Supplement of Nat. Hazards Earth Syst. Sci., 24, 3479–3495, 2024
https://doi.org/10.5194/nhess-24-3479-2024-supplement

Natural Hazards and Earth System Sciences
Open Access

Supplement of

GTDI: a game-theory-based integrated drought index implying hazard-causing and hazard-bearing impact change

Xiaowei Zhao et al.

Correspondence to: Hongbo Zhang (hbzhang@chd.edu.cn)

Contents of this file

Introduction

This supplementary material includes the calculation methods for the Standardized Precipitation Evapotranspiration Index (SPEI) and the Standardized Soil Moisture Index (SSMI). We also introduce the construction of the entropy theory-based drought index (ETDI) in detail here.

S1 Calculation method of the SPEI

The Standardized Precipitation Evapotranspiration Index (SPEI) combines precipitation and temperature data to characterize the drought state of a region (Vicente Serrano et al., 2010). The calculation process for the SPEI is as follows:

Step 1: The calculation of the potential evapotranspiration (PET). We followed the approach to calculate PET (Thornthwaite, 1948), which has the advantage of only requiring data on monthly-mean temperature. The monthly *PET* (mm) is obtained by:

$$PET_i = 16K \times \left(\frac{10T_i}{H} \right)^A \tag{1}$$

$$A = 6.75 \times 10^{-7} H^3 - 7.71 \times 10^{-5} H^2 + 1.792 \times 10^{-2} H + 0.49 \tag{2}$$

where T_i is the monthly-mean temperature (°C); *K* is a correction coefficient computed as a function of the latitude and month, and its value is taken as 1 in this study; *H* is the annual heat index, which is calculated as the sum of 12 monthly heat index H_i.

$$H = \sum_{i=1}^{12} H_i \tag{3}$$

$$H_i = \left(\frac{T_i}{5} \right)^{1.514} \tag{4}$$

Step 2: The difference between the precipitation *P* and *PET* for the month *i* is calculated using:

$$D_i = P_i - PET_i \tag{5}$$

where P_i is the monthly precipitation (mm); PET_i is the monthly potential evapotranspiration (mm).

Step 3: Normalize the data series D_i, calculate the SPEI index corresponding to each value, and use the log-logistic probability density function with three parameters to fit the constructed data series. The cumulative function of the log-logistic probability distribution is:

$$F(x) = \left[1 + \left(\frac{\alpha}{x - \gamma} \right)^\beta \right]^{-1} \tag{6}$$

where α, β, and γ are scale, shape, and origin parameters, respectively, for D_i values.

$$\alpha = \frac{\left(W_0 - 2W_1\right)\beta}{\Gamma(1+1/\beta)\Gamma(1-1/\beta)} \tag{7}$$

$$\beta = \frac{2W_1 - W_0}{6W_1 - W_0 - 6W_2} \tag{8}$$

$$\gamma = W_0 - \alpha\Gamma(1+1/\beta)\Gamma(1-1/\beta) \tag{9}$$

where $\Gamma(\beta)$ is the gamma function of β; W_1, W_2 and W_3 are the probability weighted moments of series D_i.

$$W_s = \frac{1}{N}\sum_{i=1}^{N}(1-F_i)^s D_i \tag{10}$$

$$F_i = \frac{i - 0.35}{N} \tag{11}$$

where N is the number of months involved in the calculation; s =1, 2, 3.

Step 4: Perform standard normal distribution transformation on the series to obtain the corresponding SPEI values:

$$P = 1 - F(x) \tag{12}$$

When the cumulative probability $P \leq 0.5$:

$$\omega = \sqrt{-2\ln(P)} \tag{13}$$

$$\text{SPEI} = \omega - \frac{c_0 - c_1\omega + c_2\omega^2}{1 + d_1\omega + d_2\omega^2 + d_3\omega^3} \tag{14}$$

When the cumulative probability $P > 0.5$:

$$P = 1 - P \tag{15}$$

$$\text{SPEI} = \frac{c_0 - c_1\omega + c_2\omega^2}{1 + d_1\omega + d_2\omega^2 + d_3\omega^3} - \omega \tag{16}$$

where the constants are c_0 = 2.515517, c_1 = 0.802853, c_2 = 0.010328, d_1 = 1.432788, d_2 = 0.189269, d_3 = 0.001308.

S2 Calculation method of the SSMI

In this study, the probability function adopted for the Standardized Soil Moisture Index (SSMI) is the log-logistic used by Oertel et al. (2018), which is similar to the probability function used for the calculation of the SPEI. The difference lies in replacing the fitting object D_i in formulas (6) to (16) with the soil moisture data, and subsequently changing the calculation results in formulas (14) and (16) to reflect the SSMI values.

S3 Construction of the entropy theory-based drought index (ETDI)

The entropy weight method is an objective weight calculation method based on information entropy theory (Huang et al., 2015). In this study, the entropy weight method was applied to determine the weights of the SPEI and SSMI when developing the integrated drought index (ETDI). In the problem of developing an integrated drought index by m single-type drought indices with n samples, the raw data matrix is represented as $X = \left(x_{ij}\right)_{m\times n}$, and the computation process of ETDI at each grid point is as follows:

Step 1: Standardization of the raw data matrix.

$$X = \left(x_{ij}\right)_{m\times n} \to R = \left(r_{ij}\right)_{m\times n}, r_{ij} \in [0,1] \tag{17}$$

$$r_{ij} = \frac{x_{ij} - \min_j(x_{ij})}{\max_j(x_{ij}) - \min_j(x_{ij})} \tag{18}$$

Step 2: The entropy of the i-th single-type drought index is expressed as follows:

$$H_i = -k\sum_{j=1}^{n} f_{ij}\ \ln f_{ij}\left(i = 1, 2, ..., m\right) \tag{19}$$

$$k = 1/\ln n \tag{20}$$

$$f_{ij} = r_{ij} \Big/ \sum_{j=1}^{n} r_{ij} \tag{21}$$

where f_{ij} denotes the proportion of the j-th sample value in the i-th single-type drought index. Suppose when $f_{ij} = 0$, $f_{ij}\ln f_{ij} = 0$.

Step 3: The weight of entropy of the i-th single-type drought index can be calculated as follows:

$$w_i = \frac{1 - H_i}{m - \sum_{i=1}^{m} H_i} \tag{22}$$

where $0 \le w_i \le 1, \sum_{i=1}^{m} w_i = 1$.

Step 4: Calculate the entropy theory-based drought index (ETDI):

$$I_{ETDI} = w_1 I_{SPEI} + w_2 I_{SSMI} \tag{23}$$

where I_{ETDI} is the value of ETDI at a grid point, I_{SPEI} and I_{SSMI} are the values of SPEI and SSMI, and w_1 and w_2 are the weight coefficients of SPEI and SSMI, respectively.

References

Vicente-Serrano, S.M., Beguería, S., and López-Moreno, J.I.: A multiscalar drought index sensitive to global warming: the standardized precipitation evapotranspiration index, J. Clim., 23, 1696-1718, https://doi.org/10.1175/2009JCLI2909.1, 2010.

Thornthwaite, C.W.: An approach toward a rational classification of climate, Geogr. Rev., 38, 55-94, https://doi.org/10.2307/210739, 1948.

Oertel, M., Meza, F.J., Gironás, J., Scott, C.A., Rojas, F., and Pineda-Pablos, N.: Drought propagation in semi-arid river basins in Latin America: lessons from Mexico to the Southern Cone, Water, 10, 1564, https://doi.org/10.3390/w10111564, 2018.

Huang, S., Chang, J., Leng, G., and Huang, Q.: Integrated index for drought assessment based on variable fuzzy set theory: a case study in the Yellow River basin, China, J. Hydrol., 527, 608-618, https://doi.org/10.1016/j.jhydrol.2015.05.032, 2015.