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Brief communication: A magma depletion alternative for vent distribution in volcanic fields

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Overview

This supplement provides the model fitting results for the Auckland Volcanic Field (AVF) with and without the most recent eruption at Rangitoto, (S1), the derivations for each of the non-trivial normalisation factors, Z (S2), and the AVF vent locations and volumes (S3).

S1 Model goodness of fit for the Auckland Volcanic Field

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Table S1: Model loglikelihoods, Akaike Information Criterion (AIC), and fitted parameters with and [without] Rangitoto (most recent AVF eruption)

Spatial Intensity Model	Log-Likelihood	AIC	Fitted Parameters
Isotropic Gaussian (Eq. 1 & 2)	-322.4 [-311.3]	646.8 [624.6]	h = 2.367 km [h = 1.829 km]
Disk-based depletion (Eq. 5 & 6)	-318.6 [-310.4]	643.2 [626.8]	h = 1.538 km, α = 6.683, l = 0.0053 km [h = 1.710 km, α = 14.246, l = 0.156 km]
Inverse-volume weighted (Eq. 7 & 8)	-321.4 [-311.0]	646.8 [626]	h = 2.395 km, α = 0.248 [h = 1.846 km, α = 0.127]
Distance-multiplied (Eq. 9 & 10)	-324.0 [-312.9]	652 [629.8]	h = 11.65 km, α = 0.456 [h = 2.928 km, α = 0.158]

S2 Derivations of the normalisation factor, Z

Disk-based depletion

$$\boldsymbol{K = \frac{1}{Zn2\pi h^2} exp\left(\frac{-r^2}{2h^2}\right) \exp\left(-\alpha I|r < R|\right.}$$

where Z is a constant to be determined. Since the kernel term must integrate to $1/n$

$$Z = \int_0^{2\pi}\int_0^{\infty} rKdrd\theta = \frac{1}{h^2}\left(e^{-\alpha}\int_0^{R} r\, exp\left(-\frac{r^2}{2h^2}\right)dr + \int_R^{\infty} r\, exp\left(-\frac{r^2}{2h^2}\right)dr\right)$$

Integrating directly

$$Z = \frac{1}{h^2}\left(e^{-\alpha}\left[-h^2\exp\left(-\frac{r^2}{2h^2}\right)\right]_0^R + \left[-h^2\exp\left(-\frac{r^2}{2h^2}\right)\right]_R^{\infty}\right) = e^{-\alpha}\left(1 - exp\left(-\frac{R^2}{2h^2}\right)\right) + exp\left(-\frac{R^2}{2h^2}\right).$$

Distance-weighted kernel

$$K = \frac{r}{Zn2\pi h^2} exp\left(\frac{-r^2}{2h^2 v^{2\alpha}}\right)$$

where Z is a constant to be determined. Since the kernel term must integrate to $1/n$

$$Z = \int_0^{2\pi}\int_0^{\infty} rKdrd\theta = \frac{2\pi v^{2\alpha}}{2\pi X}\int_0^{\infty} r^2 exp\left(-\frac{r^2}{2X}\right)dr$$

Where $X = (hv^{\alpha})^2$. Integrating by parts

$$\int_0^{\infty} r^2 exp\left(-\frac{r^2}{2X}\right)dr = \left[-rX\, exp\left(-\frac{r^2}{2X}\right)\right]_0^{\infty} + \int_0^{\infty} X\, exp\left(-\frac{r^2}{2X}\right)dr$$

The first term is zero by l'Hôpital's rule. The second term is

$$\int_0^{\infty} X\, exp\left(-\frac{r^2}{2X}\right)dr = X\sqrt{2\pi X}\int_0^{\infty}\frac{1}{\sqrt{2\pi X}} exp\left(-\frac{r^2}{2X}\right)dr$$

And recognizing the last integral as half of a Gaussian pdf, we arrive at

$$Z = \frac{v^{2\alpha}}{X} X\sqrt{\frac{\pi X}{2}} = \frac{\sqrt{\pi} h v^{3\alpha}}{\sqrt{2}}.$$

S3 Auckland Volcanic Field data

Data from Bebbington & Cronin (2011) and Bebbington (2015). Coordinate Reference System for Easting and Northing is New Zealand Transverse Mercator (NZTM).

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Vent ID	Easting	Northing	X (km)	Y (km)	Magmatic volume (10^6 m^3)	Tephra volume (10^6 m^3)
1	1756181	5924977	9.9	20.7	11.37	2.1145
2	1757579	5920232	9.2	22.1	27.762	0.4326
3	1757607	5928065	18.1	22.1	2.197	0.6163
4	1766149	5909705	7.9	26.6	2.623	0.7685
5	1766181	5920880	8.2	27.3	5.873	2.0637
6	1761609	5918333	9.3	29.5	46.664	10.0554
7	1757445	5916708	12.4	24.7	0	0.4419
8	1761284	5917420	12.2	24.5	2.223	3.8738
9	1761110	5905482	13.3	24.4	2.611	0.6143
10	1756412	5925658	5.1	17.6	19.917	4.7157
11	1758303	5919280	6.5	15.2	13.058	2.068
12	1758644	5916579	16.4	15.8	7.145	2.7727
13	1755508	5904295	16.1	13.1	18.13	5.8981
14	1758864	5914736	15.4	12.7	2.638	5.1406
15	1768974	5922244	15.6	11.1	27.665	0.6015
16	1753273	5915896	17.7	11.5	2.478	0.8654
17	1764009	5905900	9.4	8	10.989	1.1463
18	1762608	5905060	7.1	6.2	9.681	1.3117
19	1758952	5911461	8.1	7.4	32.761	3.5062
20	1755541	5907563	7.9	7.1	3.263	0.6554
21	1761600	5922703	15.8	4	5.307	1.4853
22	1768766	5908893	17.7	5.1	7.154	0.9011
23	1764760	5914075	15.5	7.7	0.076	0.0262

24	1754400	5912969	14.2	7	7.238	2.5505
25	1766684	5918558	16.9	4.6	23.714	4.0868
26	1766169	5903192	7.2	9.5	15.492	1.338
27	1764452	5911388	21.1	17.4	15.397	4.7226
28	1760438	5922853	18.4	20.2	3.038	1.0795
29	1760704	5923087	12.7	7.4	4.891	0.5084
30	1765349	5902700	13.1	14.4	9.188	3.5514
31	1763819	5911028	20.8	12.4	11.076	3.5172
32	1756327	5914459	20.7	11.9	0.366	0.1229
33	1757213	5917320	20.5	11.2	10.616	2.4323
34	1759186	5917193	20.1	10.5	1.602	1.1016
35	1757572	5906217	10.1	10.2	2.012	0.6509
36	1769587	5915767	10.5	11	2.028	0.3736
37	1763988	5909259	10.4	16.4	31.2	22.5083
38	1758263	5908419	8.2	16	254.736	8.5558
39	1769034	5909312	10.6	13.3	66.308	4.506
40	1756387	5905487	9.2	18.5	0.856	0.1542
41	1758765	5909233	9	19	28.067	0.12
42	1769042	5910110	10.3	18.4	83.833	8.9165
43	1761449	5912653	11	18.8	5.475	1.7997
44	1756114	5905230	12.9	19.1	1.207	0.749
45	1769218	5910525	13.3	20.1	6.703	1.8873
46	1764226	5902036	16.4	17.6	1.659	0.144
47	1764632	5916054	16.2	17.3	79.264	5.4914
48	1764537	5915399	20.5	24	3.243	2.2964
49	1765975	5927171	17.4	28.9	657.299	64.7299
50	1761464	5908537	13.2	10.2	0.322	0.092
51	1763842	5904672	15.6	6.2	0.242	0.0688

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