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Bayesian hierarchical modelling of sea-level extremes in the Finnish coastal region

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S1 Model details and prior distributions for the statistical models

This document provides more details on the priors chosen for the models. The choice of prior distribution is mostly conventional. We use Gaussian and log-Gaussian distributions with relatively large variances, so that the posterior distributions are not sensitive to the priors. This has been tested by sensitivity analyses in which prior distribution standard deviations were decreased or increased by an order of magnitude in comparison to the values present here. For the shape parameter and Gaussian process kernel parameter priors the values were either halved or doubled. These tests showed that using a narrower prior for the shape parameter had the largest impact on posterior distributions. Also using a narrower prior for the random walk parameters in the Spline model and for the Gaussian process kernel parameters affected the posterior distributions of the location and scale parameter to some extent. However, setting the priors wider did not have any noticeable impact on the posterior distributions, which supports our original prior choices. We note that for the parameters of the Gaussian process kernel, a more informative prior has to be used due to identifiability problems. This is a typical feature in spatial models. In the spline model, similar regularizing effect comes from the choice of number of spline knots.

More specifically, for Separate model, we used weakly informative Gaussian priors for the three GEV parameters with the following specifications:

$$\begin{aligned} &y_i \sim GEV(\mu_i, \sigma_i, \xi_i) \\ &\mu_i \sim N(800, 10000^2) \\ &\sigma_i \sim N(200, 10000^2)1_{\{\sigma_i>0\}} \\ &\xi_i \sim N(0, 0.5^2) \end{aligned} \tag{S1}$$

In Common model, tide gauge specific GEV parameters are assumed to come from the same joint Gaussian distribution. We therefore need to define hyper-priors for six additional parameters and the corresponding hyper parameters in our model:

$$\begin{aligned} &y_i \sim GEV(\mu_i, \sigma_i, \xi_i) \\ &\boldsymbol{\mu} \sim N(\mu_\mu, \sigma_\mu^2) \\ &\boldsymbol{\sigma} \sim N(\mu_\sigma, \sigma_\sigma^2) \\ &\boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2) \\ &\mu_\mu \sim N(800, 10000^2) \\ &\sigma_\mu \sim N(100, 200^2)1_{\{\sigma_\mu>0\}} \\ &\mu_\sigma \sim N(250, 1000^2) \\ &\sigma_\sigma \sim N(20, 100^2)1_{\{\sigma_\sigma>0\}} \\ &\xi_\mu \sim N(0, 0.5^2) \\ &\xi_\sigma \sim \log N(\log(0.05), 0.5^2) \end{aligned} \tag{S2}$$

Posterior means and basic summary statistics of the posterior distributions of the hyper parameters for the Common model are shown in Table S1.

Table S1. Statistics for the posterior estimates of the hyperparameters of Common model.

	Mean	SE_{mean}	StD	2.5%	25%	50%	75%	97.5%	n_{eff}	Rhat
μ_μ	793.06	0.61	48.84	693.91	762.90	792.90	823.38	888.89	6475.53	1
σ_μ	166.19	0.48	40.51	105.79	137.51	159.74	187.24	265.55	7175.01	1
μ_σ	208.02	0.17	12.43	184.36	200.17	207.78	215.82	233.94	5548.52	1
σ_σ	37.28	0.17	11.28	20.52	29.38	35.60	42.94	64.98	4184.71	1
μ_ξ	-0.16	0.00	0.02	-0.21	-0.18	-0.16	-0.15	-0.11	2479.67	1
σ_ξ	0.04	0.00	0.01	0.02	0.03	0.03	0.04	0.07	1977.66	1

In Spline, we used penalised cubic B-splines with first order random walk priors for the spline coefficients, expressed (e.g.) for the spline coefficients of μ as $\alpha_j = \alpha_{j-1} + u_j,\ j > 1$, where $u_j \sim N(0, \tau^2)$ and $\alpha_1 \propto$ const. Let $\boldsymbol{B}$ denote $K \times L$ matrix, where K is the number of stations and $L = 12$ the number of B-spline basis functions. Spline model is expressed as

$$\begin{aligned}
&y_i \sim GEV(\mu_i, \sigma_i, \xi_i)\\
&\boldsymbol{\mu} = \boldsymbol{B\alpha}\\
&\boldsymbol{\sigma} = \boldsymbol{B\beta}\\
&\boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2)\\
&\alpha_1 \sim N(1000, 10000^2)\\
&\beta_1 \sim N(200, 1000^2)\\
&\alpha_j \sim N(\alpha_{j-1}, \tau_\alpha^2), \quad j = 2, \dots, L\\
&\beta_j \sim N(\beta_{j-1}, \tau_\beta^2), \quad j = 2, \dots, L\\
&\tau_\alpha, \tau_\beta \overset{\text{ind}}{\sim} N(0, 100^2) 1_{\{\tau_\alpha, \tau_\beta > 0\}}\\
&\mu_\xi \sim N(0, 0.5^2)\\
&\sigma_\xi \sim \log N(\log(0.05), 0.5^2)
\end{aligned} \tag{S3}$$

Statistics of the posterior distributions for the spline model hyper parameters estimates are shown in Table S2.

Table S2. Statistics for the posterior estimates of the hyper parameters of Spline model.

	Mean	$\mathrm{SE}_{\mathrm{mean}}$	StD	2.5%	25%	50%	75%	97.5%	$\mathrm{n}_{\mathrm{eff}}$	Rhat
τ_α	140.84	0.53	33.68	88.90	117.33	136.18	159.76	219.18	4020.51	1
τ_β	40.07	0.31	15.10	19.03	29.48	37.28	47.60	76.42	2305.17	1
μ_ξ	-0.16	0.00	0.02	-0.20	-0.17	-0.16	-0.14	-0.11	2117.97	1
σ_ξ	0.04	0.00	0.01	0.01	0.03	0.03	0.04	0.07	1737.15	1

For the GP model we use squared exponential kernel K based on distance d described earlier. The model specification is the following:

$$\begin{aligned}
&y_i \sim GEV(\mu_i, \sigma_i, \xi_i)\\
&\boldsymbol{\mu} \sim GP(m_\mu, K(\alpha_\mu, \rho_\mu))\\
&\boldsymbol{\sigma} \sim GP(m_\sigma, K(\alpha_\sigma, \rho_\sigma))\\
&\boldsymbol{\xi} \sim N(\mu_\xi, \sigma_\xi^2)\\
&m_\mu \sim N(800, 10000^2)\\
&\alpha_\mu \sim \log N(\log(40), 0.5^2)\\
&\rho_\mu \sim \log N(\log(100), 0.1^2)\\
&m_\sigma \sim N(200, 1000^2)\\
&\alpha_\sigma \sim \log N(\log(30), 0.5^2)\\
&\rho_\sigma \sim \log N(\log(100), 0.1^2)\\
&\mu_\xi \sim N(0, 0.5^2)\\
&\sigma_\xi \sim \log N(\log(0.05), 0.5^2)
\end{aligned} \tag{S4}$$

Statistics of the posterior estimates of the Gaussian process parameters are given in table S3.

Table S3. Statistics for the posterior estimates of the hyper parameters of GP model.

	Mean	SE_{mean}	StD	2.5%	25%	50%	75%	97.5%	n_{eff}	Rhat
m_μ	822.87	0.61	54.10	715.45	788.74	823.25	857.78	929.28	7853.80	1
α_μ	113.52	0.27	22.81	78.00	97.45	110.42	126.35	167.53	7335.41	1
ρ_μ	106.16	0.13	9.85	88.34	99.25	105.74	112.28	127.14	5418.45	1
m_σ	214.07	0.21	16.64	180.75	203.65	213.61	224.49	247.79	6158.97	1
α_σ	33.11	0.14	9.04	19.36	26.85	31.69	37.96	53.99	4232.81	1
ρ_σ	104.09	0.16	10.41	85.38	96.81	103.48	110.75	126.08	4082.79	1
μ_ξ	-0.16	0.00	0.02	-0.20	-0.18	-0.16	-0.14	-0.11	2126.22	1
σ_ξ	0.04	0.00	0.01	0.01	0.03	0.03	0.04	0.07	2192.81	1