of each other (i.e., Cov = 0), the variance of $Y$ can be calculated as follows:

$$\mathrm{Var}(Y) \approx \sum_{i=1}^{n}\left[\left(\frac{\partial Y}{\partial X_i}\right)^2 \mathrm{Var}(X_i)\right] \tag{4}$$

## 4 FOSM-based seismic hazard analysis

Before introducing the new analytical framework, it should be worth giving some background about seismic hazard analysis. It should be noted that such an analysis is not

to assess the casualty or economic loss caused by earthquakes as the word "hazard" in the title could imply. Instead, the analysis is to best estimate, for example, the annual rate of a given ground motion level (e.g., PGA > 0.2 g), with the earthquake evidence around the site. Seismic hazard can be presented differently depending on which method is used, such as Probabilistic Seismic Hazard Analysis (PSHA) or Deterministic Seismic Hazard Analysis (DSHA).

Ground motion models are one of the underlying pieces of information needed in a seismic hazard analysis. Similarly, the new analysis was developed with an attenuation relationship. Take a rock-site model for Taiwan for instance, PGA (in unit g) can be estimated as follows (Cheng et al., 2007):

$$\ln \text{PGA} = -3.25 + 1.075 M_{\text{w}} - 1.723 \ln\left(D + 0.156 \exp(0.624 M_{\text{w}})\right) + \varepsilon_{\text{M}} \quad (5)$$

where $D$ denotes source-to-site distance (km); $\varepsilon_{\text{M}}$ is the model's error, whose standard deviation is equal to 0.577 (mean = 0). Combining Eqs. (1) and (5), the governing equation of this study becomes:

$$
\begin{aligned}
\ln \text{PGA} = & -3.25 + 1.075 \exp\left(\frac{M_{\text{L}} + 2.09 + \varepsilon}{4.53}\right) \\
& - 1.723 \ln\left(D + 0.156 \exp\left(0.624 \exp\left(\frac{M_{\text{L}} + 2.09 + \varepsilon}{4.53}\right)\right)\right) + \varepsilon_{\text{M}}
\end{aligned}
\quad (6)
$$

As a result, this study aims to solve this function with the involvement of four random variables (i.e., $M_{\text{L}}$, $D$, $\varepsilon$, $\varepsilon_{\text{M}}$), two of them related to the uncertainty of major earthquakes (detailed in the next section), and the other two related to the errors of two empirical models. After the mean and standard deviation of lnPGA in Eq. (6) are solved, the exceedance probability against a given value $y^*$ can be computed with fundamentals of probability (Ang and Tang, 2007), as follows:

$$\Pr(\text{PGA} > y^*) = \Pr(\ln \text{PGA} > \ln y^*) = 1 - \Pr(\ln \text{PGA} \le \ln y^*) = 1 - \Phi\left(\frac{\ln y^* - \mu_{\ln \text{PGA}}}{\sigma_{\ln \text{PGA}}}\right) \quad (7)$$

where $\Phi$ denotes the standard normal (mean = 0 and standard deviation = 1) cumulative probability function; $\mu_{\ln PGA}$ and $\sigma_{\ln PGA}$ are the mean and standard deviation of lnPGA. Explicitly, this calculation involves an analytical presumption that earthquake ground motion (e.g., PGA) follows a lognormal distribution (Kramer, 1996).

## 5 Seismic hazard assessment for a site in North Taiwan

The seismic hazard at the Lungmen nuclear power plant, under construction, was then evaluated with the new analysis. Extracting from the earthquake catalog containing more than 55 000 events since 1900 (Fig. 1), there are a total of 142 major events with $M_L$ greater than 6.0 occurring within 200 km from the site. Figure 2 shows the epicenters of the major earthquakes. Accordingly, Fig. 3a shows the histogram of earthquake size, with the mean and standard deviation equal to $M_L$ = 6.43 and 0.46, respectively. Likewise, Fig. 3b shows the histogram of source-to-site distance, with the mean and standard deviation equal to 114 and 42 km, respectively.

With the statistics of the four random variables, summarized in Table 1, the mean and standard deviation of lnPGA associated with their uncertainties were −4.5 and 1.09, respectively. They are equivalent to 0.02 g (mean) and 0.03 g (standard deviation) after conversion (Ang and Tang, 2007). With the two, Fig. 4 shows the probability density function of PGA at the site. Accordingly, there is a 0.15 % probability that PGA could exceed 0.28 g at the site when a major earthquake occurs within 200 km around the site.

Following the framework of PSHA, the annual rate of motion of exceedance (e.g., PGA > 0.28 g) can be calculated with the exceedance probability (Eq. 7) multiplying the earthquake rate ($v$), as follows:

$$\lambda_{PGA>y^*} = v \times \left(1 - \Phi\left(\frac{\ln y^* - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right)\right) \tag{8}$$

Since the rate of major earthquakes around the site is around 1.3 per year, the rate of PGA > 0.28 g, for example, is equal to 0.002 per year. Also following another analytical presumption adopted in PSHA, we used the Poisson model to calculate the recurrence probability within a given period of time (Kramer, 1996).

As a result, Fig. 5 shows both the annual rate of motions of exceedance, and their recurrence probability in 50 yr. Accordingly, there is a 10 % probability that PGA at the site could exceed 0.28 g in 50 yr because of major earthquakes, given the uncertainties of their size and location, in addition to the errors in the empirical models for conducting magnitude conversion and calculating ground motion.

## 6  Spreadsheet calculation

Like a few geoscience studies (Mayborn and Lesher, 2011; Wang and Huang, 2012; Wang et al., 2013a), we utilized an Excel spreadsheet, as shown in Fig. 6, for the computation in this study. Some detail about the spreadsheet is given in the figure's caption. Note that the calculation of $\frac{\partial Y}{\partial X_i}$ in Eq. (4) was assisted with the finite difference approximation of the derivative (US Army Corps of Engineers, 1997). Take $X_1$ for example, $\frac{\partial Y}{\partial X_1}$ can be approximated as follows:

$$\frac{\partial Y}{\partial X_1} = \frac{g(\mu_1 + \sigma_1, \mu_2, \ldots, \mu_n) - g(\mu_1 - \sigma_1, \mu_2, \ldots, \mu_n)}{2\sigma_1} \qquad (9)$$

where $\mu_1$ and $\sigma_1$ denote the mean and standard deviation of $X_1$, respectively.

## 7  Discussions

### 7.1  Seismic hazards at the study site

Cheng et al. (2007) presented a PSHA hazard map for Taiwan in 10 % exceedance probability within 50 yr. Looking up the map, we found that the PGA estimate at the

site is around $0.3 \sim 0.32$ g, comparable to this study's estimate in 0.28 g at the same exceedance probability given the same period of time.

Although the seismic hazards at the site were found comparable in the two studies, it must be noted that the two analyses are of fundamental difference. One study (our study) utilized a common probabilistic analysis in which four sources of uncertainties were accounted for, including the one during earthquake magnitude conversion. In contrast, the other was a case study with an existing method, accounting for three sources of earthquake uncertainties that were also taken into account in our analysis.

## 7.2 The controversy of seismic hazard analysis

Although seismic hazard analysis is considered a viable solution to seismic hazard mitigation, given the fact that earthquakes can be hardly predicted (Geller et al., 1997), some discussion over its methodological robustness has been reported (e.g., Castanos and Lomnitz, 2002; Bommer, 2003; Krinitzsky, 2003; Mualchin, 2011). One of the possibilities causing this controversy, yet resolved, is that seismic hazard estimates could not be verified with the ground motion data measured in the field (Musson, 2012a,b; Wang, 2012).

Therefore, it is a logical perspective that not a seismic hazard analysis should be perfect without challenge, given our limited understandings of the random earthquake process (Mualchin, 2005). Under the circumstance, some suggest that the key to a robust seismic hazard study is the transparent and repeatable analysis (Klugel, 2008; Wang et al., 2012a). On the other hand, decision makers need to fundamentally understand the difference from one analysis to another, before making the decision about which approach is suitable for their application (Mualchin, 2005). During the analysis, they need to ask hard questions about any number used in the calculation, making a seismic hazard estimate as transparent as possible (Krinitzsky, 2003).

### 7.3 Logic-tree analysis

Logic-tree analysis has become a common procedure to account for the so-called epistemic uncertainty during a seismic hazard assessment. Basically, it can be considered a weighted-average method. For example, when three ground motion models are all considered suitable, the calculation will repeated with each model. Then the final estimate is equal to the summation of each subordinate estimate multiplying its weight. Obviously, we did not perform such analysis in this study (although it can be easily accomplished), because this paper aims to focus on the uncertainty of earthquake magnitude conversion in a seismic hazard analysis.

### 7.4 Are earthquake variables independent of each other?

Like most seismic hazard analyses (Cheng et al., 2007; Wang et al., 2012a), the earthquake variables considered in this analysis are assumed to be independent of each other, e.g., earthquake size and location. More studies should be worth conducting to offer some more concrete evidence about possible correlations between earthquake variables.

### 8 Conclusions

This study conducted a seismic hazard assessment for a site in North Taiwan, where a nuclear power plant is located. Unlike others, this study integrated the uncertainty during earthquake magnitude conversion into the assessment utilizing a common probabilistic analysis. The result shows that the mean and standard deviation of PGA at the site could be 0.02 g and 0.03 g, given a major earthquake with uncertain size and location occurring around the site, and the errors of the empirical models used for earthquake magnitude conversion and ground motion calculation. As a result, there is a 10 % probability that PGA at the site could exceed 0.28 g within 50 yr.

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**Table 1.** Summary of the statistics of the four earthquake variables and the resulting PGA.

| Parameters | Mean value | Standard deviation |
|---|---|---|
| Major earthquake size | $M_L = 6.43$ | $M_L = 0.46$ |
| Source-to-site distance | 114 km | 42 km |
| Magnitude conversion model error (Wu et al., 2001) | 0 | 0.14 |
| Ground motion model error (Cheng et al., 2007) | 0 | 0.577 |
| PGA (output) | 0.02 g | 0.03 g |

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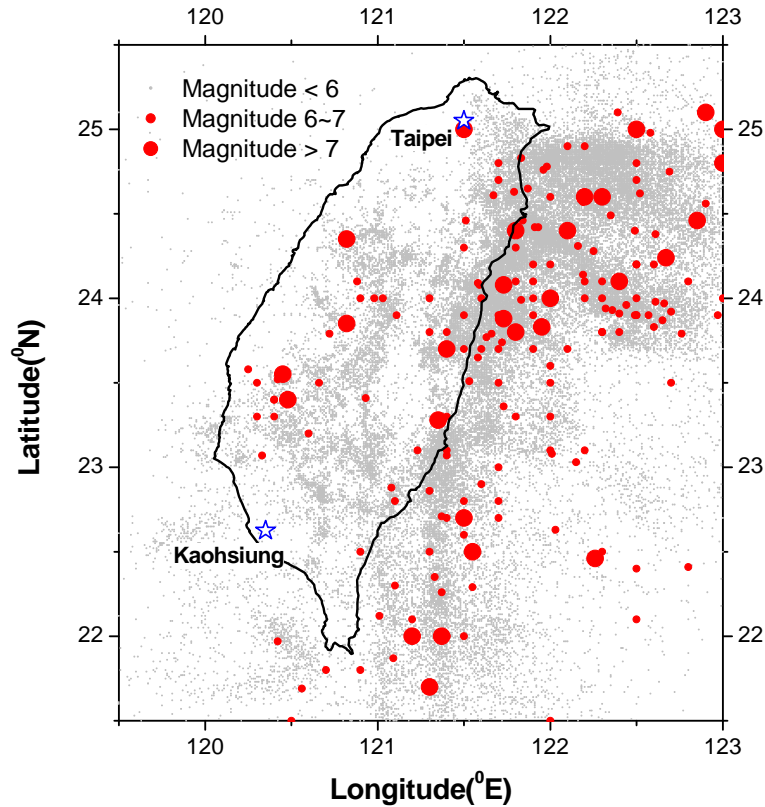


**Fig. 1.** The spatial distribution of more than 55 000 earthquakes since 1900 around Taiwan.

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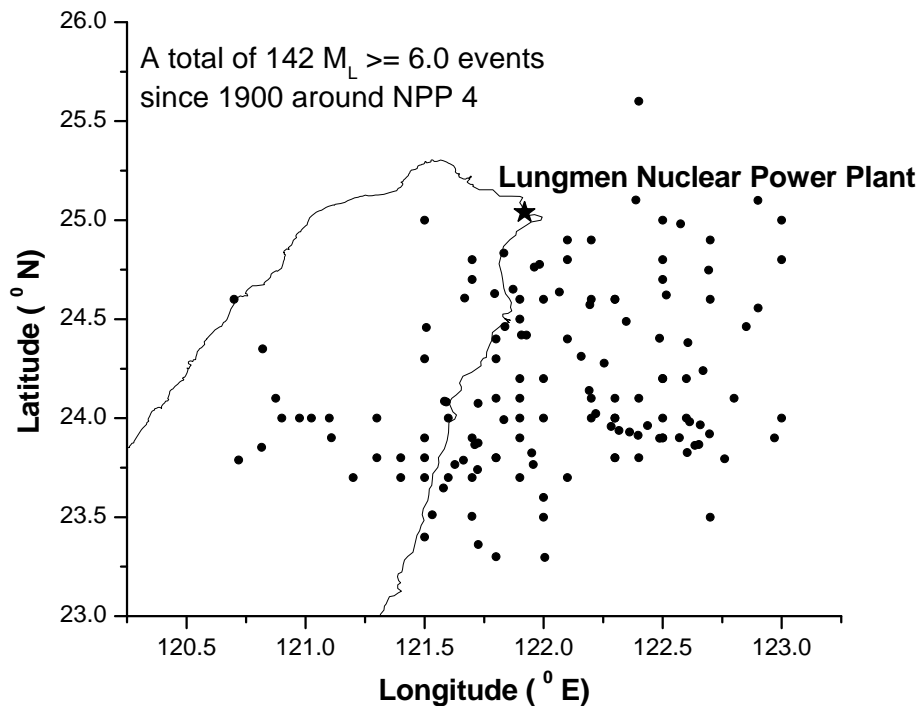Printer-friendly Version

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**Fig. 2.** The spatial distribution of $M_L \geq 6.0$ events occurring within a distance of 200 km from the Lungmen nuclear power plant in North Taiwan.
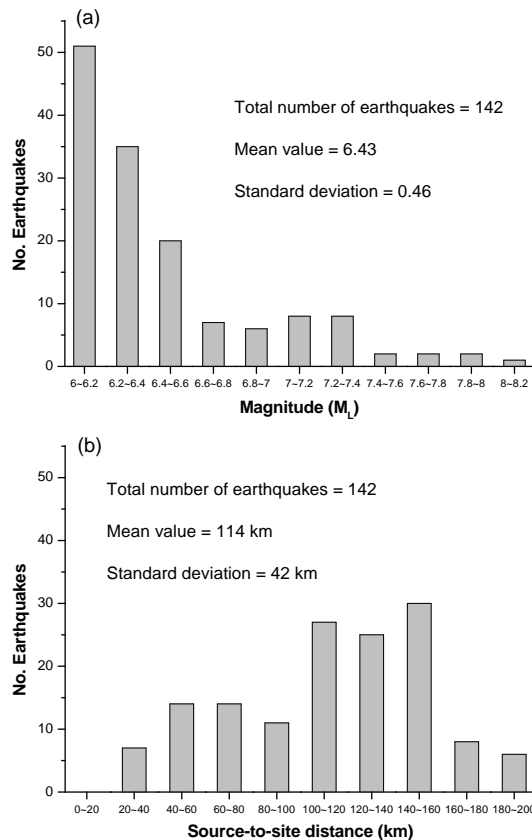
**Fig. 3.** Histograms of earthquake size and source-to-site distance for 142 major earthquakes.

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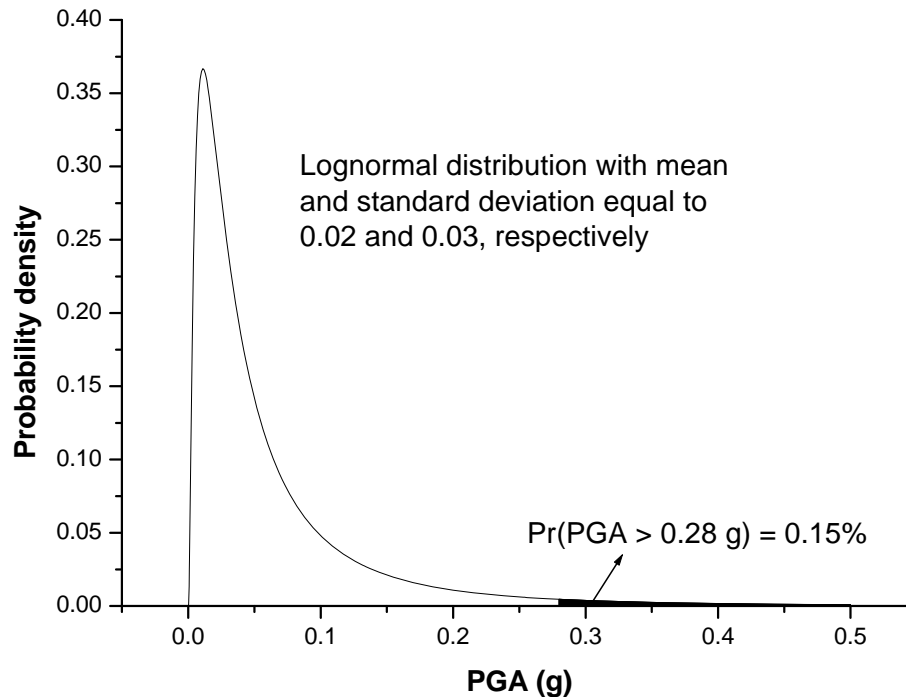Printer-friendly Version

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**Fig. 4.** Probability density function for PGA given its mean and standard deviation equal to 0.02 g and 0.03 g, when a major earthquake occurs around the site.

Within the figure:
Lognormal distribution with mean and standard deviation equal to 0.02 and 0.03, respectively

Pr(PGA > 0.28 g) = 0.15%

Axis labels: Probability density (y-axis), PGA (g) (x-axis)

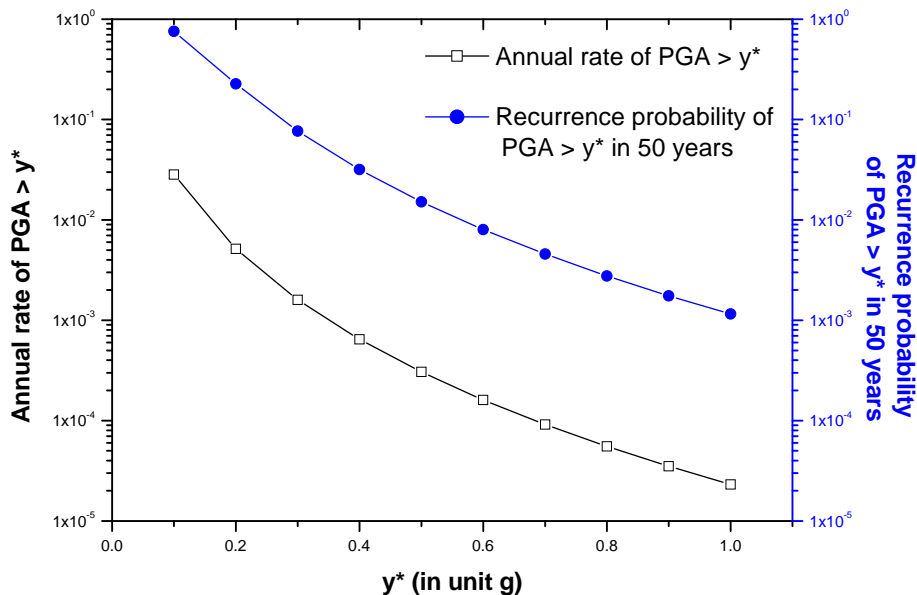**Fig. 5.** The annual rate of PGA > $y^*$ and the recurrence probability of PGA > $y^*$ in 50 yr at the study site in North Taiwan, associated with the uncertainties of major earthquakes and with the errors of two empirical earthquake models.

| | G | H | I | J | K |
|---|---|---|---|---|---|
| 21 | | X1 | X2 | X3 | X4 |
| 22 | Delta Y (+) | -3.81 | -5.00 | -4.69 | -3.92 |
| 23 | Magnitude in ML | 6.89 | 6.43 | 6.43 | 6.43 |
| 24 | Distance in km | 114.00 | 156.00 | 114.00 | 114.00 |
| 25 | Conversion model error | 0.00 | 0.00 | 0.14 | 0.00 |
| 26 | Ground motion model error | 0.00 | 0.00 | 0.00 | 0.58 |
| 27 | Delta Y (-) | -5.13 | -3.78 | -4.29 | -5.07 |
| 28 | Magnitude in ML | 5.97 | 6.43 | 6.43 | 6.43 |
| 29 | Distance in km | 114.00 | 72.00 | 114.00 | 114.00 |
| 30 | Conversion model error | 0.00 | 0.00 | -0.14 | 0.00 |
| 31 | Ground motion model error | 0.00 | 0.00 | 0.00 | -0.58 |
| 32 | Delta Y | 1.32 | -1.22 | -0.40 | 1.15 |
| 33 | (partical Y / partical X)^2 | 1.75 | 1.49 | 0.16 | 1.33 |
| 34 | | | | | |
| 35 | Variance of Y | 1.18 | | | |

**Fig. 6.** The spreadsheet created for such a FOSM calculation in this study; texts in red are the input cells. Cells in the same color of background means the same function programmed. In other words, Column H was programmed only because the rest could be easily achieved with the common operation in computer: copy-and-paste.

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